# MAT3021

# EXPERIMENTAL DESIGN

## Do colours affect a person's ability to retain information they have seen?


**Daven Dela Cruz**
**Amiel Fabrigas**
**Anmol Gurung**
**Marjoline Quila**


16th December 2019

# Table of Contents

# Table of figures

# 1. Objective

We aim to investigate whether colours affect the ability to retain information, which could possibly improve revision technique for students to help aid memorisation. For the duration of this study, we will expose our subjects to coloured sets of randomised words within a certain time frame, and subsequently ask them to recall these words.

# 2. Pilot Study Experiment

The procedure for the experiment will consist of seven subjects being exposed to a set of 50 randomly generated words of the same colour for 30 seconds, all of which are displayed against a white background.

The students will then have a one-minute break, representing a long-term recollection of information after revision, before attempting to recall the words by writing them on a blank piece of paper. Their measure of success would be indicated by the number of words they can remember within a two-minute time frame.

There will be seven colours: *Red, Orange, Yellow, Green, Blue, Purple and Black.* We will also be using *randomwordgenerator.com*. If there exists the same word within two different sets of randomly generated words, we will generate until a distinct set of words is acquired.

To reduce the chances of the subjects learning a technique to complete the task and becoming too familiar with it, we have chosen the block size of 4 (*k=4*). So, we require each subject to be exposed to 4 different coloured sets.

To minimise human error, we will be using Microsoft PowerPoint to transition the slides automatically, allowing us to set the time accurately.

Results will be collected over four consecutive days, where each subject will complete one set each day to give a fairer reflection of the effects of each treatment.

## 2.1 Analysis of Pilot Study

The pilot study was carried out on two subjects. Over four consecutive days, both subjects were exposed to four distinct treatments; one coloured set per day between 7-8pm, allowing us to stop the environment from being a nuisance factor. In order to give a fairer reflection of the effects of each treatment, we decided not to have our subjects completing more than one set on the same day. This was to avoid giving them too much input (i.e. too many words at once) than what their brain can sort through and process. The treatments were randomly allocated using Excel (using a similar randomisation process explained in detail later).

The results were as follows:

| Subject 1 | Subject 2 |
|---|---|
| Red = 4 | Blue = 3 |
| Yellow = 6 | Green = 5 |
| Black = 4 | Red = 5 |
| Green = 5 | Black = 3 |

*Figure 1 - Pilot Study result*

This suggests that colours do affect one's ability to retain information, as brighter colours such as yellow had the highest number of words that were recalled, whilst darker colours like black and blue had the lowest count.

Upon completion, there were a few changes and new additions we implemented in the procedure of our main study to ensure that we perform the experiment as accurate as possible.

The following are:

1. To make sure the length of characters and the complexity of the words were not nuisance factors, we ensured that our selection of words was only six character long and universal. We found out that six is approximately the average number of characters per word in English language (Diuna LSP, 2019).
2. We decided to use the same screen size (15" inch) to further eliminate the effect of any nuisance source of variability, in order to produce a reliable and consistent result.
3. The total number of words per treatment was reduced from 50 to 40 after evaluating the participants' feedback that, 50 random words were overwhelming and too many for them to process for the given time.
4. We increased the word memorisation time from 30 seconds to 1 minute, as the subjects found the initial time of 30 seconds too short to properly observe the words shown in the screen.
5. We decided to give the subjects 30 seconds of not doing anything, followed by 30 seconds of playing noughts and crosses, instead of 1 minute of noughts and crosses immediately after the word memorisation step. Generally, after acquiring new information, one would require time to store and retain the new data; hence, 30 second rest represents this process (Verywell Mind, 2019)
6. We reduced the time for writing down the memorised words from 2 minutes to 1 minute because the subjects only required, on average, 35 seconds out of 2 minutes to write down what they recollected.
7. Before the main experiment, we decided to add a trial run with smaller set of words (20 words with 30 seconds of memorisation, followed by 15 seconds of rest, 15 seconds of noughts and crosses, and lastly, 30 seconds of writing down) to give them a taste of what to expect. This is to ensure that the time given for the experiment was used effectively to help produce reliable comparisons among the treatments.

## 3. Construction of Balanced Incomplete Block Design (BIBD)

Some properties of the of a BIBD are defined below:

- Equi-replicate: each treatment occurs **r** times.
- Proper: each block contains **k** experimental units.
- Concurrence: each pair of treatments occurs $\boldsymbol{\lambda}$ times.

<u>Construction 1</u>

Let $\boldsymbol{p}$ be a prime number of the form $\mathbf{4n+3.}$

To obtain a symmetric balanced design with parameters:

$\boldsymbol{t}=\boldsymbol{b}=\boldsymbol{p},\ \boldsymbol{r}=\boldsymbol{k}=\frac{p-1}{2}$, and $\boldsymbol{\lambda}=\ \frac{p-3}{4}$

Take the first block to be the set of non-zero squares (modulo *p*) and construct the other blocks using cyclic constructions.

The selection of colours (treatment t) will be:

- Purple (**A**)
- Black **(B)**
- Red **(C)**
- Yellow **(D)**
- Green **(E)**
- Blue **(F)**
- Orange **(G)**

We decided to use the seven rainbow colours as our treatment factors, with the exception that we replaced indigo with black due to indigo being very similar to blue. We chose black as a control variable as it is commonly used as a font colour. After researching whether colours have any relation to our memory, we found that colours, as part of the electromagnetic spectrum, has its own magnetic frequency that can affect neurological pathways in our brain (i.e. every colour has a specific wavelength which affects our body and brain in a different way), (Shiftelearning.com, 2019). Using this knowledge as our baseline, it allowed our investigation to be more representative of the student population, as vast majority of students use different coloured pens, highlighters or some form of resource affiliated with colours to help with their revision.

For our blocking factor, we decided that the students themselves would be ideal as this ensured that their natural ability of memorising was considered as a factor. By exposing multiple students to words of the same colour randomly, it reduces sources of variability; thus, leading to greater precision on the statistical comparisons between treatments.

Thus, we have our treatments (colours) and blocking factor (students) as 7. As 7 is a prime number, $\boldsymbol{p}$, which is in the form $\mathbf{4n+3}\ with\ \boldsymbol{n}=1$; we can obtain a symmetric balanced design with the parameters using Construction 1: $\boldsymbol{t}=\boldsymbol{b}=\boldsymbol{p}=7, \boldsymbol{r}=\boldsymbol{k}=3,$ and $\boldsymbol{\lambda}=1.$

Hence, we have our equi-replicate as 3 and concurrence as 1.

The squares modulo of 7 are:

$$1^2 = 1;\ 2^2 = 4;\ 3^2 = 9 \equiv 2; 4^2 = 16 \equiv 2;\ 5^2 = 25 \equiv 4;\ 6^2 = 36 \equiv 1$$

This gives us the set of non-zero squares $\{1,2,4\}$, which will be used as the first block for our BIBD. The other blocks are then made using cyclic construction. If we assign, $A = 1, B = 2, \dots, G = 7$, we have our construction as:

| BLOCKS | TREATMENT ALLOCATION |
|---|---|
| BLOCK 1 | A,B,D |
| BLOCK 2 | B,C,E |
| BLOCK 3 | C,D,F |
| BLOCK 4 | D,E,G |
| BLOCK 5 | E,F,A |
| BLOCK 6 | F,G,B |
| BLOCK 7 | G,A,C |

*Figure 2 - BIBD construction*

## 4. Randomisation Process

To randomly assign a block to a person we used the following MS Excel commands:

- *RAND()* - returns a random number between 0 and 1.
- *RANK.EQ()* - returns the rank of a number in a list of numbers.
- *MATCH()* - searches for a specified item in a range of cells, and then returns the relative position of that item in the range.
- *INDEX()* - gets a value from a list or table based on its location.

| BLOCK ALLOCATION RANDOMIZER | |
|---|---|
| =RANK.EQ(C14,C$14:C$20) | =RAND() |
| =RANK.EQ(C15,C$14:C$20) | =RAND() |
| =RANK.EQ(C16,C$14:C$20) | =RAND() |
| =RANK.EQ(C17,C$14:C$20) | =RAND() |
| =RANK.EQ(C18,C$14:C$20) | =RAND() |
| =RANK.EQ(C19,C$14:C$20) | =RAND() |
| =RANK.EQ(C20,C$14:C$20) | =RAND() |

*Figure 3 - Block allocation randomizer I*

| BLOCK ALLOCATION RANDOMIZER | |
|---|---|
| 4 | 0.520845 |
| 7 | 0.052588 |
| 1 | 0.748883 |
| 2 | 0.682879 |
| 5 | 0.470468 |
| 6 | 0.440934 |
| 3 | 0.553465 |

*Figure 4 – Randomized Ranking Table Example*

On the second column of *Figure 3*, we use the command *RAND()* to generate a random number between 0 and 1; this is then ranked on the first column using *RANK.EQ()* command. This table generates a randomised ranking for us to use onto the block's column of *Figure 2*.

Using the *MATCH()* command from *Figure 5,* we can search for the subject's number and receive its relative position in the randomised ranking table from *Figure 4*. This is then used for the *INDEX()* command (*Figure 5*), where it gives us the block number from *Figure 2* and assign it onto a person.

For example, we search for Subject 1 in the match command, we would then receive 3 from *Figure 4*, as that is its relative position in the range we've selected. This would then give us block 3 from *Figure 2* using the index command and assign it to Subject 1, hence, in this randomisation process we get block 3 to be assigned to Subject 1.

| SUBJECT | BLOCK ALLOCATION |
|---|---|
| 1 | =INDEX($E$14:$E$20,MATCH(1,$B$14:$B$20,0)) |
| 2 | =INDEX($E$14:$E$20,MATCH(2,$B$14:$B$20,0)) |
| 3 | =INDEX($E$14:$E$20,MATCH(3,$B$14:$B$20,0)) |
| 4 | =INDEX($E$14:$E$20,MATCH(4,$B$14:$B$20,0)) |
| 5 | =INDEX($E$14:$E$20,MATCH(5,$B$14:$B$20,0)) |
| 6 | =INDEX($E$14:$E$20,MATCH(6,$B$14:$B$20,0)) |
| 7 | =INDEX($E$14:$E$20,MATCH(7,$B$14:$B$20,0)) |

*Figure 5 - Block allocation randomizer II*

And so, by using this randomisation process for the treatments and blocks we obtain the following:

| TREATMENT LABEL | TREATMENTS |
|---|---|
| A | PURPLE |
| B | BLACK |
| C | RED |
| D | YELLOW |
| E | GREEN |
| F | BLUE |
| G | ORANGE |

*Figure 7 - Treatment allocation*

| SUBJECT | BLOCK ALLOCATION |
|---|---|
| 1 | BLOCK 6 |
| 2 | BLOCK 1 |
| 3 | BLOCK 2 |
| 4 | BLOCK 3 |
| 5 | BLOCK 7 |
| 6 | BLOCK 4 |
| 7 | BLOCK 5 |

*Figure 6 - Subject/Block allocation*

# 5. Main Experiment

The main experiment was carried out similarly to the Pilot Study Experiment section, while considering the improvements and additions stated from the previous section.

As per the changes, we reduced the number of words to 40. We also randomly assigned our labels *(A-G)* to the treatments and our blocks to each subject *(see Randomisation Process section)*.

280 *(7×40)* words were generated using *randomwordgenerator.com* to rid the possibility of duplicates across sets. Each word was limited to six characters to avoid any biased responses.

Each coloured set was displayed to each subject using timed PowerPoint slides for 1 minute.

| | | | |
|---|---|---|---|
| dilute | thread | cotton | wander |
| tumble | tenant | throat | affair |
| master | charge | budget | borrow |
| appear | grudge | europe | shower |
| elapse | middle | bother | degree |
| closed | belong | kettle | source |
| length | carbon | prefer | mother |
| extort | bomber | kidney | system |
| prison | forest | polite | exempt |
| facade | proper | racism | marble |

| | | | |
|---|---|---|---|
| scream | rhythm | corpse | advice |
| expose | indoor | lonely | choice |
| clique | barrel | defeat | record |
| theory | topple | adjust | visual |
| rotate | linger | career | colony |
| screen | killer | writer | unique |
| height | needle | freeze | pocket |
| linear | sunday | endure | offend |
| desert | mature | square | exotic |
| flavour | stitch | virtue | attack |

| | | | |
|---|---|---|---|
| labour | forget | ethics | family |
| cancel | inject | trance | margin |
| series | coerce | tactic | patrol |
| praise | church | suburb | relief |
| timber | reward | artist | define |
| candle | annual | manner | banner |
| suntan | kidnap | bottle | notion |
| please | sleeve | defend | betray |
| supply | jockey | sailor | thrust |
| centre | mirror | accent | flawed |

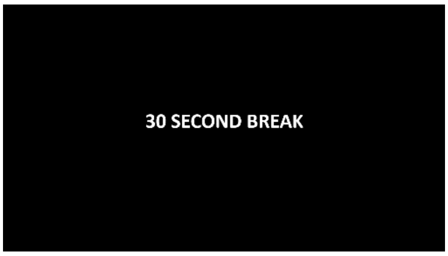
| | | | |
|---|---|---|---|
| treaty | stroke | policy | remind |
| modest | expand | cancer | agenda |
| redeem | border | growth | cherry |
| ignore | gutter | resign | spider |
| ritual | animal | avenue | favour |
| smooth | survey | sodium | stable |
| affect | format | mosque | safari |
| breeze | spring | action | pillow |
| choose | basket | tycoon | finger |
| install | regret | behave | double |

| | | | |
|---|---|---|---|
| retain | moment | bridge | create |
| dinner | common | powder | coffee |
| mobile | decade | expect | listen |
| rocket | asylum | cinema | harass |
| friend | office | arrest | result |
| clinic | glance | letter | studio |
| dollar | player | jacket | finish |
| summit | lawyer | heaven | script |
| hotdog | detail | absent | ignite |
| cellar | fossil | temple | orange |

| | | | |
|---|---|---|---|
| likely | foster | unlike | nuance |
| empire | couple | trench | manage |
| porter | ribbon | squash | market |
| poison | viable | sacred | launch |
| absorb | afford | course | cheese |
| remain | ladder | muscle | access |
| banana | change | regard | strike |
| bounce | native | worker | bucket |
| deadly | threat | census | window |
| volume | polish | agency | poetry |

| | | | |
|---|---|---|---|
| prince | hiccup | bloody | season |
| insert | cheque | hammer | dragon |
| garlic | revoke | frozen | gossip |
| suffer | doctor | speech | vacuum |
| pardon | review | legend | horror |
| accept | morsel | embark | estate |
| reject | option | factor | bronze |
| future | quaint | banish | appeal |
| column | shiver | answer | makeup |
| escape | helmet | flight | person |

*Figure 8 - PowerPoint slides I*




To simulate long-term collection of information post-revision, the subjects were then given a 30 second break to reflect on the words.


To recall words stored in the subject's long-term memory, each subject played 30 seconds of noughts and crosses against the overseeing member to divert their attention. This was intended to limit the effect of what the subject could readily remember in the short term.


The subjects finished the experiment by being given a minute to write all the words they could remember.

*Figure 9 - PowerPoint slides II*

## 5.1 Analysis of the Main Experiment

We have our BIBD and results below:

| | | | | | | |
|---|---|---|---|---|---|---|
| A<br>**9** | B<br>**10** | C<br>**8** | D<br>**5** | E<br>**6** | F<br>**6** | G<br>**4** |
| B<br>**4** | C<br>**7** | D<br>**7** | E<br>**6** | F<br>**10** | G<br>**8** | A<br>**5** |
| D<br>**7** | E<br>**10** | F<br>**8** | G<br>**5** | A<br>**9** | B<br>**12** | C<br>**7** |

*Figure 10 - BIBD*

| Colours | Score 1 | Score 2 | Score 3 | Average | Range |
|---|---|---|---|---|---|
| Green | 10 | 6 | 10 | 8.66666667 | 4 |
| Blue | 8 | 8 | 9 | 8.33333333 | 1 |
| Orange | 12 | 7 | 5 | 8 | 7 |
| Red | 7 | 8 | 5 | 6.66666667 | 3 |
| Black | 6 | 4 | 10 | 6.66666667 | 6 |
| Yellow | 7 | 7 | 5 | 6.33333333 | 2 |
| Purple | 9 | 4 | 6 | 6.33333333 | 5 |

*Figure 11 - Remarks on BIBD*

As you can see from *Figure 11*, green has the highest average of number of words that were recalled, which could suggest that colours do affect the ability of people to retain information. This could mean that people would be more likely to remember words written using green.

Range is the measure of dispersion between our data. It has a very crude measurement of the spread of data because it is extremely sensitive to outliers, and as a result, this can affect our overall analysis. For example, we can see from *Figure 11* that Orange has the highest range which could mean that 12 could be a potential outlier, given that it was the highest score achieved by our Subjects. We will further investigate this in the later section.

| | **Blocks** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Treatments** | **Subject 1** | **Subject 1** | **Subject 3** | **Subject 4** | **Subject 5** | **Subject 6** | **Subject 7** |
| **A** | | 9 | | | 4 | | 6 |
| **B** | 6 | 4 | 10 | | | | |
| **C** | | | 7 | 8 | 5 | | |
| **D** | | 7 | | 7 | | 5 | |
| **E** | | | 10 | | | 6 | 10 |
| **F** | 8 | | | 8 | | | 9 |
| **G** | 12 | | | | 7 | 5 | |
| **Total** | **26** | **20** | **27** | **23** | **16** | **16** | **25** |

*Figure 12 - Total word count recalled by each subject*

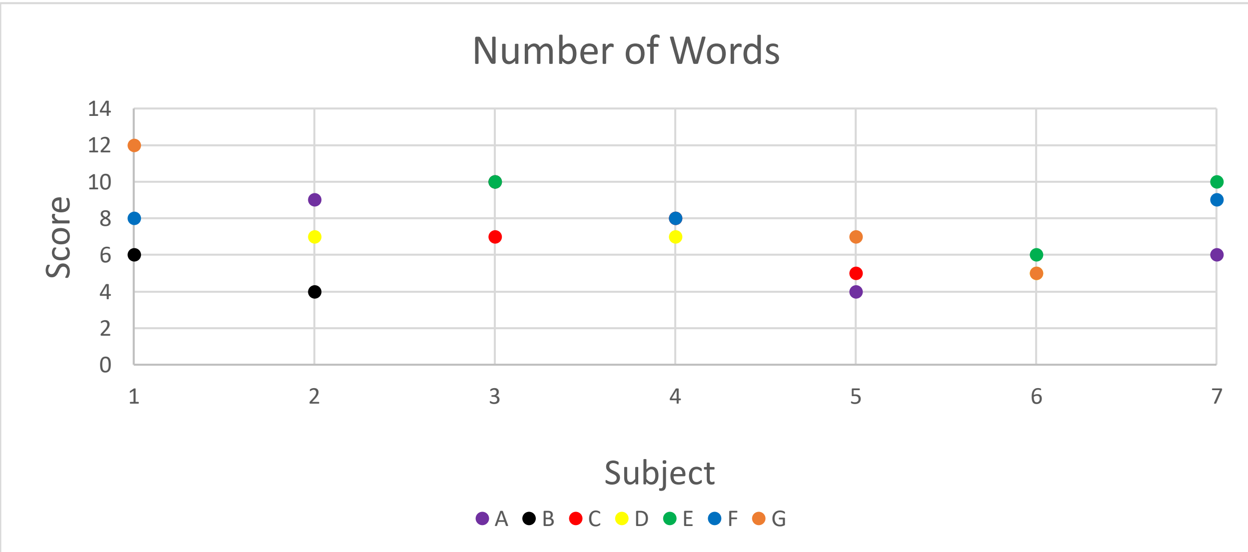



*Figure 13 - Graph of total word count by each subject*

From *Figures 12 and 13*, however, we can see that Subjects 3, 4, 5 and 6 consistently recalled similar amount of words regardless of the colour. This suggesting that colours doesn't have any influence over memory retention. However, Subjects 1, 2 and 7 have larger spread of outcomes in comparison, indicating that certain colours can affect our Subjects' ability to recollect information.

To be able to form an accurate analysis of our data, we need to further analyse these using a statistical software – R.

To do this, we read our obtained data into R, as shown in *Figure 14*:

```
> rainbow.dat<-read.table("C:/Users/rf00212/Downloads/colours.txt",header=T)
> rainbow.dat
   Colours Score Subject
1        F     8       1
2        G    12       1
3        B     6       1
4        A     9       2
5        B     4       2
6        D     7       2
7        B    10       3
8        C     7       3
9        E    10       3
10       C     8       4
11       D     7       4
12       F     8       4
13       G     7       5
14       A     4       5
15       C     5       5
16       D     5       6
17       E     6       6
18       G     5       6
19       E    10       7
20       F     9       7
21       A     6       7
```

*Figure 14 - Data processed into R*

```
> y<-rainbow.dat$Score
> y
 [1]  8 12  6  9  4  7 10  7 10  8  7  8  7  4  5  5  6  5 10  9  6
> Colour<-factor(rainbow.dat$Colours)
> Colour
 [1] F G B A B D B C E C D F G A C D E G E F A
Levels: A B C D E F G
> Subject<-factor(rainbow.dat$Subject)
> Subject
 [1] 1 1 1 2 2 2 3 3 3 4 4 4 5 5 5 6 6 6 7 7 7
Levels: 1 2 3 4 5 6 7
```

*Figure 15 - Data processed into R with variables defined*

All 21 observations have been converted and read into R. Our response variable, Scores, are the number of words recalled by the Subject, which we label as *y.* We also define Colour as our treatment factor and Subject as our blocking factor. These are shown above in *Figure 15*:

The model we use for BIBD is:

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$$

where $y_{ij}$ is the observation $i^{th}$ treatment on $j^{th}$ block; $i = 1, \dots, t$; $j = 1, \dots, b$. $t$ is the number of treatments and $b$ is the number of blocks which, from our experiment, are 7 and 3 respectively. $\mu$ is the mean, $\tau_i$ is the $i^{th}$ treatment effect, and $\beta_j$ is the $j^{th}$ blocking effect. $\varepsilon_{ij}$ is the noise variable which we assume to be identically, independently distributed, with a normal distribution $\sim N(0, \sigma^2)$. This implies that the mean is 0 and variance is $\sigma^2$.

We label and assign *rainbow1* to fit our linear model, linking *y* to our blocking and treatment factor. Therefore, the appropriate model for our experiment is:

```
> rainbow1<-lm(y~Subject+Colour)
> rainbow1

Call:
lm(formula = y ~ Subject + Colour)

Coefficients:
(Intercept)     Subject2     Subject3     Subject4     Subject5     Subject6     Subject7      ColourB      ColourC
     8.0000      -1.0000       0.8571      -0.5714      -3.4286      -4.2857      -0.5714      -1.2857      -0.2857
    ColourD      ColourE      ColourF      ColourG
     0.2857       2.0000       0.7143       2.5714
```

*Figure 16 – Linear model for the experiment*

Note that the order in which the factors are entered is important. By entering the blocking factor ‘Subject’, before the treatment factor ‘Colour’, the unadjusted Subject sum of squares and the Colour sum of squares adjusted for Subject are calculated.

Our Analysis of Variance (ANOVA) table is given below, which we use to test our hypothesis:

```
> anova(rainbow1)
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value Pr(>F)
Subject    6 42.286  7.0476  2.0772 0.1669
Colour     6 24.857  4.1429  1.2211 0.3858
Residuals  8 27.143  3.3929
```

*Figure 17 - ANOVA table*

| Source | Df | Sum Sq. | Mean Sq. | F value |
|---|---|---|---|---|
| Subject | *b-1* | $SS_{BL(un)}$ | $SS_{BL(un)}/_{b-1}$ | $SS_{BL(un)}/_{b-1}/_{s^2}$ |
| Colour | *t-1* | $SS_{TRT(adj)}$ | $SS_{TRT(adj)}/_{t-1}$ | $SS_{TRT(adj)}/_{t-1}/_{s^2}$ |
| Residuals | *(tr-1)- (t-1)-(b-1)* | $SS_{RESID}$ | $SS_{RESID}/(tr-1)-(t-1)-(b-1) = \boldsymbol{s^2}$ | |
| **Total** | ***tr-1*** | $\boldsymbol{SS_{TOTAL}}$ | | |

*Figure 18 – Calculation procedure for ANOVA table*

We have four columns in our table, which are calculated as shown from *Figure 18:*

i. $Df$ is the degrees of freedom. Since we have $t=7$, $b=7$ and $r=3$, our degrees of freedom for Subject, Colours and Residuals are 6, 6 and 8 respectively.
ii. *Sum Sq.* denotes the Sums of Squares for our blocks, treatments, and residuals, which are calculated using the formulas below.
The treatment sum of squares, adjusted for blocks, is

$$SS_{TRT(adj)} = \frac{\lambda t}{k}\sum_{i=1}^{t}\hat{\tau}_i^2 = \frac{1}{k\lambda t}\sum_{i=1}^{t}(T_i^*)^2$$

where $T_i^*$ is the sum of all observations in blocks containing treatment $i$.
$\hat{\tau}_i$ is the maximum likelihood estimator of $\tau_i$ given by

$$\hat{\tau}_i = \frac{1}{\lambda t}T_i^* = \frac{1}{\lambda t}\left\{kT_i - \sum_{j=1}^{b} n_{ij}B_j\right\} \qquad i = 1, \quad \cdots, \quad t$$

Both the total sum of squares and the block sum of squares, shown below, are calculated using the Mean Correction (M.C.), given by

$$M.C. = N\bar{y}_{..}^{\,2}$$

where $N$ is the number of observations and $\bar{y}_{..}$ is the average of all $N$ observations.

$$SS_{BL(un)} = \frac{1}{k}\sum_{j=1}^{b} B_j^2 - M.C.$$

$$SS_{TOTAL=}\sum_{i=1}^{t}\sum_{j=1}^{b} y_{ij}^2 - M.C.$$

The residual sum of squares is calculated as:

$$SS_{RESID} = SS_{TOTAL} - SS_{TRT(adj)} - SS_{BL(un)}$$

iii. *Mean Sq.* is calculated by dividing the *Sum Sq.* values by their corresponding *Df*, as shown in *Figure 10.* This also equates to the unbiased estimator of our variance, $\sigma^2$.
iv. *F value* is what we pay attention to the most on our table, as we use this to test our hypothesis. These are calculated by dividing the *Mean Sq.* by $s^2$.

## 5.2 Hypothesis Test

Our hypotheses are:

$H_0: \tau_A = \tau_B = \tau_C = \tau_D = \tau_E = \tau_F = \tau_G = 0$

versus

$H_1: not\ all\ \tau_i\ are\ equal$

Our null hypothesis, $H_0$, implies that our treatments have no effect on memory retention, and our alternative hypotheses, $H_1$, means that our treatments have an effect.

The $F$ statistic is calculated using

$$\frac{SS_{TRT(adj)}/(t-1)}{SS_{RESID}/(tr-1)-(b-1)-(t-1)}$$

From the ANOVA table in Figure 17, our test statistic is the $F$ value 1.2211. Under $H_0$, 1.2211 is a random observation from $F_8^6$. From table 12(b) of the New Cambridge Statistical Tables, the 95th percentile of $F_8^6$ is 3.581. As $1.2211 < 3.581$, we conclude that there aren't enough grounds to reject $H_0$. Our $p$-value, 0.3858, is greater than 0.05, indicating weak evidence against the null hypothesis, which supports are initial conclusion. Therefore, we conclude that colours do not affect the ability to memorise information.

Finally, we also need to determine whether blocking with Subjects has been effective.

```
> rainbow2<-lm(y~Colour)
> anova(rainbow2)
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value Pr(>F)
Colour     6 18.286  3.0476  0.5614 0.7541
Residuals 14 76.000  5.4286
```

*Figure 19 - ANOVA table without blocking factor*

The ANOVA table on *Figure 19* demonstrates the results we would have obtained, if blocking had not been included in the model. Blocking is an important design technique and can be used to systematically eliminate the effect of a nuisance source of variability, on the statistical comparisons among treatments. Since our p-value, 0.7541, is greater than the 5% significance level, we have weak evidence against our $H_0$. Therefore, we can conclude that blocking with Subjects has been indeed effective.

## 5.3 Model Adequacy

Results from our model showed that there was insufficient evidence to suggest colours had any effect memory retention. Consequently, we call in to question the assumptions we made during the construction of our model namely, the noise terms $\varepsilon_{ij}$ whereby we assumed constant variance.

To check our assumption, we plot the standardised residuals against the fitted values to identify any systemic trend.

Shown are the standardised residuals followed by the fitted values:

```
> sres<-rstandard(rainbow1)
> sres
         1          2          3          4          5          6          7          8          9         10         11
-0.6282809  1.2565617 -0.6282809  1.7591864 -1.5078741 -0.2513123  2.1361549 -1.3822179 -0.7539370  0.7539370 -0.6282809
        12         13         14         15         16         17         18         19         20         21
-0.1256562 -0.1256562 -0.5026247  0.6282809  0.8795932  0.2513123 -1.1309056  0.5026247  0.7539370 -1.2565617
> fit<-fitted(rainbow1)
> fit
        1         2         3         4         5         6         7         8         9        10        11        12
 8.714286 10.571429  6.714286  7.000000  5.714286  7.285714  7.571429  8.571429 10.857143  7.142857  7.714286  8.142857
       13        14        15        16        17        18        19        20        21
 7.142857  4.571429  4.285714  4.000000  5.714286  6.285714  9.428571  8.142857  7.428571
```

```
> plot(fit,sres,xlab="Fitted Values",ylab="Standard Residuals",ylim=c(-4,4))
> abline(0,0)
```

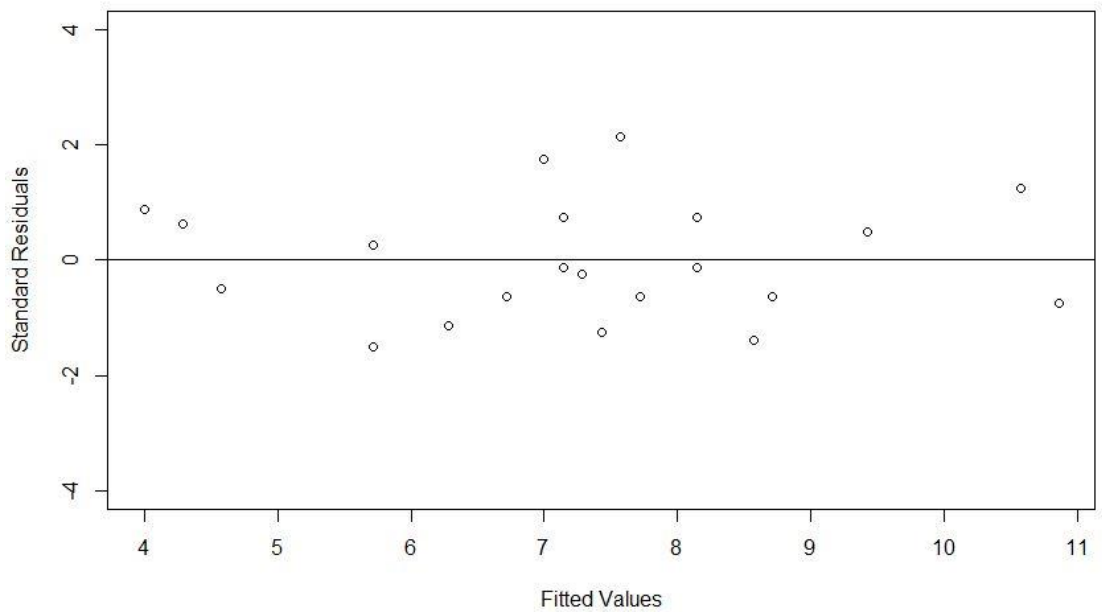



*Figure 20 - Graph for fitted values against standard residuals*

Here we used abline to plot a line through (0,0) to be able to discern any systemic trend. Though there is a slight cluster grouped in the middle, overall there is a random scatter which suggests that the assumption of equal variances is reasonable.

We also check the normality assumption of $\varepsilon_{ij}$ by obtaining the normal probability plot.

```
> qqnorm(sres)
> qqline(sres)
```

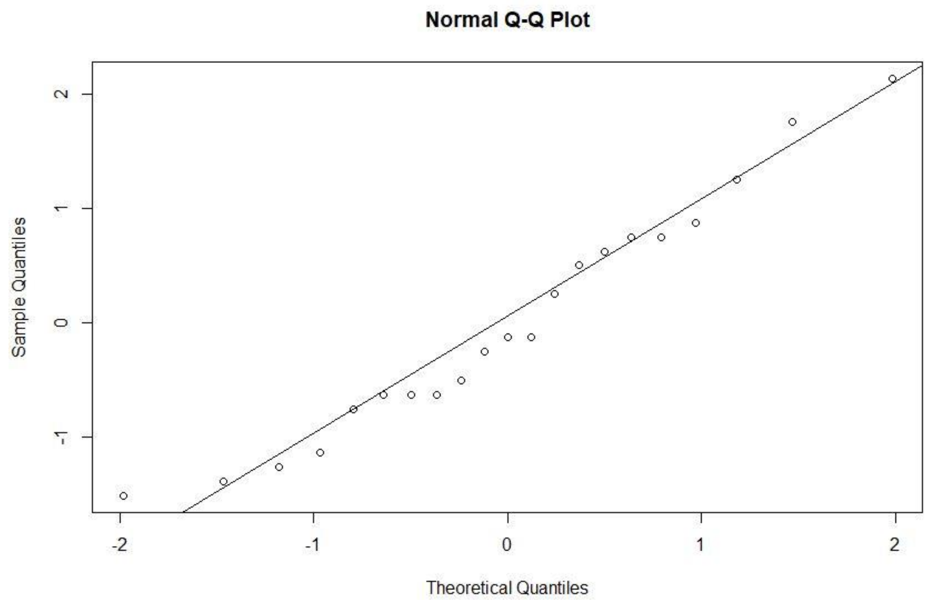



*Figure 21 - Normal probability graph*

Overall, most of the residuals fit the Q-Q plot. The plot looks roughly linear which would suggest the normality assumption is reasonable. The first plot is most notable and seems to be the main outlier. Given the opportunity to investigate further, perhaps this value could be omitted.

## 5.4 Outliers

```
> plot(sres,ylab="Standardised Residuals",ylim=c(-4,4))
> abline(-2,0)
> abline(2,0)
```

*Figure 22 - Standardised Residuals in R*

By considering the potential for outliers, we look again to our standardised residuals. We compare the absolute size of the residuals and check any notably big values. Any standardized residual >|2| should be deemed an outlier. The 7$^{th}$ value has absolute value 2.1361549 and so, can be considered an outlier. With further investigation, the outlier could be omitted, and the subjects could be retested with a different set of words using the same colour.

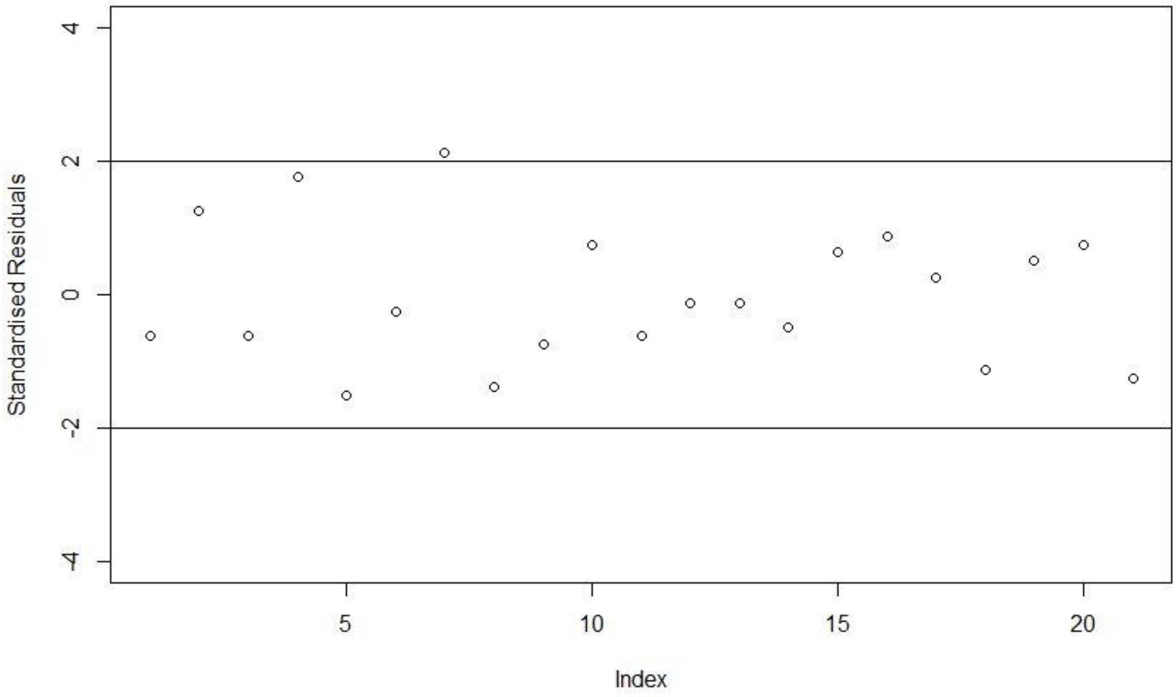



*Figure 23 - Standardised residuals graph*

## 5.5 Linear Contrasts

Although our hypothesis test suggests that colours have no influence against memory retention, we still wish to investigate the orthogonal contrasts used to compare some treatments to each other to gain a full understanding of how colours of fonts can affect memory. And so, we have decided to do four comparisons:

1) Black against other colours
2) Complementary colours (i.e. Blue & Green against Red & Orange)
3) Bright Analogous colours (i.e. Red against Orange)
4) Dark Analogous colours (i.e. Blue against Green)

The reasoning behind the comparison (2) is that in general cases brighter colours are much more visible and noticeable than darker colours, potentially having an influence over memory retention. And so, the comparisons (3) and (4) are made to test brighter colours against each other and likewise for darker colours. Therefore, we are comparing B against A, C, D, E, F, G; E and F against C and G; C against G; and E against F.

These comparisons can be represented by these linear contrast vectors:

i) $\underline{c_1} = (-1, 6, -1, -1, -1, -1, -1)^T$
ii) $\underline{c_2} = (0, 0, 1, 0, -1, -1, 1)^T$
iii) $\underline{c_3} = (0, 0, 1, 0, 0, 0, -1)^T$
iv) $\underline{c_4} = (0, 0, 0, 0, 1, -1, 0)^T$

These contrast vectors are valid to use as $1_7^T \underline{c_i} = 0 \; for \; i = 1,2,3,4$ (i.e. in $\underline{c_1}$ we have, $-1 + 6 - 1 - 1 - 1 - 1 - 1 = 0$); and because they are all orthogonal to each other i.e. $\underline{c_i}.\underline{c_j} = 0$

If $\sum_{i=1}^{t} c_i \tau_i, \cdots, \sum_{i=1}^{t} c_{i,t-1} \tau_i$ are t-1 mutually orthogonal contrasts, we have

$$SS_{TRT(ADJ)} = q_1 + \cdots + q_{t-1}$$

Where

$$q_j = \frac{1}{k\lambda t} \frac{\left(\sum_{i=1}^{t} c_{ij} T_i^*\right)^2}{\sum_{i=1}^{t} c_{ij}^2}$$

With 1 ≤ j ≤ t-1

For our experiment, the Treatment totals are:

$T_A = 9 + 4 + 6 = 19,\ T_B = 20,\ T_C = 20,\ T_D = 19,\ T_E = 26, T_F = 25,\ T_G = 24.$

Our corresponding Block totals, $B_i$, are:

$B_1 = 6 + 8 + 12 = 26, B_2 = 20, B_3 = 27, B_4 = 23, B_5 = 16, B_6 = 16, B_7 = 25.$

The corresponding $T_i^*$are as follows:

$T_A^* = 3 * T_A - (B_2 + B_5 + B_7) = 3 * 19 - (20 + 16 + 25) = -4$

$T_B^* = 3 * T_B - (B_1 + B_2 + B_3) = 3 * 20 - (26 + 20 + 27) = -13$

$T_C^* = 3 * T_C - (B_3 + B_4 + B_5) = 3 * 20 - (27 + 23 + 16) = -6$

$T_D^* = 3 * T_D - (B_2 + B_4 + B_6) = 3 * 19 - (20 + 23 + 16) = -2$

$T_E^* = 3 * T_E - (B_3 + B_6 + B_7) = 3 * 26 - (27 + 16 + 25) = 10$

$T_F^* = 3 * T_F - (B_1 + B_4 + B_7) = 3 * 25 - (26 + 23 + 25) = 1$

$T_G^* = 3 * T_G - (B_5 + B_6 + B_7) = 3 * 24 - (26 + 16 + 16) = 14$

Our $T_i^*$ must sum to 0 so to check, we will add them all up:

$$-4 - 13 - 6 - 2 + 10 + 1 + 14 = 0$$

Thus, the $T_i^*$ are valid

This leads us to our first comparison:

### 5.5.1 Comparison 1: B VS A, C, D, E, F, G

We wish to compare our control variable, Black, against the other colours to see if colours affect our memory retention at all.

Here, we use $\underline{c_1}$, which gives us $q_1$:

$$q_1 = \frac{1}{3 * 1 * 7} \frac{(-T_A^* + 6T_B^* - T_C^* - T_D^* - T_E^* - T_F^* - T_G^*)^2}{(-1)^2 + (6)^2 + (-1)^2 + (-1)^2 + (-1)^2 + (-1)^2 + (-1)^2}$$

$$q_1 = \frac{1}{21} \frac{8281}{42} = 9.38$$

The hypothesis test for this comparison is

$H_0$: $-\tau_A + 6\tau_B - \tau_C - \tau_D - \tau_E - \tau_F - \tau_G = 0$ vs $H_1$: $-\tau_A + 6\tau_B - \tau_C - \tau_D - \tau_E - \tau_F - \tau_G \neq 0$

The test statistic is $\frac{q_1}{s^2}$ where $s^2$ is our Mean Sum of Squares of the Residuals from our ANOVA table earlier, which is 3.3929. Thus, our test statistic is $\frac{9.38}{3.3929} = 2.2767216508$. Under $H_0$, $\frac{q_1}{s^2}$ is a random observation from $F_8^1$. As the 95th percentile of $F_8^1$ is 5.318, our test statistic is small compared to it, which no significant reason to reject $H_0$, so we conclude that there is no evidence of difference between Black and other colours.

### 5.5.2 Comparison 2: E and F VS C and G

Here, we use $\underline{c_2}$, which gives us q2:

$$q_2 = \frac{1}{3*1*7}\frac{(T_C^* - T_E^* - T_F^* + T_G^*)^2}{+(1)^2 + (-1)^2 + (-1)^2 + (1)^2}$$

$$q_1 = \frac{1}{21}\frac{9}{4} = 0.10714285$$

The hypothesis test for this comparison is

H0: $\tau_C - \tau_E - \tau_F + \tau_G = 0$ vs H1: $\tau_C - \tau_E - \tau_F + \tau_G \neq 0$

Our test statistic is $\frac{q_2}{s^2} = \frac{0.10714285}{3.3929} = 0.03157854848$. As the 95th percentile of $F_8^1$ is greater than our test statistic, we can conclude there are insufficient reason to reject H0, thus we accept it and conclude that there is no difference between complementary colours.

### 5.5.3 Comparison 3: *C VS G*

Here, we use $\underline{c_3}$, which gives us q3:

$$q_3 = \frac{1}{3*1*7}\frac{(T_C^* - T_G^*)^2}{1^2 + (-1)^2}$$

$$q_3 = \frac{1}{21}\frac{400}{2} = 9.523809$$

The hypothesis test for this comparison is

H0: $\tau_A - \tau_G = 0$ vs H1: $\tau_A - \tau_G \neq 0$

Our test statistic is $\frac{q_3}{s^2} = \frac{9.523809}{3.3929} = 2.806982087$. As the 95th percentile of $F_8^1$ is greater than our test statistic, we can conclude there are insufficient grounds to reject H0, thus we accept it and conclude that there is no difference between Red and Orange.

### 5.5.4 Comparison 4: *C VS F*

Here, we use $\underline{c_4}$, which gives us q4:

$$q_4 = \frac{1}{3*1*7}\frac{(T_E^* - T_F^*)^2}{1^2 + (-1)^2}$$

$$q_4 = \frac{1}{21}\frac{81}{2} = 1.9285714$$

The hypothesis test for this comparison is

H0: $\tau_E - \tau_F = 0$ vs H1: $\tau_E - \tau_F \neq 0$

Our test statistic is $\frac{q_4}{s^2} = \frac{1.9285714}{3.3929} = 0.5684138727$. As the 95th percentile of $F_8^1$ is greater than our test statistic, we can conclude there are insufficient grounds to reject H0, thus we accept it and conclude that there is no difference between Blue and Green

# 6. Conclusion

Our initial objective was to see whether colours impact the ability to retain information, which could potentially be used to improve revision techniques for students. From our study, we see that changing the font/word colours have no influence over memory retention, supporting our null hypothesis $H_0$; despite our pilot study suggesting otherwise. This was then further supported when we investigated the linear contrasts; suggesting that writing notes in a coloured pen wouldn't be any different than writing it with a black pen. However, this excludes the idea that highlighting key words within a paragraph written in black would improve memorisation, which is what students would typically do. This idea is different from having a paragraph only written with a coloured pen. Perhaps in future, we could refine our experiment or design a new experiment altogether, which would be able test out such an idea.

# 7. Reference

Diuna LSP. (2019). *Length of words - average number of characters in a word - Diuna LSP*. [online] Available at: https://diuna.biz/length-of-words-average-number-of-characters-in-a-word/ [Accessed 11 Dec. 2019].

Shiftelearning.com. (2019). *The Psychology of Color: How Do Colors Influence Learning?* [online] Available at: https://www.shiftelearning.com/blog/how-do-colors-influence-learning [Accessed 11 Dec. 2019].

Verywell Mind. (2019). *Take a Deeper Look Into Human Memory*. [online] Available at: https://www.verywellmind.com/what-is-memory-2795006 [Accessed 11 Dec. 2019].