

Planar Groups

August 31, 2025

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1 Planar Groups

1.1 Group 2

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
matrix	det	except	

1.2 Group 3

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$x_0 x_1$	$(x_0 + 1)x_1 + x_0 + 1$	$(x_0 - 1)x_1 - x_0 + 1$
matrix	det	except	
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$x_0 x_1$	$\{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$	

1.3 Group 4

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0x_1	$(x_0 + 1)x_1 + x_0 + 1$	$(x_0 - 1)x_1 - x_0 + 1$
matrix	det	except	
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0x_1	$\{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$	
matrix		conds	
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$		$-2\,a_0 \in \mathbb{Z} \quad \frac{1}{2}\,x_0 - \frac{1}{2} \in \mathbb{Z}$	

1.4 Group 5

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_0 - 2x_1 & 0 \\ x_1 & x_0 \end{pmatrix}$	$(x_0 - 2x_1)x_0$	$x_0^2 - 2(x_0 + 1)x_1 + 2x_0 + 1$	$x_0^2 - 2(x_0 - 1)x_1 - 2x_0 + 1$
matrix	det	except	
$\begin{pmatrix} x_0 - 2x_1 & 0 \\ x_1 & x_0 \end{pmatrix}$	$(x_0 - 2x_1)x_0$	$\{(2t_0 - 1, t_0), (-1, n_1), (-1, -1), (1, 1), (1, n_1), (2t_0 + 1, t_0), (-1, -1)\}$	

1.5 Group 6

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} 0 & x_0 \\ x_1 & 0 \end{pmatrix}$	$-x_0x_1$	$-x_0x_1 + 1$	$-x_0x_1 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	x_0x_1	$(x_0 + 1)x_1 + x_0 + 1$	$(x_0 - 1)x_1 - x_0 + 1$
matrix	det	except	
$\begin{pmatrix} 0 & x_0 \\ x_1 & 0 \end{pmatrix}$	$-x_0x_1$	$\{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$	
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	x_0x_1	$\{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$	

1.6 Group 7

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0x_1	$(x_0 + 1)x_1 + x_0 + 1$	$(x_0 - 1)x_1 - x_0 + 1$
matrix	det	except	
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0x_1	$\{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$	
matrix	conds		
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	$-2\,a_0 \in \mathbb{Z} \quad -2\,a_1 \in \mathbb{Z} \quad -2\,a_0 - \frac{1}{2}\,x_1 + \frac{1}{2} \in \mathbb{Z} \quad \frac{1}{2}\,x_1 - \frac{1}{2} \in \mathbb{Z}$ $-2\,a_1 \in \mathbb{Z}$		

1.7 Group 8

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & -1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$x_0 x_1$	$(x_0 + 1)x_1 + x_0 + 1$	$(x_0 - 1)x_1 - x_0 + 1$
$\begin{pmatrix} 0 & x_1 \\ x_0 & 0 \end{pmatrix}$	$-x_0 x_1$	$-x_0 x_1 + 1$	$-x_0 x_1 + 1$

matrix	det	except
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$x_0 x_1$	$\{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$
$\begin{pmatrix} 0 & x_1 \\ x_0 & 0 \end{pmatrix}$	$-x_0 x_1$	$\{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$

matrix	conds
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$-2a_0 \in \mathbb{Z} \quad -2a_1 \in \mathbb{Z} \quad -2a_0 - \frac{1}{2}x_0 + \frac{1}{2} \in \mathbb{Z} \quad \frac{1}{2}x_1 - \frac{1}{2} \in \mathbb{Z}$ $\frac{1}{2}x_0 - \frac{1}{2} \in \mathbb{Z} \quad -2a_1 + \frac{1}{2}x_1 - \frac{1}{2} \in \mathbb{Z}$
$\begin{pmatrix} 0 & x_1 \\ x_0 & 0 \end{pmatrix}$	$-2a_0 \in \mathbb{Z} \quad -2a_1 \in \mathbb{Z} \quad \frac{1}{2}x_1 - \frac{1}{2} \in \mathbb{Z} \quad -2a_1 - \frac{1}{2}x_0 - \frac{1}{2} \in \mathbb{Z}$ $-2a_0 + \frac{1}{2}x_1 + \frac{1}{2} \in \mathbb{Z} \quad \frac{1}{2}x_0 - \frac{1}{2} \in \mathbb{Z}$

1.8 Group 9

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} -x_0 & -2x_0 \\ x_1 & x_0 \end{pmatrix}$	$-x_0^2 + 2x_0 x_1$	$-x_0^2 + 2x_0 x_1 + 1$	$-x_0^2 + 2x_0 x_1 + 1$
$\begin{pmatrix} x_0 - 2x_1 & 0 \\ x_1 & x_0 \end{pmatrix}$	$(x_0 - 2x_1)x_0$	$x_0^2 - 2(x_0 + 1)x_1 + 2x_0 + 1$	$x_0^2 - 2(x_0 - 1)x_1 - 2x_0 + 1$

matrix	det	except
$\begin{pmatrix} -x_0 & -2x_0 \\ x_1 & x_0 \end{pmatrix}$	$-x_0^2 + 2x_0 x_1$	$\{(-1, -1), (-1, 0), (1, 0), (1, 1)\}$
$\begin{pmatrix} x_0 - 2x_1 & 0 \\ x_1 & x_0 \end{pmatrix}$	$(x_0 - 2x_1)x_0$	$\{(2t_0 - 1, t_0), (-1, n_1), (-1, -1), (1, 1), (1, n_1), (2t_0 + 1, t_0), (-1, -1)\}$

1.9 Group 10

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + x_1^2$	$x_0^2 + x_1^2 + 2x_1 + 1$	$x_0^2 + x_1^2 - 2x_1 + 1$
$\begin{pmatrix} -x_0 & x_1 \\ x_1 & x_0 \end{pmatrix}$	$-x_0^2 - x_1^2$	$-x_0^2 - x_1^2 + 1$	$-x_0^2 - x_1^2 + 1$
matrix	det	except	
$\begin{pmatrix} x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + x_1^2$	$\{(-1, 0), (0, -1), (0, 1), (1, 0)\}$	
$\begin{pmatrix} -x_0 & x_1 \\ x_1 & x_0 \end{pmatrix}$	$-x_0^2 - x_1^2$	$\{(-1, 0), (0, -1), (0, 1), (1, 0)\}$	

1.10 Group 11

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$2x_0^2 + 2x_0 + 1$	$2x_0^2 - 2x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0^2	$x_0^2 + 2x_0 + 1$	$x_0^2 - 2x_0 + 1$
$\begin{pmatrix} -x_0 & x_0 \\ x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$-2x_0^2 + 1$	$-2x_0^2 + 1$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$2x_0^2 + 2x_0 + 1$	$2x_0^2 - 2x_0 + 1$
$\begin{pmatrix} 0 & x_0 \\ -x_0 & 0 \end{pmatrix}$	x_0^2	$x_0^2 + 1$	$x_0^2 + 1$
$\begin{pmatrix} -x_0 & -x_0 \\ -x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$-2x_0^2 + 1$	$-2x_0^2 + 1$
$\begin{pmatrix} -x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$

matrix	det	except
$\begin{pmatrix} x_0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$\{\}$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & x_0 \\ x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$\{\}$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$\{\}$
$\begin{pmatrix} 0 & x_0 \\ -x_0 & 0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & -x_0 \\ -x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$\{\}$
$\begin{pmatrix} -x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$

1.11 Group 12

Generators of group:

$$\left[\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & -1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$2x_0^2 + 2x_0 + 1$	$2x_0^2 - 2x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0^2	$x_0^2 + 2x_0 + 1$	$x_0^2 - 2x_0 + 1$
$\begin{pmatrix} -x_0 & x_0 \\ x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$-2x_0^2 + 1$	$-2x_0^2 + 1$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$2x_0^2 + 2x_0 + 1$	$2x_0^2 - 2x_0 + 1$
$\begin{pmatrix} 0 & x_0 \\ -x_0 & 0 \end{pmatrix}$	x_0^2	$x_0^2 + 1$	$x_0^2 + 1$
$\begin{pmatrix} -x_0 & -x_0 \\ -x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$-2x_0^2 + 1$	$-2x_0^2 + 1$
$\begin{pmatrix} -x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$

matrix	det	except
$\begin{pmatrix} x_0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$\{\}$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & x_0 \\ x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$\{\}$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$\{\}$
$\begin{pmatrix} 0 & x_0 \\ -x_0 & 0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & -x_0 \\ -x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$\{\}$
$\begin{pmatrix} -x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$

matrix	conds
$\begin{pmatrix} x_0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$-2a_0 \in \mathbb{Z}$ $-a_0 + a_1 + \frac{1}{2} \in \mathbb{Z}$ $-a_0 - a_1 + x_0 - \frac{1}{2} \in \mathbb{Z}$ $-x_0 - \frac{1}{2} \in \mathbb{Z}$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$-2a_0 \in \mathbb{Z}$ $-2a_0 - \frac{1}{2}x_0 + \frac{1}{2} \in \mathbb{Z}$ $\frac{1}{2}x_0 - \frac{1}{2} \in \mathbb{Z}$ $-a_0 + a_1 - \frac{1}{2}x_0 + \frac{1}{2} \in \mathbb{Z}$
$\begin{pmatrix} -x_0 & x_0 \\ x_0 & x_0 \end{pmatrix}$	$-2a_0 \in \mathbb{Z}$ $-a_0 + a_1 + x_0 + \frac{1}{2} \in \mathbb{Z}$ $-a_0 - a_1 - \frac{1}{2} \in \mathbb{Z}$ $-2a_0 + \frac{1}{2} \in \mathbb{Z}$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$-2a_0 \in \mathbb{Z}$ $-a_0 - a_1 - x_0 - \frac{1}{2} \in \mathbb{Z}$ $-a_0 + a_1 + \frac{1}{2} \in \mathbb{Z}$ $-2a_0 + \frac{1}{2} \in \mathbb{Z}$
$\begin{pmatrix} 0 & x_0 \\ -x_0 & 0 \end{pmatrix}$	$-2a_0 \in \mathbb{Z}$ $\frac{1}{2}x_0 - \frac{1}{2} \in \mathbb{Z}$ $-2a_0 + \frac{1}{2}x_0 + \frac{1}{2} \in \mathbb{Z}$ $-a_0 - a_1 - \frac{1}{2}x_0 - \frac{1}{2} \in \mathbb{Z}$
$\begin{pmatrix} -x_0 & -x_0 \\ -x_0 & x_0 \end{pmatrix}$	$-2a_0 \in \mathbb{Z}$ $-a_0 - a_1 - \frac{1}{2} \in \mathbb{Z}$ $-a_0 + a_1 - x_0 + \frac{1}{2} \in \mathbb{Z}$ $x_0 - \frac{1}{2} \in \mathbb{Z}$
$\begin{pmatrix} -x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$-2a_0 \in \mathbb{Z}$ $-2a_0 + \frac{1}{2}x_0 + \frac{1}{2} \in \mathbb{Z}$ $-\frac{1}{2}x_0 - \frac{1}{2} \in \mathbb{Z}$ $-a_0 - a_1 + \frac{1}{2}x_0 - \frac{1}{2} \in \mathbb{Z}$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-2a_0 \in \mathbb{Z}$ $\frac{1}{2}x_0 - \frac{1}{2} \in \mathbb{Z}$ $-2a_0 + \frac{1}{2}x_0 + \frac{1}{2} \in \mathbb{Z}$ $-a_0 + a_1 - \frac{1}{2}x_0 + \frac{1}{2} \in \mathbb{Z}$

1.12 Group 13

Generators of group:

$$\left[\begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} -x_1 & x_0 + x_1 \\ x_0 & x_1 \end{pmatrix}$	$-(x_0 + x_1)x_0 - x_1^2$	$-x_0^2 - x_0x_1 - x_1^2 + 1$	$-x_0^2 - x_0x_1 - x_1^2 + 1$
$\begin{pmatrix} x_0 + x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + (x_0 + x_1)x_1$	$x_0^2 + (x_0 + 2)x_1 + x_1^2 + x_0 + 1$	$x_0^2 + (x_0 - 2)x_1 + x_1^2 - x_0 + 1$
matrix	det	except	
$\begin{pmatrix} -x_1 & x_0 + x_1 \\ x_0 & x_1 \end{pmatrix}$	$-(x_0 + x_1)x_0 - x_1^2$	$\{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$	
$\begin{pmatrix} x_0 + x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + (x_0 + x_1)x_1$	$\{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$	

1.13 Group 14

Generators of group:

$$\left[\begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	x_0^2	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	x_0^2	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0^2	$x_0^2 + 2x_0 + 1$	$x_0^2 - 2x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ x_0 & -x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$

matrix	det	except
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} x_0 & 0 \\ x_0 & -x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$

1.14 Group 15

Generators of group:

$$\left[\begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	x_0^2	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	x_0^2	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0^2	$x_0^2 + 2x_0 + 1$	$x_0^2 - 2x_0 + 1$
$\begin{pmatrix} -x_0 & 0 \\ -x_0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$

matrix	det	except
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & 0 \\ -x_0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$

1.15 Group 16

Generators of group:

$$\left[\begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} -x_1 & x_0 + x_1 \\ x_0 & x_1 \end{pmatrix}$	$-(x_0 + x_1)x_0 - x_1^2$	$-x_0^2 - x_0x_1 - x_1^2 + 1$	$-x_0^2 - x_0x_1 - x_1^2 + 1$
$\begin{pmatrix} x_0 + x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + (x_0 + x_1)x_1$	$x_0^2 + (x_0 + 2)x_1 + x_1^2 + x_0 + 1$	$x_0^2 + (x_0 - 2)x_1 + x_1^2 - x_0 + 1$
matrix	det	except	
$\begin{pmatrix} -x_1 & x_0 + x_1 \\ x_0 & x_1 \end{pmatrix}$	$-(x_0 + x_1)x_0 - x_1^2$	$\{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$	
$\begin{pmatrix} x_0 + x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + (x_0 + x_1)x_1$	$\{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$	

1.16 Group 17

Generators of group:

$$\left[\begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} -x_0 & \frac{1}{2}x_0 \\ -\frac{1}{2}x_0 & x_0 \end{pmatrix}$	$-\frac{3}{4}x_0^2$	$-\frac{3}{4}x_0^2 + 1$	$-\frac{3}{4}x_0^2 + 1$
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	x_0^2	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	x_0^2	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0^2	$x_0^2 + 2x_0 + 1$	$x_0^2 - 2x_0 + 1$
$\begin{pmatrix} -x_0 & 2x_0 \\ x_0 & x_0 \end{pmatrix}$	$-3x_0^2$	$-3x_0^2 + 1$	$-3x_0^2 + 1$
$\begin{pmatrix} -x_0 & 0 \\ -x_0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} \frac{1}{2}x_0 & \frac{1}{2}x_0 \\ -\frac{1}{2}x_0 & x_0 \end{pmatrix}$	$\frac{3}{4}x_0^2$	$\frac{3}{4}x_0^2 + \frac{3}{2}x_0 + 1$	$\frac{3}{4}x_0^2 - \frac{3}{2}x_0 + 1$
$\begin{pmatrix} -x_0 & 2x_0 \\ -2x_0 & x_0 \end{pmatrix}$	$3x_0^2$	$3x_0^2 + 1$	$3x_0^2 + 1$
$\begin{pmatrix} -x_0 & -x_0 \\ -2x_0 & x_0 \end{pmatrix}$	$-3x_0^2$	$-3x_0^2 + 1$	$-3x_0^2 + 1$
$\begin{pmatrix} 2x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$3x_0^2$	$3x_0^2 + 3x_0 + 1$	$3x_0^2 - 3x_0 + 1$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$

matrix	det	except
$\begin{pmatrix} -x_0 & \frac{1}{2}x_0 \\ -\frac{1}{2}x_0 & x_0 \end{pmatrix}$	$-\frac{3}{4}x_0^2$	$\{\}$
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	x_0^2	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & 2x_0 \\ x_0 & x_0 \end{pmatrix}$	$-3x_0^2$	$\{\}$
$\begin{pmatrix} -x_0 & 0 \\ -x_0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} \frac{1}{2}x_0 & \frac{1}{2}x_0 \\ -\frac{1}{2}x_0 & x_0 \end{pmatrix}$	$\frac{3}{4}x_0^2$	$\{\}$
$\begin{pmatrix} -x_0 & 2x_0 \\ -2x_0 & x_0 \end{pmatrix}$	$3x_0^2$	$\{\}$
$\begin{pmatrix} -x_0 & -x_0 \\ -2x_0 & x_0 \end{pmatrix}$	$-3x_0^2$	$\{\}$
$\begin{pmatrix} 2x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$3x_0^2$	$\{\}$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$

num	srdegrees
3	$\{x_0x_1 x_0, x_1 \in \mathbb{Z}\} / \{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$
4	$\{x_0x_1 x_0, x_1 \in \mathbb{Z}\} / \{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$
5	$\{(x_0 - 2x_1)x_0 x_0, x_1 \in \mathbb{Z}\} / \{(2t_0 - 1, t_0), (-1, n_1), (-1, -1), (1, 1), (1, n_1), (2t_0 + 1, t_0), (-1, 0), (1, 0)\}$
6	$\{-x_0x_1 x_0, x_1 \in \mathbb{Z}\} / \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$ $\{x_0x_1 x_0, x_1 \in \mathbb{Z}\} / \{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$
7	$\{x_0x_1 x_0, x_1 \in \mathbb{Z}\} / \{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$
8	$\{x_0x_1 x_0, x_1 \in \mathbb{Z}\} / \{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$ $\{-x_0x_1 x_0, x_1 \in \mathbb{Z}\} / \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$
9	$\{-x_0^2 + 2x_0x_1 x_0, x_1 \in \mathbb{Z}\} / \{(-1, -1), (-1, 0), (1, 0), (1, 1)\}$ $\{(x_0 - 2x_1)x_0 x_0, x_1 \in \mathbb{Z}\} / \{(2t_0 - 1, t_0), (-1, n_1), (-1, -1), (1, 1), (1, n_1), (2t_0 + 1, t_0), (-1, 0), (1, 0)\}$
10	$\{x_0^2 + x_1^2 x_0, x_1 \in \mathbb{Z}\} / \{(-1, 0), (0, -1), (0, 1), (1, 0)\}$ $\{-x_0^2 - x_1^2 x_0, x_1 \in \mathbb{Z}\} / \{(-1, 0), (0, -1), (0, 1), (1, 0)\}$
11	$\{2x_0^2 x_0 \in \mathbb{Z}\}$ $\{2x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-2x_0^2 x_0 \in \mathbb{Z}\}$ $\{-2x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{2x_0^2 x_0 \in \mathbb{Z}\}$ $\{2x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-2x_0^2 x_0 \in \mathbb{Z}\}$ $\{-2x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$
12	$\{2x_0^2 x_0 \in \mathbb{Z}\}$ $\{2x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-2x_0^2 x_0 \in \mathbb{Z}\}$ $\{-2x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{2x_0^2 x_0 \in \mathbb{Z}\}$ $\{2x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-2x_0^2 x_0 \in \mathbb{Z}\}$ $\{-2x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$
13	$\{-(x_0 + x_1)x_0 - x_1^2 x_0, x_1 \in \mathbb{Z}\} / \{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$ $\{x_0^2 + (x_0 + x_1)x_1 x_0, x_1 \in \mathbb{Z}\} / \{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$
14	$\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$
15	$\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$
16	$\{-(x_0 + x_1)x_0 - x_1^2 x_0, x_1 \in \mathbb{Z}\} / \{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$ $\{x_0^2 + (x_0 + x_1)x_1 x_0, x_1 \in \mathbb{Z}\} / \{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$
17	$\{-\frac{3}{4}x_0^2 x_0 \in \mathbb{Z}\}$ $\{-\frac{3}{4}x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-3x_0^2 x_0 \in \mathbb{Z}\}$ $\{-3x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$ $\{\frac{3}{4}x_0^2 x_0 \in \mathbb{Z}\}$ $\{\frac{3}{4}x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{3x_0^2 x_0 \in \mathbb{Z}\}$ $\{3x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{-3x_0^2 x_0 \in \mathbb{Z}\}$ $\{-3x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{3x_0^2 x_0 \in \mathbb{Z}\}$ $\{3x_0^2 x_0 \in \mathbb{Z}\} / \{\}$ $\{-x_0^2 x_0 \in \mathbb{Z}\} / \{(-1), (1)\}$