

# Planar Groups

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## 1 Planar Groups

### 1.1 Group 2

Generators of group:

$$\left[ \left( \begin{array}{ccc} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array} \right) \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
matrix	det	except	

## 1.2 Group 3

Generators of group:

$$\left[ \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$x_0 x_1$	$(x_0 + 1)x_1 + x_0 + 1$	$(x_0 - 1)x_1 - x_0 + 1$
matrix	det	except	
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$x_0 x_1$	$\{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$	

## 1.3 Group 4

Generators of group:

$$\left[ \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1		
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0 x_1$	$(x_0 + 1)x_1 + x_0 + 1$	$(x_0 - 1)x_1 - x_0 + 1$		
matrix	det	except			
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0 x_1$	$\{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$			
matrix	conds				
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	$-2 a_0 \in \mathbb{Z} \quad \frac{1}{2} x_0 - \frac{1}{2} \in \mathbb{Z}$				

## 1.4 Group 5

Generators of group:

$$\left[ \begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_0 - 2 x_1 & 0 \\ x_1 & x_0 \end{pmatrix}$	$(x_0 - 2 x_1)x_0$	$x_0^2 - 2(x_0 + 1)x_1 + 2x_0 + 1$	$x_0^2 - 2(x_0 - 1)x_1 - 2x_0 + 1$
matrix	det	except	
$\begin{pmatrix} x_0 - 2 x_1 & 0 \\ x_1 & x_0 \end{pmatrix}$	$(x_0 - 2 x_1)x_0$	$\{(2t_0 - 1, t_0), (-1, n_1), (-1, -1), (1, 1), (1, n_1), (2t_0 + 1, t_0), (-1, -n_1)\}$	

## 1.5 Group 6

Generators of group:

$$\left[ \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} 0 & x_0 \\ x_1 & 0 \end{pmatrix}$	$-x_0x_1$	$-x_0x_1 + 1$	$-x_0x_1 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$x_0x_1$	$(x_0 + 1)x_1 + x_0 + 1$	$(x_0 - 1)x_1 - x_0 + 1$
matrix	det	except	
$\begin{pmatrix} 0 & x_0 \\ x_1 & 0 \end{pmatrix}$	$-x_0x_1$	$\{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$	
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$x_0x_1$	$\{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$	

## 1.6 Group 7

Generators of group:

$$\left[ \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1		
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0x_1$	$(x_0 + 1)x_1 + x_0 + 1$	$(x_0 - 1)x_1 - x_0 + 1$		
matrix	det	except			
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0x_1$	$\{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$			
matrix	conds				
$\begin{pmatrix} x_1 & 0 \\ 0 & x_0 \end{pmatrix}$	$-2a_0 \in \mathbb{Z}$ $-2a_1 \in \mathbb{Z}$ $-2a_0 - \frac{1}{2}x_1 + \frac{1}{2} \in \mathbb{Z}$ $\frac{1}{2}x_1 - \frac{1}{2} \in \mathbb{Z}$ $-2a_1 \in \mathbb{Z}$				

## 1.7 Group 8

Generators of group:

$$\left[ \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & -1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$x_0 x_1$	$(x_0 + 1)x_1 + x_0 + 1$	$(x_0 - 1)x_1 - x_0 + 1$
$\begin{pmatrix} 0 & x_1 \\ x_0 & 0 \end{pmatrix}$	$-x_0 x_1$	$-x_0 x_1 + 1$	$-x_0 x_1 + 1$

matrix	det	except
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$x_0 x_1$	$\{(-1, n_1), (-1, -1), (-1, 1), (1, 1), (1, n_1), (1, -1), (n_1, -1), (n_1, 1)\}$
$\begin{pmatrix} 0 & x_1 \\ x_0 & 0 \end{pmatrix}$	$-x_0 x_1$	$\{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$

matrix	conds
$\begin{pmatrix} x_0 & 0 \\ 0 & x_1 \end{pmatrix}$	$-2 a_0 \in \mathbb{Z}$ $-2 a_1 \in \mathbb{Z}$ $-2 a_0 - \frac{1}{2} x_0 + \frac{1}{2} \in \mathbb{Z}$ $\frac{1}{2} x_1 - \frac{1}{2} \in \mathbb{Z}$ $\frac{1}{2} x_0 - \frac{1}{2} \in \mathbb{Z}$ $-2 a_1 + \frac{1}{2} x_1 - \frac{1}{2} \in \mathbb{Z}$
$\begin{pmatrix} 0 & x_1 \\ x_0 & 0 \end{pmatrix}$	$-2 a_0 \in \mathbb{Z}$ $-2 a_1 \in \mathbb{Z}$ $\frac{1}{2} x_1 - \frac{1}{2} \in \mathbb{Z}$ $-2 a_1 - \frac{1}{2} x_0 - \frac{1}{2} \in \mathbb{Z}$ $-2 a_0 + \frac{1}{2} x_1 + \frac{1}{2} \in \mathbb{Z}$ $\frac{1}{2} x_0 - \frac{1}{2} \in \mathbb{Z}$

## 1.8 Group 9

Generators of group:

$$\left[ \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} -x_0 & -2 x_0 \\ x_1 & x_0 \end{pmatrix}$	$-x_0^2 + 2 x_0 x_1$	$-x_0^2 + 2 x_0 x_1 + 1$	$-x_0^2 + 2 x_0 x_1 + 1$
$\begin{pmatrix} x_0 - 2 x_1 & 0 \\ x_1 & x_0 \end{pmatrix}$	$(x_0 - 2 x_1)x_0$	$x_0^2 - 2(x_0 + 1)x_1 + 2 x_0 + 1$	$x_0^2 - 2(x_0 - 1)x_1 - 2 x_0 + 1$

  

matrix	det	except
$\begin{pmatrix} -x_0 & -2 x_0 \\ x_1 & x_0 \end{pmatrix}$	$-x_0^2 + 2 x_0 x_1$	$\{(-1, -1), (-1, 0), (1, 0), (1, 1)\}$
$\begin{pmatrix} x_0 - 2 x_1 & 0 \\ x_1 & x_0 \end{pmatrix}$	$(x_0 - 2 x_1)x_0$	$\{(2 t_0 - 1, t_0), (-1, n_1), (-1, -1), (1, 1), (1, n_1), (2 t_0 + 1, t_0), (-1, -n_1)\}$

## 1.9 Group 10

Generators of group:

$$\left[ \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + x_1^2$	$x_0^2 + x_1^2 + 2x_1 + 1$	$x_0^2 + x_1^2 - 2x_1 + 1$
$\begin{pmatrix} -x_0 & x_1 \\ x_1 & x_0 \end{pmatrix}$	$-x_0^2 - x_1^2$	$-x_0^2 - x_1^2 + 1$	$-x_0^2 - x_1^2 + 1$

  

matrix	det	except
$\begin{pmatrix} x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + x_1^2$	$\{(-1, 0), (0, -1), (0, 1), (1, 0)\}$
$\begin{pmatrix} -x_0 & x_1 \\ x_1 & x_0 \end{pmatrix}$	$-x_0^2 - x_1^2$	$\{(-1, 0), (0, -1), (0, 1), (1, 0)\}$

## 1.10 Group 11

Generators of group:

$$\left[ \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$2x_0^2 + 2x_0 + 1$	$2x_0^2 - 2x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0^2$	$x_0^2 + 2x_0 + 1$	$x_0^2 - 2x_0 + 1$
$\begin{pmatrix} -x_0 & x_0 \\ x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$-2x_0^2 + 1$	$-2x_0^2 + 1$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$2x_0^2 + 2x_0 + 1$	$2x_0^2 - 2x_0 + 1$
$\begin{pmatrix} 0 & x_0 \\ -x_0 & 0 \end{pmatrix}$	$x_0^2$	$x_0^2 + 1$	$x_0^2 + 1$
$\begin{pmatrix} -x_0 & -x_0 \\ -x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$-2x_0^2 + 1$	$-2x_0^2 + 1$
$\begin{pmatrix} -x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$

matrix	det	except
$\begin{pmatrix} x_0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$2x_0^2$	{}
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & x_0 \\ x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	{}
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$2x_0^2$	{}
$\begin{pmatrix} 0 & x_0 \\ -x_0 & 0 \end{pmatrix}$	$x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} -x_0 & -x_0 \\ -x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	{}
$\begin{pmatrix} -x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$

## 1.11 Group 12

Generators of group:

$$\left[ \left( \begin{array}{ccc} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{ccc} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{ccc} -1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array} \right) \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & -1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} x_0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$2x_0^2 + 2x_0 + 1$	$2x_0^2 - 2x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0^2$	$x_0^2 + 2x_0 + 1$	$x_0^2 - 2x_0 + 1$
$\begin{pmatrix} -x_0 & x_0 \\ x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$-2x_0^2 + 1$	$-2x_0^2 + 1$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$2x_0^2$	$2x_0^2 + 2x_0 + 1$	$2x_0^2 - 2x_0 + 1$
$\begin{pmatrix} 0 & x_0 \\ -x_0 & 0 \end{pmatrix}$	$x_0^2$	$x_0^2 + 1$	$x_0^2 + 1$
$\begin{pmatrix} -x_0 & -x_0 \\ -x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	$-2x_0^2 + 1$	$-2x_0^2 + 1$
$\begin{pmatrix} -x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$

matrix	det	except
$\begin{pmatrix} x_0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$2x_0^2$	{}
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0^2$	{(-1), (1)}
$\begin{pmatrix} -x_0 & x_0 \\ x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	{}
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$2x_0^2$	{}
$\begin{pmatrix} 0 & x_0 \\ -x_0 & 0 \end{pmatrix}$	$x_0^2$	{(-1), (1)}
$\begin{pmatrix} -x_0 & -x_0 \\ -x_0 & x_0 \end{pmatrix}$	$-2x_0^2$	{}
$\begin{pmatrix} -x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	{(-1), (1)}
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	{(-1), (1)}

## 1.12 Group 13

## Generators of group:

$$\left[ \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} -x_1 & x_0 + x_1 \\ x_0 & x_1 \end{pmatrix}$	$-(x_0 + x_1)x_0 - x_1^2$	$-x_0^2 - x_0x_1 - x_1^2 + 1$	$-x_0^2 - x_0x_1 - x_1^2 + 1$
$\begin{pmatrix} x_0 + x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + (x_0 + x_1)x_1$	$x_0^2 + (x_0 + 2)x_1 + x_1^2 + x_0 + 1$	$x_0^2 + (x_0 - 2)x_1 + x_1^2 - x_0 + 1$
matrix	det	except	
$\begin{pmatrix} -x_1 & x_0 + x_1 \\ x_0 & x_1 \end{pmatrix}$	$-(x_0 + x_1)x_0 - x_1^2$	$\{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$	
$\begin{pmatrix} x_0 + x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + (x_0 + x_1)x_1$	$\{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$	

## 1.13 Group 14

Generators of group:

$$\left[ \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$x_0^2$	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	$x_0^2$	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0^2$	$x_0^2 + 2x_0 + 1$	$x_0^2 - 2x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ x_0 & -x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
matrix	det	except	
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$x_0^2$	$\{(-1), (1)\}$	
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	$x_0^2$	$\{(-1), (1)\}$	
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0^2$	$\{(-1), (1)\}$	
$\begin{pmatrix} x_0 & 0 \\ x_0 & -x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$	
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$	
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$	

## 1.14 Group 15

Generators of group:

$$\left[ \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

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$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$x_0^2$	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	$x_0^2$	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0^2$	$x_0^2 + 2x_0 + 1$	$x_0^2 - 2x_0 + 1$
$\begin{pmatrix} -x_0 & 0 \\ -x_0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
matrix	det	except	
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$x_0^2$	$\{(-1), (1)\}$	
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	$x_0^2$	$\{(-1), (1)\}$	
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0^2$	$\{(-1), (1)\}$	
$\begin{pmatrix} -x_0 & 0 \\ -x_0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$	
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$	
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$\{(-1), (1)\}$	

## 1.15 Group 16

Generators of group:

$$\left[ \left( \begin{array}{rrr} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{rrr} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{rrr} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{rrr} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array} \right) \right]$$

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$$\left[ \left( \begin{array}{rrr} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{rrr} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{rrr} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{rrr} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{rrr} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right), \left( \begin{array}{rrr} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{array} \right) \right]$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} -x_1 & x_0 + x_1 \\ x_0 & x_1 \end{pmatrix}$	$-(x_0 + x_1)x_0 - x_1^2$	$-x_0^2 - x_0x_1 - x_1^2 + 1$	$-x_0^2 - x_0x_1 - x_1^2 + 1$
$\begin{pmatrix} x_0 + x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + (x_0 + x_1)x_1$	$x_0^2 + (x_0 + 2)x_1 + x_1^2 + x_0 + 1$	$x_0^2 + (x_0 - 2)x_1 + x_1^2 - x_0 + 1$
matrix	det	except	
$\begin{pmatrix} -x_1 & x_0 + x_1 \\ x_0 & x_1 \end{pmatrix}$	$-(x_0 + x_1)x_0 - x_1^2$	$\{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$	
$\begin{pmatrix} x_0 + x_1 & -x_0 \\ x_0 & x_1 \end{pmatrix}$	$x_0^2 + (x_0 + x_1)x_1$	$\{(-1, 0), (-1, 1), (0, -1), (0, 1), (1, -1), (1, 0)\}$	

## 1.16 Group 17

Generators of group:

$$\left[ \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

SNoT

$$\left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right],$$

matrix	det	eigenvalue != 1	eigenvalue != -1
$\begin{pmatrix} -x_0 & \frac{1}{2}x_0 \\ -\frac{1}{2}x_0 & x_0 \end{pmatrix}$	$-\frac{3}{4}x_0^2$	$-\frac{3}{4}x_0^2 + 1$	$-\frac{3}{4}x_0^2 + 1$
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$x_0^2$	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	$x_0^2$	$x_0^2 + x_0 + 1$	$x_0^2 - x_0 + 1$
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0^2$	$x_0^2 + 2x_0 + 1$	$x_0^2 - 2x_0 + 1$
$\begin{pmatrix} -x_0 & 2x_0 \\ x_0 & x_0 \end{pmatrix}$	$-3x_0^2$	$-3x_0^2 + 1$	$-3x_0^2 + 1$
$\begin{pmatrix} -x_0 & 0 \\ -x_0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$
$\begin{pmatrix} \frac{1}{2}x_0 & \frac{1}{2}x_0 \\ -\frac{1}{2}x_0 & x_0 \end{pmatrix}$	$\frac{3}{4}x_0^2$	$\frac{3}{4}x_0^2 + \frac{3}{2}x_0 + 1$	$\frac{3}{4}x_0^2 - \frac{3}{2}x_0 + 1$
$\begin{pmatrix} -x_0 & 2x_0 \\ -2x_0 & x_0 \end{pmatrix}$	$3x_0^2$	$3x_0^2 + 1$	$3x_0^2 + 1$
$\begin{pmatrix} -x_0 & -x_0 \\ -2x_0 & x_0 \end{pmatrix}$	$-3x_0^2$	$-3x_0^2 + 1$	$-3x_0^2 + 1$
$\begin{pmatrix} 2x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$3x_0^2$	$3x_0^2 + 3x_0 + 1$	$3x_0^2 - 3x_0 + 1$
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	$-x_0^2 + 1$	$-x_0^2 + 1$

matrix	det	except
$\begin{pmatrix} -x_0 & \frac{1}{2}x_0 \\ -\frac{1}{2}x_0 & x_0 \end{pmatrix}$	$-\frac{3}{4}x_0^2$	{}
$\begin{pmatrix} 0 & x_0 \\ -x_0 & x_0 \end{pmatrix}$	$x_0^2$	{(-1), (1)}
$\begin{pmatrix} x_0 & -x_0 \\ x_0 & 0 \end{pmatrix}$	$x_0^2$	{(-1), (1)}
$\begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}$	$x_0^2$	{(-1), (1)}
$\begin{pmatrix} -x_0 & 2x_0 \\ x_0 & x_0 \end{pmatrix}$	$-3x_0^2$	{}
$\begin{pmatrix} -x_0 & 0 \\ -x_0 & x_0 \end{pmatrix}$	$-x_0^2$	{(-1), (1)}
$\begin{pmatrix} -x_0 & x_0 \\ 0 & x_0 \end{pmatrix}$	$-x_0^2$	{(-1), (1)}
$\begin{pmatrix} \frac{1}{2}x_0 & \frac{1}{2}x_0 \\ -\frac{1}{2}x_0 & x_0 \end{pmatrix}$	$\frac{3}{4}x_0^2$	{}
$\begin{pmatrix} -x_0 & 2x_0 \\ -2x_0 & x_0 \end{pmatrix}$	$3x_0^2$	{}
$\begin{pmatrix} -x_0 & -x_0 \\ -2x_0 & x_0 \end{pmatrix}$	$-3x_0^2$	{}
$\begin{pmatrix} 2x_0 & -x_0 \\ x_0 & x_0 \end{pmatrix}$	$3x_0^2$	{}
$\begin{pmatrix} 0 & x_0 \\ x_0 & 0 \end{pmatrix}$	$-x_0^2$	{(-1), (1)}



