Lab work 4

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1 Task1

$$P = \{(x,y) \in \mathbf{R} : |x-5| \leq 3, |y-3| \leq 2\}$$

$$f(x,y) = 2xy + y^2 \in C(P)$$
$$f'_y = 2x + 2y \in C(P) \Rightarrow L = \max_{(x,y)\in P} |f'_y(x,y)| = f'_y(8,5) = 26$$

$$M = \max_{(x,y)\in P} |f(x,y)| = f(8,5) = 105$$

$$h = \min(a, \frac{b}{M}) = \frac{2}{105}$$

Thus, by the Picard's theorem on $I_h = [5 - \frac{2}{105}, 5 + \frac{2}{105}]$ $\exists\,!$ solution of the given Cauchy's problem:

$$y_0(x) = 3$$
$$y_1(x) = 3 + \int_5^x f(s, y_0(s))ds = \int_5^x (6s+9)ds = 3x^2 + 9x - 30$$
$$y_2(x) = 3 + \int_5^x (2s(3s^2+9s-30)+(3s^2+9s-30)^2)ds = \frac{9x^5}{5} + 15x^4 - 27x^3 - 300x^2 + 900x - 8622$$
$$y_3 = 3 + \int_5^x (2s(\frac{9s^5}{5}+15s^4-27s^3-300s^2+900s-8622)+(\frac{9s^5}{5}+15s^4-27s^3-300s^2+900s-8622)^2)ds =$$
$$= \frac{81x^{11}}{275} + \frac{27x^{10}}{5} + \frac{71x^9}{5} - \frac{945x^8}{4} - \frac{3591x^7}{5} + \frac{10159x^6}{5} -$$
$$-\frac{217314x^5}{5} - 18753x^4 + 1995000x^3 - 7768422x^2 + 74338884x - \frac{11330888405}{44}$$

$$\Delta_3 \leq \frac{M}{L}\frac{(Lh)^{n+1}}{(n+1)!} = \frac{105}{26}\frac{(26*2/105)^4}{26*4!} = \frac{1352}{3472875} \approx 0.000389302 < 10^{-3}$$

2 Task2

$$K = \{(x, y) \in \mathbf{R}^2 | x^2 + y^2 \le 9\}$$
$$y' = g(y) = -\sqrt{y},\ y(2) = 0$$

1. Given Cauchy problem doesn't satisfy the conditions of Peano existence theorem because of $g(y) \notin C(K)$ (y must be non-negative), but $g(y) \in C(K_1)$ where $K_1 = K \cap \{(x, y) \in \mathbf{R}^2 : y > 0\}$. For K_1 Peano existence theorem guarantees that solution exists on $[2-h, 2+h], h = \min(-3, \frac{3}{M})$.

$$M = \max_{K1} |g(y)| = |g(3)| = \sqrt{3}$$

   Thus, solution exists on $[-1, 5]$

2. $g(y)$ satisfy the Lipschitz condition on $K1$ because of $g'(y) = -\frac{1}{2\sqrt{y}}$

3. After the integration of equation we have

$$x = -\frac{\sqrt{y}}{2} - C$$
$$y = 4(x+C)^2,\ x < -C$$

   Obvious, that there are no maximum and minimum solutions.

4. Domain, where there are graphics of solutions is (x, y) — y ¿ 0

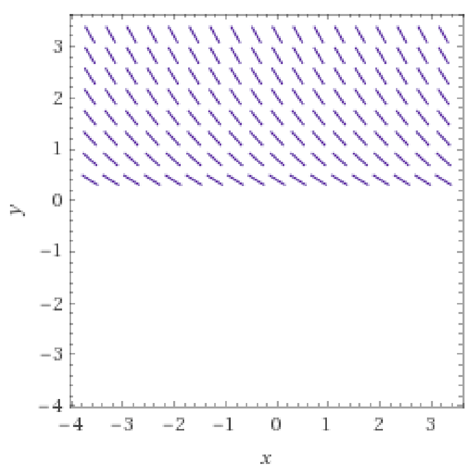

Figure 1: Domain

3 Task 3

$$y' = y + x + 1,\ y(0) = 0$$

1. Using Picard approximation method find the solution.

$$f(x,y) = y + x + 1 \in C(\mathbf{R}), f'_y(x,y) = 1 \in C(\mathbf{R}) \Rightarrow L = 1$$

$$y_n = y_0 + \int_0^x f(s, y_{n-1}(s))ds$$

$$y_0 = 0$$

$$y_1 = \int_0^x (1+s)ds = \frac{x^2}{2} + x$$

$$y_2 = \int_0^x (\frac{s^2}{2} + s + s + 1)ds = \frac{x^3}{6} + x^2 + x$$

$$y_3 = \int_0^x (\frac{s^3}{6} + s^2 + s + s + 1)ds = \frac{x^4}{24} + \frac{x^3}{3} + x^2 + x$$

$$y_4 = \int_0^x (\frac{s^4}{24} + \frac{s^3}{3} + s^2 + s + s + 1)ds = \frac{x^5}{120} + \frac{x^4}{12} + \frac{x^3}{3} + x^2 + x$$

Thus,

$$y_n = \frac{x^n}{n!} + \sum_{k=3}^{n} \frac{2x^n}{n!} + x^2 + x$$

$$\lim_{n\to\infty} y_n(x) = \lim_{n\to\infty} \left(\frac{x^n}{n!} + \sum_{k=3}^{n} \frac{2x^n}{n!} + x^2 + x \right) = \sum_{n=3}^{\infty} \frac{2x^n}{n!} + x^2 + x = 2e^x - x - 2$$

2. Using Lagrange method find the solution.

$$y(x) = e^{\int_0^x 1dx} \left(0 + \int_0^x e^{-\int_0^s 1dt}(1+s)ds \right) =$$

$$= e^x \left(-e^{-x} + 1 + \int_0^x e^{-s} sds \right) = e^x(-e^{-x} + 1 - xe^{-x} - e^{-x} + 1) = 2e^x - x - 2$$

3. See the *.m files

4. See the *.m files