1. $Reservation(sID,\ age,\ length,\ sName,\ day,\ cName,\ rating,\ cID)$
$Function\ Dependencies,\ S\ =\ \{sID\ \rightarrow\ sName,\ rating,\ age\ ;\ cID\ \rightarrow\ cName,\ length\}$

a.

sID	age	length	sName	day	cName	rating	cID
4	24	8	Davendra	04-07-2021 13:00:00	craft_one	3	2
4	24	8	Davendra	11-09-2021 10:00:00	craft_one	3	2

b. <u>Using BCNF Decomposition</u>

Step 1	$Reservation(sID,\ age,\ length,\ sName,\ day,\ cName,\ rating,\ cID)$ $S\ =\ \{sID\ \rightarrow sName,\ rating,\ age\ ;\ cID\ \rightarrow cName,\ length\}$ Let us find the closure of sID: $sID^{+} = \{sID,\ sName,\ rating,\ age\ \}\ \Rightarrow Violates\ BCNF$ Create relation R1: $R1(sID,\ sName,\ rating,\ age)$ Create relation R2: $X^{+}\ =\ \{sID,\ sName,\ rating,\ age\}$ $X\ =\ \{sID\}$ $R2\ =\ Reservation\ -\ (X^{+} - X)$ $R2\ = Reservation\ -\ \{sName,\ rating,\ age\}$ $R2(sID,\ cID,\ cName,\ length,\ day)$
Step 2	Project Functional Dependencies (FD) onto R1 and R2: $R1(sID,\ sName,\ rating,\ age)$ $sID^{+} = \{sID,\ sName,\ rating,\ age\ \}\ \Rightarrow Is\ a\ key$ $sID\ \rightarrow sName,\ rating,\ age\ \Rightarrow Strongest\ FD$ $R2(sID,\ cID,\ cName,\ length,\ day)$ $sID^{+} = \{sID,\ sName,\ rating,\ age\ \}\ \Rightarrow Already\ have\ sID\ \rightarrow sName,\ rating,\ age$ $cID^{+} = \{cID,\ cName,\ length\ \}\ \Rightarrow We\ get,\ cID\ \rightarrow cName,\ length$ $cID\ \rightarrow cName,\ length\ \Rightarrow Violates\ BCNF$
Step 3	Now, decompose R2 from step 2: $R2(sID,\ cID,\ cName,\ length,\ day)$ Create a new space called S' : $S'\ =\ \{\ cID\ \rightarrow cName,\ length\}$ $cID^{+} = \{cID,\ cName,\ length\ \}$ $X^{+}\ =\ \{cID, cName,\ length\}$ $X\ =\ \{cID\}$ Given R1',

	$R1'(cID,\ cName,\ length)$ Determine R2' to be: $R2' = R2 - (X^{+} - X)$ $R2' = R2 - \{cName,\ length\}$ $R2'(sID,\ cID,\ day)$
Step 4	Project FDs in S' onto R1' and R2' $R1'(cID,\ cName,\ length)$ $cID^{+} = \{cName,\ length\ \} \Rightarrow Strongest\ FD:\ cID \rightarrow cName,\ length$ $R2'(sID,\ cID,\ day)$ $cID^{+} = \{cName,\ length\ \} \Rightarrow Strongest\ FD:\ cID \rightarrow cName,\ length$ $sID^{+} = \{\ sID\}$
Step 5	Finally we get three relations after the BCNF decomposition: $R1(sID, sName,\ rating,\ age)$ $R2(cID,\ cName,\ length)$ $R3(sID,\ cID,\ day)$

2. Relation F has attributes $R = \{K\ L\ M\ N\ O\ P\ Q\ R\ S\}$and Functional Dependencies (FD) $G = \{KOQ \rightarrow PS,\ L \rightarrow KN,\ KQ \rightarrow RS\}$
   a. Now, let us find the closures:
      $KOQ^{+} = KOPQRS \Rightarrow Not\ a\ Superkey,\ violates\ BCNF$
      $L^{+} = KLN \Rightarrow Not\ a\ Superkey,\ violates\ BCNF$
      $KQ^{+} = KQRS \Rightarrow Not\ a\ Superkey,\ violates\ BCNF$
      Therefore all FD's violate BCNF:
      $KOQ \rightarrow PS$
      $KQ \rightarrow RS$
      $L \rightarrow KN$
   b.

Step 1	Take $KOQ \rightarrow PS$and compute KOQ^{+} $KOQ^{+} = KOPQRS$
Step 2	Replace with two relations given relation *R(K L M N O P Q R S)* $R1 = \{KOPQRS\}$ $X^{+} = KOQ^{+} = KOPQRS$ $X = KOQ$ $X^{+} - X = \{K\ O\ P\ Q\ R\ S\} - \{K\ O\ Q\}$ $X^{+} - X = \{PR\ S\}$

	R2 = R - (X⁺- X) *R2 = {K L M N O P Q R S}- {P R S}* *R2 = {K L M N O Q}*
Step 3	Project the FD's on R1(K O P Q R S) $K^+ = K \Rightarrow nothing$ $O^+ = O \Rightarrow nothing$ $P^+ = P \Rightarrow nothing$ $Q^+ = Q \Rightarrow nothing$ $R^+ = R \Rightarrow nothing$ $S^+ = S \Rightarrow nothing$ $KO^+ = KO \Rightarrow nothing$ $KP^+ = KP \Rightarrow nothing$ $KQ^+ = KQRS \Rightarrow KQ \rightarrow RS,$ $which\ violates\ BCNF\ since\ KQ\ is\ not\ a\ superkey$ ***Therefore, stop projection and decompose R1 further.***
Step 4	Decompose R1 using $KQ \rightarrow RS$ $KQ^+ = KQRS$ *R3(KQRS)* $X^+ = KQ^+ = KQRS$ $X = KQ$ $X^+ - X = \{K\ Q\ R\ S\} - \{K\ Q\}$ $X^+ - X = \{R\ S\}$ *R4 = R1 - (X⁺- X)* *R4 = {K O PQ R S}- {R S}* *R4 = {K O P Q}*
Step 5	Project FD's onto R3(K Q R S) $L = \{KQRS\},\ S = \{KQ \rightarrow RS\}$ $K^+ = K \Rightarrow nothing$ $Q^+ = Q \Rightarrow nothing$ $R^+ = R \Rightarrow nothing$ $S^+ = S \Rightarrow nothing$ $KQ^+ = KQRS \Rightarrow Superkey\ of\ R3\ so\ we\ can\ ignore\ supersets\ of\ KQ$ $KR^+ = KR \Rightarrow nothing$ $KS^+ = KS \Rightarrow nothing$ $KRS^+ = KRS \Rightarrow nothing$ $QRS^+ = QRS \Rightarrow nothing$ *Only KQ → RS follows and it is a superkey* ***Therefore R3 satisfies BCNF***
Step 6	Project FD's onto R4(K O P Q)

	$K^+ = K \Rightarrow nothing$ $O^+ = O \Rightarrow nothing$ $P^+ = P \Rightarrow nothing$ $Q^+ = Q \Rightarrow nothing$ $KO^+ = KO \Rightarrow nothing$ $KP^+ = KP \Rightarrow nothing$ $KQ^+ = KQRS \Rightarrow nothing$ $OP^+ = OP \Rightarrow nothing$ $OQ^+ = OQ \Rightarrow nothing$ $PQ^+ = PQ \Rightarrow nothing$ $KOP^+ = KOP \Rightarrow nothing$ $KOQ^+ = KOPQRS \Rightarrow KOQ \rightarrow P is\ a\ superkey\ of\ R4$ $KPQ^+ = KPQ \Rightarrow nothing$ $OPQ^+ = OPQ \Rightarrow nothing$ ***Since KOQ$^+$ is a superkey, R4 satisfies BCNF***
Step 7	Project FD's onto R2(K L M N O Q) $K^+ = K \Rightarrow nothing$ $L^+ = KLN \Rightarrow L \rightarrow KN\ is\ not\ a\ superkey\ of\ R2,\ violates\ BCNF$ *Cancel projection and decompose R2 using L → KN*
Step 8	Decompose R2(K L M N O Q) using FD *L ⟶ KN* $L^+ = KLN$ *R5(KLN)* $X^+ = L^+ = LKN$ $X = L$ $X^+ - X = \{L\ K\ N\} - \{L\}$ $X^+ - X = \{KN\}$ *R6 = R2 - (X$^+$- X)* *R6 =* {*K L M N O Q*}*- {K N}* *R6 = {L M O Q}*
Step 9	Project FD's onto R5(K L N) $K^+ = K \Rightarrow nothing$ $L^+ = KLN \Rightarrow L \rightarrow KN\ is\ a\ superkey\ of\ R5,\ ignore\ supersets\ of\ L$ $N^+ = N \Rightarrow nothing$ $KN^+ = KN \Rightarrow nothing$ ***With FD L⟶ KN, R5(KLN) satisfies BCNF***
Step 10	Project FD's onto R6(L M O Q) $L^+ = KLN$

	$M^{+} = M \Rightarrow nothing$ $O^{+} = O \Rightarrow nothing$ $Q^{+} = Q \Rightarrow nothing$ $LM^{+} = KLMN \Rightarrow nothing$ $LO^{+} = KLNQRS \Rightarrow nothing$ $MO^{+} = MO \Rightarrow nothing$ $MQ^{+} = MQ \Rightarrow nothing$ $OQ^{+} = OQ \Rightarrow nothing$ $LMO^{+} = KLMNO \Rightarrow nothing$ $LMQ^{+} = KLMNQRS \Rightarrow nothing$ $LOQ^{+} = KLNOQRS \Rightarrow nothing$ $MOQ^{+} = MOQ \Rightarrow nothing$ *No FD's, **R6 satisfies BCNF***
Step 11	From R(K L M N O P Q R S) we get: $\Rightarrow R3(KQRS),\ KQ \rightarrow RS$ $\Rightarrow R4(KOPQ),\ KOQ \rightarrow P$ $\Rightarrow R5(KLN), L \rightarrow KN$ $\Rightarrow R6(LMOQ),\ no\ FD's$

c. For the relation *R(K L M N O P Q R S)*, the Functional Dependencies are: *G = {KOQ → PS, L → KN, KQ → RS}*
When decomposed into R3, R4, R5, R6:
- *KQ → RS* from R3 is preserved
- *L → KN* from R5 is preserved
- *KOQ → S* has been given by a stronger FD from R3 i.e *KQ → S*

Thus, the final schema preserves the dependencies.

d. Recall *R(K L M N O P Q R S)* was decomposed into:
- *R3(K Q R S)*
- *R4(K O P Q)*
- *R5(K L N)*
- *R6(L M O Q)*

Let $t = < k, l, m, n, o, p, q, r, s >$ be a tuple in the join of projections of some tuples of, one for each *R3, R4, R5, R6.*
Use FD's to show that one of the tuples must be *t*.

	K	*L*	*M*	*N*	*O*	*P*	*Q*	*R*	*S*
R3	***k***	*l1*	*m1*	*n1*	*o1*	*p1*	***q***	***r***	***s***
R4	***k***	*l2*	*m2*	*n2*	***o***	***p***	***q***	*r2*	*s2*

R5	**k**	**l**	m3	**n**	o3	p3	q3	r3	s3
R6	k4	**l**	**m**	n4	**o**	p4	**q**	r4	s4

Apply L → KN:

	K	L	M	N	O	P	Q	R	S
R3	**k**	l1	m1	n1	o1	p1	**q**	**r**	**s**
R4	**k**	l2	m2	n2	**o**	**p**	**q**	r2	s2
R5	**k**	**l**	m3	**n**	o3	p3	q3	r3	s3
R6	**k**	**l**	**m**	**n**	**o**	p4	**q**	r4	s4

Apply KOQ → PS:

	K	L	M	N	O	P	Q	R	S
R3	**k**	l1	m1	n1	o1	p1	**q**	**r**	**s**
R4	**k**	l2	m2	n2	**o**	**p**	**q**	r2	**s**
R5	**k**	**l**	m3	**n**	o3	p3	q3	r3	s3
R6	**k**	**l**	**m**	**n**	**o**	**p**	**q**	r4	**s**

Apply KQ → RS:

	K	L	M	N	O	P	Q	R	S
R3	**k**	l1	m1	n1	o1	p1	**q**	**r**	**s**
R4	**k**	l2	m2	n2	**o**	**p**	**q**	**r**	**s**
R5	**k**	**l**	m3	**n**	o3	p3	q3	r3	s3
R6	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**

The green indicates a completely unsubscripted row.
→ Any tuple in the project-join is in the original relation R

→ **From the Chase Test, BCNF guarantees a lossless join**

3. R has attributes ABCDEFGH

S = {ACDE → B, B → CF, CD → AF, BCF → AD, ABF → H}

a.

<table>
<tr><td>Step 1</td><td>Split the RHS to get our initial set of FD's, S':
(a) $ACDE \rightarrow B$
(b) $B \rightarrow C$
(c) $B \rightarrow F$
(d) $CD \rightarrow A$
(e) $CD \rightarrow F$
(f) $BCF \rightarrow A$
(g) $BCF \rightarrow D$
(h) $ABF \rightarrow H$</td></tr>
<tr><td>Step 2</td><td>For each FD, try to reduce the LHS using closures over the set S'.
(a) $ACDE \rightarrow B$
$A^+ = A$
$C^+ = C$
$D^+ = D$
$E^+ = E$
$AC^+ = AC$
$AD^+ = AD$
$AE^+ = AE$
$CD^+ = ACDF$
$CE^+ = CE$
$DE^+ = DE$
$ACD^+ = ACDF$
$ACE^+ = ACE$
$ADE^+ = ADE$
$CDE^+ = ACDEF$
None of the LHS yields B so therefore we cannot reduce the LHS of this FD
(b) $B \rightarrow C$
There is nothing to reduce since we look at where $\lvert x \rvert \geq 2$ in FD $x \rightarrow y$
(c) $B \rightarrow F$
There is nothing to reduce here
(d) $CD \rightarrow A$
$C^+ = C$
$D^+ = D$
No singleton LHS yields anything, thus we cannot reduce the LHS of this FD
(e) $CD \rightarrow F$</td></tr>
</table>

	$C^{+} = C$ $D^{+} = D$ ***No singleton LHS yields anything, thus we cannot reduce the LHS of this FD*** *(f) BCF → A* $B^{+} = ABCDFH$ ***We can reduce the LHS of the FD, yielding the new FD to: B → A*** *(g) BCF → D* $B^{+} = ABCDFH$ ***By the same argument as above, we can reduce this FD to: B → D*** *(h) ABF → H* $A^{+} = A$ $B^{+} = ABCDFH$ ***We can reduce this FD to: B → H*** Now our new set of FD's S'': *(a) ACDE → B* *(b) B → C* *(c) B → F* *(d) CD → A* *(e) CD → F* *(f) B → A* *(g) B → D* *(h) B → H*
Step 3	Try to eliminate the redundant FD's: *(a)* $ACDE^{+}_{S''-(a)} = ACDEF \Rightarrow We\ need\ this\ FD$ *(b)* $B^{+}_{S''-(b)} = ABDFH \Rightarrow We\ need\ this\ FD$ *(c)* $B^{+}_{S''-(c)} = ABCDFH \Rightarrow We\ can\ remove\ FD\ B \rightarrow F$ *(d)* $CD^{+}_{S''-\{(c),(d)\}} = CDF \Rightarrow We\ need\ this\ FD$ *(e)* $CD^{+}_{S''-\{(c),(e)\}} = ACD \Rightarrow We\ need\ this\ FD$ *(f)* $B^{+}_{S''-\{(c),(f)\}} = ABCDH \Rightarrow We\ can\ remove\ FD\ B \rightarrow A$ *(g)* $B^{+}_{S''-\{(c),(f),(g)\}} = BCH \Rightarrow We\ need\ this\ FD$ *(h)* $B^{+}_{S''-\{(c),(f),(h)\}} = ABCDF \Rightarrow We\ need\ this\ FD$ The final set FD's (renumbered) which is the minimal basis S''' is: *(a) ACDE → B* *(b) B → C* *(c) B → D* *(d) B → H* *(e) CD → A* *(f) CD → F*

b. If an attribute is not found anywhere in the FD's, it has to be in every key. Also, if an attribute appears in the FD but never appears on the RHS, it has to be in every key. On the other hand, if an attribute appears only on the RHS of the FD's, never the left, it does not help in computing any key since it can be removed and will still get the same closure.
Given our attributes: A B C D E F G H and considering the minimal basis
$S''' = \{ACDE \rightarrow B,\ B \rightarrow C,\ B \rightarrow D,\ B \rightarrow H,\ CD \rightarrow A,\ CD \rightarrow F\}$

- G is not found anywhere hence, G must be in every key
- E appears in an FD but never on the RHS thus, E must be in every key
- H and F appear only on the RHS of the FD's, but never the left, thus, H and F are not in any key

To summarize:

Attribute	Appears on		Conclusion
	LHS	RHS	
G	—	—	Must be in every key
E	✓	—	Must be in every key
H, F	—	✓	Not in any key
A, B, C, D	✓	✓	Must check

We only have to consider all combinations of A, B, C, D. For each combination, we must add E and G as they must be in every key.

$AEG^+ = AEG \Rightarrow does\ not\ yield\ every\ attribute,\ therefore\ not\ a\ key$

$BEG^+ = ABCDEFGH \Rightarrow Yields\ every\ attribute,\ therefore\ BEG\ is\ a\ key$

$\Rightarrow can\ ignore\ supersets\ of\ BEG\ as\ they\ will\ be\ superkeys,\ but\ not\ minimal$

$CEG^+ = CEG \Rightarrow does\ not\ yield\ every\ attribute,\ therefore\ not\ a\ key$

$DEG^+ = DEG \Rightarrow does\ not\ yield\ every\ attribute,\ therefore\ not\ a\ key$

$ACEG^+ = ACEG \Rightarrow does\ not\ yield\ every\ attribute,\ therefore\ not\ a\ key$

$ADEG^+ = ADEG \Rightarrow does\ not\ yield\ every\ attribute,\ therefore\ not\ a\ key$

$CDEG^+ = ABCDEFGH \Rightarrow Yields\ every\ attribute,\ therefore\ CDEG\ a\ key$

Keys for R: BEG, CDEG

c. Recall the minimal FD's : $S_5 = \{ACDE \rightarrow B,\ B \rightarrow CDH,\ CD \rightarrow AF\}$

Step 1	Construct a minimal basis, which is $S_5 = \{ACDE \rightarrow B,\ B \rightarrow CDH,\ CD \rightarrow AF\}$

Step 2	For each FD $X \to Y \in S_5$, define a new schema $X \cup Y$ The resulting relations are: ○ R1(A B C D E) ○ R2(B C D H) ○ R3(A C D F)
Step 3	If no relation is a superkey for L = A, B, C, D, E, F, G, H add a relation whose schema is some key. None of the relations in Step 2 contains the key of relation R, so we add a new relation R4(B E G) The final set of relations are: ○ R1(A B C D E) ○ R2(B C D H) ○ R3(A C D F) ○ R4(B E G)

d. Clearly, $CD \to AF$ will be projected onto R1 and $CD^{+} = ACDF$
$\Rightarrow CD\ is\ not\ a\ superkey\ for\ R1\ (violates\ BCNF) \Rightarrow FD's\ with\ a\ non-superkey\ on\ the\ LHS$
$\Rightarrow Therefore\ the\ schema\ allows\ for\ redundancy$