MECE 5397: Scientific Computing for Mechanical Engineers

Solution of the discretized diffusion equation in two spatial dimensions with several time integration schemes

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# Abstract

Several different time integration schemes are explored in the pursuit of solving the diffusion equation in two spatial dimensions. Grid independence as well as error analyses are carried out for several relevant time steps in the domain of interest. Explorations are undertaken to determine the effective CPU run time for each discretization as the number of discrete grid points in the discretization is increased. Finally, optimization in the MATLAB code used to determine the solution is discussed.

# Problem Statement

Write a computer code to solve the two-dimensional diffusion equation

on the domain

Subject to

# Discretization

For the purposes of this assignment, the solution to the differential equation will be denoted by *u* and the exact solution will be noted by *uexact*. For compactness, the solution at time coordinate *tn* and spatial coordinates *xj* and *yk* will be given by

The second order centered discretization of the second derivatives in space will be denoted,

With this notation in place, the Crank-Nicolson discretization of the diffusion equation is a two-level time scheme given by

Which we know is unconditionally stable for Dirichlet boundary conditions and second order accurate in time and space.

What about Neumann bcs?

While the Crank-Nicolson method will certainly solve the differential equation given in (1), its implementation in two spatial dimensions will require the solution of a matrix with five diagonals. There is no simple algorithm for solving such a system, so we might resort to LU factorization to achieve this end. The factorization need only be found a single time, however, at which point the unknown side of the matrix equation could be changed repeatedly which would