CS51 FINAL PROJECT: IMPLEMENTING MINIML

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Contents

The final project is due Wednesday, April 27, 2016 at 5:00pm EST.

The culminative final project for CS51 is the implementation of a small subset of an OCaml-like language. Unlike OCaml and the ML programming language it was derived from, the language includes only a subset of constructs, is only very weakly typed, and does no type inference. On the other hand, the language is "Turing-complete", as expressive as any other programming language in the sense specified by the Church-Turing thesis. (Take CS121 for more on this fundamental and fascinating computational idea.) Because the language is so small, we refer to it as MiniML (pronounced "minimal").

This document will introduce the idea of specifying the semantics of a programming language and implementing that specification. The EXERCISES herein are to test your understanding. We recommend that you do the exercises, but you won't be turning them in. The STAGES provide a sequence of stages to implement MiniML. It's the result of working on these stages that you will be turning in and on which the project grade will be based.

EXERCISES

STAGES

The implementation of this OCaml subset MiniML is in the form of an interpreter for expressions of the language written in OCaml itself, a METACIRCULAR INTERPRETER. Actually, you will implement a series of interpreters that vary in the semantics they manifest. The first is based on the substitution model; the second

METACIRCULAR INTERPRETER

Date: April 23, 2016.

a dynamically scoped environment model; and the third, a version of the second implementing one or more extensions of your choosing, with lexical scoping being a simple and highly recommended option.

This project specification is divided into three sections (corresponding to the section numbers marked below):

Substitution model (Section 3): Implementation of a MiniML interpreter using the substitution semantics for the language.

Dynamic scoped environment model (Section 4): Implementation of a MiniML interpreter using the environment model and manifesting dynamic scoping.

Extensions (Section 5): Implementation of one or more extensions to the basic MiniML language of your choosing. Special attention is paid below to an extension to the environment model manifesting lexical scoping (Section 5.2).

Projects are to be done individually (the sole exception described in Section 7), and will be graded based on: correctness of the implementation of the first two stages, design and style of the submitted code, and scope of the project as a whole as demonstrated by a short paper describing your work.

It may be that you are unable to complete all parts of the final project. You should make sure to keep versions at appropriate milestones so that you can always roll back to a working partial project to submit. Git will be especially useful for this version tracking if used properly.

Some students or groups might prefer to do a different final project on a topic of their own divising. For students who have been doing exceptionally well in the course to date, this may be possible. See Section 7 for further information.

1. Programming language semantics

How do we know that a certain program computes a certain value? When we present OCaml with the expression `3 + 4 * 5`, how do we know that it will calculate `23` as the result? This kind of question is the province of a programming language SEMANTICS.[1]

SEMANTICS

We'll give the semantics of a language by defining a set of rules that provide a schematic recipe for transforming one expression into another simpler expression. Because the output of each rule is intended to be simpler than the input, the rules are called REDUCTION RULES. The input expression to be transformed is the REDEX, and the output its REDUCTION.

REDUCTION RULES
REDEX
REDUCTION

[1] Syntax constitutes the well-formed structures of a language, semantics the meaning of those structures.

The idea is probably already familiar to you, as it is implicitly used as early as grade school for providing the semantics of arithmetic expressions. For instance, a sequence of reductions provides a value for the arithmetic expression $3 + 4 \cdot 5$:

$$3 + \underline{4 \cdot 5} \longrightarrow \underline{3 + 20} \longrightarrow 23$$

Here, I've underlined each redex for clarity. The first redex is $4 \cdot 5$, for which we substitute its value 20, implicitly appealing to a reduction rule

$$4 \cdot 5 \longrightarrow 20$$

resulting in an expression which is a redex in its entirety, $3 + 20$; that redex further reduces to 23 according to a reduction rule

$$3 + 20 \longrightarrow 23 \quad .$$

You may think these reduction rules seem awfully particular. We'd need an infinite number of them, one for $1 + 1$, one for $1 + 2$, one for $2 + 1$, and so forth. We will shortly generalize them to more schematic rules.

The reduction approach applies to OCaml arithmetic expressions in exactly the same manner, more or less undergoing a mere change of font.

```
3 + 4 * 5 ⟶ 3 + 20 ⟶ 23
```

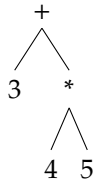
1.1. **Abstract and Concrete Syntax.** In order to apply the reduction rules properly, we need to have an understanding of the subparts of an expression. The following is an improper application of the reduction rules, because the bit being substituted for, the putative redex, is not a proper subpart of the expression:

```
3 + 4 * 5 ⟶ 7 * 5 ⟶ 35
```

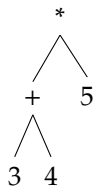
Though expressions are notated as *sequences* of tokens – `3`, `+`, `4`, `*`, ... – they are really structured hierarchically as *trees* of subexpressions. We distinguish between the ABSTRACT SYNTAX of the language, the tree structures that make up well-formed hierarchical expressions, and the CONCRETE SYNTAX of the language, the token sequences that notate those tree structures. Implicitly, the concrete syntax `3 + 4 * 5` notates the abstract syntax that might be conveyed by the following tree:

ABSTRACT SYNTAX

CONCRETE SYNTAX



The alternative structure expressed in concrete syntax as `(3 + 4) * 5` would then be



Notice how the abstract syntax has (and needs) no parentheses. Parentheses are a notion of concrete syntax used to explicate the abstract syntax of an expression. From here on, we will use the concrete syntax of OCaml to notate the abstract syntax where convenient, assuming that no confusion should result.

1.2. **Implementing abstract syntax.** For the purpose of writing an interpreter for a language, it is useful to be able to specify the abstract syntax of an expression. An appropriate abstract data type definition for the abstract syntax of an arithmetic expression language might be the following:

```
# type expr =
#   | Num of int
#   | Unop of string * expr
#   | Binop of string * expr * expr
# ;;
 type expr =
     Num of int
   | Unop of string * expr
   | Binop of string * expr * expr
```

This allows us to construct abstract syntax trees such as

```
# Binop ("+", Num 3, Binop ("*", Num 4, Num 5)) ;;
 - : expr = Binop ("+", Num 3, Binop ("*", Num 4, Num 5))
```

(Section 5.3 discusses how a computer system like a compiler or interpreter converts from concrete syntax to abstract syntax, a process known as parsing.)

Exercise 1. *Draw the abstract syntax trees for the following concrete arithmetic expressions.*

(1) `- 5 * 3`
(2) `3 - 4 - 5`
(3) `1 - 2 / 3 + 4`
(4) `(1 - 2) / (3 + 4)`
(5) `((1 - 2) / (3 + 4))`

(6) `(1 - 2) / 3 + 4`

□

Exercise 2. *Write a function* `exp_to_string : expr -> string` *that converts an abstract syntax tree of type* `expr` *to a concrete syntax string.* □

1.3. **Schematic reduction rules.** Returning to the reduction semantics for OCaml arithmetic expressions, the reduction rules governing integer arithmetic covering a few binary and unary operators might be

$$\begin{aligned}
\underline{m}\ \texttt{+}\ \underline{n} &\longrightarrow \underline{m+n} \\
\underline{m}\ \texttt{*}\ \underline{n} &\longrightarrow \underline{m \cdot n} \\
\underline{m}\ \texttt{-}\ \underline{n} &\longrightarrow \underline{m-n} \\
\underline{m}\ \texttt{/}\ \underline{n} &\longrightarrow \underline{\lfloor m/n \rfloor} \\
\texttt{\~{}-}\ \underline{n} &\longrightarrow \underline{-n}
\end{aligned}$$

and so forth. Here, we use the notation $\underline{m}$ for the OCaml numeral token that encodes the number m. (The FLOOR notation $\lfloor x \rfloor$ might be unfamiliar to you. It specifies the integer obtained by rounding a real number down, so $\lfloor 2.71828 \rfloor = 2$.) FLOOR

These rules may look trivial, but they are not. The first rule specifies that the + operator in OCaml when applied to two numerals has the property of generating the numeral representing their sum. The fourth rule specifies that the / operator in OCaml when applied to two numerals specifies the integer portion of their ratio. It could have been otherwise.[2] The language might have used a different operator for integer division,

$$\underline{m}\ \texttt{//}\ \underline{n} \longrightarrow \underline{\lfloor m/n \rfloor}$$

(as happens to be used in Python 3 for instance), or could have defined the result differently, say

$$\underline{m}\ \texttt{/}\ \underline{n} \longrightarrow \underline{\lceil m/n \rceil}$$

Nonetheless, there is not *too* much work being done by these rules, and if that were all there was to defining a semantics, there would be little reason to go to the

[2]What may be mind-boggling here is the role of the mathematical notation used on the right-hand side of the rule. How is it that we can make use of notations like $\lfloor m/n \rfloor$ in defining the semantics of the / operator? Doesn't appeal to that kind of mathematical notation beg the question? Or at least call for its own semantics? Yes, it does, but since we have to write down the semantics of constructs somehow or other, we use commonly accepted mathematical notation applied in the context of natural language (in the case at hand, English). You may think that this merely postpones the problem of giving OCaml semantics by reducing it to the problem of giving semantics for mathematical notation and English. You would be right, and the problem is further exacerbated when the semantics makes use of mathematical notation that is not so familiar, for instance, the substitution notation to be introduced shortly. But we have to start somewhere.

trouble. Things get more interesting, however, when additional constructs such as function application are considered, which we turn to next.

2. Substitution semantics

Consider, for instance, the application of a function to an argument, which would be expressed by a redex of the form `(fun` ⟨*var*⟩ `->` ⟨*body*⟩`)` ⟨*arg*⟩. This allows programs like

```
(fun x -> x + x) (3 * 4)
```

The reduction rule for this construct might be something like

$$\texttt{(fun}\ x\ \texttt{->}\ P\texttt{)}\ \ Q \longrightarrow P[x \mapsto Q]$$

where x stands for an arbitrary variable, and P and Q for arbitrary expressions, and $P[x \mapsto Q]$ is the SUBSTITUTION of Q for x in P. Because of the central place of substitution in providing the semantics of the language, this approach to semantics is called SUBSTITUTION SEMANTICS.

SUBSTITUTION

SUBSTITUTION SEMANTICS

Exercise 3. *Give the expressions (in concrete syntax) specified by the following substitutions:*

(1) $(x + x)[x \mapsto 3]$
(2) $(x + x)[y \mapsto 3]$
(3) $(x * x)[x \mapsto 3 + 4]$

□

Let's use this new rule to evaluate the example above:

```
(fun x -> x + x) (3 * 4)
⟶ 3 * 4 + 3 * 4
⟶ 12 + 3 * 4
⟶ 12 + 12
⟶ 24
```

Of course, there is an alternate derivation.

```
(fun x -> x + x) (3 * 4)
⟶ (fun x -> x + x) 12
⟶ 12 + 12
⟶ 24
```

In this case, it doesn't matter which order we apply the rules, but later, it will become important. We can mandate a particular order of reduction by introducing

a new concept, the VALUE. An expression is a value if no further reduction rules apply to it. Numerals, for instance, are values. We'll also specify that `fun` expressions are values; there aren't any reduction rules that apply to a `fun` expression, and we won't apply any reduction rules within its body. We can restrict the function application rule to apply only when its argument expression is a value, that is,

VALUE

$$\texttt{(fun } x \texttt{ -> } P\texttt{)} \ v \longrightarrow P[x \mapsto v]$$

(We use the schematic expression v to range only over values.) In that case, the step

$$\underline{\texttt{(fun x -> x + x) (3 * 4)}} \longrightarrow \texttt{3 * 4 + 3 * 4}$$

doesn't hold because `3 * 4` is not a value. Instead, the second derivation is the only one that applies.

What about OCaml's local naming construct, the `let ... in ...` form? Once we have function application in place, we can give a simple semantics to variable definition by reducing it to function application as per this rewrite rule:

$$\texttt{let } x \texttt{ = } v \texttt{ in } P \longrightarrow \texttt{(fun } x \texttt{ -> } P\texttt{)} \ v$$

Exercise 4. *Use the reduction rules developed so far to reduce the following expressions to their values.*

(1)

```
let x = 3 * 4 in
x + x
```

(2)

```
let f = fun x -> x in
f (f 5)
```

□

Exercise 5. *Show that the rule for the* `let` *construct could have been written instead as*

$$\texttt{let } x \texttt{ = } v \texttt{ in } P \longrightarrow P[x \mapsto v]$$

From here on, we'll use this rule instead of the previous rule for `let`*.* □

Let's try a derivation using all these rules.

```
let double = fun x -> 2 * x in double (double 3)
⟶ (fun x -> 2 * x) ((fun x -> 2 * x) 3)
⟶ (fun x -> 2 * x) (2 * 3)
⟶ (fun x -> 2 * x) 6
⟶ 2 * 6
⟶ 12
```

Exercise 6. *Carry out similar derivations for the following expressions:*

(1)

```
let square = fun x -> x * x in
let y = 3 in
square y
```

(2)

```
let id = fun x -> x in
let square = fun x -> x * x in
let y = 3 in
id square y
```

□

You may have noticed in Exercise 6 that some care must be taken when substituting. Consider the following case:

```
let x = 3 in let double = fun x -> x + x in double 4
```

If we're not careful, we'll get a derivation like this:

```
let x = 3 in let double = fun x -> x + x in double 4
⟶ let double = fun x -> 3 + 3 in double 4
⟶ (fun x -> 3 + 3) 4
⟶ 3 + 3
⟶ 6
```

or even worse

```
let x = 3 in let double = fun x -> x + x in double 4
⟶ let double = fun 3 -> 3 + 3 in double 4
⟶ (fun 3 -> 3 + 3) 4
⟶ hunh?
```

It appears we must be very careful in how we define this substitution operation $Q[x \mapsto P]$. In particular, we don't want to replace *every* occurrence of the token x in Q, only the *free* occurrences. A variable being introduced in a `fun` should definitely not be replaced, nor should any occurrences of x within the body of a `fun` that also introduces x.

More precisely, we can define the set of FREE VARIABLES in an expression P, notated $FV(P)$ through the inductive definition in Figure 1. By way of example, the definition says that the free variables in the expression `fun y -> f (x + y)` are just `f` and `x`, as shown in the following derivation:

$$\begin{aligned} FV(\texttt{fun y -> f (x + y)}) &= FV(\texttt{f (x + y)}) - \{\texttt{y}\} \\ &= FV(\texttt{f}) \cup FV(\texttt{x y}) - \{\texttt{y}\} \\ &= \{\texttt{f}\} \cup FV(\texttt{x}) \cup FV(\texttt{y}) - \{\texttt{y}\} \\ &= \{\texttt{f}\} \cup \{\texttt{x}\} \cup \{\texttt{y}\} - \{\texttt{y}\} \\ &= \{\texttt{f}, \texttt{x}, \texttt{y}\} - \{\texttt{y}\} \\ &= \{\texttt{f}, \texttt{x}\} \end{aligned}$$

Exercise 7. *Use the definition of FV to derive the free variables in the expressions*

(1) `let x = 3 in let y = x in f x y`
(2) `let x = x in let y = x in f x y`
(3) `let x = fun y -> x in x`

□

Exercise 8. *The definition of FV in Figure 1 is incomplete, in that it doesn't specify the free variables in a* `let rec` *expression. Add appropriate rules for this construct of the language, being careful to note that in an expression like* `let rec x = fun y -> x in x`*, the variable* `x` *is not free. (Compare with Exercise 7(3).)* □

A good definition of the substitution operation is given by the following recursive definition, which replaces only free occurrences of variables:

$$\begin{aligned} \underline{m}[x \mapsto P] &= \underline{m} \\ x[x \mapsto P] &= P \\ y[x \mapsto P] &= y \qquad \text{where } x \not\equiv y \\ (\texttt{~-}\ Q)[x \mapsto P] &= \texttt{~-}\ Q[x \mapsto P] \end{aligned}$$

and similarly for other unary operators

$$(Q\ \texttt{+}\ R)[x \mapsto P] = Q[x \mapsto P]\ \texttt{+}\ R[x \mapsto P]$$

and similarly for other binary operators

$$\texttt{(fun } x \texttt{ -> } Q\texttt{)}[x \mapsto P] = \texttt{fun } x \texttt{ -> } Q$$
$$\texttt{(fun } y \texttt{ -> } Q\texttt{)}[x \mapsto P] = \texttt{fun } y \texttt{ -> } Q[x \mapsto P] \qquad \text{where } x \not\equiv y$$

Exercise 9. *Use the definition of the substitution operation above to verify your answers to Exercise 3.* □

2.1. **More on capturing free variables.** But there is still a problem in our definition of substitution. Consider the following expression: `let x = y in (fun y -> x)`. Intuitively speaking, this expression seems ill-formed; it defines `x` to be an unbound variable `y`. But using the definitions that we have given so far, we would have the following derivation:

```
let x = y in (fun y -> x)
⟶ (fun x -> (fun y -> x)) y
⟶ (fun y -> x)[x ↦ y]
= fun y -> x[x ↦ y]        ⇐
= fun y -> y
```

The problem happens in the line marked ⇐. We're sneaking a `y` inside the scope of the variable `y` bound by the `fun`. That's not kosher. We need to change the definition of substitution to make sure that such VARIABLE CAPTURE doesn't occur. The following rules work by replacing the bound variable `y` with a new freshly minted variable, say `z` that doesn't occur elsewhere, renaming all occurrences of `y` accordingly.

VARIABLE CAPTURE

$$\texttt{(fun } x \texttt{ -> } Q\texttt{)}[x \mapsto P] = \texttt{fun } x \texttt{ -> } Q$$
$$\texttt{(fun } y \texttt{ -> } Q\texttt{)}[x \mapsto P] = \texttt{fun } y \texttt{ -> } Q[x \mapsto P]$$
$$\text{where } x \not\equiv y \text{ and } y \notin FV(P)$$
$$\texttt{(fun } y \texttt{ -> } Q\texttt{)}[x \mapsto P] = \texttt{fun } z \texttt{ -> } Q[y \mapsto z][x \mapsto P]$$
$$\text{where } x \not\equiv y \text{ and } y \in FV(P) \text{ and } z \text{ is a fresh variable}$$

Exercise 10. *Carry out the derivation for* `let x = y in (fun y -> x)` *as above but with this updated definition of substitution.* □

Exercise 11. *What should the corresponding rule or rules defining substitution on* `let ...in ...` *expressions be? That is, how should the following rule be completed? You'll*

want to think about how this construct reduces to function application in determining your answer.

$$(\texttt{let } y = Q \texttt{ in } R)[x \mapsto P] = \ldots$$

Try to work out your answer before checking it with the full definition of substitution in Figure 1. □

Exercise 12. *Use the definition of the substitution operation above to determine the results of the following substitutions:*

(1) $(\texttt{fun } x \texttt{ -> } x + x)[x \mapsto 3]$
(2) $(\texttt{fun } x \texttt{ -> } y + x)[x \mapsto 3]$
(3) $(\texttt{let } x = y * y \texttt{ in } x + x)[x \mapsto 3]$
(4) $(\texttt{let } x = y * y \texttt{ in } x + x)[y \mapsto 3]$

□

3. Implementing a substitution semantics for MiniML

You'll start your implementation with a substitution semantics for MiniML, a simple ML-like language. The abstract syntax of the language is given by the following type definition:

```
type expr =
  | Var of varid                          (* variables *)
  | Num of int                            (* integers *)
  | Bool of bool                          (* booleans *)
  | Unop of varid * expr                  (* unary operators *)
  | Binop of varid * expr * expr          (* binary operators *)
  | Conditional of expr * expr * expr     (* if then else *)
  | Fun of varid * expr                   (* function definitions *)
  | Let of varid * expr * expr            (* local naming *)
  | Letrec of varid * expr * expr         (* recursive local naming *)
  | Raise                                 (* exceptions *)
  | Unassigned                            (* (temporarily) unassigned *)
  | App of expr * expr                    (* function applications *)
and varid = string ;;
```

This type definition can be found in the partially implemented `Expr` module in the files `expr.ml` and `expr.mli`. You'll notice that the module signature requires additional functionality that hasn't been implemented, including functions to find the free variables in an expression, to generate a fresh variable name, and to substitute expressions for free variables.

To get things started, we also provide a parser for the MiniML language, which takes a string in a concrete syntax and returns a value of this type `expr`; you'll likely extend the parser in a later part of the project (Section 5.3). The compiled parser and a read-eval-print loop for the language are available in the following files:

evaluation.ml: The future home of anything needed to evaluate expressions to values. Currently, provides a trivial "evaluator" that merely returns the expression unchanged.

miniml.ml: Runs a read-eval-print loop for MiniML, using the Evaluation module that you will write.

miniml_lex.ml: A lexical analyzer for MiniML. (You should never need to look at this.)

miniml_parse.ml: A parser for MiniML. (Ditto.)

What's left to implement is the `Evaluation` module in `evaluation.ml`.

Start by familiarizing yourself with the code. You should be able to compile `miniml.ml` and get the following behavior.

```
# ocamlbuild miniml.byte
Finished, 13 targets (12 cached) in 00:00:00.
# ./miniml.byte
Entering miniml.byte...
<== 3 ;;
Fatal error: exception Failure("exp_to_string not implemented")
```

Stage 1. *Implement the function* `exp_to_string : expr -> string` *to convert abstract syntax trees to strings and test it thoroughly. If you did Exercise 2, the experience may be helpful here.* □

Once you write the function `exp_to_string`, you should have a functioning read-eval-print loop, except that the evaluation part is missing. It just prints out the abstract syntax tree of the input concrete syntax:

```
# ./miniml.byte
Entering miniml.byte...
<== 3 ;;
==> Num(3)
<== 3 4 ;;
==> App(Num(3), Num(4))
<== (((3) ;;
xx> parse error
<== let f = fun x -> x in f f 3 ;;
==> Let(f, Fun(x, Var(x)), App(App(Var(f), Var(f)), Num(3)))
```

```
<== let rec f = fun x -> if x = 0 then 1 else x * f (x - 1) in f 4 ;;
==> Letrec(f, Fun(x, Conditional(Binop(=, Var(x), Num(0)), Num(1),
    Binop(*, Var(x), App(Var(f), Binop(-, Var(x), Num(1))))),
    App(Var(f), Num(4)))
<== Goodbye.
```

To actually get evaluation going, you'll need to implement a substitution semantics, which requires completing the functions in the `Expr` module.

Stage 2. *Start by writing the function* `free_vars` *in* `expr.ml`*, which takes an expression (*`expr`*) and returns a representation of the free variables in the expression. Test this function completely.* □

Stage 3. *Next, write the function* `subst` *that implements substitution as defined in Figure 1. In some cases, you'll need the ability to define new fresh variables in the process of performing substitutions. Something like the* `gensym` *function that you wrote in lab might be useful for that. Once you've written* `subst` *make sure to test it completely.* □

You're actually quite close to having your first working interpreter for MiniML. All that is left is writing a function `eval_s` (the '`s`' is for *substitution semantics*) that evaluates an expression using the substitution semantics rules. (Those rules were described informally in lecture 7. The lecture slides may be helpful to review.) The `eval_s` function walks an abstract syntax tree of type `expr`, evaluating subparts recursively where necessary and performing substitutions when appropriate. The recursive traversal bottoms out when you get to primitive values like numbers or booleans or in applying primitive functions like the unary or binary operators to values.

Stage 4. *Implement the* `eval_s` *function in* `evaluation.ml`*. (You can ignore the signature and implementation of the Env module for the time being. That comes into play in later sections.) We recommend that you implement it in stages, from the simplest bits of the language to the most complex. You'll want to test each stage thoroughly using unit tests as you complete it. Keep these unit tests around so that you can easily unit test the later versions of the evaluator that you'll develop in future sections.* □

Using the substitution semantics, you should be able to handle evaluation of all of the MiniML language. If you want to postpone handling of some parts while implementing the evaluator, you can always just raise the `EvalError` exception, which is intended just for this kind of thing, when a runtime error occurs. Another place `EvalError` will be useful is when a runtime type error occurs, for instance, for the expressions `3 + true` or `3 4` or `let x = true in y`.

Notice at the bottom of `miniml.ml` the definition of `evaluate`, which is the function that the REPL loop in `miniml.ml` calls. Replace the definition with the one calling `eval_s`, and you should get behavior like this:

```
# miniml_soln.byte
Entering miniml_soln.byte...
<== 3 ;;
==> 3
<== 3 + 4 ;;
==> 7
<== 3 4 ;;
xx> evaluation error: (3 4) bad redex
<== (((3) ;;
xx> parse error
<== let f = fun x -> x in f f 3 ;;
==> 3
<== let rec f = fun x -> if x = 0 then 1 else x * f (x - 1) in f 4 ;;
xx> evaluation error: not yet implemented: let rec
<== Goodbye.
```

Some things to note about this example:

- The parser that we provide will raise an exception `Parsing.Parse_error` if the input doesn't parse as well-formed MiniML. The REPL handles the exception by printing an appropriate error message.
- The evaluator can raise an exception `Evaluation.EvalError` at runtime if a (well-formed) MiniML expression runs into problems when being evaluated.
- You might also raise `Evaluation.EvalError` for parts of the evaluator that you haven't (yet) implemented, like the tricky `let rec` construction in the example above.

Stage 5. *After you've changed* `evaluate` *to call* `eval_s`*, you'll have a complete working implementation of MiniML. You should save a snapshot of this – using a git commit might be a good idea – so that if you have trouble down the line you can always roll back to this version to submit it.* □

4. Implementing an environment semantics for MiniML

The substitution semantics is sufficient for all of MiniML because it is a pure functional programming language. But binding constructs like `let` and `letrec` are awkward to implement, and extending the language to handle references, mutability, and imperative programming is impossible. For that, you'll extend

the language semantics to make use of an ENVIRONMENT that stores a mapping from variables to their values. You will want to be able to replace the values of variables dynamically – you'll see why shortly – so that these variable values need to be mutable. We've provided a type signature for environments. It stipulates types for environments and values, and functions to create an empty environment (which we've already implemented for you), to extend an environment with a new mapping of a variable to its (mutable) value, and to look up the value associated with a variable.

ENVIRONMENT

Stage 6. *Implement the various functions involved in the* `Env` *module and test them thoroughly.* □

How will this be used? For atomic literals like numerals and truth values, they evaluate to themselves as usual, independently of the environment. But to evaluate a variable in an environment, we look up the value that the environment assigns to it and return that value.

A slightly more complex case involves function application, as in this example:

```
(fun x -> x + x) 5
```

The abstract syntax for this expression is an application of one expression to another.

To evaluate an application $P\ Q$ in an environment ρ,

(1) Evaluate P in ρ to a value v_P, which should be a function `fun x -> B`. If v_P is not a function, raise an evaluation error.
(2) Evaluate Q in the environment ρ to a value v_Q.
(3) Evaluate B in the environment ρ extended with a binding of `x` to v_Q.

In the example: (1) `fun x -> x + x` is already a function, so evaluates to itself. (2) The argument `5` also evaluates to itself. (3) The body `x + x` is evaluated in an environment that maps `x` to `5`.

For `let` expressions, a similar evaluation process is used. Consider

```
let x = 3 * 4 in x + 1 ;;
```

The abstract syntax for this `let` expression has a variable name, a definition expression, and a body. To evaluate this expression in, say, the empty environment, we first evaluate (recursively) the definition part in the same empty environment, presumably getting the value `12` back. We then extend the environment to associate that value with the variable `x` to form a new environment, and then evaluate the body `x + 1` in the new environment. In turn, evaluating `x + 1` involves recursively evaluating `x` and `1` in the new environment. The latter is straightforward.

The former involves just looking up the variable in the environment, retrieving the previously stored value `12`. The sum can then be computed and returned as the value of the entire `let` expression.

For recursion, consider this expression, which makes use of an (uninteresting) recursive function:

```
let rec f = fun x -> if x = 0 then x else f (x - 1) in f 2 ;;
```

Again, the `let rec` expression has three parts: a variable name, a definition expression, and a body. To evaluate it, we ought to first evaluate the definition part, but using what environment? If we use the incoming (empty) environment, then what will we use for a value of `f` when we reach it? We should use the value of the definition, but we don't have it yet. In the interim, we'll store a special value, `Unassigned`, which you'll have noticed in the `expr` type but which is never generated by the parser. We evaluate the definition in this extended environment, hopefully generating a value. (The definition part better not ever evaluate the variable name though, as it is unassigned; doing so should raise an `EvalError`. An example of this problem might be `let rec x = x in x`.) The value returned for the definition can then *replace* the value for the variable name (thus the need for a mutable environment) and that environment passed on to the body for evaluation.

In the example above, we augment the empty environment with a binding for `f` to `Unassigned` and evaluate `fun x -> if x = 0 then x else f (x - 1)` in that environment. Since this is a function, it is already a value, and the environment can be updated to have `f` have this function as a value. Finally, the body `f 2` is evaluated in this environment. The body, an application, evaluates `f` by looking it up in this environment yielding `fun x -> if x = 0 then x else f (x - 1)` and evaluates `2` to itself, then evaluates the body of the function

Stage 7. *Implement another evaluation function* `eval_d` *(the '*`d`*' is for* dynamically scoped environment semantics*), which works along the lines just discussed. Make sure to test it on a range of tests exercising all the parts of the language.* □

5. Extending the language

In this final part of the project, you will extend MiniML in one or more ways of your choosing.

5.1. **Extension ideas.** Here are a few ideas for extending the language, very roughly in order from least to most ambitious. Especially difficult extensions are marked with ❣ symbols.

(1) Add additional atomic types (floats, strings, unit, etc.) and corresponding literals and operators.

(2) Modify the environment semantics to manifest lexical scope instead of dynamic scope (Section 5.2).
(3) Add lists to the language.
(4) Add records to the language.
(5) Add references to the language, by adding operators `ref`, `!`, and `:=`.
(6) Add laziness to the language (by adding refs and syntactic sugar for the `lazy` keyword). If you've also added lists, you'll be able to build infinite streams.
(7) ❣ Add simple type checking to the language. For this extension, the language would be extended so that *every* introduction of a bound variable (in a `let`, `let rec`, or `fun` construct) is accompanied by its (monomorphic) type. The abstract syntax would need to be extended to store those types, and you would write a function to walk the tree to verify that every expression in the program is well typed. This is a quite ambitious project.
(8) ❣❣ Add type inference to the language, so that (as in OCaml) types are inferred even when not given explicitly. This is *extremely ambitious*, not for the faint of heart.

Most of the extensions (in fact, all except for (2)) require extensions to the concrete syntax of the language. We provide information about extending the concrete syntax in Section 5.3. Many other extensions are possible. Don't feel beholden to this list. Be creative!

Most importantly: It is better to do a great job (clean, elegant design; beautiful style; well thought-out implementation) on a smaller extension, than a mediocre job on an ambitious extension. That is, the scope aspect of the project will be weighted substantially less than the design and style aspects. Caveat emptor.

5.2. **A lexical scope environment semantics.** Consider the following OCaml expression:

```
let x = 1 in
let f = fun y -> x + y in
let x = 2 in
f 3 ;;
```

Exercise 13. *What should this expression evaluate to? Test it in the OCaml interpreter. Try this expression using your* `eval_s` *and* `eval_d` *evaluators. Which ones accord with OCaml's evaluation?* □

The `eval_d` evaluator that you've implemented so far is DYNAMICALLY SCOPED. The values of variables are governed by the dynamic ordering in which they are

DYNAMICALLY SCOPED

evaluated. But OCaml is LEXICALLY SCOPED. The values of variables are governed by the lexical structure of the program. In the case above, when the function `f` is applied to `3`, the most recent assignment to `x` is of the value `2`, but the assignment to the `x` that lexically outscopes `f` is of the value `1`. Thus a dynamically scoped language calculates the body of `f`, `x + y`, as `2 + 3` (that is, `5`) but a lexically scoped language calculates the value as `1 + 3` (that is, `4`).

LEXICALLY SCOPED

The substitution semantics manifests lexical scope, as it should, but the dynamic semantics does not. To fix the dynamic semantics, we need to handle function values differently. When a function value is computed (say the value of `f`, `fun y -> x + y`), we need to keep track of the lexical environment in which the function occurred so that when the function is eventually applied to an argument, we can evaluate the application in the lexical environment – the environment when the function was *defined* – rather than the dynamic environment – the environment when the function was *called*.

The technique to enable this is to package up the function being defined with a snapshot of the environment at the time into a data structure called a CLOSURE. There is already provision for closures in the `env` module. You'll notice that the `value` type has two constructors, one for normal values (like numbers, booleans, and the like) and one for closures. The `Closure` constructor just puts together a function with its lexical environment.

CLOSURE

Stage 8. *Make a copy of your* `eval_d` *evaluation function and call it* `eval_l` *(the '*`l`*' for* lexically scoped environment semantics*). Modify the code so that the evaluation of a function returns a closure containing the function itself and the current environment. Modify the function application part so that it evaluates the body of the function in the lexical environment from the corresponding closure (appropriately updated). As usual, test it thoroughly. If you've carefully accumulated good unit tests for the previous evaluators, you should be able to fully test this new one with just a single function call.*

The copy-paste recommendation for building `eval_l` *from* `eval_d` *makes for simplicity in the process, but will undoubtedly leave you with redundant code. You may want to think about merging the two implementations so that they share as much code as possible. Various of the abstraction techniques you've learned in the course could be useful here.* □

Stage 9. *Write up your extensions in a short paper describing your work and demonstrating any extensions and how you implemented them. Use Markdown or LaTeX format, and name the file* `writeup.md` *or* `writeup.tex`*. You'll submit both the source file and a rendered PDF file.* □

5.3. **The MiniML parser.** We provided you with a MiniML parser that converts the concrete syntax of MiniML to an abstract syntax representation

using the `expr` type. But to extend the implemented language, you'll typically need to extend the parser. The parser we provided was implemented using `ocamllex` and `ocamlyacc`, programs designed to build lexical analyzers and parser for programming languages. Documentation for them can be found at http://caml.inria.fr/pub/docs/manual-ocaml/lexyacc.html, and tutorial material is archived at https://web.archive.org/web/20150921125456/http://plus.kaist.ac.kr/~shoh/ocaml/ocamllex-ocamlyacc/ocamllex-tutorial and https://web.archive.org/web/20150907071659/http://plus.kaist.ac.kr/~shoh/ocaml/ocamllex-ocamlyacc/ocamlyacc-tutorial/.

In summary, `ocamllex` takes a specification of the tokens of a programming language in a file, say, `miniml_lex.mll`. The `ocamlbuild` system knows how to use `ocamllex` to turn such files into OCaml code for a lexical analyzer in the file `miniml_lex.ml`. Similarly, an `ocamlyacc` specification of a parser in a file `miniml_parse.mly` will be transformed by `ocamlyacc` (automatically with `ocamlbuild`) to a parser in `miniml_parse.ml`. By modifying `miniml_lex.mll` and `miniml_parse.mly`, you can modify the concrete syntax of the MiniML language, which may be useful for many of the extensions you might be interested in.

6. Submission of the project

In addition to submitting the code implementing MiniML to Vocareum, you should submit the `writeup.md` or `writeup.tex` file and the rendered PDF file `writeup.pdf` as well.

7. Alternative final projects

Students who have been doing exceptionally well in the course to date can petition to do alternative final projects of their own devising.

(1) Alternative final projects can be undertaken individually or in groups of up to four.
(2) The implementation language for the project must be OCaml.
(3) You will need to submit a proposal for the project by April 13. The proposal should describe what the project goals are, how you will go about implementing the project, and how the work will be distributed among the members of the group (if applicable).
(4) You will receive notification around April 15 as to whether your request has been approved. Approval will be based on performance in the course to date and the appropriateness of the project.
(5) You will submit a progress report by April 21, including a statement of progress, any code developed to date, and any changes to the expected scope of the project.

(6) You will submit the project results, including all code, a demonstration of the project system in action, and a paper describing the project and any results, by April 27.
(7) You will be scheduled to perform a presentation and demonstration of your project for course staff during reading period.
(8) The group as a whole may drop out of the process at any time. Individual members of the group would then submit instead the standard final project.

$$
\begin{aligned}
FV(\underline{m}) &= \emptyset && \text{(integers and other literals)}\\
FV(x) &= \{x\} && \text{(variables)}\\
FV(\texttt{\~{}-}\ Q) &= FV(Q) && \text{(and similarly for other unary operators)}\\
FV(P\ \texttt{+}\ Q) &= FV(P) \cup FV(Q) && \text{(and similarly for other binary operators)}\\
FV(P\ Q) &= FV(P) \cup FV(Q) && \text{(applications)}\\
FV(\texttt{fun}\ x\ \texttt{->}\ P) &= FV(P) - \{x\} && \text{(functions)}\\
FV(\texttt{let}\ x\ \texttt{=}\ P\ \texttt{in}\ Q) &= (FV(Q) - \{x\}) \cup FV(P) && \text{(binding)}
\end{aligned}
$$

$$
\begin{aligned}
\underline{m}[x \mapsto P] &= \underline{m}\\
x[x \mapsto P] &= P\\
y[x \mapsto P] &= y \qquad \text{where } x \not\equiv y\\
(\texttt{\~{}-}\ Q)[x \mapsto P] &= \texttt{\~{}-}\ Q[x \mapsto P]\\
&\qquad \text{and similarly for other unary operators}\\
(Q\ \texttt{+}\ R)[x \mapsto P] &= Q[x \mapsto P]\ \texttt{+}\ R[x \mapsto P]\\
&\qquad \text{and similarly for other binary operators}\\
(\texttt{fun}\ x\ \texttt{->}\ Q)[x \mapsto P] &= \texttt{fun}\ x\ \texttt{->}\ Q\\
(\texttt{fun}\ y\ \texttt{->}\ Q)[x \mapsto P] &= \texttt{fun}\ y\ \texttt{->}\ Q[x \mapsto P]\\
&\qquad \text{where } x \not\equiv y \text{ and } y \notin FV(P)\\
(\texttt{fun}\ y\ \texttt{->}\ Q)[x \mapsto P] &= \texttt{fun}\ z\ \texttt{->}\ Q[y \mapsto z][x \mapsto P]\\
&\qquad \text{where } x \not\equiv y \text{ and } y \in FV(P) \text{ and } z \text{ is a fresh variable}\\
(\texttt{let}\ x\ \texttt{=}\ Q\ \texttt{in}\ R)[x \mapsto P] &= \texttt{let}\ x\ \texttt{=}\ Q[x \mapsto P]\ \texttt{in}\ R\\
(\texttt{let}\ y\ \texttt{=}\ Q\ \texttt{in}\ R)[x \mapsto P] &= \texttt{let}\ y\ \texttt{=}\ Q[x \mapsto P]\ \texttt{in}\ R[x \mapsto P]\\
&\qquad \text{where } x \not\equiv y \text{ and } y \notin FV(P)\\
(\texttt{let}\ y\ \texttt{=}\ Q\ \texttt{in}\ R)[x \mapsto P] &= \texttt{let}\ z\ \texttt{=}\ Q[x \mapsto P]\ \texttt{in}\ R[y \mapsto z][x \mapsto P]\\
&\qquad \text{where } x \not\equiv y \text{ and } y \in FV(P) \text{ and } z \text{ is a fresh variable}
\end{aligned}
$$

FIGURE 1. Definitions of free variables and substitution