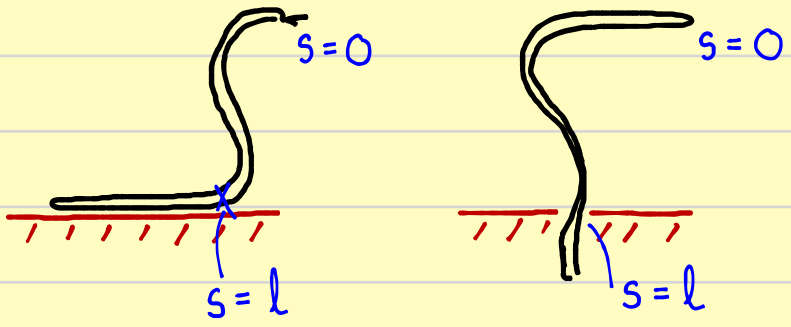
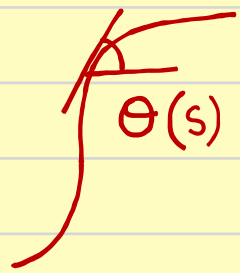
Posture stability & dynamics in Eels, Snakes & Soft Robots

January 6, 2021





$\underline{t} = (\cos\theta, \sin\theta) = (\partial x/\partial s, \partial y/\partial s)$ ②

$\underline{f} = H\underline{e}_x + V\underline{e}_y$

$m = B(\partial\theta/\partial s + m^a/B)$ total moment

$\left.\begin{aligned} \partial m/\partial s + \underline{t} \times \underline{f} &= \emptyset \\ \partial \underline{f}/\partial s + \rho \underline{g} &= \emptyset \end{aligned}\right\}$ ② equilibrium — ② force, torque

boundary conditions: $s = 0$;

$\underline{f} = 0$; $B\partial\theta/\partial s + m^a = 0$

[torque equilibrium $\not\Rightarrow \partial\theta/\partial s \neq \emptyset$]

& $\theta = 0$ (4 bc)

$s = l$

Snake :- $\theta = 0$; $x = y = 0$; $m(l) \le m_a^M$; $\partial\theta/\partial s\big|_l \ge 0$

Eel :- $x = y = 0$; $\theta = \pi/2$ (3 bc)

m_a^M ← max. active torque!

& $l \le L$ ← max. length!

6th order bvp w/ 7 unknowns ∵ length l is also a parameter. & $m_a(s)$?

$m^a(s)$ - active torque. Work $= \frac{1}{2}\int_0^l m_a \cdot \partial\theta/\partial s \cdot ds$, Max. $y(0)$

Complete problem :

Min. W | subject to ① - ⑥ + (7 bc) + global torque balance
& Max. $y(0)$ |

note that : $\partial/\partial s(\underline{r} \times \underline{f}) = \underline{t} \times \underline{f} + \underline{r} \times \partial\underline{f}/\partial s$

$= -\partial m^e/\partial s + r \times \partial \underline{f}/\partial s$

$\Rightarrow \int_0^l \underline{r} \times \partial \underline{f}/\partial s \, ds = m(l) - m(0)$

$\int_0^l \underline{r} \times \left[\partial \underline{f}/\partial s + \rho \underline{g}\right] ds = 0$

$\Rightarrow \int_0^l \underline{r} \times \rho \underline{g} \, ds = -m(l) + m(0)$

$\Rightarrow \left\{\int_0^l \rho g \times ds = -m(l) \le m_a^M\right\}$

$x_s = \cos\theta$ ① ; $H_s = 0$ ③ ; $(B\theta_s + m^a)_s + H\cos\theta - V\sin\theta = 0$ ⑤

$y_s = \sin\theta$ ② ; $V_s = -\rho g$ ④ ;

B.C. $x(l) = y(l) = 0$; $\theta(l) = \frac{\pi}{2}$ eel (7 bc.)

$m(0) = H(0) = V(0) = 0$ $\quad = 0$ snake

& $\theta(0) = 0$ ← look forward b.c.

Min. $_{\theta, x, y, m, \ldots}$ $(1-\alpha)\int_0^l \frac{1}{2} m^a \theta_s \, ds - \alpha y(0)$ ⑥

muscular work ; tip height

⊕ $l \le L$, $|m| \le m^M$, subject to (1-6) &

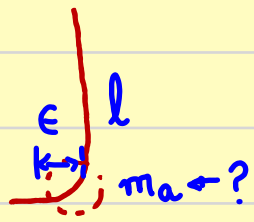
$\int_0^l \rho g \times ds = m(0) \le m^M$ ⑦

parameters : $l_g = (B/\rho g)^{1/3}$

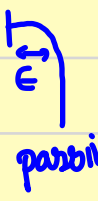
$M(s) = m^a l_g/B$; $h(s) = H l_g^2/B$, $v(s) = V l_g^2/B$, $\hat{s} = s/l_g$

⑤ $\Rightarrow$ $(\theta_{\hat{s}} + M^a)_{\hat{s}} + h\cos\theta - v\sin\theta = 0$

④ $\Rightarrow$ $v_{\hat{s}} = -1$, all other eqns. remain as they are!



$\kappa \sim 1/\epsilon$; B/ϵ



$\rho g l \epsilon r^2 \sim B/\epsilon \sim E r^4/\epsilon$

$\epsilon \sim (B/\rho g r^2 l)^{1/2} \sim (l_g^3/l)^{1/2}$

$B/\epsilon \sim B(l/l_g^3)^{1/2} \sim m_a$?

$l \sim m_a^2 \cdot l_g^3 / B^2$