

# D

## Statistical Tables

**TABLE A4.1** Cumulative Normal Distribution. The Area or Cumulative Distribution,  $F(z)$ , Under the Standardized Normal Distribution Curve for  $z \leq z_F$  Such that the Probability  $P(z < z_F) = F(z)$ . For Example,  $P(z < z_F = 1.21) = 0.8869$ , and  $P(z > z_F = 1.21) = 1 - 0.8869 = 0.1131$

$$F(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441

(Continued)

**TABLE A4.1** Cumulative Normal Distribution. The Area or Cumulative Distribution,  $F(z)$ , Under the Standardized Normal Distribution Curve for  $z \leq z_F$ . Such that the Probability  $P(z < z_F) = F(z)$ . For Example,  $P(z < z_F = 1.21) = 0.8869$ , and  $P(z > z_F = 1.21) = 1 - 0.8869 = 0.1131$

$$F(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

(cont'd)

<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

*Adapted from Introductory Statistical Analysis by D. L. Harnett and J. L. Murphy, Addison-Wesley, 1976.*

**TABLE A4.2** Cumulative Chi-square Distribution. The Area or Cumulative Distribution,  $F(\chi^2)$ , Under the  $\chi^2$  Distribution Curve for Different Degrees of Freedom,  $\nu$ , Such that the Probability  $P(\chi_\nu^2 < \chi_{\nu;F}^2) = F(\chi^2)$ . For Example, for  $\nu = 16$ , the Probability  $P(\chi_{16}^2 < \chi_{16;F}^2 = 26.3) = F(26.3) = 0.950$ . Consequently,  $P(\chi_{16}^2 > \chi_{16;F}^2 = 26.3) = 1 - F(26.3) = 0.050$

$$F(\chi^2) = \int_0^{\chi^2} \frac{x^{(\nu-2)/2} e^{-x/2} dx}{2^{\nu/2} [(v-2)/2]!}$$

$\nu$	$F$												
	0.005	0.010	0.025	0.050	0.100	0.250	0.500	0.750	0.900	0.950	0.975	0.990	0.995
1	0.0 <sup>4</sup> 393	0.0 <sup>3</sup> 157	0.0 <sup>3</sup> 982	0.0 <sup>2</sup> 393	0.0158	0.102	0.455	1.32	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	0.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6
3	0.0717	0.115	0.216	0.352	0.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8
4	0.207	0.297	0.484	0.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9
5	0.412	0.554	0.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7
6	0.676	0.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5
7	0.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6

(Continued)

**TABLE A4.2** Cumulative Chi-square Distribution. The Area or Cumulative Distribution,  $F(\chi^2)$ , Under the  $\chi^2$  Distribution Curve for Different Degrees of Freedom,  $\nu$ , Such that the Probability  $P(\chi^2_\nu < \chi^2_{\nu;F}) = F(\chi^2)$ . For Example, for  $\nu = 16$ , the Probability  $P(\chi^2_{16} < \chi^2_{16;F} = 26.3) = F(26.3) = 0.950$ . Consequently,  $P(\chi^2_{16} > \chi^2_{16;F} = 26.3) = 1 - F(26.3) = 0.050$

$$F(\chi^2) = \int_0^{\chi^2} \frac{x^{(\nu-2)/2} e^{-x/2} dx}{2^{\nu/2}[(\nu-2)/2]!}$$

(cont'd)

$\nu$	$F$												
	0.005	0.010	0.025	0.050	0.100	0.250	0.500	0.750	0.900	0.950	0.975	0.990	0.995
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7

Adapted from *Introductory Statistical Analysis* by D.L. Harnett and J.L. Murphy, Addison-Wesley, 1976; abridged from *Tables of percentage points of the incomplete beta function and of the chi-square distribution* C.M. Thompson, *Biometrika*, Vol. 32 (1941).

**TABLE A4.3A** Cumulative  $t$ -distribution. The Area or Cumulative Distribution,  $F(t)$ , Under the  $t$ -distribution Curve for Different Degrees of Freedom,  $\nu$ , Such that the Probability  $P(t_\nu < t_{\nu;F}) = F(t)$ . The example here is for  $n = 20$ . For Example, for  $\nu = 9$ , the Probability  $P(t_9 < t_{9;F} = 2.262) = F(2.262) = 0.975$  and  $P(t_9 > t_{9;F} = 2.262) = 1 - F(2.262) = 0.025$ , Corresponding to the 95% Confidence Interval ( $F = F_{0.025}$ ). Note that  $F_{0.100}$ ,  $F_{0.50}$ , and  $F_{0.005}$  Correspond to the 80, 90, and 99% Levels, Respectively

$$F(t) = \int_{-\infty}^t \frac{\left(\frac{\nu-1}{2}\right)!}{\left(\frac{\nu-2}{2}\right)! \sqrt{\pi n} \left(1 + \frac{t^2}{\nu}\right)^{(\nu+1)/2}} dt$$

$\nu$	$F$						
	0.75	0.90	0.95	0.975	0.99	0.995	0.9995
1	1.000	3.078	6.314	12.706	31.821	63.657	636.615
2	0.816	1.886	2.920	4.303	6.965	9.925	31.598
3	0.765	1.638	2.353	3.182	4.541	5.841	12.941
4	0.741	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	6.859
6	0.718	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	5.405
8	0.706	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.767
24	0.685	1.318	1.711	2.064	2.492	2.797	3.745

(Continued)

**TABLE A4.3A** Cumulative *t*-distribution. The Area or Cumulative Distribution,  $F(t)$ , Under the *t*-distribution Curve for Different Degrees of Freedom,  $\nu$ , Such that the Probability  $P(t_\nu < t_{\nu;F}) = F(t)$ . The example here is for  $n = 20$ . For Example, for  $\nu = 9$ , the Probability  $P(t_9 < t_{9;F} = 2.262) = F(2.262) = 0.975$  and  $P(t_9 > t_{9;F} = 2.262) = 1 - F(2.262) = 0.025$ , Corresponding to the 95% Confidence Interval ( $F = F_{0.025}$ ). Note that  $F_{0.100}$ ,  $F_{0.50}$ , and  $F_{0.005}$  Correspond to the 80, 90, and 99% Levels, Respectively

$$F(t) = \int_{-\infty}^t \frac{\left(\frac{\nu-1}{2}\right)!}{\left(\frac{\nu-2}{2}\right)! \sqrt{\pi n} \left(1 + \frac{t^2}{\nu}\right)^{(\nu+1)/2}} dt$$

(cont'd)

$\nu$	<i>F</i>						
	0.75	0.90	0.95	0.975	0.99	0.995	0.9995
25	0.684	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	3.373
$\infty$	0.674	1.282	1.645	1.960	2.326	2.576	3.291

Adapted from *Introductory Statistical Analysis* by D. L. Harnett and J. L. Murphy, Addison-Wesley, 1976; abridged from the *Statistical tables* of R.A. Fisher and Frank Yates, Oliver & Boyd, Edinburgh and London, 1938.

**TABLE A4.3B** Cumulative  $t$ -distribution (Two-tailed Tests). Similar to Table A4.3A Except that Values Give Cumulative Distribution,  $F(t)$ , Under the  $t$ -distribution Curve for Different Degrees of Freedom,  $\nu$ , Regardless of Sign, Such that the Probability  $P(|t_\nu| > |t_{\nu,F}|) = F(t)$ . The example here is for  $n = 20$ . For Example, for  $\nu = 9$ , the Probability  $P(|t_9| > |t_{9,F}| = 2.262) = F(2.262) = 0.05$  and  $P(|t_9| < |t_{9,F}| = 2.262) = 1 - F(2.262) = 0.95$ , Corresponding to the 95% Confidence Interval. Note that  $F_{0.200}$ ,  $F_{0.100}$ , and  $F_{0.010}$  Correspond to the 80, 90, and 99% Levels, Respectively

$\nu$	$F$ Probability of a Larger Value, Sign Ignored								
	0.500	0.400	0.200	0.100	0.050	0.025	0.010	0.005	0.001
1	1.000	1.376	3.078	6.314	12.706	25.452	63.657		
2	0.816	1.061	1.886	2.920	4.303	6.205	9.925	14.089	31.598
3	0.765	0.978	1.638	2.353	3.182	4.176	5.841	7.453	12.941
4	0.741	0.941	1.533	2.132	2.776	3.495	4.604	5.598	8.610
5	0.727	0.920	1.476	2.015	2.571	3.163	4.032	4.773	6.859
6	0.718	0.906	1.440	1.943	2.447	2.969	3.707	4.317	5.959
7	0.711	0.896	1.415	1.895	2.365	2.841	3.499	4.029	5.405
8	0.706	0.889	1.397	1.860	2.306	2.732	3.355	3.832	5.041
9	0.703	0.883	1.383	1.833	2.262	2.685	3.250	3.690	4.781
10	0.700	0.879	1.372	1.812	2.228	2.634	3.169	3.581	4.587
11	0.697	0.876	1.363	1.796	2.201	2.593	3.106	3.497	4.437
12	0.695	0.873	1.356	1.782	2.179	2.560	3.055	3.428	4.318
13	0.694	0.870	1.350	1.771	2.160	2.533	3.012	3.372	4.221
14	0.692	0.868	1.345	1.761	2.145	2.510	2.977	3.326	4.140
15	0.691	0.866	1.341	1.753	2.131	2.490	2.947	3.286	4.073
16	0.690	0.865	1.337	1.746	2.120	2.473	2.921	3.252	4.015
17	0.689	0.863	1.333	1.740	2.110	2.458	2.898	3.222	3.965
18	0.688	0.862	1.330	1.734	2.101	2.445	2.878	3.197	3.922
19	0.688	0.861	1.328	1.729	2.093	2.433	2.861	3.174	3.883
20	0.687	0.860	1.325	1.725	2.086	2.423	2.845	3.153	3.850
21	0.686	0.859	1.323	1.721	2.080	2.414	2.831	3.135	3.819
22	0.686	0.858	1.321	1.717	2.074	2.406	2.819	3.119	3.792
23	0.685	0.858	1.319	1.714	2.069	2.398	2.807	3.104	3.767
24	0.685	0.857	1.318	1.711	2.064	2.391	2.797	3.090	3.745
25	0.684	0.856	1.316	1.708	2.060	2.385	2.787	3.078	3.725
26	0.684	0.856	1.315	1.706	2.056	2.379	2.779	3.067	3.707

(Continued)

**TABLE A4.3B** Cumulative  $t$ -distribution (Two-tailed Tests). Similar to Table A4.3A Except that Values Give Cumulative Distribution,  $F(t)$ , Under the  $t$ -distribution Curve for Different Degrees of Freedom,  $\nu$ , Regardless of Sign, Such that the Probability  $P(|t_\nu| > |t_{\nu,F}|) = F(t)$ . The example here is for  $n = 20$ . For Example, for  $\nu = 9$ , the Probability  $P(|t_9| > |t_{9,F}| = 2.262) = F(2.262) = 0.05$  and  $P(|t_9| < |t_{9,F}| = 2.262) = 1 - F(2.262) = 0.95$ , Corresponding to the 95% Confidence Interval. Note that  $F_{0.200}$ ,  $F_{0.100}$ , and  $F_{0.010}$  Correspond to the 80, 90, and 99% Levels, Respectively (*cont'd*)

$\nu$	$F$ Probability of a Larger Value, Sign Ignored								
	0.500	0.400	0.200	0.100	0.050	0.025	0.010	0.005	0.001
27	0.684	0.855	1.314	1.703	2.052	2.373	2.771	3.056	3.690
28	0.683	0.855	1.313	1.701	2.048	2.368	2.763	3.047	3.674
29	0.683	0.854	1.311	1.699	2.045	2.364	2.756	3.038	3.659
30	0.683	0.854	1.310	1.697	2.042	2.360	2.750	3.030	3.646
35	0.682	0.852	1.306	1.690	2.030	2.342	2.724	2.996	3.591
40	0.681	0.851	1.303	1.684	2.021	2.329	2.704	2.971	3.551
45	0.680	0.850	1.301	1.680	2.014	2.319	2.690	2.952	3.520
50	0.680	0.849	1.299	1.676	2.008	2.310	2.678	2.937	3.496
55	0.679	0.849	1.297	1.673	2.004	2.304	2.669	2.925	3.476
60	0.679	0.848	1.296	1.671	2.000	2.299	2.660	2.915	3.460
70	0.678	0.847	1.294	1.667	1.994	2.290	2.648	2.899	3.435
80	0.678	0.847	1.293	1.665	1.989	2.284	2.638	2.887	3.416
90	0.678	0.846	1.291	1.662	1.986	2.279	2.631	2.878	3.402
100	0.677	0.846	1.290	1.661	1.982	2.276	2.625	2.871	3.390
120	0.677	0.845	1.289	1.658	1.980	2.270	2.617	2.860	3.373
$\infty$	0.6745	0.8416	1.2816	1.6448	1.9600	2.214	2.5758	2.8070	3.2905



**TABLE A4.4A** Critical Values of the  $F$ -distribution for  $\alpha = 0.05$ . The Distributions Represent the Area Exceeding the Value of  $F_{0.05, \nu_1, \nu_2}$ , and  $F_{0.01, \nu_1, \nu_2}$  as Shown by the Shaded Area in the Figure for Different Degrees of Freedom,  $\nu$ . For Example, if  $\nu_1 = 15$  and  $\nu_2 = 20$ , then the Critical Value for  $\alpha = 0.05$  is 2.20

		Values of $F_{0.05, \nu_1, \nu_2}$																		
		$\nu_1 = \text{Degrees of freedom for numerator}$																		
$\nu_2/\nu_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$	
$\nu_2 = \text{Degrees of freedom for denominator}$	1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
	2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
	3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
	4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
	5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
	6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
	7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
	8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
	9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
	11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
	12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
	13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
	14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
	15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07

(Continued)

**TABLE A4.4A** Critical Values of the  $F$ -distribution for  $\alpha = 0.05$ . The Distributions Represent the Area Exceeding the Value of  $F_{0.05, \nu_1, \nu_2}$ , and  $F_{0.01, \nu_1, \nu_2}$  as Shown by the Shaded Area in the Figure for Different Degrees of Freedom,  $\nu$ . For Example, if  $\nu_1 = 15$  and  $\nu_2 = 20$ , then the Critical Value for  $\alpha = 0.05$  is 2.20 (*cont'd*)

		Values of $F_{0.05, \nu_1, \nu_2}$																		
		$\nu_1 = \text{Degrees of freedom for numerator}$																		
$\nu_2/\nu_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$	
$\nu_2 = \text{Degrees of freedom for denominator}$	16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
	17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
	18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
	19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
	20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
	21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
	22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
	23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
	24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
	25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25	
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00	

$\nu_1$  = Degrees of freedom for numerator.  
 $\nu_2$  = Degrees of freedom for denominator.  
 $P(F > 2.20) = 0.05$ .  
 $P(F < 2.20) = 0.95$ .

**TABLE A4.4B** Critical Values of the  $F$ -distribution for  $\alpha = 0.01$  The Distributions Represent the Area Exceeding the Value of  $F_{0.05, \nu_1, \nu_2}$ , and  $F_{0.01, \nu_1, \nu_2}$  as Shown by the Shaded Area in the Figure for Different Degrees of Freedom,  $\nu$ . For Example, if  $\nu_1 = 15$  and  $\nu_2 = 20$ , then the Critical Value for  $\alpha = 0.01$  is 3.09

		Values of $F_{0.01, \nu_1, \nu_2}$																		
		$\nu_1 = \text{Degrees of freedom for numerator}$																		
$\nu_2 = \text{Degrees of freedom for denominator}$	$\nu_1/\nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1		4052	5000	5403	5625	5764	5859	5928	5982	6023	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2		98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5
3		34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
4		21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6	13.5
5		16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6		13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7		12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8		11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9		10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10		10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11		9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12		9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13		9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14		8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15		8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87

(Continued)

**TABLE A4.4B** Critical Values of the  $F$ -distribution for  $\alpha = 0.01$  The Distributions Represent the Area Exceeding the Value of  $F_{0.05, \nu_1, \nu_2}$ , and  $F_{0.01, \nu_1, \nu_2}$  as Shown by the Shaded Area in the Figure for Different Degrees of Freedom,  $\nu$ . For Example, if  $\nu_1 = 15$  and  $\nu_2 = 20$ , then the Critical Value for  $\alpha = 0.01$  is 3.09 (*cont'd*)

		Values of $F_{0.01, \nu_1, \nu_2}$																		
		$\nu_1 = \text{Degrees of freedom for numerator}$																		
$\nu_1/\nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$	
$\nu_2 = \text{Degrees of freedom for denominator}$	16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
	17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
	18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
	19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
	20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
	21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
	22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
	23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
	24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
	25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.53	2.45	2.36	2.27	2.17
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01	
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80	
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60	
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38	
$\infty$	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00	

$\nu_1$  = Degrees of freedom for numerator.

$\nu_2$  = Degrees of freedom for denominator.

$P(F > 3.09) = 0.01$ .

$P(F < 3.09) = 0.99$ .