Informetis Assessment Q1

There are two dice. A person rolls the first die. If the number is higher than 2, they roll the first die again. Otherwise, they roll a second die. If the number on the second die is greater than 5, they score 1 point. If the number on the second die is less than or equal to 5, they score 2 points. What is the expectation value and variance if this game is played many times?

Answer

There are two dices (Dice *a* and Dice *b*) and three scenarios (x_1, x_2 and x_3), each with their respective scores and probabilities.

Scenarios

Scenario	Description	Probability	Points Scored
x1	Dice *a* > 2	$4/6 = 0.67$	0
x2	Dice *a* ≤ 2 & Dice *b* > 5	$2/6 * 1/6 = 0.056$	1
x3	Dice *a* ≤ 2 & Dice *b* ≤ 5	$2/6 * 5/6 = 0.278$	2

Note that the Score for scenario x_1 is assumed to be 0, as it's not explicitly stated in the question.

Expected Value

The **expected value** (or mean) of X, where X is a discrete random variable, is a weighted average of the possible values that X can take, each value being weighted according to the probability of that event occurring.

$$E[X] = \sum x \cdot p$$

So the expected value is the sum of: [(each of the possible outcomes) × (the probability of the outcome occurring)]. Note, here we're assuming that it is the score that we're getting the expected value of and NOT the number on the dice (1 to 6).

Hence, the expected value $E = (0.67 * 0) + (0.056 * 1) + (0.278 * 2) = 11/18 = 0.61$

Variance

The variance of a random variable tells us something about the spread of the possible values of the variable. For a discrete random variable X, the variance of X is written as Var(X).

$$Var(X) = \sum x^2 p \ - \ \mu^2$$

Where μ is the expected value of 0.61 calculated earlier. Hence, calculating variance for score $Var(X) = 0^2 * 0.67 + 1^2 * 0.056 + 2^2 * 0.278 - 0.61 = 5/9 = 0.56$