Mathematics 352
Quiz 8

April 12, 2013; 10 minutes Name: ____________________

This quiz is *open-note*, but no books or calculators.

1. Use the method of undetermined coefficients to find the general solution of the differential equation
$$y'' - 2y' - 3y = 3e^{2t}.$$

> **Solution:** The characteristic polynomial is $r^2 - 2r - 3 = (r-3)(r+1)$, and so the complementary solution y_c has the form
> $$y_c = c_1 e^{3t} + c_2 e^{-t}.$$
> Therefore, we can look for a solution Y of the form
> $$Y = Ae^{2t}.$$
> Differentiating, we find that $Y' = 2Ae^{2t}$ and $Y'' = 4Ae^{2t}$, and therefore that if Y is a solution to the equation, we have
> $$(4Ae^{2t}) - 2(2Ae^{2t}) - 3Ae^{2t} = 3e^{2t},$$
> which implies that $A = -1$.
>
> Therefore, the general solution of the differential equation is $y_c + Y$, which we can write as
> $$c_1 y_1 e^{3t} + c_2 y_2 e^{-t} - e^{2t}.$$

2. Use the method of undetermined coefficients to find the general solution of the differential equation
$$y'' - 2y' - 3y = -3te^{-t}.$$

> **Solution:** The characteristic polynomial is the same as in the previous problem, so that the complementary solution y_c takes the same form:
> $$y_c = c_1 e^{3t} + c_2 e^{-t}.$$
> Observe that the right-hand side $g(t)$ is the product of a polynomial of degree one with an etponential. The general yoga of undetermined coefficients tells us that in this case, we should first look for a solution of the form $(At + B)e^{-t}$.

However, the same yoga advises us that when *part of the inhomogeneous term appears in the general solution, we must multiply our guess by t*. This is because the entire Be^{-t} from above is a solution of the associated homogeneous equation. Therefore we are looking for a solution of the form

$$Y = At^2e^{-t} + Bte^{-t}.$$

Differentiating, we find that $Y' = (-At^2 + (2A-B)t + B)e^{-t}$ and $Y'' = (At^2 + (-4A+B)t + (2A-2B))e^{-t}$. Substituting, we find that

$$-8Ate^{-t} + 2Ae^{-t} - 4Be^{-t} = -3te^{-t}.$$

This yields the two algebraic equations

$$\begin{aligned} -8A &= -3 \\ 2A - 4B &= 0. \end{aligned}$$

Therefore, $A = 3/8$ and $B = 3/16$, and the general solution is

$$c_1e^{3t} + c_2e^{-t} + \frac{3}{8}t^2e^{-t} + 3/16te^{-t}.$$