Mathematics 352
Repeated roots and reduction of order

March 20, 2013 Name: _____

Due: March 22, 2013

Introduction. In this worksheet, you will see what happens when the method of characteristic polynomials produces only a 1-dimensional family of solutions. The rest are generated by a method called *reduction of order*.

Consider the equation

$$y'' + 4y' + 4 = 0. \tag{1}$$

1. Check that the characteristic polynomial of this equation has only one root. Write down the exponential solution of Equation 1 that corresponds to this root; call it y_1.

2. Since Equation 1 is linear and homogeneous, cy_1 is a solution for each real c. Let us look for a second type of solution of the form $v(t)y_1(t) = vy_1$. Remembering that each of v and y_1 is a function of t, find the derivative of vy_1.

3. Find the second derivative of vy_1.

4. Substitute vy_1 and its derivatives into Equation 1 and simplify. When the dust settles, you should have a new (first-order) differential equation you can solve for v. What is this equation?

5. Find the general solution v of your new differential equation, and multiply it by y_1 to deduce a new solution y of Equation 1. *Hint.* Your exponential solution y_1 of Equation 1 should be "included" in y.