Mathematics 352
Undetermined Coefficients

April 5, 2013 Name: ______________________________

Due: April 10, 2013

Introduction. In this activity you will get some practice using the method of undetermined coefficients to solve inhomogeneous differential equations $ay'' + by' + cy = g(t)$ in which the inhomogeneous term $g(t)$ is a combination of polynomial, exponential, or trigonometric functions.

Recall that the general solution to the inhomogeneous differential equation $ay''+by'+cy = g(t)$ is y_c+Y, where the *complementary solution* $y_c = c_1y_1 + c_2y_2$ is the general solution of the *associated homogeneous equation* and Y is *any* particular solution of the inhomogeneous equation.

1. Find the *complementary solution* of $y'' - 4y' - 12y = 3e^{5t}$. This is always the first step. You should always do this *before* attempting to find a particular solution Y.

2. Use the method of undetermined coefficients to find a particular solution Y of $y'' - 4y' - 12y = 3e^{5t}$.

3. Check, by direct substitution, that your function Y really is a particular solution.

4. No further calculation is necessary to write down the general solution of the inhomogeneous equation: do so below.

5. Use the method of undetermined coefficients to find a particular solution Y of $y'' - 4y' - 12y = \sin(2t)$.

6. Verify, by direct substitution, that Y really is a particular solution.

7. Use the method of undetermined coefficients to find a particular solution Y of $y'' - 4y' - 12y = 2t^3 - t + 3$.

8. Verify, by direct substitution, that Y really is a particular solution.

9. Write down the form (including the undetermined coefficients) of initial guesses for the particular solutions if $g(t)$ is the indicated function.
   (a) $g(t) = 16e^{7t}\sin(10t)$
   (b) $g(t) = (9t^2 - 103t)\cos(t)$
   (c) $g(t) = -e^{-2t}(3-5t)\cos(9t)$

10. Solve the initial value problem

$$y'' - 4y' - 12y = 3e^{5t}, \quad y(0) = 18/7,\ y'(0) = -1/7$$