Separation of variables; introduction to modeling

Math 352 Differential Equations

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Recap/warmup: linear equations

In Workshop 01, you saw how to solve some linear equations of the form

$$y' + ay = g(t).$$

In your groups, use the same technique to solve this equation:

- $y' + (2/t)y = 4t$
- Be careful when you are finding $\mu(t)$!

In general

- To solve a linear equation, put it in standard form:

$$y' + p(t)y = g(t)$$

- and use the integrating factor $\mu(t) = \exp(\int p(t)\, dt)$.
- You're free to memorize the formula in the text, but I'm not sure that it is easier.

Separable equations

- The next class of differential equations are the *separable* ones.
- These are the easiest to solve of all.
- Standard form: $M(x) + N(y)y' = 0$.

The trick: a hidden u-substitution

Watch what happens when we integrate both sides dx:

$$M(x) + N(y)y' = 0$$

$$\int M(x)\,dx + \int N(y)\frac{dy}{dx}\,dx = C$$

Now let's do a u-sub with $u = y$ (so $du = y'dx = \frac{dy}{dx}\,dx$)

$$\int M(x)\,dx + \int N(y)\,dy = C$$

The same idea, but with "separating"

Just write the equation this way, "multiplying through" by "dx":

$$M(x) + N(y)\,\frac{dy}{dx} = 0 \implies N(y)\,dy = -M(x)\,dx$$

- Now integrate and solve for y (if possible).
- Often, you will not be able to isolate y.

Example

$$\frac{dy}{dx} = \frac{x^2}{1 - y^2}$$

- The equation is separable, with standard form

$$x^2\, dx = (1 - y^2)\, dy$$

- Integrating, we find

$$\frac{x^3}{3} + C = y - \frac{y^3}{3}$$

- Not possible to solve for y: solution curves are $-x^3 + 3y - y^3 = c$

Work together

Just like with linear equations, you want to practice this procedure until it feels natural.

- $y' = x^2/y$
- $y' + y^2 \sin x = 0$
- $y' = x^2/(y + yx^3)$