Math 352 **Workshop 05**

March 12, 2014 Due: March 14 Name: ______________________________

Second-order linear equations

1. Find the roots of the characteristic equation for the differential equation $y'' + 9y = 0$. Note that the discriminant is negative, so they are complex. Call them r_1 and r_2.

2. Evaluate $(r_1 + r_2)/2$ and $(r_1 - r_2)/2i$. What do you notice about these numbers?

3. What complex exponential functions solve the differential equation? Use the roots you found and ape the exponential trick from before. (You want two different functions, u_1 and u_2, with no c_1 or c_2.)

4. Write these functions using Euler's formula. Use the identities $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$ to simplify.

5. Write down $y_1 = (u_1 + u_2)/2$ and $y_2 = (u_1 - u_2)/2i$ and simplify. What do you notice?

6. Apply the method to each differential equation, obtaining two independent *real-valued* solutions. (Independent means the Wronskian is nonzero.) Remember:

$$e^{(x+iy)t} = e^{xt} e^{iyt}$$

   (a) $y'' + 2y' + 8y = 0$

   (b) $3y'' + 6y' + 4y = 0$

   (c) $6y'' - 8y' + 10y = 0$