Math 352 **Workshop 06**

March 14, 2014 Due: March 17 Name: ______________________

Second-order linear equations: discriminant zero

In this workshop, you'll examine the last remaining case of the second-order linear differential equation: discriminant 0. Then, you'll meet another way of writing the general solution of a special class of second-order equations.

1. Consider the differential equation $y'' + 2y' + y = 0$. What is the discriminant of its characteristic polynomial?

1. ______________________

2. Use the characteristic polynomial to find an exponential solution of the equation above. That is, find a solution of the form $y_1 = e^{rt}$.

2. ______________________

Experience, general principles, and your sense of mathematical justice indicate there should be another solution y_2, evidently not of exponential type. It should satisfy $W(y_1, y_2) = 0$ with y_1 from above.

3. You are visited in a dream by a spirit who suggests you try to find a solution of the form $y_2(t) = u(t)y_1(t)$, where y_1 is the exponential solution you found. Follow your dream, write small, and simplify as you go.

3. ______________________

4. Check that the Wronskian $W(y_1, y_2)$ is nonzero and write down the general solution of $y'' + 2y' + y = 0$.

4. ______________________

5. On a separate piece of paper, repeat the above procedure for the following differential equations. Use the initial conditions, when given, to find particular solutions. Be careful with your calculus.

   (a) $y'' - 4y' + 4y = 0$

   (b) $4y'' - 12y' + 9y = 0,\ y(0) = 1,\ y'(0) = -1$

   (c) $169y'' - 234y' + 81y = 0,\ y(0) = 0,\ y'(0) = 4$

Special cases: zero linear term

6. Let a and b be some numbers. Solve the initial value problem $y'' + y = 0,\ y(0) = a,\ y'(0) = b$.

6. ______________________________

7. Now solve the initial value problem $y'' - y = 0,\ y(0) = a,\ y'(0) = b$.

7. ______________________________

8. Recall that the *hyperbolic trigonometric functions* are given by the formulas

$$\cosh t = \frac{e^t + e^{-t}}{2}, \quad \sinh t = \frac{e^t - e^{-t}}{2}.$$

   Check that the Wronskian $W(\cosh t, \sinh t)$ is nonzero (you may want to do some of the work on a separate page). Explain why this means that every solution of $y'' - y = 0$ can *also* be written as a linear combination of $\cosh t$ and $\sinh t$.

9. Using the new formulation of the general solution of $y'' - y = 0$,

$$y = c_1 \cosh t + c_2 \sinh t,$$

   solve the initial value problem $y'' - y = 0,\ y(0) = a,\ y'(0) = b$ from above.

9. ______________________________