Introduction to modeling, II

Math 352 Differential Equations

February 21, 2014

Predator–prey equations

If x and y are populations of two different organisms, then one model for their interaction is the *Lotka–Volterra equations*, sometimes known as the *predator–prey* equations. These are:

$$\frac{dx}{dt} = x(\alpha - \beta y)$$
$$\frac{dy}{dt} = -y(\gamma - \delta x)$$

Here, α, β, γ, and δ are parameters describing the nature of the interspecies interaction.

This model was the gold standard in biomathematics for a generation, but may have been supplanted in the late 80s by an alternative model.

Don't jump to the solutions

- Start with differential equations
- Resist the urge to jump straight to solutions
- Understand the model before you begin working out the consequences

Interpret and predict, even w/o solution

Like we did in class: - Logistic direction field - What will happen if $P(0) > K$? if $P(0) < K$? - Are the results consistent with prior understanding of reality?

Modeling project assignment

- Modeling projects will be *guided*
  - Projects come with instructions
- Submit a written paper (5–8 pages) on the last day of class
- Group presentations May 14 (finals week)
  - 15 minutes + 3 minutes for questions
- Rubrics will be provided
- Presentation counts: your paper and presentation should be professional
  - Use Prezi, PowerPoint, LaTeX Beamer, etc.
  - Papers should include properly formatted mathematics, images where appropriate, and a bibliography

Timeline:

- Monday, February 24: Groups submit top three choices and are assigned a project
- By spring break, have the requirements completed
- Spend a few weeks writing the paper and developing the presentation
  - Drafts of papers will be collected (tentatively, April 18)
  - I will make suggestions and corrections to your drafts
  - Drafts are not graded, but if you skip this step you are missing out on the opportunity to get feedback on your paper
- Polish your paper and finish the presentation during the last 3 weeks

Aerodynamics and aeronautics

- Project 1: Modeling Airborne Situations (paratroopers, D.B. Cooper)
- Project 2: Aircraft Flight Strategies (flight operations)

Statics

- Project 3: Modeling Deflection in a Rigid Beam

Vibrations and waves

- Project 4: A Bungee Jumping Problem
- Project 5: Another Bungee Jumping Problem
- Project 15: Resonance (requirements 1 and 3 only)
- Project 16: A Vibrating Chain

Project 16 requires some extra investigation (Bessel's equations). Project 15 will require some extra investigation (Fourier series) to complete requirements 2 and 4.

Fluid dynamics

- Project 6: The Fifth Labor of Hercules

Heat transfer

- Project 7: A Partially Insulated Rod

Forensic mathematics

- Project 8: Murder at the Mayfair

Dynamics of human (?) relationships

- Project 9: Gnomeo and Juliet

Technique: Laplace transforms

- Project 10: Hercules meets Laplace
- Project 11: Laplace's Equation in Spherical Coordinates

These projects will require more investigation and are mostly mathematical in nature.

Epidemiology

- Project 12: Modeling Malaria in Central America

Population dynamics

- Project 14: Introduction to Predator/Prey Problems
- Project 18: Immortal Differential Equations

Vibrations in 2 dimensions

- Project 17: A Vibrating Square Membrane

This project will require more investigation and represents a more serious time commitment.