Math 352 **Workshop 09**

April 16, 2014 Due: Not collected Name: ________________________________

Matrices and vectors in Sage

For each of the following systems, make a conjecture about whether the system has zero, one, or infinitely many solutions. Then use Sage to solve each system. Record your answers and make any notes you feel are necessary to help you remember the Sage procedures.

1.
$$\begin{array}{rcrcrcr} 3x & + & 3y & + & 12z & = & 6 \\ x & + & y & + & 4z & = & 2 \\ 2x & + & 5y & + & 20z & = & 10 \\ -x & + & 2y & + & 8z & = & 4 \end{array}$$

2.
$$\begin{array}{rcrcrcr} 2x & + & 10y & + & 2z & = & 6 \\ x & + & 5y & + & 2z & = & 6 \\ x & + & 5y & + & z & = & 3 \\ -3x & - & 15y & + & 3z & = & -9 \end{array}$$

3.
$$\begin{array}{rcrcrcrcr} 2x & + & y & - & z & + & 2w & = & -6 \\ 3x & + & 4y & & & + & w & = & 1 \\ x & + & 5y & + & 2z & + & 6w & = & -3 \\ 5x & + & 2y & - & z & - & w & = & 3 \end{array}$$

4.
$$\begin{array}{rcrcrcrcr} x & + & 2y & + & z & + & 4w & = & 11 \\ 3x & + & 6y & + & 5z & + & 12w & = & 30 \\ x & + & 3y & - & 3z & + & 2w & = & -5 \\ 6x & - & y & - & z & + & w & = & -9 \end{array}$$

5.
$$\begin{array}{rcrcrcrcr} x & + & y & + & z & + & w & = & 0 \\ 2x & + & 3y & + & z & - & 2w & = & 0 \\ 3x & + & 5y & + & z & & & = & 0 \end{array}$$

6.
$$\begin{array}{rcrcrcrcr} x & + & 2y & + & z & + & 3w & = & 0 \\ x & - & y & & & + & w & = & 0 \\ & & 5y & - & z & + & 2w & = & 0 \end{array}$$

7.
$$\begin{array}{rcrcrcrcr} x_1 & - & x_2 & - & 2x_3 & + & 3x_4 & = & -7 \\ 2x_1 & - & x_2 & + & 6x_3 & + & 6x_4 & = & -2 \\ -2x_1 & + & x_2 & - & 4x_3 & - & 3x_4 & = & 0 \\ 3x_1 & - & 2x_2 & + & 9x_3 & + & 10x_4 & = & -5 \end{array}$$

8.
$$\begin{array}{rcrcrcrcrcr} 2x_1 & - & 2x_2 & - & x_3 & + & 6x_4 & - & 2x_5 & = & 1 \\ x_1 & - & x_2 & + & x_3 & + & 2x_4 & - & x_5 & = & 1 \\ 4x_1 & - & 4x_2 & + & 5x_3 & + & 7x_4 & - & x_5 & = & 1 \end{array}$$

9.
$$\begin{array}{rcrcrcrcrcr} 3x_1 & - & x_2 & + & 2x_3 & + & 4x_4 & + & x_5 & = & 2 \\ x_1 & - & x_2 & + & 2x_3 & + & 3x_4 & + & x_5 & = & -1 \\ 2x_1 & - & 3x_2 & + & 6x_3 & + & 9x_4 & + & 4x_5 & = & -5 \\ 7x_1 & - & 2x_2 & + & 4x_3 & + & 8x_4 & + & x_5 & = & 6 \end{array}$$

10.
$$\begin{array}{rcrcrcrcrcr} 2x_1 & - & & + & 3x_3 & + & & - & 4x_5 & = & 5 \\ 3x_1 & - & 4x_2 & + & 8x_3 & + & 3x_4 & - & & = & 8 \\ x_1 & - & x_2 & + & 2x_3 & + & x_4 & - & x_5 & = & 2 \\ -2x_1 & + & 5x_2 & - & 9x_3 & + & -3x_4 & - & 5x_5 & = & -8 \end{array}$$

Matrix multiplication and linear combinations

Not every matrix-vector product is defined. For example, an $m \times n$ matrix can be multiplied (on the right) only by an $n \times 1$ (column) vector. Observe that the number of columns of the matrix is the same as the number of entries in the vector.

This is necessary because the definition of the product $A\mathbf{x}$ is

$$A\mathbf{x} = x_1\mathbf{A}_1 + x_2\mathbf{A}_2 + \cdots + x_n\mathbf{A}_n,$$

where the x_i are the entries of $\mathbf{x}$ and $\mathbf{A}_i$ is the ith column of A regarded as a (column) vector. This is what people mean when they say that the definition of matrix multiplication encodes linear combinations.

11. Use Sage to evaluate the products.

    (a) $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix} =$

    (b) $\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -3 \\ 4 & 1 & 2 \end{pmatrix} =$

    (c) $\begin{pmatrix} -2 & -3 \end{pmatrix}\begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & -1 \end{pmatrix} =$

    (d) Why won't this work? Try, and note Sage's complaint.
    $$\begin{pmatrix} 1 & 0 & -3 \\ 4 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & -1 \end{pmatrix}$$

12. Use Sage to invert the following matrices.

    (a) $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}^{-1} =$

    (b) $\begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{pmatrix}^{-1} =$

    (c) $\begin{pmatrix} 1 & -1/2 & 3/4 \\ 3/2 & 1/2 & -2 \\ 1/4 & 1 & 1/2 \end{pmatrix}^{-1} =$

    (d) $\begin{pmatrix} 2 & 4 & 5/2 \\ -3/4 & 2 & 1/4 \\ 1/4 & 1/2 & 2 \end{pmatrix}^{-1} =$

13. Use Sage and matrix inversion to solve the system.
$$\begin{array}{rcrcrcrcrcr} x_1 & + & 2x_2 & - & x_3 & + & 3x_4 & - & x_5 & = & -3 \\ x_1 & - & 3x_2 & + & x_3 & + & 2x_4 & - & x_5 & = & -3 \\ 2x_1 & + & x_2 & + & x_3 & - & 3x_4 & + & x_5 & = & 6 \\ x_1 & - & x_2 & + & 2x_3 & + & x_4 & - & x_5 & = & 2 \\ 2x_1 & + & x_2 & - & x_3 & + & 2x_4 & + & x_5 & = & -3 \end{array}$$

14. Same as above.
$$\begin{array}{rcrcrcrcrcrcr} 4x_1 & - & 2x_2 & + & 4x_3 & + & 2x_4 & - & 5x_5 & - & x_6 & = & 1 \\ 3x_1 & + & 6x_2 & - & 5x_3 & - & 6x_4 & + & 3x_5 & + & 3x_6 & = & -11 \\ 2x_1 & - & 3x_2 & + & x_3 & + & 3x_4 & - & x_5 & - & 2x_6 & = & 0 \\ -x_1 & + & 4x_2 & - & 4x_3 & - & 6x_4 & + & 2x_5 & + & 4x_6 & = & -9 \\ 3x_1 & - & x_2 & + & 5x_3 & + & 2x_4 & - & 3x_5 & - & 5x_6 & = & 1 \\ -2x_1 & + & 3x_2 & - & 4x_3 & - & 6x_4 & + & x_5 & + & 2x_6 & = & -12 \end{array}$$