Math 352 **Workshop 13**

April 28, 2014 Due: Monday, April 28 Name: ______________________

Plotting solutions in the phase plane

Sage commands that will be useful include

`plot_vector_field()`, `parametric_plot()`, and `A.eigenspaces_right()` (where `A` is a square matrix).

Remember, you can use the `help()` command to get information on any of these commands.

Last time, we saw that if the 2×2 matrix A has real eigenvalues r_1 and r_2 with corresponding eigenvectors $\xi^{(1)}$ and $\xi^{(2)}$, then

$$\xi^{(1)} e^{r_1 t}, \quad \xi^{(2)} e^{r_2 t}$$

are (vector-valued) solutions of the homogeneous system $\mathbf{x}' = A\mathbf{x}$.

1. For each of the 2×2 systems, use Sage to plot solutions in the phase plane. Make some notes about how the solutions appear. The last two will be rather different from the others.

(a)

$$\begin{aligned} x_1' &= 3x_1 - 2x_2 \\ x_2' &= 2x_1 - 2x_2 \end{aligned}$$

(b)

$$\begin{aligned} x_1' &= x_1 - 2x_2 \\ x_2' &= 3x_1 - 4x_2 \end{aligned}$$

(c)

$$\begin{aligned} x_1' &= 2x_1 - x_2 \\ x_2' &= 3x_1 - 2x_2 \end{aligned}$$

(d)

$$\begin{aligned} x_1' &= x_1 + x_2 \\ x_2' &= 4x_1 - 2x_2 \end{aligned}$$

(e)

$$\begin{aligned} x_1' &= -2x_1 + x_2 \\ x_2' &= x_1 - 2x_2 \end{aligned}$$

(f)

$$\begin{aligned} x_1' &= \frac{5}{4}x_1 + \frac{3}{4}x_2 \\ x_2' &= \frac{3}{4}x_1 + \frac{5}{4}x_2 \end{aligned}$$

(g)

$$\begin{aligned} x_1' &= 4x_1 - 3x_2 \\ x_2' &= 8x_1 - 6x_2 \end{aligned}$$

(h)

$$\begin{aligned} x_1' &= 3x_1 + 6x_2 \\ x_2' &= -x_1 - 2x_2 \end{aligned}$$