Math 352 **Workshop 14**

May 2, 2014 Not collected Name: _____

Complex eigenvalues, limit cycles, spiral points

Sage commands that will be useful include

`plot_vector_field()`, `parametric_plot()`, and `A.eigenspaces_right()`

(where `A` is a square matrix). Remember, you can use the `help()` command to get information on any of these commands, e.g. `help(parametric_plot)`.

1. For each of the systems of equations below, complete the following steps.

   i. Use the method of eigenvectors to find the general (real-valued) solution of the system.
   ii. Use Sage to make a phase plot that shows three trajectories.
   iii. Make note of any limit cycles or equilibrium solutions.

   (a)

   $$x_1' = x_2$$
   $$x_2' = x_1$$

   (b)

   $$x_1' = 3x_1 - 2x_2$$
   $$x_2' = 4x_1 - x_2$$

   (c)

   $$x_1' = 2x_1 - (5/2)x_2$$
   $$x_2' = (9/5)x_1 - x_2$$

   (d)

   $$x_1' = 2x_1 - 5x_2$$
   $$x_2' = x_1 - 2x_2$$

2. These systems contain a parameter α. For each system,

   i. Determine the eigenvalues in terms of α.
   ii. Find the critical value(s) of α where the qualitative nature of the phase portrait of the system changes.
   iii. Make phase portraits for a value of α slightly below, and for another value slightly above, each critical value.

(a)
$$\mathbf{x}' = \begin{pmatrix} \alpha & 1 \\ -1 & \alpha \end{pmatrix} \mathbf{x}$$

(b)
$$\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ \alpha & -2 \end{pmatrix} \mathbf{x}$$

(c)
$$\mathbf{x}' = \begin{pmatrix} -1 & \alpha \\ -1 & -1 \end{pmatrix} \mathbf{x}$$

(d)
$$\mathbf{x}' = \begin{pmatrix} \alpha & 10 \\ -1 & -4 \end{pmatrix} \mathbf{x}$$