Math 352 **Second-order linear equations**

March 7, 2014 Due: March 10 Name: ____________________

1. Consider the differential equation
$$y'' = -y.$$
Think about the familiar functions until you spot a solution. Compare with the people around you. You should be able to find two really "different" solutions. Call them y_1 and y_2. Remember, when we say that y_1 is a *solution* to the differential equation, we mean that substitution of y_1 for y makes the equation true.

2. Check (by substitution) that $c_1 y_1$ and $c_2 y_2$ for arbitrary constants c_1 and c_2 are also solutions to $y'' = -y$, for the functions y_1 and y_2 defined above.

3. Substitute to check that $c_1 y_1 + c_2 y_2$ is yet another solution to $y'' = -y$.

4. Initial value problems look a little different for second-order equations. Since there is a 2-dimensional family of solutions, we need 2 initial conditions to pick one out. These most frequently take the form
$$y(0) = y_0, \quad y'(0) = y'_0.$$
Find a solution to the initial value problem $y'' = -y$, $y(0) = 1$, $y'(0) = 0$ by plugging in $c_1 y_1 + c_2 y_2$ for y and using the initial conditions to find c_1 and c_2. How many solutions of this form are there?

5. Let $r \neq 0$ and consider the function $y = e^{rt}$. Find y' and y'', and write $ay'' + by' + cy$ in terms of y (i.e., find an equivalent expression with no primes). You should be able to factor y out of the expression you find.

6. Now consider the differential equation $ay'' + by' + cy = 0$. Assume that this equation has a solution of the form $y = e^{rt}$, and use the result of the previous part to express r in terms of a, b, c.

7. Find two really "different" solutions to the differential equation $y'' - y' - 6y = 0$. Call them y_1 and y_2.

8. The linear combination $c_1 y_1 + c_2 y_2$ is called the general solution of $y'' - y' - 6y = 0$. Can you use it to solve the initial value problem $y'' - y' - 6y = 0$, $y(0) = 2$, $y'(0) = -5$?