Math 352 **Workshop 12**

April 25, 2014 Due: Monday, April 28 Name: ______________________

Eigenvectors, phase portraits, and stability

Sage commands that will be useful include

`plot_vector_field()`, `parametric_plot()`, and `A.eigenspaces_right()`

(where `A` is a square matrix). Remember, you can use the `help()` command to get information on any of these commands, e.g. `help(parametric_plot)`.

Last time, we saw that if the 2×2 matrix A has real eigenvalues r_1 and r_2 with corresponding eigenvectors $\vec{\xi}^{(1)}$ and $\vec{\xi}^{(2)}$, then

$$\vec{\xi}^{(1)} e^{r_1 t}, \quad \vec{\xi}^{(2)} e^{r_2 t}$$

are (vector-valued) solutions of the homogeneous system $\vec{x}' = A\vec{x}$.

1. Explain in one sentence, with no mathematical symbols, what it means for a vector to be an eigenvector of some matrix.
2. List the eigenvalues and eigenvectors of the following matrices. Make sure you preserve the correspondence between an eigenvalue and its eigenvector(s).

   (a) $\begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}$

   (b) $\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$

   (c) $\begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}$

   (d) $\begin{pmatrix} -6 & 2 \\ 3 & -1 \end{pmatrix}$

   (e) $\begin{pmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

   (f) $\begin{pmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

(g) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{pmatrix}$

(h) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

3. For each of the matrices A above, write down all the solutions of exponential type (of $\vec{x}' = A\vec{x}$) you can find.

(a) $\begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}$

(d) $\begin{pmatrix} -6 & 2 \\ 3 & -1 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

(f) $\begin{pmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

(g) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{pmatrix}$

(h) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$