Math 352 **Workshop 10**

April 18, 2014 Due: Monday, April 21 Name: ________________________________

Matrices as transformations

This workshop will give you practice in visualizing matrix multiplication geometrically. It will make extensive use of the following fact about matrix multiplication: multiplication by a matrix A *preserves linear combinations*, that is,

$$A(c_1\mathbf{x}_1 + c_2\mathbf{x}_2 + \cdots + c_n\mathbf{x}_n) = c_1 A\mathbf{x}_1 + c_2 A\mathbf{x}_2 + \cdots + c_n A\mathbf{x}_n,$$

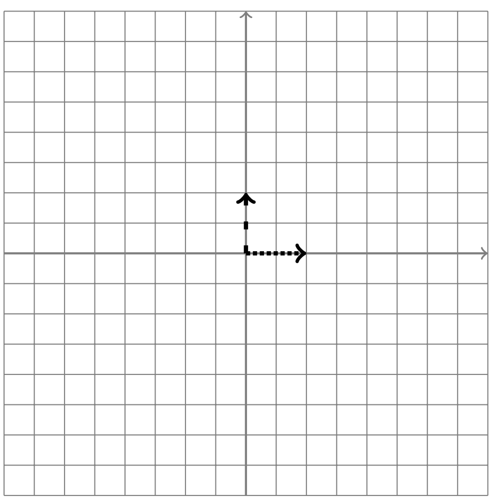
for any vectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ and coefficients $c_1, c_2, \ldots, c_n$.

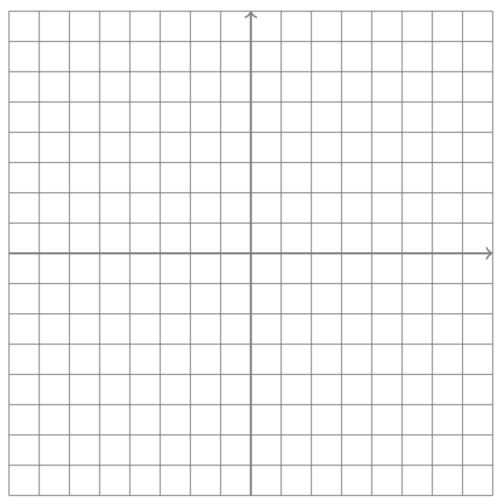
1. Let A be the matrix

$$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}.$$

This is an example of a *diagonal* matrix. We say that the plane $\mathbf{R}^2$ is *spanned* by the standard unit vectors $\mathbf{i} = \langle 1, 0\rangle$ and $\mathbf{j} = \langle 0, 1\rangle$. These vectors are drawn for you below. On the right, sketch the vectors $A\mathbf{i}$ and $A\mathbf{j}$. Remember,

$$A\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1\mathbf{A}_1 + x_2\mathbf{A}_2.$$





Now use colored pencils or shading/crosshatching to demarcate some 1×1 "tiles" on the left. Using the same colors or shading, fill in appropriate shapes for the *images* of your squares on the right.

The matrix A gives rise to a function $L_A(\mathbf{x}) = A\mathbf{x}$, called *left multiplication* by A. You have drawn a picture of this function that is analogous to the graph of an ordinary function. The actual graph of L_A is a surface in $\mathbf{R}^4$, which is difficult to draw.

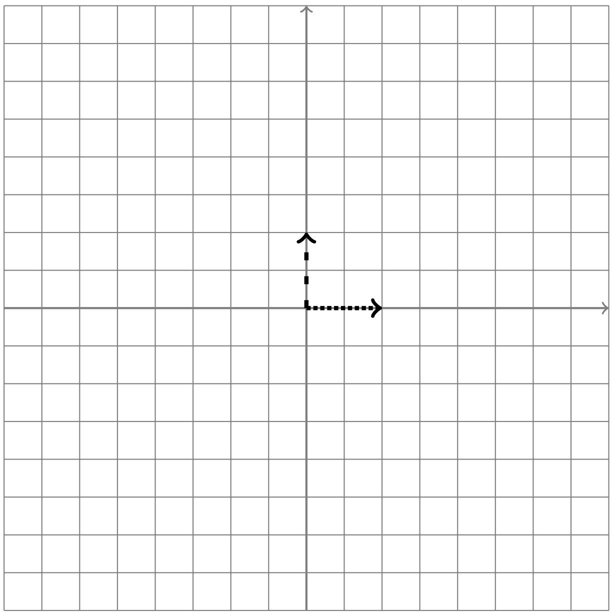
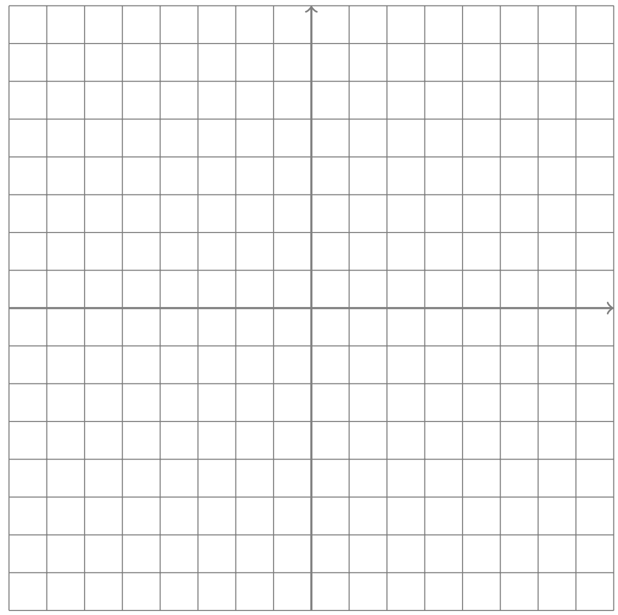
(a) Describe, in a sentence without mathematical notation, the "action" of the matrix A on the squares.

(b) What effect does A have on area? On angles?

2. On the right, sketch the vectors $A\mathbf{i}$ and $A\mathbf{j}$ where

$$A = \begin{pmatrix} 3 & -1 \\ 2 & -2 \end{pmatrix},$$

and color or shade grids as before to construct a picture of $L_A(\mathbf{x})$.

   (a) Does L_A preserve orientation? That is, is the counterclockwise angle from $L_A(\mathbf{i})$ to $L_A(\mathbf{j})$ acute?

   (b) What effect does L_A have on area?

   (c) What is the determinant $\det A$?

3. Using just one grid for each, repeat the exercises for the matrices

$$A = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 4 \\ -1 & -2 \end{pmatrix}.$$