# Recap of second-order homogeneous equations

Math 352 Differential Equations

March 17, 2014

# Second-order homogeneous linear equations

We have completed the solution of the equation

$$ay'' + by' + cy = 0.$$

It proceeds by analysis of the characteristic polynomial $ar^2 + br + c = 0$.

# Positive discriminant

If the discriminant $b^2 - 4ac$ is positive, then the characteristic polynomial has two real roots, $r_1$ and $r_2$. Thus

$$y_1 = e^{r_1 t}, \quad y_2 = e^{r_2 t}$$

are solutions of the equation.

The Wronskian $W(y_1, y_2)$ is nonzero everywhere, and it follows from section 3.2 that the general solution is

$$c_1 e^{r_1 t} + c_2 e^{r_2 t}.$$

# Negative discriminant

If the discriminant $b^2 - 4ac$ is negative, then the characteristic polynomial are:

$$r = \lambda \pm i\mu.$$

(Complex roots of real polynomials occur in conjugate pairs.)

We used Euler's formula to interpret the complex exponentials and obtained real-valued solutions

$$y_1 = e^{\lambda t} \cos \mu t, \quad y_2 = e^{\lambda t} \sin \mu t.$$

The Wronskian of these is again nonzero, so they generate the general solution.

## Zero discriminant

If the discriminant $b^2 - 4ac$ is 0, then the characteristic polynomial has a single real root $-b/2a$. We worked out an example in Workshop 06 showing that in addition to the exponential solution coming from this root, there is another:

$$y_2 = te^{(-b/2a)t}$$

The general solution is therefore

$$c_1 e^{(-b/2a)t} + c_2 te^{(-b/2a)t}$$