Mathematics 352 Differential Equations

The College of Idaho

Spring 2014

I didn't discover curves; I only uncovered them.

MAE WEST

Quick Reference

Homework is assigned from the textbook and on WeBWorK. You are free to work together on problems, but all work you submit must be your own. Weeklies may not be assigned every week, but you will have about a week to work on the problems. Projects will be presented during the final exam period.

GRADING

Tier	Weight	Date
WeBWorK	0.10	continual
Workshops	0.05	continual
Weeklies	0.05	continual
Quizzes	0.35	1.5x/week
Project	0.06	May 14
Midterm 1	0.13	March 5
Midterm 2	0.13	April 9
Midterm 3	0.23	May 5

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Website `https://zeus.collegeofidaho.edu/academics/MathPhysics/courses/MAT-352/`

WeBWorK `https://webwork.collegeofidaho.edu/webwork2/MAT352_S14`

Twitter `@daverosoff`

Prerequisite

The prerequisite for this course is MAT-251.[1]

[1] Or equivalent.

Preface: Learning outcomes

This course is designed to provide certain experiences, called "learning outcomes", to students who successfully complete it. These outcomes are enumerated in the margin.[2] I explicitly include these outcomes in the syllabus so that it is clear why I have chosen the various course components (each of which is described below.) Each learning outcome is addressed by one or more components of the course: quizzes, WeBWorK exercises, longer weekly problem sets, group research projects, presentations to the class, and exams. See the *Grading* section below for more information.

[2] LEARNING OUTCOMES:

1. Recognize and describe fundamental ideas from course content described below.
2. Illustrate ideas from the course with examples.
3. Improve their mathematical writing skills.
4. Appreciate the historical context of differential equations and their role in the physical sciences.
5. Plan, organize, and combine arguments to solve new problems, as appropriate.
6. Solve certain types of differential equations or initial value problems, or verify that a particular function is a solution.
7. Recognize phenomena related to differential equations (e.g., resonance or stability) and explain how they arise.

Introduction

If you are considering a life as any kind of engineer, physical scientist, or applied mathematician, life is differential equations. The laws of the universe are, without exception, differential equations: problems involving fluid flow, current in electric circuits, population dynamics, pricing of financial derivatives,[3] quantum mechanics, heat transfer, relativity, etc., are all formulated in this language. They are ubiquitous and of paramount importance for the understanding of any chemical, physical, or economic phenomena. I hope this is reason enough for you to be interested.

[3] Financial derivatives are unrelated to the derivatives of calculus.

THE RECENT HISTORY OF SCIENTIFIC PROGRESS, say the last 300–400 years, is characterized by a succession of *models* of various phenomena. These models tend to be increasingly mathematical with time. Classical physics, for example, has been thoroughly mathematized since the time of Newton (1642–1727), and quantum physics could not have been invented without it, while the mathematization of biology is still in its nascent phase.

Mathematical models of phenomena that change in time (or space, or in regard to any other measurable quantity) make essential use of derivatives. Without derivatives, it is difficult to imagine a really serious formulation of any phenomenon that is not the same in all places, at all times. The very idea of changeability implies the necessity of *rates of change*, because most of the phenomena of importance in life appear to be *continuous* ones. Moreover the ways in which these phenomena change are evidently also continuous.

A mathematical model can be something as simple as an equation. In fact, they usually are. For example, balances remaining on outstanding student loans can be modeled with exponential functions. Exactly what this means is somewhat nebulous, because no exponential function is equal to the actual balance at all times (since real loans are neither paid continuously nor does their interest accumulate continuously). Yet somehow the essential behavior of the balance function is captured by the simpler exponential model.

Similarly, the level of the tide in the Bay of Fundy is not exactly given by any sinusoidal function, but sinusoidal models of the tide and tides around the world seem to be close enough to inform our understanding of these phenomena.

These effective models arise from differential models of the underlying rates. The tidal system in a narrow bay is described by just a few rates. These rates, starting from physical first principles, are related in a way that is algebraically quite simple. The relationship of the rates—the derivatives—is a differential equation, and the solutions of this equation are functions describing possible tidal behavior.

Similarly, while most students have encountered the exponential model of a loan or investment as a limiting phenomenon,[4] it is perhaps better understood in terms of arising from a specific differential

[4] Continuous interest is the limit of ordinary interest as the length of the compounding period approaches 0.

model. Let us suppose that the accumulation of interest is the only way in which money enters or leaves the account. Then, since interest is a constant multiple of principal,[5] and the rate of increase of the account balance is identical with the accumulation of interest, we must have $dP/dt = rP$, where P is the principal and r is the interest rate.

[5] The constant of proportionality is called the *interest rate*.

HERE IS A BIT OF MATHEMATICAL philosophy. Numbers and points are the same. Ordinary numbers are points on a line; points in other spaces are different kinds of numbers. If the space the points live in is a Euclidean space like $\mathbf{R}^2$ or $\mathbf{R}^3$, these numbers are often called *vectors*. You have spent many years solving equations involving both constants and variables whose values are ordinary numbers, and maybe a few weeks solving equations involving vectors. Usually, we refer to this kind of work as *algebra*.

Points, and therefore numbers, are zero-dimensional.[6] They are pure locations. They do not have extent of any kind. A one-dimensional number is called a *curve*.[7] You can also think of a curve as a family of points, as we often do in calculus. These points are related—as are the members of any family. Often, they are related by an equation defining the curve. Many algebraic problems involve finding a special point on a curve: a special member of a family.

[6] For points in the vector spaces $\mathbf{R}^2$ or $\mathbf{R}^3$, think of the heads of the vectors, not the whole arrow.

[7] On the other hand, a vector (the whole arrow) is another kind of one-dimensional number. Are vectors and curves the same? Not really—but curves (at least differentiable ones) have tangent vectors, so they seem to be related.

Differential equations also involve both constants and variables and need to be solved. But here, the constants, variables, and solutions are all potentially one-dimensional: they are curves! The solutions are called the *integral curves* of the differential equation or equations. The solution of differential equations is the understanding of the algebra of curves.[8] Geometrically, the plane will be partitioned into a bunch of related curves that fit together in a *family*—related, just as in algebra, by an equation (now, a differential equation) defining the family. Many problems in differential equations involve finding a special curve in a family.[9] These are called *initial value problems*. As in algebra, most equations cannot be solved. Equations that admit exact solutions are very special. We will investigate some of the elementary equations and classify them according to their form. We will also discuss systems of linear differential equations, which are solved via linear-algebraic techniques involving matrices.

[8] This is not a merely casual metaphor. The subject of *differential geometry* involves a precise formulation of the algebra of curves (and higher-dimensional surfaces and hypersurfaces).

[9] Compare to the above description of solution of algebraic equations.

Catalog description

"A study of the solution and applications of ordinary differential equations including systems of equations using matrix algebra."

Text

The text is *Elementary Differential Equations* by Boyce and DiPrima, ninth edition. This book and its cousins[10] have been the standard for two generations, so the world is awash in earlier editions (as well as one much more expensive later edition). I don't recommend you use an edition earlier than the eighth. If you have an earlier edition of the text than the ninth, that is fine: but ascertaining the differences in the texts, especially differently numbered or new homework problems, *will be your sole responsibility*. The same caveat applies to any international version. Websites exist that catalog differences between versions.

[10] In particular, *Elementary Differential Equations and Boundary Value Problems*, which is an acceptable substitute for our class.

Grading

Scores[11] are computed as a weighted average, with the weights indicated in the margin. Descriptions of the various tiers follow. Observe that the weights sum to 1 = 100%. The exact determination of letter grades from these scores depends on the final distribution of scores in the class, but you can expect a C for earning 73% of the points, a C+ for 77%, a B– for 80%, and so on. I may adjust these cutoffs downward, but I will not adjust them upward.

[11] TABLE OF WEIGHTS

Tier	Weight
WeBWorK	0.10
Workshops	0.05
Weeklies	0.05
Quizzes	0.35
Project	0.06
Midterm 1	0.13
Midterm 2	0.13
Midterm 3	0.23

Homework

Both WeBWorK and traditional pencil-and-paper homework will be assigned. WeBWorK exercises are shorter and more procedural, helping you assess your own understanding of the ideas of the course. The weekly homework sets, which are longer, are more involved problems. Some problems may take you several hours, or require some work with a computer algebra system such as Sage, Mathematica, or Matlab.[12] My goal is as follows: every student who successfully completes all of the homework problems and *understands all the solutions* should be able to earn an A in this course. All quizzes and exams are designed with this in mind. Therefore, I hope you will agree that it is in your very best interest to complete all of the assigned work, regardless of whether it is turned in for credit. The course is designed so that you will do best if you work at a modest but constant pace throughout the term. Cramming might work too, but not as well—and not as permanently, which is really the point.

[12] I believe that in fact it is possible to do all of the problems by hand, but some would be very laborious without computer assistance.

A note on written homework

Even it you are not turning it in, it is important to pay attention to the style of your writing and your presentation. Good mathematical writing is essential for anyone who wishes to think clearly about mathematics—sloppy writing *invariably* reflects underlying sloppy thinking.[13] The process of making your ideas and reasoning *clear, complete, and unambiguously correct* is the greatest amplifier of mathematical power there is. Hence your solutions should be composed in brilliant English prose.[14] You must explain what is happening as the action unfolds. You should also avoid falling into a "two-column" format that you may have learned in a high-school geometry class. It is stilted, artificial, and not easier to read than a pile of equations. Weave text and equations together for a gentle presentation that doesn't leave the reader guessing.

[13] Since mathematics is one of the hardest things to think about, its careful study is the best training for proficiency in any kind of thought.

[14] This means conforming to accepted scientific usage, more or less correct grammar and spelling, and above all *complete sentences*, sprinkled with equations here and there. Solutions in the popular "pile-of-equations" style are to be avoided and will not get much credit.

Quizzes

Quizzes[15] will be given frequently to help you make sure you are staying on top of the material. Quiz problems will come directly, or nearly so, from the assigned daily WeBWorK problems. Quizzes begin promptly at 8:00 and cannot be made up.

[15] Quizzes constitute the single largest component of the course grade. It is very difficult to get a high grade in the class without getting a high quiz grade. Since quizzes are so frequent, this is eminently possible, but it is extremely advisable to stay current with reading and WeBWorK in order to effect it.

Workshops

Most of you are familiar with the workshop format[16] from Calculus III. Workshops are primarily in-class activities (although some will need to be finished at home) in which you will work through some aspects of a problem with other students in the class. Along with the homework, the workshops are the heart of the course.

[16] Workshops are generally marked for full credit if it appears you made an honest effort to complete them.

Exam dates

As specified above, you will take three hourlong midterm exams. Let me know *immediately* if you foresee a conflict. See below also for information regarding make-up exams.

- Exam 1 (tentative): Wednesday, March 5
- Exam 2 (tentative): Wednesday, April 9
- Exam 3 (tentative): Wednesday, May 5 (week 12, not finals week)[17]

[17] Contrary to occasionally expressed opinion, the last week of class is very much alive.

Make-ups

I will only consider make-up exams with a *documented, compelling reason* and sufficient (two weeks is always enough) notice; otherwise, remaining exams will be reweighted. Quizzes can not be made up.

Academic integrity

Students are expected to complete all graded work in accordance with the College Honor Code. Plagiarism, cheating, or borrowing without proper credit will not be tolerated. Violations of academic honesty can result in loss of credit on an assignment, failure on an exam, or failure in the course. A referral may be made to the Vice President for Academic Affairs for all parties involved in academic dishonesty.

A note on studying math

This class is all about practice. Learning to recognize the different types of equations and quickly and accurately recall the procedures needed to solve them is a matter of rote technique. If you have practiced enough, you will remember how to do it.[18] If not, something else may happen. Of course, there are deep ideas lurking beneath the computational techniques. Awareness and understanding of these ideas also comes with practice.

[18] This is the definition of "enough".

Disability statement

Students with documented disabilities as addressed by the Americans With Disabilities Act and who need any test or course materials to be furnished in an alternative format should notify me immediately (during the first week of class). Reasonable efforts will be made to accommodate the needs of such students.

GOOD LUCK THIS SEMESTER!