Mathematics 352
Final Exam

May 15, 2013; 60 minutes   Name: _____

This exam is closed book; you can use a calculator (*not* a cell-phone calculator) but no other electronic aids or printed references. *If the wording or intent of any question is unclear, please ask me to clarify.* I am not trying to confuse you with the problem statements.

You can use your own paper or the provided blank copy paper. Please write your name on each page you hand in. Show all your reasoning and all pertinent calculations. *Give all answers in exact form. Decimal approximations of any accuracy will not receive full credit.*

When you have finished the exam, place this cover sheet on top of it and fold the packet in half *the long way, with your name facing out.*

Question	Points	Score
1	25	
2	15	
3	15	
4	0	
5	0	
Total:	55	

Good luck!

1. A tank holds 200 L of brine that contains 10 kg of salt. Now pure water runs into the tank at a rate of 5 L/min and the well-stirred mixture drains out of the tank at the same rate.

    (a) (15 points) Determine the amount of salt in the tank after 30 min.

    (b) (10 points) How long will it take for the amount of salt to drop to 1 kg?

2. (15 points) Use Euler's method with step size $h = 0.1$ to approximate $y(0.2)$, where y is the unique solution of the initial value problem

$$y' = 3y - ty^2, \quad y(0) = 1.$$

    You may express your answer as a decimal, but keep three digits of accuracy if you do.

3. (15 points) Find a fundamental set of solutions of the differential equation $4y'' - 4y' + y = 0$. Justify your answer by computing an appropriate Wronskian.

4. Use whatever method you like to find the general solution to the equation

$$y'' - 10y' + 25y = e^{5t}.$$

5. Find the general solution of the equation

$$ty'' - (t+1)y' + y = t^2 e^{2t}$$

    given that $y_1 = e^t$ and $y_2 = t+1$ are solutions of the associated homogeneous equation. *Hint.* Use the method of variation of parameters.