Math 352 **Workshop 11**

April 21, 2014 Due: Wednesday, April 23 Name: ______________________

Phase portraits

When you submit this worksheet, attach printouts including the graphics (you don't need to include your whole Sage session).

1. Consider an undamped spring-mass system whose position function satisfies the differential equation

$$9u'' + 4u = 0.$$

Find a fundamental set of solutions for this equation.

2. Write $u_1 = u$ and $u_2 = u'$. Then the equation above becomes

$$\begin{aligned} u_1' &= u_2 \\ u_2' &= -\frac{4}{9}u_1. \end{aligned}$$

Write this system in matrix–vector form: that is, in the form $\mathbf{u}' = A\mathbf{u}$.

The Sage functions `plot_vector_field` and `parametric_plot` are very helpful in visualizing the solutions of systems of differential equations.

3. In Sage, initialize variables `x` and `y` and a matrix `A` containing your coefficients from above. Then use the `plot_vector_field` command to plot the vector field $A\langle x, y\rangle$.

4. It is simple to overlay plots in Sage. Instead of just calling the `plot_vector_field` function, store its output in a variable. Then use the `parametric_plot` function to store the phase plot of a particular solution. Invoke `show` on the *sum* of your plots to superimpose them.

5. Add a few more trajectories to your image. If you are feeling adventurous, use a `for` loop to automate the procedure.

6. Choose a damping coefficient $\gamma > 0$ and repeat the process. Note the differences you observe below.

7. Repeat the process for the matrix

$$A = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}.$$

   Can you complete the analysis? Note the difficulties below.