

Pythagorean means

In this activity we explore the three different means of the ancient Greeks.

The arithmetic mean

The *arithmetic mean* is the good-old mean that we are all familiar with.

Question 1 What is the mean that we are all familiar with? Explain how to compute the mean of a_1, a_2, \dots, a_n . Give some examples.

The geometric mean

The *geometric mean* is a bit different. The geometric mean of a_1, a_2, \dots, a_n is given by:

$$\left(\prod_{i=1}^n a_i \right)^{1/n}$$

Question 2 Explain an analogy between the arithmetic mean and the geometric mean.

Question 3 Can you explain the geometric mean in terms of geometry? First do it for 2 numbers. Next do it for three.

The harmonic mean

The *harmonic mean* might be the most mysterious of all. The harmonic mean of a_1, a_2, \dots, a_n is given by:

$$\frac{n}{\sum_{i=1}^n \frac{1}{a_i}}$$

Exploration 4 Can you find a connection between the harmonic mean and music?

Question 5 In the United States, the fuel efficiency of a car is usually given in the units:

$$\frac{\text{miles}}{\text{gallon}}$$

However, in Europe, the fuel efficiency of a car is usually given in the units:

$$\frac{\text{liters}}{100\text{km}}$$

Learning outcomes:
Author(s):

Give some examples of fuel efficiency (both efficient and inefficient) with each set of units.

Question 6 Now suppose that a car gets $60 \frac{\text{miles}}{\text{gallon}}$ and another car gets $20 \frac{\text{miles}}{\text{gallon}}$. What is the average fuel efficiency?

Question 7 Now suppose that a car gets $4 \frac{\text{liters}}{100\text{km}}$ and another car gets $20 \frac{\text{liters}}{100\text{km}}$. What is the average fuel efficiency?

Exploration 8 Compare your answers to the last two questions. Something fishy is going on, what is it?