

September 10, 2013

Due: Friday, September 12

Name: _____

1 Slicing spheres

This workshop will confirm what is, to varying degrees for various people, intuitively clear: that all the nonempty slices of a sphere by planes are circles (we adopt the severely reasonable convention that a circle of radius 0 is a point).

1.1 Warm-up

Recall that the *unit sphere* in \mathbf{R}^3 is the sphere of radius 1 centered at $(0,0,0)$. Its equation is

$$x^2 + y^2 + z^2 = 1.$$

1. Give equations of all the planes parallel to the (y,z) -plane that miss the unit sphere. *Hint.* Use an auxiliary variable.
2. How many planes parallel to the (y,z) -plane meet the sphere at one point? Give equations for all of them.

1.2 The unit sphere and a horizontal plane

1. Draw a big, beautiful picture of the unit sphere sliced by a horizontal plane. Remember the drawing tips:
 - Get the angles right. Make the parallelogram of your plane parallel to the x - and y -axes.
 - Draw a *big* picture. Big pictures have more room for labels.
 - The (x,y) -plane itself does slice the sphere, but it's very special. Pick a more generic slicing plane.
 - Don't be afraid to erase.

2. Now, we're still solving two equations. What are they? (Hint: one from the sphere and one from the slicing plane.)
3. Explain why this situation is algebraically preferable to the brute-force solution outlined above.
4. Write down two equations that all the points in the intersection of the sphere and the plane satisfy. (Hint: one equation involves x and y only, and the other involves z only.) These aren't the same as the equations above, but they aren't too terribly different either.
5. Identify the flaw in this attempted use of WLOG.

Consider the cylinder of radius 1 with axis the x -axis and the (y, z) -plane. The intersection is evidently the circle of radius 1, centered at $(0, 0, 0)$, and contained in the (y, z) -plane.

Therefore, we can say that WLOG, whenever a cylinder is cut by a plane, the intersection is a circle. This is because we can always “rotate and zoom” to place the cylinder so that its radius is 1 and its axis is the x -axis.