Math 251 **Workshop 08: Introducing the double integral**

October 18, 2013

Due: Not collected Name: ______________________________

1 Workshop 08: Introducing the double integral

While the double integral is defined as the limit of a Riemann sum, just like ordinary definite integrals are, they are usually computed using Fubini's Theorem, which tells us how to write them as *iterated integrals*. These are expressions of the form

$$\int_a^b \left(\int_c^d f(x,y)\,dy \right) dx.$$

Usually, we omit the parentheses with this understood grouping of operations. Notice that the innermost differential dy is matched with the innermost integral. A notation that is less ambiguous would be

$$\int_{x=a}^b \left(\int_{y=c}^d f(x,y)\,dy \right) dx.$$

1.1 Evaluating iterated integrals

Since $f(x,y)$ in the above expressions is a function of x and y, if we perform a definite integral of this function over x, we get a function of y alone. It's analogous to partial differentiation: if you're integrating dx, y is like a constant.

1. Evaluate the iterated integral. *Answer*: $40(e^4 - e^2)$

   $\int_{x=2}^4 \left(\int_{y=1}^9 ye^x\,dy \right) dx =$

2. Evaluate the iterated integral. *Answer*: 84

$$\int_2^6 \int_1^4 x^2 \, dx \, dy =$$

3. Evaluate the iterated integral. *Answer*: 4/3

$$\int_{-1}^1 \int_0^\pi x^2 \sin y \, dy \, dx =$$

4. Evaluate the iterated integral.

$$\int_{-1}^1 \int_0^\pi x^2 \sin y \, dx \, dy =$$