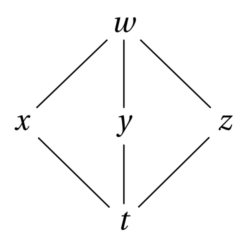
Math 251 **Workshop 06: The chain rule**

October 7, 2013

Due: Wednesday, October 9 Name: ______________________________

1 Workshop 06: The chain rule



The chain rule, just as in one-variable calculus, tells us how to find the derivative of a composite function in terms of the derivatives of the composition factors.

Complications arise, because functions of different *valence* may be composed. For example, we might have $x = x(t)$, $y = y(t)$, and $z = z(t)$, each an ordinary one-variable function, and $w = f(x, y, z)$. It's somewhat of our choice whether we want to view w as a function of x, y, and z, or as a function of t. The chain rule helps us relate the various derivatives through pictures like this. There is only one composite derivative to find in the pictured scenario, and it is evidently

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}.$$

1.1 Warm-up

1. Let $f(x, y, z) = xy + z$. Suppose that $x(s, t) = s^2$, $y(s, t) = st$, $z(s, t) = t^2$. Find $\partial f/\partial s$.

2. Let $f(x, y) = e^{xy}$. Evaluate $\partial f/\partial t$ at $(s, t, u) = (2, 3, -1)$, if $x(s, t, u) = st$, $y(s, t, u) = s - ut^2$.

1.2 A little tougher

3. Suppose $x = r\cos\theta$ and $y = r\sin\theta$. (This is the so-called *polar coordinate transformation*, which we will meet again when we discuss integration.) Let $f(x, y)$ be a function. Give formulas for $\partial f/\partial r$ and $\partial f/\partial\theta$ in terms of the other derivatives.

4. Apply the result from the previous problem to the function $f(x, y) = x^2 y$.

5. Let $x = \rho\sin\varphi\cos\theta$, $y = \rho\sin\varphi\sin\theta$, and $z = \rho\cos\varphi$. (This is the so-called *spherical* coordinate transformation.) Find formulas for the derivatives of a general function $f(x, y, z)$ with respect to ρ, φ, and θ.