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Due: Monday, October 7	Name:	

## 1 Workshop 05: Gradient vectors

This workshop motivates and introduces the fundamental idea of gradient vectors to functions.

## 1.1 Warm-up

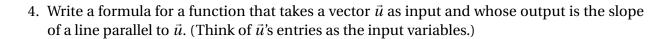
You may remember from a previous math experience that lines in the plane are orthogonal precisely when the product of their slopes is -1. Here you will use vectors to get a simple proof of this fact.

1. Consider the line y = 3x + 1. Find a vector  $\vec{v}$  (of any length) parallel to this line. (Your vector will have just 2 entries.)

2. Now find a vector that is orthogonal to  $\vec{v}$ , and call it  $\vec{w}$ . Hint. The easiest way to generate  $\vec{w}$  is to choose its entries in such a way that guarantees  $\vec{v} \cdot \vec{w} = 0$ .

3. What is the slope of a line parallel to  $\vec{w}$ ? Is the product of the slopes -1 as claimed?

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5. Now let  $f(x) = x^3 + 3x + 1$ . Find the tangent line to the graph of f at x = 0. Write its equation in the form mx - y = -b. What do you notice about the vector  $\langle m, -1 \rangle$  in relation to the tangent line?

6. The general form of a line in the plane is Ax + By = C (not every line has a slope). If  $B \ne 0$ , this line does admit a slope-intercept form. Find it, and show that  $\langle A, B \rangle$  is parallel to  $\langle m, -1 \rangle$ .

## 1.2 General functions

In the previous section you saw that  $\langle f'(a), -1 \rangle$  is normal to the tangent line to the graph of f at x = a. This is true in general.

7. Use the point-slope formula to write the tangent line to the graph of an arbitrary differentiable function f at x = a. Get the line into the standard form Ax + By = C.

8. Check that  $\langle f'(a), -1 \rangle$  is orthogonal to the tangent line.