Mathematics 275 Multivariable Calculus

The College of Idaho

Fall 2014

The pursuit of knowledge, brother, is the askin' of many questions.

RAYMOND CHANDLER

Farewell, My Lovely

Quick Reference

You have both online WeBWorK problems and written portfolio problems. See below for more details. You are free to work together on problems, but all work you submit must be your own. Portfolios will be collected at least four times. We have frequent quizzes and in-class activities that cannot be made up.

GRADING

Tier	Weight	Date
WeBWorK	0.08	continual
Workshops	0.10	continual
Portfolios	0.10	continual
Presentations	0.08	continual
Quizzes	0.32	1.5/week
Midterm 1	0.08	September 30
Midterm 2	0.08	November 11
Final Exam	0.16	December 10

Instructor Dr. Dave Rosoff

Office Boone Hall 102C

Office hours MW 9:10–10:10, T 10:20–11:20, F 2:10–3:10, or by appointment

Email drosoff@collegeofidaho.edu

Twitter `@daverosoff`

WeBWorK `https://webwork.collegeofidaho.edu/webwork2/MAT275_F14`

Other web resources Moodle

Preface: Learning outcomes

This course is designed to provide certain experiences, called "learning outcomes", to students who successfully complete it. These outcomes are enumerated in the margin.[1] I explicitly include these outcomes in the syllabus so that it is clear why I have chosen the various course components (each of which is described below.) Each learning outcome is addressed by one or more components of the course: quizzes, WeBWorK exercises, workshops, portfolios, presentations to the class, and exams. See the *Grading* section below for more information.

[1] LEARNING OUTCOMES:

1. Recognize and describe fundamental ideas from course content described below.
2. Illustrate these ideas with examples and translate them into everyday terminology.
3. Demonstrate the use of calculus on specific numerical problems.
4. Classify the techniques of calculus according to their use.
5. Improve their mathematical writing skills.
6. Develop their communication skills by presenting mathematical material.
7. Effectively discuss and solve mathematical problems in group settings.
8. Infer connections made in arguments without specific direction.
9. Explain why arguments making use of calculus are sound or not, as appropriate.
10. Plan, organize, and combine arguments to solve new problems, as appropriate.
11. Generalize or modify to create new arguments, as appropriate.

Introduction

Welcome to MAT-275, Multivariable Calculus. I am very pleased to be teaching the course once more. It is the first college mathematics course that is designed specifically for people intending to study higher mathematics, and as such will form the foundation of your continued work in the mathematical and physical sciences. It is more modern than your previous courses in calculus. The machinery and terminology of *vectors*, in particular, is quite young, dating back to the work of Gibbs and Hamilton in the late 19th century—less than 150 years ago, as compared to five centuries or more for the foundational results of single-variable calculus.[2]

The name of the course is traditional, and at one time all colleges and universities throughout this country had very similar offerings called "Calculus 3", but matters have changed in recent years. Quick and reliable hand calculation was once a skill of paramount importance, but is less so today. It is more important that you have a good working understanding of the fundamental concepts of calculus. This understanding will lead to asking the correct questions of software, literature searches, and your eventual colleagues.

[2] Newton and Leibniz tend to get all the credit for inventing calculus in the late 1600s, but some key notions go all the way back to Archimedes, who lived about 23 centuries ago.

One fundamental idea and theme of the course is what is called "parametrization".[3] A main goal of the course is for you to have an enduring understanding of this idea that permits you to recognize, employ, and interpret parametrization in your future mathematical enterprises, and understand how it binds the other fundamental concepts of calculus and geometry together.

[3] Briefly, to parametrize a collection of objects is to put them in correspondence with some kind of numbers. When we list items, or rank preferences, we are parametrizing them with the positive integers (the collection that *indexes* our list). Many different kinds of numbers can be used to parametrize collections, such as real or complex numbers, vectors, matrices, tensors, and more.

We generalize the main ideas and results of single-variable calculus to multivariate situations, with particular attention to applications and modeling. There are two ways to proceed. First, we allow the domains (inputs) of the functions at hand to be 2-, 3-, or higher-dimensional, while maintaining the restriction that the function's values are real numbers in the usual sense. Here we will meet the essential concepts of *partial derivative* and *total* or *Jacobian derivative* and revisit the familiar themes of differential calculus (rates of change, chain rule, optimization, etc.). We will introduce the powerful modern algebraic framework of *vectors* in which the rest of the theory is cast. At this level of abstraction there are many opportunities to discuss mathematical modeling, of which we avail ourselves as appropriate.

Just as functions of several variables may be differentiated via their partial derivatives, so too can they be integrated over appropriate *regions* (other than intervals) in their domains. We study the important change-of-variable theorem that allows integrals over complicated regions to be reckoned in terms of simpler ones (e.g., rectangles) and examine some applications.

The dual part of our investigation is to let the codomains of our functions be higher-dimensional Euclidean spaces while keeping the domains 1-dimensional. This approach entails an investigation into fundamental ideas like *parametrization*.

We conclude with a study of the important field of *differential geometry*. The laws of electromagnetism, Maxwell's equations, are formulated in this language, as are the laws of aerodynamics. We will make as much progress as we can toward the fundamental theorems of G. Green, C. F. Gauss, and G. Stokes[4]. All three are vast generalizations of the usual Fundamental Theorem of Calculus, and all have extremely important implications for physics, engineering, and the rest of mathematics.

[4] Famously, Stokes's theorem is due not to Stokes, but to Lord Kelvin; Stokes merely *assigned* it, in 1854.

Catalog description

"This course is an extension of calculus to higher-dimensional spaces. Main topics include differentiation of functions of two and three variables, an introduction to vector analysis and parameterization, and a study of definite integration in both rectangular and curved coordinate systems. Topics may include a review of functions of several variables, vector geometry of 3-dimensional space, partial derivatives, gradient vectors, optimization techniques, multiple integration in the three classical curvilinear coordinate systems, parametric equations, vector fields, line integrals and Green's Theorem, and the other classical integral theorems of differential geometry."

Text

The text is *Calculus: Early Transcendentals* by Jon Rogawski, second edition. [5]

[5] It is OK if you have the first edition, although your section numbers may be different.

If you choose to use a different edition than the recommended one, it is your responsibility to make sure the problem numbers are correct (important for presentations/portfolio problems, see below).

Grading

Scores are computed as a weighted average, with the following weights: WeBWorK 0.08, presentations 0.08, portfolios 0.10, workshops 0.10, quizzes 0.32, two in-class exams 0.08 and 0.08, and final exam 0.16. Observe that the weights sum to 1 = 100%.[6] The exact determination of letter grades from these scores depends on the final distribution of scores in the class, but you can expect a C for earning 75% of the points, a C+ for 78%, a B– for 80%, and so on.

[6] Observe that the exams are relatively lightly weighted, at 32% in total, the same as the quizzes! It is my hope that this reduces the incentives and payoffs to cram and realigns them in the appropriate direction, supporting steady incremental effort.

Workshops

Experience has taught me that this course functions much better as an interactive course than as a pure lecture. I will use what I call *workshops* to introduce many new ideas and guide you through examples.[7]

[7] Workshops address learning outcomes 1, 2, 3, and 7, and sometimes others.

Instead of doing these at home, we will use class time to work on them. Occasionally you may need to finish them outside of class, if we don't have enough time to finish. They have proved to be very valuable in helping students get main ideas before tackling WeBWorK or portfolio problems. The workshops and the homework are the heart of the course.[8]

[8] When workshops are graded, they are usually graded for completion. You will have plenty of time in class to make sure you understand properly.

Homework

Homework in this class comprises both online and traditional written assignments.

1. WeBWorK[9] assigned almost-daily, available at `https://webwork.collegeofidaho.edu/webwork2/MAT275_F14/`
2. *Portfolio problems*, assigned in weekly batches and collected regularly throughout the term.[10] These problems mostly come from the textbook and are intended to stretch your thinking beyond the level of the online homework. It is these problems that you have opportunities to present at the board. Note that you may record classmates' solutions to these problems as they present. Your portfolio problems should be stored and submitted in a three-ring binder whose contents are well organized. For more details on the style and format of the portfolio, see the section *How to do homework*.

[9] WeBWorK problems address learning outcomes 1, 2, 3, and 4.

[10] Portfolio problems address learning outcomes 5, 8, 9, and 11.

Quizzes

To make sure each student is keeping up with the online problems, we will have frequent quizzes.[11] Quiz problems are simple variants of WeBWorK problems and all quizzes are open-note (but not open-book).[12] Therefore, if you have solved the WeBWorK problems prior to the quiz day, a score of 100% is very possible. Note that quizzes cannot be made up, so it is very important that you come to class each day. Quizzes begin promptly at 8:00.

[11] Quizzes address learning outcomes 1, 3, 4, and 9.

[12] I would advise you to write up your WeBWorK solutions for reference during quizzes. The more thorough your write-up, the more it will help you.

Presentations

Each student should attempt to earn 60 *presentation points* during the semester. Presentations of portfolio problems are worth up to 10 points each. A presentation consists of two or three active presenters who provide an explanation, a solution, and any justification of the solution requested by the class.[13] I will maintain a list of problems and their presentation metadata online for your reference.

[13] Presentations address learning outcomes 6, 7, 9, 10, and 11.

Exams

Two exams are given in class.[14] Exams comprise both portfolio problems and new problems intended to stretch your thinking. During the exam, you may refer freely to your portfolio. A missed exam results in an exam grade of zero. Arrangements for absences, again, must be made well in advance (two weeks suffices). *If arrangements are not made in advance, I will consider make-ups only with compelling, documented reasons.* The final exam takes place at the indicated date and time. It cannot be rescheduled *for any reason*. Make your travel plans accordingly.

[14] Dates for Exams 1 and 2 are tentative.
Exam 1 Tuesday, September 30
Exam 2 Tuesday, November 11
Final Exam: Wednesday, December 10, 8:30–11:30 am

How to do homework

Whenever you are writing a solution to a math problem, it is important to strive for the clearest exposition you can manage. Good mathematical writing is essential for anyone who wishes to think clearly about mathematics. The process of making your ideas and reasoning *clear, complete, and unambiguously correct* is the most effective amplifier of mathematical power there is. Hence your solutions should be composed in brilliant English prose. This means employing accepted scientific usage, more or less correct grammar and spelling, and above all *complete sentences*—sprinkled here and there with tangy, delicious equations. Solutions in the popular "pile-of-equations" style with little or no explanatory text will not get much credit. You must explain what is happening as the action unfolds.

The reason for all this is that the process of such writing and editing will implant understanding more firmly in your mind. Similar problems appear on exams. If you dash off a quick and dirty solution it is less likely you will recall what you did at the appropriate time.

As mentioned above, portfolio problems should be stored and submitted in a clean and well organized three-ring binder. About four times during the semester, I will collect and assess your work. While I provide extensive comments on your work, only two grades are possible on a portfolio problem: 0 points and 10 points.[15] Because of this, portfolio problems may be submitted more than once and scores are not final until the last day of class. *No portfolios will be accepted after the last day of class without a compelling, documented reason.* Your write-ups should be clear, complete, and free of typographical errors.[16] I am happy to talk about these problems in office hours with you.

[15] Do not be dismayed if your early attempts are graded at 0 points. Most people do not know how to write mathematically and it is not the kind of writing one learns to do in other classes. That is why I have this "revise-and-resubmit" approach: it allows you to focus on writing once you have solved the problem, or to reëvaluate your solution and try again if it is incomplete.

[16] If you are interested in typing your solutions, I encourage you to do so using the WriteLaTeX service. See Moodle for more information and a sign-up link, or ask me about it any time.

A note on studying math

By now you have studied enough mathematics to have learned something about how it is that the material passes through your shapely skull and into your soft, wrinkly brain. Nevertheless, you may find that this course is rather more difficult than your previous calculus courses. To really understand it, we will have to dig into subtle distinctions and nuances that no one has asked you to think about before. In the one-dimensional world, these nuances simply do not signify, so they are novel by necessity at this time. It is also more difficult to picture what is going on mentally, again owing to the presence of extra dimensions. Part of what I'm here to tell you is that while the material may seem wholly new and unfamiliar, the underlying principles of calculus (what do derivatives *do*? what are integrals *for*?) are immutable.

Academic integrity

I encourage all students to form study groups and collaborate on homework; each student is of course individually responsible for their own work. Collaborators must be acknowledged.

Students are expected to complete all graded work in accordance with the College Honor Code. Plagiarism, cheating, or borrowing without proper credit will not be tolerated. Violations of academic honesty can result in loss of credit on an assignment, failure on an exam, or failure in the course. Referrals may be made to the Vice President for Academic Affairs for any party involved in academic dishonesty.

Special accommodations

Students who have documented disabilities as addressed by the Americans With Disabilities Act and who need any test or course materials to be furnished in an alternative format should notify me immediately (during the first week of class). Reasonable efforts will be made to accommodate the needs of such students.

GOOD LUCK THIS SEMESTER!