Squares of distances and 2-variable functions

Math 251 Calculus 3

September 9, 2013

Distances in the plane

- Find the distance between $(x_1, y_1), (x_2, y_2)$ in the plane:

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Works because $(x_1, y_1), (x_2, y_2)$ are the endpoints of the hypotenuse of a right triangle.

Cleaning up the square root

- Often better to work with squares of distances

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- Because two positive numbers are equal if and only if their squares are equal.

Coordinate planes and axes

- Planes $x = 0$, $y = 0$, $z = 0$ are called the *coordinate* planes: the (y, z)-plane, (x, z)-plane, and (x, y)-plane, respectively

Intersection of the (x, z)-plane with the (y, z)-plane is a line whose points evidently all satisfy $y = x = 0$. This line is called the z-axis.

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- Intersect any pair of coordinate planes, we get a line.

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Workshop 00: Distances to axes in the plane

Measuring distance from a point to an *arbitrary* line sucks, but if the line is a coordinate axis, it's easy.

- What's the distance from $(-4, 3)$ to the x-axis?

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Measuring distance from a point to an *arbitrary* line sucks, but if the line is a coordinate axis, it's easy.

- What's the distance from $(-4, 3)$ to the x-axis?
- The y-axis?

Distances in space

If coordinates of a point in $\mathbf{R}^2$ measure distances to axes, what do coordinates of $(2, 1, 3)$ measure?

The distance from the complementary plane.

Distances from axes

It's fine that coordinates tell us distances from the coordinate planes, but what about from the axes?

> *Axes are a more familiar way of picturing points' "addresses"*

Imagine looking straight down at the (x, y)-plane, so that the positive z-axis goes right between your eyes.

Right between the eyes

This looks just like the ordinary plane! Here is the most important metamathematical technique there is. You have used it hundreds of times already.

> *Replace your problem by an easier problem that has the same solution.*

That's what we're doing when we visualize the ordinary plane as a cross-section of space this way.

The distance to the z-axis

Now what's the distance to the z-axis? Think in terms of the cross-sectional picture.

Notice how the formula has no z in it.

2-variable functions

A 2-variable function is a rule f that associates a number, called $f(x, y)$, to each point (x, y) in the ordinary plane.

```
cost = trip_cost + s*(shirt_price) + t*(trou_price)
bill = 10 + 4/100*(pages)*(copies) + 75/100*(copies)
```

$$f(x, y) = x + yx^4$$

Contour plots

One way to visualize 2-variable functions is with contour plots.

- Each (x, y) gets a value $f(x, y)$

In regions where contours are far apart, the values change slowly. If contours are closely spaces, values are changing rapidly.

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- Connect points whose values are the same

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One way to visualize 2-variable functions is with contour plots.

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- Connect points whose values are the same
- The "contours" are the connecting lines

In regions where contours are far apart, the values change slowly. If contours are closely spaces, values are changing rapidly.