Math 251 **Workshop 09: Integration over planar regions**

October 28, 2013

Due: October 30, 2013 Name: __________________________________

1 Workshop 09: Integration over planar regions

You saw in the presentation how to decompose double integrals over a triangle into iterated integrals. The same technique works for regions whose boundaries have nice algebraic expressions. We will only concern ourselves with regions whose boundaries are made up of line segments, arcs of circles, and pieces of the graphs of functions. Our regions will also all be "connected"—that is, they are all one piece.

1.1 More triangles

The simplest regions meeting our descriptions other than rectangles are triangles: specifically triangles with one side parallel to an axis. Give your answers by writing an integral of an anonymous function $f(x,y)$—that is, fill in the limits and the differentials, but don't pick a function to integrate.

1. For each triangle, find the limits of an iterated integral in each of the possible orders of integration. Draw *big, beautiful pictures.*

   (a) The triangle with vertices $(-1,2)$, $(2,2)$, and $(-1,-2)$.

   (b) The triangle with vertices $(-1,2)$, $(-1,-1)$, and $(1,1)$.

(c) The triangle with verticles $(-1,-1)$, $(3,-1)$, and $(0,2)$.

1.2 Regions with curved boundaries

2. Let R be the region whose boundary is the lines $y = 0$ and $x = 2$ and the parabola $x = y^2$. Find limits for iterated integrals over R in both orders.

3. Let U and L be the parts of the unit disk that fall in the upper and lower half-planes, respectively. Find limits for iterated integrals over U and L, in both orders.

4. Let $\mathbf{D}$ be the unit disk. Find limits for iterated integrals over $\mathbf{D}$ in both orders.

1.3 When order matters

5. Let C be the circular cap consisting of those points of $\mathbf{D}$ whose x-coordinates are at least $-1/2$. Draw C, and try to find limits for an iterated integral over it in one or the other order. Are they equivalent?

6. Let S be the parabolic sector whose boundary is the parabola $y = x^2$ and the line $y = x + 2$.

   (a) Find limits for an iterated integral over S in the order $dy\,dx$.

   (b) Find limits for an iterated integral over S in the order $dx\,dy$.