MATHEMATICS 251, FALL 2011
REVIEW

Here is a list of equations and formulas you may want to include on your exam note sheet. It is not intended to be exhaustive.

First, you should know how to draw pictures of the following plane and solid figures, given equations describing their points.

- Lines
- Planes
- Parabolas (opening in any of the four plane coordinate directions)
- Circles
- Spheres and hemispheres
- Elliptic paraboloids opening in a coordinate direction with circular cross-section
- Parallelograms and triangles; parallelepipeds (skew boxes in $\mathbf{R}^3$)

Standard parametrizations of lines, line segments, circles, and graphs of functions $y = f(x)$ or $x = g(y)$ should also be included in your notes if not engraved on your brain.

(1) Chapter 11
  - Arclength and speed formulas on parametrized curves
  - Polar coordinate transformation formulas

(2) Chapter 12
  - Length of vectors (in coordinates and in terms of dot product)
  - Normalization formula converting a nonzero vector to a unit vector pointing in the same direction
  - Triangle inequality
  - Distance formula
  - Parametric equations of a line
  - Dot product–cosine formula
  - Dot product of orthogonal vectors
  - Vector projection formula (component of $\vec{u}$ along $\vec{v}$)
  - Cross product formula
  - Geometric properties of cross product
  - Area and volume of skew boxes via cross product
  - Equation of plane in point–normal vector form and linear form
  - Cylindrical and spherical coordinate transformation formulas (I would also include pictures indicating the meaning of the coordinates)

(3) Chapter 13
  - Product rule for dot and cross products
  - Chain/product rule for products of vector and scalar functions
  - Parametric equation for the tangent line to a parametrized curve
  - Arc length of parametrized curve
  - Meaning of arc length parametrization (I won't ask you to *find* them, but you need to know how to use them)
  - Unit tangent vector to a parametrized curve
  - Three formulas for curvature (in addition to the definition)
  - Unit normal vector, binormal vector

- Relation between position, velocity, and acceleration vectors
- Solve vector differential equations of motion (Ch. 13.5) for gravitational and uniform circular acceleration
- Tangential and normal components of acceleration

(4) Chapter 14
- Traces and contour lines of a function of two variables
- Level surfaces of functions of three variables
- Continuity; composition of continuous functions is continuous
- Limits; how to rule out their existence using special paths; how to find them with the Squeeze Theorem
- Partial derivatives; gradient
- Chain rule (3 forms: for gradients; for paths; general)
- Clairaut's Theorem on the equality of mixed partials
- Differentiability: definition in terms of local linearity
- Linearization: what is its graph? What is its formula? What is its significance?
- Criterion for differentiability
- Directional derivatives and computation via gradient–dot product
- Interpretation of the gradient vector
- Local extreme values; critical points; how to find absolute extrema on closed and bounded regions
- Lagrange multipliers

(5) Chapter 15
- Double integrals over general regions; vertical and horizontal simplicity & Fubini's theorem
- Mean value theorem for double integrals
- Decomposition of region of integration into subregions
- All that stuff also for triple integrals
- Integration in general coordinates: the change of variable theorem and its special cases in polar, cylindrical, and spherical coordinates
- Jacobian determinant: how to calculate it

(6) Chapter 16
- Vector fields; decomposition into several real-valued component functions
- Cross partial condition for recognizing gradients
- Potential functions; uniqueness
- Line integrals of scalar (real-valued) and vector functions using parametrizations
- Line integrals, force, and work
- Conservative vector fields; do they have potentials? Relation with gradients
- Cross-partials and conservatism