**Mathematics 251**
**Exam 1**

September 24, 2013 Name: ________________________________

**Instructions**: This exam is closed book: you may refer to one double-sided page of handwritten notes, but no electronic aids or other printed references are permitted. Justification of all answers is required for partial credit unless otherwise noted; please $\boxed{\textbf{box}}$ your final answers. Unless specifically directed, leave all answers in **exact form**, e.g. $\sqrt{3}$ instead of 1.732 and $\pi/2$ instead of 1.57.

Show all pertinent work. *Correct answers without accompanying work will receive little or no credit.* Results from class or from homework or from class can be cited freely. It is in your interest to display your solution in a clear, readable fashion.

Do not write on the exam paper. Use your own paper, or the provided paper. *Work on the exam paper itself will be disregarded.*

Please submit problems in order. Leave a note if you get them out of order.

If you have a question, please raise your hand.

Be sure to read all questions carefully and completely.

| Question | Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 24 | |
| 3 | 24 | |
| 4 | 8 | |
| 5 | 12 | |
| 6 | 8 | |
| 7 | 12 | |
| Total: | 100 | |

*Good luck!*

1. Let $\vec{v} = \langle 0, -3, 1\rangle$ and $\vec{w} = \langle 2, -3, 4\rangle$.

   (a) (6 points) Find a vector that is orthogonal to the plane containing these vectors.

   (b) (6 points) Find an equation for the plane containing these vectors. (Either a vector equation or a scalar equation is acceptable.)

2. (a) (12 points) Explain geometrically why the equation

$$\langle 1,1,1\rangle \times \langle x,y,z\rangle = \langle 1,0,0\rangle$$

   has no solution, i.e., why the equation is false for every choice of $\langle x,y,z\rangle$.

   (b) (12 points) Find a vector $\langle x,y,z\rangle$ that satisfies the equation

$$\langle 1,1,1\rangle \times \langle x,y,z\rangle = \langle 1,-1,0\rangle.$$

   *Note.* There are infinitely many such vectors $\langle x,y,z\rangle$.

3. Let $P = (1,1,0)$, $Q = (1,-2,1)$, and $R = (3,-2,4)$.

   (a) (8 points) Find the cosine of the angle between the line segments $\overline{PQ}$ and $\overline{PR}$.

   (b) (8 points) Explain why your answer to the previous part means that the points $P$, $Q$, and $R$ are the vertices of a triangle (in other words, why the points are not collinear).

   (c) (8 points) Recall that for *any* two vectors $\vec{u}$, $\vec{v}$, the angle formed by $\vec{u}$ and $\vec{v}$ is acute (resp. obtuse) if $\vec{u}\cdot\vec{v}$ is positive (resp. negative). Are any of the angles of the triangle obtuse? Justify your answer.

4. (8 points) (Note: In this problem, no justification or explanation is required.) Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be nonzero vectors in $\mathbf{R}^3$. Identify the correct completion(s) of the sentence: The vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ are coplanar (they lie in one plane) if (select one of (a) through (g)):

   I. One of the three vectors is parallel to the cross product of the others.
   II. There exist scalars $a$ and $b$ with $\vec{w} = a\vec{u} + b\vec{v}$.
   III. $(\vec{u}\times\vec{v})\times\vec{w} = \vec{0}$.
   IV. $\vec{u}\cdot(\vec{v}\times\vec{w}) = 0$.

   a. I only
   b. II only
   c. III only
   d. II and IV only
   e. III and IV only
   f. II, III, and IV only
   g. I, II, III, and IV

5. (12 points) Suppose that $\vec{u}$ and $\vec{v}$ are orthogonal. Use facts about vectors and their dot products to verify the equation

$$\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2.$$

Do you think this equation can be true for a pair of non-orthogonal vectors? Justify your answer.

6. (8 points) Suppose that $\mathcal{P}$ is a plane in $\mathbf{R}^3$ and that $\ell$ is a line contained in $\mathcal{P}$. Let $Q$ be a point not on $\mathcal{P}$. Choose the correct relationship between $D$, the distance from $Q$ to $\ell$, and $d$, the distance from $Q$ to $\mathcal{P}$.

   A. $d \le D$.

   B. $d \ge D$.

   C. $d = D$.

   D. None of the above.

7. Figure 7 shows a contour plot of a function $f(x, y)$.

   (a) (6 points) Starting at $(2, 2)$ and moving in the negative $x$-direction, are the values of $f(x, y)$ decreasing or increasing?

   (b) (6 points) Starting at point $(2, 0)$ and moving in the positive $y$-direction, are the values of $f(x, y)$ decreasing or increasing?

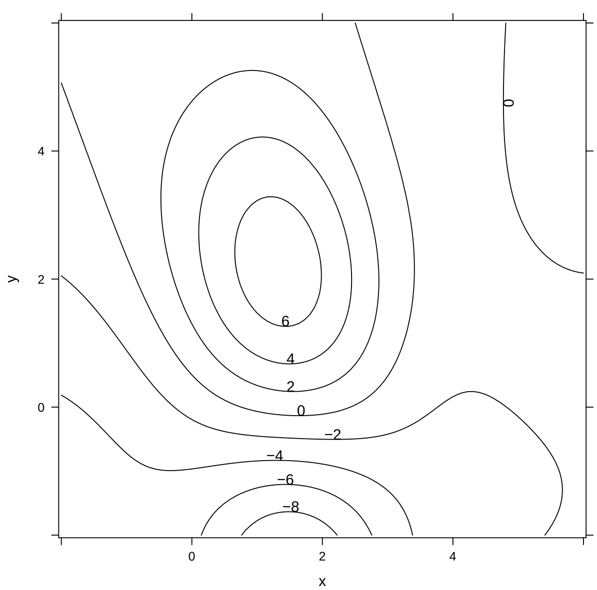


Figure 1: Contour plot for Problem 7