Mathematics 251
Exam 3

November 22, 2011 Name: ______________________________

Instructions: This exam is closed book: you may refer to one 8.5×11 page of handwritten notes, but no electronic aids or other printed references are permitted. *Justification of all answers is required for partial credit.* Unless specifically directed, leave all answers in **exact form**, e.g. $\sqrt{3}$ instead of 1.732 and $\pi/2$ instead of 1.57.

Show all pertinent work. *Correct answers without accompanying work will receive little or no credit.* Results from homework or from class can be cited freely. It is in your interest to display your solution in a clear, readable fashion. Note that standards of justification are not as high as for homework.

Budget your time wisely. It is a good idea, whenever possible, to look through the entire exam before beginning work on a particular problem.

If your work continues onto the back of another page, please indicate this. Check and make sure you have all of the pages in the exam; there should be 5, including this one. If you have a question, please raise your hand.

Be sure to read all questions carefully and completely.

Question:	1	2	3	4	5	6	Total
Points:	24	24	12	12	24	0	96
Bonus Points:	0	0	0	0	0	4	4
Score:							

Good luck!

1. Consider a differentiable function $z = f(x, y)$ under the polar coordinate transform $x = r\cos\theta$, $y = r\sin\theta$.

   (a) (12 points) Use the chain rule to express the partial derivatives $\partial z/\partial r$ and $\partial z/\partial\theta$ in terms of the partial derivatives $\partial z/\partial x$ and $\partial z/\partial y$.

   (b) (12 points) Referring to your answer above, show that

$$||\nabla f||^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$$

   Hint. Expand the left-hand side using the definition of the gradient as a vector and properties of dot products.

2. Recall that the linearization of the function $f(x, y)$ at a point (a, b) is the function $L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$. Here f_x and f_y are the partial derivatives of $f(x, y)$.

   (a) (8 points) State a condition on f_x and f_y that will ensure that the plane $z = L(x, y)$ is tangent to the graph of f. No justification is necessary.

   (b) (8 points) Find the gradient of the function $F(x, y, z) = x^2 + y^2 + z^2$, and evaluate this gradient at the point $(\sqrt{3}, \sqrt{3}, \sqrt{3})$.

   (c) (8 points) Is there a plane that is tangent to the sphere $x^2 + y^2 + z^2 = 9$ at the point $(\sqrt{3}, \sqrt{3}, \sqrt{3})$? If not, explain why. If so, find an equation for such a tangent plane. You may use the previous parts of this problem if you wish, but it is not necessary to do so.

3. (12 points) Find the critical points of the function $f(x, y) = \sin x \cos y$ in the region $0 \leq x \leq \pi$, $0 \leq y \leq \pi$ and identify the global maximum and minimum of f in the region. Justify your answers for full credit. Elementary properties of the sine and cosine functions may be cited without justification.

4. (12 points) Use Lagrange multipliers to find the maximum value of $g(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 4$. For full credit, give the coordinates of all the points on the circle where this maximum value is attained.

5. Evaluate the double integrals.

   (a) (12 points) $\iint_D 1 \, dA$, where D is the region bounded by the curve $x = y^2 - 4y/5$ and the line $y = 5x$.

   (b) (12 points) $\iint_D x^2 + 2y \, dA$, where D is the region in the first quadrant bounded by the line $y = x$ and the curve $y = x^3$. You may leave your answer as a sum of rational numbers (e.g. $7/12 - 4/59 + 12/101$ would be OK here).

6. (4 points (bonus)) I need new music to listen to. Do you have a recommendation? I like electronic music (especially IDM, jungle/drum and bass, and breakbeats), hip-hop, dub and roots reggae, straight-ahead rock, and related genres. For example: Squarepusher, Plastikman, Autechre, Blackalicious, Jurassic 5, Scientist, Dennis Brown, The White Stripes. Thanks for your suggestions.