Mathematics 251
Exam 1

November 12, 2013 Name: ______________________________

Instructions: This exam is closed book: you may refer to one double-sided page of handwritten notes, but no electronic aids or other printed references are permitted. Justification of all answers is required for partial credit unless otherwise noted; please $\boxed{\textbf{box}}$ your final answers. Unless specifically directed, leave all answers in **exact form**, e.g. $\sqrt{3}$ instead of 1.732 and $\pi/2$ instead of 1.57.

Show all pertinent work. *Correct answers without accompanying work will receive little or no credit.* Results from class or from homework or from class can be cited freely. It is in your interest to display your solution in a clear, readable fashion.

Do not write on the exam paper. Use your own paper, or the provided paper. *Work on the exam paper itself will be disregarded,* except for multiple-choice questions.

Please submit problems in order. Leave a note if you get them out of order.

If you have a question, please raise your hand.

Be sure to read all questions carefully and completely.

Question	Points	Score
1	16	
2	12	
3	8	
4	16	
5	8	
6	10	
7	12	
8	18	
Total:	100	

Good luck!

1. (16 points) Please find the indicated partial derivatives.
   (a) $f(x,y) = x^4y^3$; find f_x, f_y.
   (b) $z = \dfrac{x^2}{1+y^2}$; find $\partial z/\partial x$.
   (c) $u = 3x^2y - 6xy^4$; find u_{xx} and u_{yy}.
   (d) $x = r\cos\theta$, where $r = t^2$, $\theta = t^3$; find dx/dt.

2. (12 points) Give a good definition for what it means for the point (a,b) to be a local maximum of the function $f(x,y)$. It doesn't have to be exactly the same as the one in the text, but it should have the same extent. This means that everything that counts as a local maximum according to the text's definition should also count according to yours and vice versa.

3. (8 points) Consider the double integral

$$\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\, dx\, dy.$$

   Sketch the domain of integration. Then express the integral above in the opposite order of integration. **DO NOT ATTEMPT TO EVALUATE THE INTEGRAL.**

4. In this problem, consider the function $g(x,y) = 7 - 2xy^2$ and the point $P = (1,-1)$.
   (a) (12 points) Give a formula for the linearization $L(x,y)$ of g at P. Describe the geometric relationship between the graphs of the two functions $L(x,y)$ and $g(x,y)$ (in complete sentences).
   (b) (4 points) Use the linearization to estimate $g(0.9,-1.1)$.

5. Recall that a subset of $\mathbf{R}^2$ is said to be *closed* if it contains all of its edge points, and *bounded* if it is contained in all sufficiently large disks centered at $(0,0)$.
   (a) (4 points) Give an example of a subset X of $\mathbf{R}^2$ that is closed, but not bounded.
   (b) (4 points) Give an example of a subset X of $\mathbf{R}^2$ that is bounded, but not closed.

6. (10 points) Select the condition(s) that guarantee the existence of a plane tangent to the graph of $f(x,y)$ at the point (a,b).

   ○ $f(x,y)$ is differentiable.

   ○ $f(x,y)$ is locally linear.

   ○ The partial derivatives f_x and f_y exist at (a,b).

   ○ The partial derivatives f_x and f_y are continuous at (a,b).

   ○ The mixed partial derivatives f_{xy} and f_{yx} are equal throughout some neighborhood of (a,b).

7. (12 points) Find the maximum and minimum of $f(x,y) = xy$ subject to the constraint $x^2 + 4y^2 = 16$. Explain why both must exist.

8. Let D be the region between the circles of radius $1/2$ and 1 centered at $(0,0)$, and let $f(x,y)$ be defined on D by the formula
$$f(x,y) = \frac{x+y}{x^2+y^2}$$
   (a) (4 points) Express the region D in polar coordinates (using inequalities).

   (b) (8 points) Set up an integral (entirely in polar coordinates) of the function f over the region D. Make sure to include all limits on your integral and specify the order of integration.

   (c) (6 points) Evaluate the integral you wrote down above.