MATHEMATICS 251 FALL 2013

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The pursuit of knowledge, brother, is the askin' of many questions[1].

Greetings: Welcome to MAT 251, Calculus 3. It is a great personal honor and responsibility for me to teach this course. It is the first college mathematics course that is not designed to be approachable by students of all majors, and as such will form the foundation of your continued work in the mathematical and physical sciences. It is more modern than your previous courses in calculus. The machinery and terminology of *vectors*, in particular, is quite young, dating back to the work of Gibbs and Hamilton in the late 19th century—less than 150 years ago, as compared to five centuries or more for the foundational results of single-variable calculus[2].

The name of the course is traditional, and at one time all colleges and universities throughout this country had very similar offerings called "Calculus 3", but matters have changed in recent years. Quick and reliable hand calculation was once a skill of paramount importance, but is less so today. It is more important that you have a good working understanding of the fundamental concepts of calculus. This understanding will lead to asking the correct questions of software, literature searches, and your eventual colleagues.

The fundamental idea and theme of the course is what is called "parametrization". Too many related concepts are built into the meaning of this word to explain it more here, but the main goal of the course is for you to have an enduring understanding of this idea that permits you to recognize, employ, and interpret parametrization in your future mathematical enterprises. See the section *Course overview* below for a more comprehensive introduction.

Learning outcomes: This course is designed so that, upon successful completion, you will be able to:

- Recognize and describe fundamental ideas from multivariable calculus.
- Illustrate these ideas with examples and translate them into everyday terminology.

[1] Raymond Chandler, *Farewell, My Lovely*, 1940.

[2] Newton and Leibniz tend to get all the credit for inventing calculus in the 17th century, but some key notions go all the way back to Archimedes, who lived about 22 centuries ago.

- Demonstrate the use of calculus on specific numerical problems.
- Classify the techniques of calculus according to their use.
- Infer connections made in arguments without specific direction.
- Explain why arguments making use of calculus are sound or not, as appropriate.
- Plan, organize, and combine arguments to solve new problems, as appropriate.
- Generalize or modify to create new arguments, as appropriate.

Text: The text is *Calculus: Early Transcendentals* by Jon Rogawski, second edition. It is OK if you have the first edition, although your section numbers may be different. If you choose to use a different edition than the recommended one, it is your responsibility to make sure you submit homework assignments correctly.

A second, recommended text is *Start R with Calculus*, by Daniel Kaplan, available from Amazon.com.

Catalog description: "A study of real functions of several real variables. Topics include differentiability and continuity, differential geometry, extrema, Lagrange multipliers, multiple integration, line and surface integrals, and the theorems of Green, Gauss and Stokes."

Homework: Homework in this class comprises both online and traditional written assignments.

- WeBWorK assigned almost-daily, available at `https://webwork.collegeofidaho.edu/webwork2/MAT251_01_F13/`
- *Portfolio problems*, assigned in weekly batches and collected regularly throughout the term. These problems mostly come from the textbook and are intended to stretch your thinking beyond the level of the online homework. It is these problems that you have opportunities to present at the board.

How to do homework: Whenever you are writing a solution to a math problem, it is important to strive for the clearest exposition you can manage. Good mathematical writing is essential for anyone who wishes to think clearly about mathematics. The process of making your ideas and reasoning *clear, complete, and unambiguously correct* is the most effective amplifier of mathematical power there is. Hence your solutions should be composed in brilliant English prose. This means employing accepted scientific usage, more or less correct grammar and spelling, and above all *complete sentences*—sprinkled here and there with tangy, delicious equations. Solutions in the popular "pile-of-equations" style with little or no explanatory text will not get much credit. You must explain what is happening as the action unfolds.

The reason for all this is that the process of such writing and editing will implant understanding more firmly in your mind. Similar problems appear on exams. If you dash off a quick and dirty solution it is less likely you will recall what you did at the appropriate time.

I encourage all students to form study groups and collaborate on homework; each student is of course individually responsible for their own work. Collaborators must be acknowledged. Portfolio problems may be submitted more than once and scores are not final until the last day of class. *No portfolios will be accepted after the last day of class without a compelling, documented reason.*

Grading: Scores are computed as a weighted average, with the following weights: WeBWorK 0.10 = 10%, presentations 0.15 = 15%, portfolios 0.05 = 5%, quizzes 0.40 = 40%, two in-class exams 0.18 = 18%, and final exam 0.12 = 12%. Observe that the weights sum to 1 = 100%. The exact determination of letter grades from these scores depends on the final distribution of scores in the class, but you can expect a C for earning 75% of the points, a C+ for 78%, a B– for 80%, and so on.

Presentations: Each student should present a portfolio problem to the class six times over the semester, in a group of two or three (each presenter must have an active role in the presentation). A presentation consists of an explanation, a solution, and any justification of the solution requested by the class. I will maintain a list of problems and their presentation metadata online for your reference.

Quizzes: To make sure each student is keeping up with the online problems, we will have frequent quizzes. Quiz problems are simple variants of WeBWorK problems and all quizzes are open-note. Therefore, if you have solved the WeBWorK problems prior to the quiz day, a score of 100% is very possible.

Exams: Two exams are given in class (see below for dates). Exams comprise both portfolio problems and new problems intended to stretch your thinking. During the exam, you may refer freely to your portfolio. A missed exam results in an exam grade of zero. Arrangements for absences, again, must be made well in advance (two weeks suffices). *If arrangements are not made in advance, I will consider make-ups only with compelling, documented reasons.*

- Exam 1 (tentative): Tuesday, September 24 (week 3)
- Exam 2 (tentative): Tuesday, November 5 (week 8)
- Final Exam (cannot be rescheduled *for any reason*): Monday, December 9, 1:30–4:30

Course overview: We generalize the main ideas and results of single-variable calculus to multivariate situations, with particular attention to applications and modeling. There are two ways to proceed. First, we allow the domains (inputs) of the functions at hand to be 2-, 3-, or higher-dimensional, while maintaining the restriction that the function's values are real numbers in the usual sense. Here we will meet the essential concepts of *partial derivative* and *total* or *Jacobian derivative* and revisit the familiar themes of differential calculus (related rates, optimization, etc.). We will introduce the powerful modern algebraic framework of *vectors* in which the rest of the theory is cast. At this level of abstraction there are many opportunities to discuss mathematical modeling, of which we avail ourselves as appropriate.

Just as functions of several variables may be differentiated via their partial derivatives, so too can they be integrated over appropriate *regions* (other than intervals) in their domains. We study the important change-of-variable theorem that allows integrals over complicated regions to be reckoned in terms of simpler ones (e.g., rectangles) and examine some applications.

The dual part of our investigation is to let the codomains of our functions be higher-dimensional Euclidean spaces while keeping the domains 1-dimensional. This approach entails an investigation into fundamental ideas like *parametrization*.

We conclude with a study of the important field of *differential geometry*. The laws of electromagnetism, Maxwell's equations, are formulated in this language, as are the laws of aerodynamics. We will make as much progress as we can toward the fundamental theorems of G. Green, C. F. Gauss, and G. Stokes[3]. All three are vast generalizations of the usual Fundamental Theorem of Calculus, and all have extremely important implications for physics, engineering, and the rest of mathematics.

Academic integrity: Students are expected to complete all graded work in accordance with the College Honor Code. Plagiarism, cheating, or borrowing without proper credit will not be tolerated. Violations of academic honesty can result in loss of credit on an assignment, failure on an exam, or failure in the course. Referrals may be made to the Vice President for Academic Affairs for any party involved in academic dishonesty.

A note on studying math: By now you have studied enough mathematics to have learned something about how it is that the material passes through your shapely skull and into your soft, wrinkly brain. Nevertheless, you may find that this course is rather more difficult than your previous calculus courses. To really understand it, we will have to dig into subtle distinctions and nuances that no one has asked you to think about before. In the one-dimensional world, these nuances simply do not signify, so they are novel by necessity at this time. It is also more difficult to picture what is going on mentally, again owing to the presence of extra dimensions. Part of what I'm here to tell you is that while the material may seem wholly new and unfamiliar, the underlying principles of calculus (what do derivatives *do*? what are integrals *for*?) are immutable.

Special accommodations: Students who have documented disabilities as addressed by the Americans With Disabilities Act and who need any test or course materials to be furnished in an alternative format should notify me immediately (during the first week of class). Reasonable efforts will be made to accommodate the needs of such students.

Good luck this semester!

[3]Famously, Stokes's theorem is due not to Stokes, but to Lord Kelvin; Stokes merely *assigned* it, in 1854.