Math 251 **Quiz 09 (Group quiz: Green's theorem)**

December 2, 2013; 35 minutes Name: ____________________

This quiz is a little different from our previous ones; work together in groups to solve the problems. You'll get more practice in line integrals, solidify ideas about parametrization, and apply Green's theorem. Appoint a scribe for your group and do as much as you can **ON YOUR OWN PAPER.**

Recall that Green's theorem applies to vector fields integrated along simple closed plane curves. A curve is simple if it doesn't intersect itself (except possibly at the endpoints) and closed if it is a continuous loop in the plane. The assertion of Green's theorem in this case is

$$\oint_{\partial\mathcal{D}} P\,dx + Q\,dy = \iint_{\mathcal{D}} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA. \tag{1}$$

Here $\mathcal{D}$ is the region "enclosed" by the curve $\partial\mathcal{D}$ and we assume that $\partial\mathcal{D}$ is traversed so that $\mathcal{D}$ is always to the left. Also, $P\,dx + Q\,dy$ is synonymous with $\langle P, Q\rangle \cdot d\vec{s}$.

1. Verify Green's theorem for the line integral

$$\oint_{\mathcal{C}} xy\,dx + y\,dy,$$

   where $\mathcal{C}$ is the unit circle, oriented counterclockwise. This means: compute each side of Equation 1 and check that the two values are equal.

2. Use Green's theorem to evaluate the line integrals.

   (a) $\oint_{\mathcal{C}} y^2\,dx + x^2\,dy$, where $\mathcal{C}$ is the boundary of the square $0 \le x \le 1$, $0 \le y \le 1$, oriented counterclockwise.

   (b) $\oint_{\mathcal{C}} x^2 y\,dx$, where $\mathcal{C}$ is the unit circle with standard orientation.

3. Let $I = \oint_{\mathcal{C}} \vec{F} \cdot d\vec{s}$, where $\vec{F} = \langle y + \sin x^2, x^2 + e^{y^2}\rangle$ and $\mathcal{C}$ is the circle of radius 4 centered at the origin.

   (a) Which is easier? Evaluating I directly via a parametrization or using Green's theorem?

   (b) Carry out the evaluation, using the easier method.