Dot and cross product

Math 251 Calculus 3

September 18, 2013

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- Here, we agree to always choose $0 \leq \theta \leq \pi$.
- Suppose $\|\vec{v}\| = 3$ and $\|\vec{w}\| = 2$. What are the maximal and minimal possible values of $\vec{v} \cdot \vec{w}$?

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- Suppose $\vec{v} \cdots \vec{w} = 0$. Now what can you conclude?

Frequent uses of dot product

- Test for *orthogonality*: i.e., whether two vectors are perpendicular
- *Projection*: part 3 of Workshop 02

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    - $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + u \times \vec{w}$

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