Math 251 **Workshop 05: Gradient vectors in 1 and 2 dimensions**

October 2, 2013

Due: Monday, October 7 Name: ____________________

1 Workshop 05: Gradient vectors

This workshop motivates and introduces the fundamental idea of *gradient vectors* to functions.

1.1 Warm-up

You may remember from a previous math experience that lines in the plane are orthogonal precisely when the product of their slopes is -1. Here you will use vectors to get a simple proof of this fact.

1. Consider the line $y = 3x + 1$. Find a vector $\vec{v}$ (of any length) parallel to this line. (Your vector will have just 2 entries.)

2. Now find a vector that is orthogonal to $\vec{v}$, and call it $\vec{w}$. *Hint*. The easiest way to generate $\vec{w}$ is to choose its entries in such a way that guarantees $\vec{v} \cdot \vec{w} = 0$.

3. What is the slope of a line parallel to $\vec{w}$? Is the product of the slopes -1 as claimed?

4. Write a formula for a function that takes a vector $\vec{u}$ as input and whose output is the slope of a line parallel to $\vec{u}$. (Think of $\vec{u}$'s entries as the input variables.)

5. Now let $f(x) = x^3 + 3x + 1$. Find the tangent line to the graph of f at $x = 0$. Write its equation in the form $mx - y = -b$. What do you notice about the vector $\langle m, -1\rangle$ in relation to the tangent line?

6. The general form of a line in the plane is $Ax + By = C$ (not every line has a slope). If $B \neq 0$, this line does admit a slope-intercept form. Find it, and show that $\langle A, B\rangle$ is parallel to $\langle m, -1\rangle$.

1.2 General functions

In the previous section you saw that $\langle f'(a), -1\rangle$ is normal to the tangent line to the graph of f at $x = a$. This is true in general.

7. Use the point-slope formula to write the tangent line to the graph of an arbitrary differentiable function f at $x = a$. Get the line into the standard form $Ax + By = C$.

8. Check that $\langle f'(a), -1\rangle$ is orthogonal to the tangent line.