# 12.3.48

2013-10-11

Assume that $\vec{u}$ and $\vec{v}$ are unit vectors with $\|\vec{u}+\vec{v}\| = \sqrt{3/2}$. We calculate $\|2\vec{u}-3\vec{v}\|$ using some properties of the dot product.

Observe that $\|2\vec{u}-3\vec{v}\|^2 = (2\vec{u}-3\vec{v})\cdot(2\vec{u}-3\vec{v})$. We expand using the bilinearity of dot product:

$$\begin{aligned}\|2\vec{u}-3\vec{v}\|^2 &= (2\vec{u}-3\vec{v})\cdot(2\vec{u}-3\vec{v})\\ &= 4\vec{u}\cdot\vec{u}-6\vec{v}\cdot\vec{u}-6\vec{u}\cdot\vec{v}+9\vec{v}\cdot\vec{v}\\ &= 4\|\vec{u}\|^2-12\vec{u}\cdot\vec{v}+9\|\vec{v}\|^2,\end{aligned}$$

where in the last step we use the identity $\vec{w}\cdot\vec{w} = \|\vec{w}\|^2$ and the commutativity of dot product.

Since $\|\vec{u}\| = \|\vec{v}\| = 1$, we have shown that

$$\|2\vec{u}-3\vec{v}\|^2 = 13-12\vec{u}\cdot\vec{v}.$$

Therefore, if we can compute $\vec{u}\cdot\vec{v}$, we will be done.

We are given that $\|\vec{u}+\vec{v}\|^2 = 3/2$. This means that

$$\begin{aligned}3/2 &= (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})\\ &= \|\vec{u}\|^2+2\vec{u}\cdot\vec{v}+\|\vec{v}\|^2\\ &= 2+2\vec{u}\cdot\vec{v},\end{aligned}$$

since $\vec{u}$ and $\vec{v}$ are unit vectors. This shows that $\vec{u}\cdot\vec{v} = -1/4$.

Putting it all together, we have

$$\|2\vec{u}-3\vec{v}\| = \sqrt{13-12\vec{u}\cdot\vec{v}} = \sqrt{13+3} = 4.$$