

September 20, 2013

Due: Monday, September 30

Name: _____

1 Workshop 04: Limits and continuity

The existence of limits is more complicated for functions of several variables. In this workshop, you will investigate some techniques for *ruling out* their existence. As in the one-variable case, establishing the value of a particular limit can be tricky in general and there is no one procedure for doing it.

1.1 Ruling out: by restriction

1. Let $f_1(x, y) = x^2/(x^2 + y^2)$. Find the limit of $f_1(x, y)$ as $(x, y) \rightarrow (0, 0)$ along the x -axis. In other words, find the (ordinary, 1-dimensional) limit of a suitable "slice function" obtained from f_1 .
2. Find the limit of $f_1(x, y)$ as $(x, y) \rightarrow (0, 0)$ as $(x, y) \rightarrow (0, 0)$ along the y -axis, again using a slice function.
3. Does $\lim_{(x, y) \rightarrow (0, 0)} f_1(x, y)$ exist? Why do you think so?

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4. Now let $f_2(x, y) = xy/(x^2 + y^2)$. Show that the limit of $f_2(x, y)$ as $(x, y) \rightarrow (0, 0)$ along either coordinate axis is 0.
5. Find the limit of $f_2(x, y)$ as $(x, y) \rightarrow (0, 0)$ along the line $y = x$.
6. Does $\lim_{(x, y) \rightarrow (0, 0)} f_2(x, y)$ exist? Why do you think so?
7. Finally, let $f_3(x, y) = x^2 y/(x^4 + y^2)$. Show that the limit as $(x, y) \rightarrow (0, 0)$ along every line through the origin is 0.
8. Show that, nevertheless, $\lim_{(x, y) \rightarrow (0, 0)} f_3(x, y)$ does not exist by letting (x, y) approach the origin along a suitable curve.