Math 251 **Workshop 04: Limits and continuity for several variables**

September 20, 2013

Due: Monday, September 30 Name: ______________________________

1 Workshop 04: Limits and continuity

The existence of limits is more complicated for functions of several variables. In this workshop, you will investigate some techniques for *ruling out* their existence. As in the one-variable case, establishing the value of a particular limit can be tricky in general and there is no one procedure for doing it.

1.1 Ruling out: by restriction

1. Let $f_1(x,y) = x^2/(x^2+y^2)$. Find the limit of $f_1(x,y)$ as $(x,y) \to (0,0)$ along the x-axis. In other words, find the (ordinary, 1-dimensional) limit of a suitable "slice function" obtained from f_1.

2. Find the limit of $f_1(x,y)$ as $(x,y) \to (0,0)$ as $(x,y) \to (0,0)$ along the y-axis, again using a slice function.

3. Does $\lim\limits_{(x,y)\to(0,0)} f_1(x,y)$ exist? Why do you think so?

4. Now let $f_2(x, y) = xy/(x^2 + y^2)$. Show that the limit of $f_2(x, y)$ as $(x, y) \to (0,0)$ along either coordinate axis is 0.

5. Find the limit of $f_2(x, y)$ as $(x, y) \to (0,0)$ along the line $y = x$.

6. Does $\lim_{(x,y)\to(0,0)} f_2(x, y)$ exist? Why do you think so?

7. Finally, let $f_3(x, y) = x^2 y/(x^4 + y^2)$. Show that the limit as $(x, y) \to (0,0)$ along every line through the origin is 0.

8. Show that, nevertheless, $\lim_{(x,y)\to(0,0)} f_3(x, y)$ does not exist by letting (x, y) approach the origin along a suitable curve.