Mathematics 251
Exam 1

September 25, 2012 Name: ______________________________

Instructions: This exam is closed book: no electronic aids or printed references are permitted. Justification of all answers is required for partial credit; please $\boxed{\textbf{box}}$ your final answers. Unless specifically directed, leave all answers in **exact form**, e.g. $\sqrt{3}$ instead of 1.732 and $\pi/2$ instead of 1.57.

Show all pertinent work. *Correct answers without accompanying work will receive little or no credit.* Results from class or from homework or from class can be cited freely. It is in your interest to display your solution in a clear, readable fashion.

If you need to include more pages, staple them to the back of your exam and make sure that they are clearly labeled by problem number. Indicate in the main body of the exam that your work continues on another page.

Check and make sure you have all of the pages in the exam; there should be 6, including this one. If you have a question, please raise your hand.

Be sure to read all questions carefully and completely.

Question	Points	Score
1	18	
2	26	
3	24	
4	8	
5	12	
6	12	
Total:	100	

Good luck!

1. The vector-valued function $\mathbf{r}(t) = \langle R\cos\omega t, R\sin\omega t\rangle$ parametrizes the circle of radius R centered at $(0,0) \in \mathbb{R}^2$ (assume that R, $\omega > 0$).

   (a) (8 points) Consider a particle whose position at time t is $\mathbf{r}(t)$ as given above. Verify that the speed of this particle is constant with value $R\omega$.

   (b) (10 points) Use the formula for arc length of a parametrized curve to verify the well-known fact that the circumference of the circle is $2\pi R$. (The particle traverses the circle once as t increases from 0 to $2\pi/\omega$.) *Answers that do not make use of the arc length integral will receive very little credit.*

2. Consider a particle moving in the (x, y)-plane whose position at time t is given for $0 \leq t \leq 2$ by the parametric equations
$$c(t) = (3t - 1, 4t^2).$$

   (a) (8 points) Find dx/dt and dy/dt, the horizontal and vertical velocities of this particle, at time $t = 2$ (your answers should be numbers, not vectors).

   (b) (6 points) Let L be the line that is tangent to the path of the particle at $c(2) = (5, 16)$. Find the slope of L.

   (c) (12 points) Find functions $x(s)$ and $y(s)$ that parametrize the line L, in other words, such that the point $(x(s), y(s))$ is on L for all times s. (The letter s is used only to distinguish times in this parametrization from times in the parametrization $c(t)$. The two parametrizations don't have to be related in any way at all.)

3. Let $P = (1, 1, 0)$, $Q = (0, -1, 1)$, and $R = (1, -2, 2)$.

   (a) (6 points) Find the cosine of the angle between the line segments $\overline{PQ}$ and $\overline{PR}$.

   (b) (6 points) Explain why your answer to the previous part means that the points P, Q, and R are the vertices of a triangle (in other words, why the points are not collinear).

   (c) (6 points) Recall that for *any* two vectors $\mathbf{u}$, $\mathbf{v}$, the angle formed by $\mathbf{u}$ and $\mathbf{v}$ is acute (resp. obtuse) if $\mathbf{u} \cdot \mathbf{v}$ is positive (resp. negative). Are any of the angles of the triangle obtuse? Justify your answer.

   (d) (6 points) Find a vector that is orthogonal to the plane containing the triangle. *Hint.* To contain the triangle, a plane must contain the vectors that connect the triangle's vertices.

4. (8 points) (Note: In this problem, no justification or explanation is required.) Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be nonzero vectors in $\mathbb{R}^3$. Identify the correct completion(s) of the sentence: The vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are coplanar (they lie in one plane) if (select one of (a) through (g)):

   I. One of the three vectors is parallel to the cross product of the others.

   II. There exist scalars a and b with $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$.

   III. $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \mathbf{0}$.

   IV. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 0$.

   a. I only

   b. II only

   c. III only

   d. II and IV only

   e. III and IV only

   f. II, III, and IV only

   g. I, II, III, and IV

5. (12 points) Suppose that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. Use facts about vectors and their dot products to verify the equation

$$||\mathbf{u} + \mathbf{v}||^2 = ||\mathbf{u}||^2 + ||\mathbf{v}||^2.$$

   Do you think this equation can be true for a pair of non-orthogonal vectors? Justify your answer.

6. (12 points) Calculate the magnitude of the force (in $\mathrm{N} = \mathrm{kg} \cdot \mathrm{m/s}^2$) required for Val to push a 10 kg squirrel up a frictionless incline as pictured. Assume Val exerts force parallel to the direction of travel up the ramp, and try not to think about what might happen to her. Assume that the acceleration g due to gravity is $9.8\ \mathrm{m/s}^2$. You do not have to simplify your answer.