Displacement in several directions: vectors

Math 251 Calculus 3

September 10, 2013

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- In everyday English, we separate it into two changes: a change in the east–west direction, and a change in the north–south direction.

Two changes in one

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- Notice: it's not a sensible *address* in any city or town. Why not?
- It is a *displacement*, not a *location*.
- These are different notions!

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- If the town is laid out on a square grid (as opposed to rectangles, some other kind of parallelograms, or worse) ...
- ... walking from $(2,3)$ to $(-3,1)$ feels the same as walking from $(2013,2013)$ to $(2008,2010)$.

Displacement vectors

We call this displacement the *vector* $\langle -5, -2 \rangle$. Observe the following naïve "equations":

$$(2, 3) + \langle -5, -2 \rangle = (-3, 1)$$
$$(2013, 2013) + \langle -5, -2 \rangle = (2008, 2010)$$

Visualize the vector as an arrow: its head is 5 blocks west and 2 blocks south of its tail.

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- Seems a little dodgy... usually, we add like to like.
- Eventually, you'll ignore the distinction...
- ...but for now, vectors and points are different things.

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Displacement vector corresponds to a point:

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Displacement vector corresponds to a point:

- Given a vector, say $\langle x_0, y_0 \rangle$, place its tail at $(0, 0)$.
- Then its head is located at the *point* (x_0, y_0).
- This is called the vector's *standard position*.

Vector arithmetic

It's easy to add and subtract vectors: do it entry by entry.

$$\langle x_0, y_0 \rangle \pm \langle x_1, y_1 \rangle = \langle x_0 + x_1, y_0 + y_1 \rangle.$$

Multiply a vector by a number:

$$c\langle x_0, y_0 \rangle = \langle cx_0, cy_0 \rangle.$$

Reading for Wednesday, September 11

- Reread Module 3
- Read sections 12.1–12.2 in Rogawski
- WeBWorK is due tonight!