

Differential Equations Worksheet

Accompanies Section 1.2 in ODEP

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1 First-order separable differential equations

In this workshop, you will see how to solve (remember the technical meaning of this word) separable first-order differential equations, that is, equations of the form

$$M(x) + N(y) \frac{dy}{dx} = 0.$$

Solve the initial value problem

$$\frac{dy}{dx} = 2xy^2, \quad y(2) = 1. \quad (1.1)$$

Activity 1.1.

- (a) Divide both sides by y^2 to put the equation in standard form.
- (b) Integrate each side with respect to the evident variable. (This is justified by appeal to u -substitution: $dy = y' dx$.)
- (c) You should now have an equation that is free of derivatives, but with a constant of integration. Plug in the initial values to find the value of this constant.
- (d) If possible, solve your equation for y .
- (e) Substitute your function and its derivative into (1.1) to make sure your solution is correct.

Activity 1.2.

- (a) Solve the initial value problem

$$1 + ye^{-x}y' = 0, \quad y(0) = 1. \quad (1.2)$$

Activity 1.3.

- (a) Find solutions to the differential equation

$$y' = \frac{3x^2 + 4x + 2}{2(y-1)}. \quad (1.3)$$

Solve the differential equations.

Activity 1.4.

(a) $y' = \frac{x^2}{y}$

(b) $y' + y^2 \sin x = 0$

(c) $y' = \frac{x^2}{y+yx^3}$

Consider the initial value problem

$$y' = 5y^2, y(0) = y_0. \quad (1.4)$$

Activity 1.5.

- (a) For what values of y_0 does the solution have a vertical asymptote at $t = 6$ and a t -interval of existence $-\infty < t < 6$?