Example: Escape Velocity

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1 Example: escape velocity

The equation of motion for a mass launched from the surface of Earth (a sphere of radius R) is

$$v\frac{dv}{dx} = -\frac{-gR^2}{(R+x)^2}.$$

Let's set up the variables we need for our computation.

```
x, g, R, v_0 = var('x g R v_0')
v = function('v')(x)
```

Now let's use these variables to encode the differential equation we were given.

```
diff_eq = v * v.diff(x) == -g*R^2/(R+x)^2
diff_eq
```

Now let's use the `desolve()` function to solve the equation, making sure to capture the solution in a new Sage variable.

```
sol = desolve(diff_eq, v, ivar=x, ics=[0, v_0])
show(sol)
```

The solution, as usual, is in implicit form. Use `solve()` to solve for $v(x)$ (this is not always possible). Again, we capture the result in a new variable.

```
vels = solve(sol, v)
show(vels)
```

With our velocity function in hand, let's find the maximum height our vehicle reaches in terms of the initial velocity v_0.

```
height_eq = 0 == vels[1].rhs()
height_eq
```

Solve this equation for x to find the maximum height (the height when the velocity is 0).

```
x_max = solve(height_eq, x)[0].rhs()
x_max
```

To find the escape velocity, let $x_{\max} \to \infty$.

```
solve(x == x_max, v_0)
```

```
escape_vel = solve(x == x_max, v_0)[1].rhs().limit(x = oo) #
    oo is +infinity in this context
```

Let's check the value of the escape velocity.

```
show(escape_vel)
```