Separable Equations Worksheet

Accompanies Section 1.2 in ODEP

Dave Rosoff
Department of Mathematics and Physical Sciences
The College of Idaho

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1 First-order separable differential equations

In this workshop, you will see how to solve (remember the technical meaning of this word) separable first-order differential equations, that is, equations of the form

$$M(x) + N(y)\frac{dy}{dx} = 0.$$

Solve the initial value problem

$$\frac{dy}{dx} = 2xy^2, \quad y(2) = 1. \tag{1.1}$$

Activity 1.1.

(a) Divide both sides by y^2 to put the equation in standard form.

(b) Integrate each side with respect to the evident variable. (This is justified by appeal to u-substitution: $dy = y'\,dx$.)

(c) You should now have an equation that is free of derivatives, but with a constant of integration. Plug in the initial values to find the value of this constant.

(d) If possible, solve your equation for y.

(e) Substitute your function and its derivative into (1.1) to make sure your solution is correct.

Activity 1.2.

(a) Solve the initial value problem

$$1 + ye^{-x}y' = 0, \quad y(0) = 1. \tag{1.2}$$

Activity 1.3.

(a) Find solutions to the differential equation

$$y' = \frac{3x^2 + 4x + 2}{2(y-1)}. \tag{1.3}$$

Solve the differential equations.

Activity 1.4.

(a) $y' = \frac{x^2}{y}$

(b) $y' + y^2 \sin x = 0$

(c) $y' = \frac{x^2}{y + yx^3}$

Consider the initial value problem

$$y' = 5y^2, y(0) = y_0. \tag{1.4}$$

Activity 1.5.

(a) For what values of y_0 does the solution have a vertical asymptote at $t = 6$ and a t-interval of existence $-\infty < t < 6$?