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Elasticity of Substitution (σ) Between Capital and Labor: A Comprehensive Analysis

Introduction

The **elasticity of substitution** between capital and labor, commonly denoted σ, is a fundamental parameter in economic theory that measures how easily capital (e.g. machines, structures) can be substituted for labor (workers) in production. In essence, σ quantifies the percentage change in the capital-labor ratio (K/L) in response to a percentage change in the relative price of labor to capital (wages *w* to rental rate *r*). This concept, dating back to Hicks (1932) and formalized by Arrow et al. (1961) via the Constant Elasticity of Substitution (CES) production function, plays a crucial role in determining the distribution of income between capital and labor. A σ greater than 1 indicates that capital and labor are **gross substitutes**, while σ less than 1 means they are **gross complements**. The value of σ has far-reaching implications: it influences how technological change and factor price shifts affect factor incomes, how the labor share of output evolves, and what the impact of automation and AI might be on workers. In this report, we survey the theoretical foundations of σ, summarize empirical estimates across countries and sectors, examine the relationship between σ and the labor share of income, discuss implications for automation, and consider critiques of the CES approach alongside alternative frameworks (including task-based models). Throughout, we reference key studies – including Karabarbounis and Neiman (2014), Rognlie (2015), Oberfield and Raval (2021), and Acemoglu and Restrepo's works – to provide a scholarly and up-to-date assessment.

Theoretical Foundations of σ and the CES Production Function

In neoclassical production theory, the elasticity of substitution σ is defined within the **CES production function** framework. A typical two-factor CES production function can be written as:

$$Y = F(K, L) = \left[\alpha K^{\rho} + (1-\alpha)L^{\rho}\right]^{1/\rho},$$

where $\rho = \frac{\sigma-1}{\sigma}$. In this formulation, σ is constant by construction, and special cases include the Cobb–Douglas function (σ = 1, achieved as ρ → 0) and the Leontief (fixed-proportion) function (σ → 0 as ρ → -∞). **Intuitively, σ measures the curvature of the isoquants** – i.e. how easily a firm can trade off capital for labor. If σ = 1 (Cobb–Douglas), the share of income going to capital and labor remains constant over time (assuming neutral technological progress), as firms always spend a fixed proportion on each input regardless of relative prices. If σ ≠ 1, factor shares will vary: the **distribution parameter** α determines baseline factor shares, but changes in K/L or factor prices will shift those shares.

Why σ matters for income distribution: Under standard assumptions (constant returns to scale and competitive factor markets), the labor share of income is $s_L = \frac{wL}{Y}$ and the capital share $s_K = \frac{rK}{Y}$. When σ differs from 1, these shares respond to changes in the capital-labor ratio or relative factor prices. In general, an increase in the effective capital-labor ratio (through capital accumulation or a fall in the price of capital relative to labor) will **raise the capital share and reduce the labor share if σ > 1**, whereas **if σ < 1, the opposite occurs – capital deepening will actually lower capital's share and raise labor's share** [1] [2]. In the Cobb–Douglas case (σ = 1), such changes have no effect on factor shares (capital and labor shares remain constant). This result can be understood by considering extreme cases: if σ → ∞ (perfect substitutes), firms can swap labor for capital at fixed factor prices, so an increase in capital input (holding wages and rental rates constant) would simply increase output and **drive up capital's share** of income; conversely, if σ → 0 (Leontief, no substitutability), additional capital beyond the fixed proportion has zero marginal product, so extra capital yields no additional output and **labor would capture all income at the margin** [3]. Thus, σ > 1 implies capital is relatively easy to substitute for labor, tending to **tilt income towards the factor that becomes more abundant or cheaper**, whereas σ < 1 implies diminishing returns set in quickly for the abundant factor, tending to stabilize or even tilt income towards the relatively scarce factor.

In economic growth and distribution theory, σ plays a pivotal role. It features prominently in debates such as those spurred by **Piketty (2014)**, who argued that if σ > 1, an increase in the capital-output ratio (for example due to high saving or slow growth) could lead to a rising capital share (and falling labor share) over time – a mechanism requiring capital and labor to be sufficiently substitutable. By contrast, if σ < 1, capital accumulation on its own would *lower* capital's share (because capital's marginal product and remuneration would fall rapidly as capital grows) [1]. Indeed, a simple formula links changes in factor shares to changes in capital intensity: under a CES production function, the percentage change in the labor share is related to the percentage change in K/L by approximately:

$$d\ln s_L \approx -(1-s_L)\frac{\sigma-1}{\sigma}\,d\ln(K/L),$$

holding technology constant [2]. This equation implies that if labor's share is to decline when K/L rises, we must have either σ > 1 or some labor-saving technological change (more on the latter below). In summary, **σ > 1 (gross substitutes)** means that declines in the relative cost of capital or rapid capital deepening will reduce labor's share of income (and boost capital's share), whereas **σ < 1 (gross complements)** means capital deepening alone would raise labor's share (and compress capital's share) [1].

It is important to note that technological change can modify these outcomes. The above analysis assumes no change in technology or factor-augmenting productivity. If technology is **factor-biased** – for instance, **capital-augmenting (labor-saving)** or **labor-augmenting** – it can alter effective K/L and the marginal products of factors. Hicks (1932) classified technical change by its factor bias, and in a CES framework the effect of biased tech on factor shares depends on σ. For example, *capital-augmenting* (or labor-saving) technical progress increases the effective input of capital. In a σ > 1 world, such progress further boosts capital's share (since firms substitute even more toward the now more productive capital); in a σ < 1 world, capital-augmenting progress might not raise capital's share because capital and labor are complements – making capital more productive raises output but also the marginal productivity of labor in tandem, potentially benefiting labor's share unless the bias is very strong. In fact, with σ < 1, sustained labor-saving (capital-augmenting) technical change is needed to generate a falling labor share; pure capital deepening alone would not do it. This point will resurface when we discuss recent labor share trends.

Empirical Estimates of σ Across Countries and Sectors

Empirically pinning down σ has been a longstanding challenge, and studies have produced **mixed results**, with estimates of σ ranging from well below 1 to above 1 depending on the data, sector, time period, and methodology. Below, we survey key findings:

- **Global and Cross-Country Evidence:** Karabarbounis and Neiman (2014) conducted an influential study on the **global decline in labor's share** of income. Using data for many countries, they observed that countries experiencing larger declines in the relative price of investment goods (i.e. cheaper capital equipment) also saw larger declines in labor share. They estimate a **preferred σ of about 1.25** – significantly above unity – to fit this pattern [4]. Intuitively, an elasticity of ~1.25 implies capital and labor are sufficiently substitutable that cheaper capital leads firms to replace labor and reduce labor's share. In their quantitative model, the roughly 25% drop in the investment good price since the 1970s can explain about half of the roughly 5 percentage-point decline in the global labor share [4]. This finding of σ > 1 provided one potential explanation for falling labor shares worldwide, attributing it to **investment-specific technical change** (cheaper capital) interacting with high substitutability. *(We will later discuss alternative interpretations and critiques of this result.)* Other macro-level cross-country studies have sometimes found σ slightly above 1 as well. For instance, some earlier literature assumed Cobb-Douglas (σ = 1) as a baseline, but the observed labor share movements have pushed researchers to consider σ ≠ 1. Piketty's own work alludes to σ > 1 in order to rationalize rising capital income shares with capital accumulation, though he did not directly estimate σ.

- **Evidence for σ < 1 (Gross Complements):** On the other hand, a large number of microeconomic and sectoral studies find σ is **below 1**. For example, **Oberfield and Raval (2021)** use rich **micro data on U.S. manufacturing plants** to estimate the aggregate capital-labor elasticity. They account for both within-plant substitution and between-plant reallocation. Their estimate for the U.S. manufacturing sector is **σ in the range 0.5–0.7**, and importantly, they find this elasticity **has slightly declined since 1970** [5]. In other words, capital and labor in manufacturing appear to be gross complements (σ < 1), and substitutability has not increased over time in that sector. With such an elasticity, they argue that the dramatic decline in the manufacturing labor share (over 15 percentage points in a few decades) **cannot be explained by changes in relative factor supplies (like cheaper capital alone)** [5] – instead it points to other factors like biased technological change (favoring capital or labor-saving automation) or changes in market structure. Many other studies using different techniques also support σ < 1. For instance, a panel of 12 advanced economies from 1980–2006 found **σ ~ 0.7 on average**, implying capital-labor complementarity in aggregate [6]. This study (Bogasu et al. 2018) used a supply-side system estimation with a normalized CES function and found consistently that labor and capital are complements on average, with technical change being net labor-augmenting [6]. Similarly, older empirical work often estimated elasticities in the 0.4–0.8 range (e.g. studies by Robert Chirinko and co-authors have long found an elasticity around 0.5 for the U.S., suggesting limited substitutability).

- **Sectoral Differences:** The elasticity of substitution **may differ across sectors** of the economy. Intuitively, manufacturing—especially for standardized or routine production—might allow more substitution of machines for workers than, say, many service industries where human labor (especially involving interpersonal tasks or creativity) is harder to replace. Some research indeed suggests that σ is higher in manufacturing than in services [7]. For example, Poschke (2019) calibrates a two-sector model and finds the elasticity in manufacturing exceeds that in services, which helps explain structural change dynamics [7]. In his estimates, manufacturing had σ of roughly 0.8 while services had σ around 0.6, consistent with the notion that **services are more complement-heavy** (closer to requiring fixed labor inputs) whereas manufacturing can more readily adopt capital in place of labor. Likewise, agriculture historically saw significant substitution (mechanization replacing farm labor), though at some point diminishing returns set in. It's important to stress, however, that even within manufacturing, recent micro-evidence (like Oberfield & Raval) doesn't support extremely high σ – instead it suggests that **most industries do not have unlimited flexibility to substitute**. Sectoral variation in σ means that the aggregate economy's elasticity is some combination of these, potentially changing as the economy's sectoral composition shifts. (For instance, a shift from manufacturing toward services would, all else equal, tend to *lower* the aggregate elasticity if services have the lower σ.)

- **Across Countries and Over Time:** Estimates of σ can also vary by country and time period. Karabarbounis & Neiman's cross-country approach yielded σ > 1 in the global context, but other country-level studies show a range. Some emerging markets or developing countries might exhibit different substitution patterns depending on their stage of development and technology adoption (though data is spottier). Over long historical periods, there is some evidence that σ might not be a structural constant: in certain eras technology might enable higher substitutability. For example, the introduction of general-purpose technologies (electricity, ICT) could temporarily raise substitutability by making new machine tools feasible. Conversely, if production becomes more specialized or if easy substitutions are already exploited, σ could effectively fall. Oberfield & Raval's finding that σ in U.S. manufacturing *declined slightly* over the late 20th century suggests that as manufacturing became more capital-intensive, remaining production might be in tasks that are harder to automate (thus reducing marginal substitutability).

To summarize the empirical landscape: **there is no consensus on a single value of σ**, and results are **mixed**. Some macro-level analyses (and the assumptions of some prominent theories like Piketty's) have leaned on σ > 1, implying capital can substantially replace labor, whereas many micro-level and sectoral studies point to σ < 1, indicating inherent complementarity. A recent meta-analysis of the literature (Gechert et al. 2021) finds an average estimate around 0.9 but notes that differences in method and publication bias can skew results, and when corrected, the evidence tends to favor σ slightly below 1 in general. The differences across studies highlight the importance of methodology (estimation technique, controls for technical change, use of gross vs net output, etc.) and the difficulty of disentangling pure substitution from other concurrent changes.

σ and the Labor Share of Income

One of the most critical applications of σ is in explaining movements in the **labor share of income**, i.e. the fraction of national income paid to labor (wages and compensation) versus capital (profits, rents, etc.). In recent decades, many countries have seen a **declining labor share**, sparking debate on the causes. Changes in σ directly tie into this debate, because the labor share's response to various shocks depends on σ as discussed above. Here we examine how σ interacts with factors like technology and capital accumulation to influence the labor share.

Factor Substitution and Labor Share: In a simple scenario with no technological change, if the relative supply of capital increases (for example, the economy accumulates more capital per worker, or the price of capital goods falls), the impact on the labor share depends on σ. As noted earlier, if σ > 1, capital deepening will *lower* the labor share (since the percentage increase in output accruing to capital exceeds the percentage fall in the capital rental rate) [2]. Conversely, if σ < 1, capital deepening will *raise* the labor share (because the rental rate falls so much that capital's income share actually drops) [2]. If σ = 1, the labor share remains constant regardless of capital accumulation. Therefore, the observed decline in labor share in many economies could, in principle, be explained by two broad mechanisms: (1) **σ > 1 with capital deepening** – capital becoming cheaper and more abundant, outpacing the elasticity threshold so that labor's share falls; or (2) **σ < 1 with labor-saving technology or other forces** – because if σ < 1, pure capital deepening wouldn't cut labor share, so something else (like biased tech or institutional changes) must be shifting income away from labor.

The Case for σ > 1 in Labor Share Decline: Karabarbounis and Neiman (2014) argue that the first mechanism has been important globally. As mentioned, they find σ ~1.25 and attribute about half of the global labor share decline to the **falling relative price of capital** (ICT and equipment becoming cheaper) [4]. In their framework, because σ > 1, firms respond to cheaper capital by significantly substituting capital for labor, thus reducing the share of income paid to labor. This story aligns with the narrative that **technology (cheap computers, automation machinery, etc.) has directly undercut labor's income share** by making machines a cost-effective replacement for workers in many tasks. Some supportive evidence for this mechanism: sectors that experienced the largest declines in investment goods prices and the fastest growth in capital intensity (like manufacturing) did tend to see larger labor share declines in recent decades, consistent with high substitutability [4]. Additionally, models calibrated with σ > 1 can replicate a falling labor share under plausible shocks to technology or factor prices. Piketty's thesis also effectively relies on σ > 1: if the rate of return on capital doesn't fall too fast as capital accumulates (which requires σ > 1), then capital's share can rise when wealth grows faster than the economy. Thus a higher σ can be associated with greater inequality between capital owners and workers in the long run.

The Case for σ < 1 and Other Drivers: On the other hand, substantial evidence suggests that the recent labor share decline is **not purely a substitution story** – and might even be happening despite σ < 1, due to other forces. A prominent critique comes from **Rognlie (2015)**, who re-examined the data on factor shares. Rognlie pointed out that the oft-cited decline in aggregate labor share is **heavily driven by the housing sector** and the way housing capital income (imputed rents, etc.) is measured [8] [9]. He finds that **outside of housing**, there has been **little long-run decline in labor's share** – in fact, in the non-housing corporate sector, the labor share in many advanced countries fell in the 1970s but then *partially rebounded*, resulting in a U-shaped pattern rather than a persistent downward trend [9]. The rise in capital's share that does appear in aggregate data since the 1980s is "entirely due to housing" according to his analysis, with imputed housing rents taking a larger portion of income [9]. This is important because housing capital is not a close substitute for labor in production (housing services are a different kind of output), and including it inflates the capital share. When focusing on the productive sectors, Rognlie's findings suggest that **the pure effect of cheaper equipment on labor share is weaker than Karabarbounis & Neiman claimed**. Moreover, Rognlie argues that even within the corporate sector, the net labor share (after accounting for depreciation) hasn't fallen much; instead, what decline there is can be largely explained by **increasing markups (profit shares)** and other non-substitution factors [10]. In other words, he shifts the attention from σ to issues like **market power and housing scarcity** as drivers of the income distribution changes. If capital and labor are actually **gross complements (σ < 1)** in production, an increase in the capital-output ratio (for example, due to slower growth or higher saving) should *lower* capital's share of income, not raise it [1]. Thus a falling labor share in a σ < 1 world hints that something else is afoot – likely **labor-saving technical change** (which effectively reduces the demand for labor at any given K/L) or institutional shifts that suppress labor's compensation (such as decline in union bargaining power, or rising monopoly power capturing more income as profit).

Biased Technological Change: Indeed, one key driver that can reduce labor's share even if $\sigma < 1$ is **labor-augmenting vs. capital-augmenting technical change**. In a famous result (Uzawa's theorem), an economy on a balanced growth path with a constant labor share must have purely labor-augmenting (Harrod-neutral) technological change if $\sigma = 1$ (Cobb–Douglas). If $\sigma \neq 1$, the bias of technical change will affect the labor share: *capital-augmenting (labor-saving)* technology tends to reduce labor's share, especially if σ is not too high. Empirical studies find that recent technological change has been **labor-saving** in many cases – for example, the adoption of industrial robots, computer software, AI, etc., specifically targets tasks previously performed by labor. This can be seen as **capital-augmenting** in an aggregate production function sense (making capital more effective at replacing labor). If $\sigma < 1$, such labor-saving tech will *unambiguously* reduce labor's share, because firms do not fully counteract the effect by rebalancing toward labor (since labor is not easily substituted back in). The study by **Lawrence (2015)** makes this point: he proposes a "gross complements" explanation for the U.S. labor share decline, suggesting that $\sigma < 1$ and a **decline in the effective capital-labor ratio** (once one accounts for slowing capital accumulation and labor-augmenting tech) led to the fall in labor share [2] [11]. In that account, the labor share fell not because capital got *cheaper* relative to labor, but paradoxically because capital investment did not keep up with a growing labor force and labor-augmenting improvements – with $\sigma < 1$, insufficient capital deepening can cause labor's share to drop (labor becomes relatively abundant and less productive at the margin). Thus, depending on the nature of technical change, even a $\sigma < 1$ economy can experience a declining labor share. This underscores that **the labor share is determined by both σ *and* the direction of technological change**. A capital-biased (labor-saving) innovation will shift the functional distribution of income in favor of capital unless offset by some other force, while a labor-biased innovation would increase labor's share. For instance, evidence indicates that much of the recent tech change (IT, automation) has been labor-saving, whereas earlier 20th-century tech (e.g. electrification) created many new labor-intensive tasks (which helped keep labor share stable).

Role of Markups and Other Factors: It's also important to note that not all changes in labor share are due to technology or substitution. Changes in market structure – such as increasing monopoly/oligopoly power of firms – can reduce the labor share by increasing the profit share (since under imperfect competition, labor is not paid its full marginal product). Research by De Loecker, Eeckhout, and others finds rising markups in many industries since the 1980s, which could account for part of the labor share decline as a distributional shift from labor to profits. Even in a Cobb-Douglas world ($\sigma = 1$), **higher markups or price–cost margins will lower labor's share** because firms with market power do not pass all output gains to labor [12]. Karabarbounis & Neiman were mindful of this: they checked whether rising markups could be confounding their σ estimate. If markups increased especially in industries with cheaper capital, that could *mimic* a high σ (since labor share would fall in those industries even without substitution) [12]. After adjustments, they still found $\sigma > 1$, but acknowledged markups played a role in many countries' labor share trends [13] [14]. Rognlie (2015) similarly found that in the non-housing sector of the U.S., **rising markups were responsible for most of the change** in the capital share aside from housing [10]. Other factors like globalization (offshoring labor-intensive production) and policy changes (weaker unions, lower labor share in value-added in some industries due to outsourcing) have also been explored (e.g. Elsby et al. 2013 on offshoring). These factors are somewhat outside the pure σ framework but are crucial for a full account of labor share dynamics.

In summary, the relationship between σ and the labor share is nuanced. High σ (>1) can directly cause labor share to fall when capital becomes cheaper or more plentiful, and this has been one prevailing explanation for global labor share declines [4]. However, a growing body of evidence suggests that σ might actually be <1 for the aggregate economy, implying that something else – chiefly *labor-saving technological change* (automation) and perhaps rising markups – are the primary causes of labor's shrinking share. The labor share is therefore best understood as resulting from an interplay of elasticity of substitution and the **bias of technological and economic changes**. If technology had been neutral or labor-complementary, a σ < 1 world would have seen stable or rising labor shares; instead, we saw many labor-displacing innovations and institutional shifts that pushed labor's share down despite only modest substitutability. This debate remains active, as new evidence and methods continue to refine our understanding of σ and its impact.

Implications of σ for Automation and AI-Driven Labor Displacement

One of the most pressing economic questions today is how advancements in **automation and artificial intelligence (AI)** will affect workers. The elasticity of substitution σ is deeply relevant to this question, as it governs whether machines can readily replace human labor. Different values of σ imply very different futures for automation's impact:

- **High σ (σ > 1) – Easier Automation:** If capital and labor are highly substitutable, then automation technologies (robots, AI systems, etc.) can **readily take over tasks from humans** with relatively little loss in efficiency. A σ significantly above 1 would mean that as soon as machines can perform a task at a slightly lower cost than a human, firms will substitute machines for humans extensively. The implication is a potentially rapid displacement of labor in jobs that become automatable. In macro terms, a sustained drop in the price of "robot capital" or AI software could lead to a **sharp decline in the labor share** of income as capital replaces labor across many tasks. This scenario often underpins dire predictions of "mass technological unemployment" or a new era of capital owners accumulating most income. Indeed, if σ is very high, one could imagine an extreme where automation drives labor's share towards zero in the long run (since machines could eventually do almost everything at lower cost). Historical examples of automation do show labor share declines in specific industries when technology enabled easy substitution – for instance, the mechanization of agriculture saw capital (tractors, harvesters) largely replace farm labor, contributing to a low labor share in that sector [15]. The **displacement effect** of automation (capital taking over tasks previously done by labor) directly reduces labor's share of value added in the affected activities [15]. If the economy-wide σ is high, these effects can propagate broadly.

- **Low σ (σ < 1) – Limited or Augmented Automation:** If capital and labor are complements with σ < 1, pure automation becomes more complicated. A low elasticity means that machines cannot easily replace humans without significant efficiency losses – many tasks require a human touch or the combined input of labor and capital. In a σ < 1 world, even if AI improves, **robots do not perfectly substitute for workers** in most tasks, and diminishing returns to substituting capital set in quickly. Thus, the displacement of labor by automation would be more limited. Instead, automation might serve to *augment* human labor, making workers more productive rather than outright redundant. For example, consider advanced medical diagnostic AI: if it complements doctors (helping them diagnose faster) rather than substitutes fully, doctors remain essential and labor share may not plummet. With low σ, even a cheap robot cannot fully replace a worker because the tasks that robot can do might still require oversight or because removing the worker leads to a loss of some output that machines can't handle. As a result, the labor share might be more resilient in the face of automation; wage and employment effects would depend on how productivity gains are shared. In fact, with $\sigma < 1$, if automation increases overall productivity, it could even increase demand for labor in complementary tasks, potentially keeping labor's income share stable or causing only modest declines. However, it is important to note: even if **aggregate σ is less than 1**, automation can still reduce labor's share through a different channel – by changing the *composition of tasks* in the economy (more on this below). So low σ doesn't mean automation has no impact, but the mechanism is different from the simple substitution story.

- **Task-Based View – Heterogeneous Substitution:** Modern research emphasizes that talking about a single σ for the whole economy can be misleading when it comes to automation. **Acemoglu and Restrepo's task-based framework** is instructive here. In their model, production is conceived as a collection of tasks, some of which can be done by labor, some by capital, and some potentially by either. Automation is the process of **expanding the set of tasks that capital (machines) can perform**, i.e. shifting tasks from the labor column to the capital column. This has a clear *displacement effect:* for those tasks, labor is no longer needed, directly **reducing labor's share of value added** in that process [15] [16]. Crucially, this effect **does not rely on a high σ in the aggregate production function sense** – even if σ is less than 1 overall, if a specific task is automated, labor is displaced from that task. In fact, Acemoglu and Restrepo note that "available estimates of σ place this parameter to be less than [1]" in many contexts, implying that pure factor substitution is limited, yet we still observe labor share declines due to automation of specific tasks [17] [5]. They argue that the key to understanding automation is **not an increase in substitutability across the board, but a change in the *allocation of tasks***. If the economy were only about a fixed set of tasks with a fixed σ, automation's impact would be constrained by σ. But when new automation technologies emerge, they effectively *change the production function* by allowing capital to enter tasks it couldn't before – this has an unambiguous negative impact on labor's share of income in those tasks, regardless of aggregate σ [18] [19]. The result is a decline in labor demand in automated tasks and a downward pressure on the overall labor share (this is sometimes called a **shift in the "task content" of production against labor**).

  However, Acemoglu and Restrepo also highlight the countervailing force: **the creation of new tasks** in which labor has a comparative advantage. Historically, even as automation displaced labor in some tasks (e.g. weaving machines displacing textile weavers, or robots displacing assembly line workers), new industries and tasks emerged where labor could be employed (e.g. jobs in machine maintenance, design, new services, etc.). This creates what they call a **"reinstatement effect"** – new labor-intensive tasks increase labor demand and can **raise labor's share** in those areas [20] [21]. If the economy keeps generating new tasks for labor at a sufficient pace, it can offset the displacement from automation. Indeed, Acemoglu and Restrepo (2019) note that the reason we did not see a continuous decline in labor's share historically (and instead often had stable shares) is that **technologies that created new tasks counterbalanced the automation (displacement) effects** [22] [23]. For example, during the 19th century, even as agriculture was mechanizing, new occupations in manufacturing and services absorbed the labor freed from farms, and labor's share in the overall economy did not collapse. They document episodes like the mechanization of agriculture in the late 1800s: the labor share in agriculture fell (displacement), but simultaneously the industrial sector expanded with many new labor-intensive jobs (reinstatement), so the **aggregate labor share remained more stable** [24] [23].

The **implication for the AI age** is that whether automation leads to a sharply lower labor share (and major labor displacement) or not depends on both σ *and* the balance of these task forces. If σ is high and automation is rapid, labor share could fall very fast because displacement is easy and there may be fewer new tasks than lost tasks. If σ is low but automation still proceeds (which it will, if new technologies target tasks that *can* be automated), then labor share will still tend to decline *unless new labor-intensive tasks arrive to offset the loss*. In a low-σ economy, the pure substitution effect might be small (capital doesn't drastically replace labor in existing tasks because of diminishing returns), but the **reallocation of tasks can still reduce labor's share** – essentially a form of biased technical change. Acemoglu and Restrepo's task model can be thought of as an extension of the CES idea: instead of a single σ, there is a distribution of substitution possibilities – some tasks are almost perfect substitutes with machines ($\sigma \to \infty$ in those tasks), others are not substitutable at all ($\sigma = 0$ in essential human tasks). AI likely increases σ in specific subdomains (e.g. data processing, certain routine cognitive tasks) to very high levels, enabling near-complete automation there, while other domains remain with low substitutability.

- **AI and the Future of Labor Share:** If advanced AI (e.g. machine learning systems) can perform a wide range of tasks that were previously non-automatable, we are effectively increasing the overall substitutability of capital for labor. Some economists warn that we may be heading toward a higher-σ world as AI improves – meaning the set of tasks where capital can replace labor expands dramatically, pushing σ upward. In such a case, even if initially σ was below 1, it could rise above 1 as technology changes the production function. This could usher in a period of falling labor share and potentially stagnant wages, especially if new task creation (entrepreneurship that finds new roles for human labor) does not keep up. On the other hand, if AI also opens up new industries and tasks for humans (for example, entirely new job categories we can't yet imagine, or increases the value of uniquely human skills), then the reinstatement effect could soften the blow. **Acemoglu and Restrepo (2018, 2019)** suggest that policy and innovation efforts should perhaps be directed not just at automation for its own sake, but at *"direction of technology"* choices that encourage labor-friendly innovation – essentially, to ensure there are plenty of new tasks where labor remains indispensable.

In conclusion, the impact of automation and AI on labor depends critically on σ and on the task composition effects. If we treat σ as fixed and high, we expect significant labor displacement and a declining labor share as machines underbid humans. If σ is fixed and low, automation alone would have less effect, but in reality automation changes the production environment by *endogenously increasing the substitutability* in certain tasks. The lesson from history, reinforced by task-based models, is that **automation's effect on labor share is "unambiguously" negative at the task level** [18] **, but the aggregate outcome hinges on whether other innovations reintegrate labor into new productive opportunities** [20] . Thus, while a high σ makes it easier for labor's share to fall, even with a moderate σ the labor share can fall if technology is predominantly labor-replacing. This perspective moves beyond the one-dimensional view of σ and emphasizes the **importance of technological dynamics** (automation vs new task creation) in shaping the future of work and income distribution.

Critiques of the CES Approach and Alternative Frameworks

The widespread use of the CES production function to model capital-labor substitution has been valuable for analytical tractability, but it has also faced several theoretical and empirical critiques, especially in the context of modern economies:

- **Is a Single σ Realistic?** One critique is that the assumption of a **constant, economy-wide σ** is overly simplistic. Modern economies are highly complex, composed of numerous sectors, industries, and tasks, each with its own possibilities (or lack thereof) for substituting capital for labor. The notion that there is one aggregate production function with a single elasticity parameter may obscure more than it reveals. For instance, as we saw, **housing capital** behaves very differently from productive capital in terms of substitutability with labor. Aggregating housing and factories into "one K" can mislead analyses of income distribution. *Rognlie (2015)* effectively demonstrated this by separating housing from non-housing: the aggregate data seemed to imply a big shift toward capital income (which some might interpret via a high σ), but once housing is taken out, the picture changes – the non-housing labor share had no clear downward trend, undermining the high-σ explanation [9]. This suggests that a single-sector CES was not capturing the **heterogeneity across asset types**. Likewise, as noted, manufacturing vs services have different substitutability features; lumping them could give a false "average" σ that doesn't truly apply to either sector's dynamics. **Multi-sector models** or **factor-specific analyses** are often more appropriate. In multi-sector models, one might use different σ's for different sectors or nest a CES structure (e.g. allow substitution among certain types of capital vs labor, etc.). These approaches often fit the data better; for example, separating equipment (which is more substitutable) from structures (less substitutable) can yield a more accurate picture of factor shares over time.

- **Variable Elasticity and Technological Change:** Another critique concerns the **constancy of σ**. The CES function assumes σ is a structural parameter, but in reality, σ could change with technology. Some economists have explored **variable elasticity** production functions or flexible forms like the **translog production function** (which does not impose a constant elasticity). These can capture how substitutability might evolve as the capital-labor ratio changes or as new technologies emerge. For instance, early in the computer revolution, substituting computers for clerical workers might have been hard (low effective σ), but after decades of improvement in software, that substitutability is much higher. A standard CES with fixed σ cannot capture such evolution unless one explicitly models it as technology shifts the parameter. Research by Klump, McAdam, Willman and others on the *normalized CES* shows that by allowing factor-augmenting technical change, one can in principle fit any time path of factor shares with an appropriate specification – which means a high estimated σ might partly reflect unaccounted biased tech change and vice versa [25] [26]. In simpler terms, if one doesn't properly control for capital-augmenting vs labor-augmenting tech, the σ estimate can be biased. The meta-analyses suggest that many older estimates that found σ near 1 or slightly below might have been influenced by how they handled (or didn't handle) technological trends. Recent works (like the NBP study cited earlier) explicitly include factor-specific technical progress in the estimation and tend to find σ < 1 with **net labor-augmenting technical progress** [6] – meaning technological change has favored labor efficiency, while substitution possibilities remain limited. All of this points to caution in treating σ as a deep, unchanging parameter. It could be context-dependent and endogenous to innovation.

- **Task-Based and Non-Parametric Frameworks:** As discussed, **task-based models** (Acemoglu & Autor, 2011; Acemoglu & Restrepo, 2018/2019; Zeira, 1998) offer an alternative to the aggregate CES. These models do not reduce the production process to a smooth function of K and L with fixed elasticity, but rather consider a set of discrete tasks that can shift between being done by labor and capital. In these models, outcomes like the labor share are not governed by a single σ, but by the range of tasks that capital can do vs the range that labor does, and how that changes. One advantage of the task approach is that it can more naturally account for **the introduction of new technologies and tasks**, which is awkward to force into a fixed functional form. The CES function, by design, cannot create new tasks – it just reallocates intensity between K and L continuously. Thus it misses the richness of how economies evolve (e.g. the rise of entirely new industries). Task models can also incorporate *task-specific* elasticities: for example, in one task (data entry) a computer might be almost a perfect substitute for a human (σ very high for that task), whereas in another (childcare) a robot is a very poor substitute (σ very low for that task). Aggregating these gives some effective elasticity, but it may not be constant and can shift if the composition of tasks changes. As automation progresses, the effective aggregate σ could rise (if more tasks move into the high-substitutability category), a dynamic that a fixed-σ model would miss. **Acemoglu and Restrepo's frameworks** emphasize how the *balance* between automation and new task creation drives labor demand and the labor share, rather than a fixed substitution parameter. They even show how, with a constant elasticity production function alone, you cannot easily generate a declining labor share unless you assume $\sigma > 1$ or specific biases – but in their task model, you can get a declining labor share *even if* σ *is* < 1 by letting automation reallocate tasks [18] [19]. This is a conceptual critique of relying solely on CES: it may be too restrictive to capture the reality of technological disruptions.

- **Empirical Limitations:** Empirically, estimating a CES production function often involves strong assumptions (perfect competition, constant returns, common technology across firms, etc.). Some critics argue that the aggregate production function is not even well-defined theoretically due to **aggregation problems** – known from the Cambridge capital controversy – i.e. you can't sum up heterogeneous capital goods into a single "K" without assuming what you're trying to prove. While in practice economists proceed by deflating and summing investment, the theoretical aggregation issues mean that σ estimated at the macro level might not have a clear micro foundation. For example, if different industries have different σ's, the aggregate might shift depending on their weights (as structural change occurs). This is an ongoing philosophical point, but one implication is we should be careful about interpreting an aggregate σ as a technological constant, rather than as a shorthand that may change if the structure of the economy changes.

- **Alternatives and Enhancements:** Apart from task-based models, researchers have considered other production function forms. A **Translog production function** is a second-order approximation that allows elasticity of substitution to vary with K/L (in fact, the elasticity becomes a function of K/L). This can capture non-constant elasticity in a flexible way and often fits data better, but it introduces many parameters and less clear intuition. Another extension is to include **more factors** – for instance, differentiate between **equipment vs structures vs labor**, or between **skilled vs unskilled labor vs capital**, each with its own elasticity of substitution (e.g. the **"capital-skill complementarity"** literature following Griliches suggests equipment capital is more substitutable for unskilled labor than for skilled labor). Such nested CES models might have one σ for capital vs unskilled labor, another for that aggregate vs skilled labor, etc. These can reflect reality more accurately by not treating labor as homogeneous. Acemoglu and Autor (2011) in their task-based view effectively do this by saying routine tasks (often mid-skill jobs) are easily substituted by computers, whereas abstract tasks (high-skill) and manual service tasks (low-skill but non-routine) are less so – effectively different σ's with computers for each type of labor. **Dynamic models** that allow technology to respond to factor scarcities also complicate the picture: if labor gets expensive, firms may innovate more in labor-saving technologies (as some models suggest), which could make σ appear higher in the long run because technology endogenously increases substitutability.

In light of these critiques, many economists advocate a multi-faceted approach to understanding capital-labor relations. The CES production function with a fixed σ is a useful tool for many purposes – it gives clear intuition and has been the workhorse in growth and distribution theory. But when it comes to **modeling modern economies with rapid technological change, task reorganization, and shifting market structures**, one often needs to go beyond a simple CES. The **task-based frameworks of Acemoglu & Restrepo** and others are one such way, providing insight into how automation can reshape factor shares even outside the confines of a fixed-σ world. Empirically, researchers are combining micro data (to estimate substitution at the firm or industry level) with macro analysis (to see the big-picture effects) – as done by Oberfield & Raval (2021) – to reconcile differences. And policy analysts are keenly interested in these issues because the value of σ (and the nature of tech change) determines how strongly policy should respond – e.g. if σ >> 1 and tech is racing, one might need more aggressive intervention to retrain workers or redistribute income; if σ < 1, improving productivity via complementary investments (education, etc.) could raise both wages and profits without a zero-sum share fight.

Conclusion

The elasticity of substitution between capital and labor, σ, is a cornerstone parameter in understanding income distribution and the potential impacts of technology on the economy. Theoretically, σ dictates how the capital–labor mix adjusts to price changes and how capital deepening affects labor's share of output – with σ > 1 leading to capital share gains at labor's expense, and σ < 1 leading to the opposite (or requiring other forces for labor share to fall) [1]. Empirically, the evidence on σ is **divided**: some research finds σ modestly above 1 [4], giving credence to substitution-driven narratives for the declining labor share, while a great deal of micro evidence points to σ below 1 [5] [6], implying inherent complementarity and forcing us to seek explanations in biased technical change, task reallocation, or institutional shifts. This tension is exemplified by the debate between Karabarbounis & Neiman's (2014) thesis of a high σ enabling cheap capital to undercut labor [4], and Rognlie's (2015) finding that the non-housing labor share hasn't persistently fallen, suggesting other factors at work [9]. Newer contributions like Oberfield & Raval (2021) reconcile some of this by showing low σ in manufacturing but emphasizing technology's labor-saving bias to explain labor share declines [5].

The labor share of income is thus not governed by σ alone, but by **σ in combination with the nature of technological and market changes**. When thinking about automation and AI, it is not enough to ask "is σ greater or less than 1?"; we must also ask *"what tasks can AI do, and how does that change the effective substitutability of capital for labor?"*. Acemoglu and Restrepo's task-based frameworks remind us that even with a given σ, automation (a form of capital-augmenting change) can reduce labor's share unless balanced by new opportunities for labor [20] [23]. Their work, along with other critiques, suggests caution in applying a simple CES model to the complex modern economy. Alternatives like multi-sector models, variable-elasticity formulations, and task-based analyses provide a more nuanced understanding that can accommodate phenomena like the creation of entirely new job categories or the rise of superstar firms and markups that alter factor shares without changing σ.

In conclusion, the value of σ – whether above or below unity – has powerful implications for distribution: **if σ > 1**, the market forces of substitution tend to amplify the share of whoever's factor is growing (recently, capital), raising concerns about inequality and the erosion of labor's earning power [4]. **If σ < 1**, the economy on its own would not naturally hollow out labor's share from substitution alone, but ongoing labor-share declines signal that **technology and other forces are actively shifting the balance** (through automation, biased innovation, and institutions). The scholarly consensus is still evolving, but there is a recognition that we may be witnessing a convergence of high substitution in certain domains (AI automating specific tasks) with overall complementarity elsewhere – a scenario that requires moving beyond one-size-fits-all models. Policymakers and researchers alike are therefore paying close attention to σ as well as the *context* in which substitution occurs, knowing that this will shape the future of work and how the economic pie is divided between workers and owners of capital.

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