CSC 212

Data Structures and Abstractions

Spring 2020

Prof. Marco Alvarez

Building a Heap in Linear Time and Heapsort

Performance

	Sorted Array/List	Unsorted Array/ List	Heap
insert	O(n)	O(1)	O(log n)
removeMax	O(1)	O(n)	O(log n)
max	O(1)	O(n)	O(1)
insert N	$O(n^2)$	O(n)	O(n)**

(**) assuming we know the sequence in advance (**buildHeap**)

buildHeap

Problem

Build a heap by inserting a sequence of **n** elements

- ‘easy’: call insert **n** times

Problem

Build a heap by inserting a sequence of **n** elements

‘easy’: call insert **n** times **O(n log n)**

Problem

Build a heap by inserting a sequence of **n** elements

'easy': call insert **n** times **O(n log n)**

Can we do it in **linear time**?

buildHeap

Place **n** items into the tree (array) in any order

keeps structure property

buildHeap

Place **n** items into the tree (array) in any order

keeps structure property

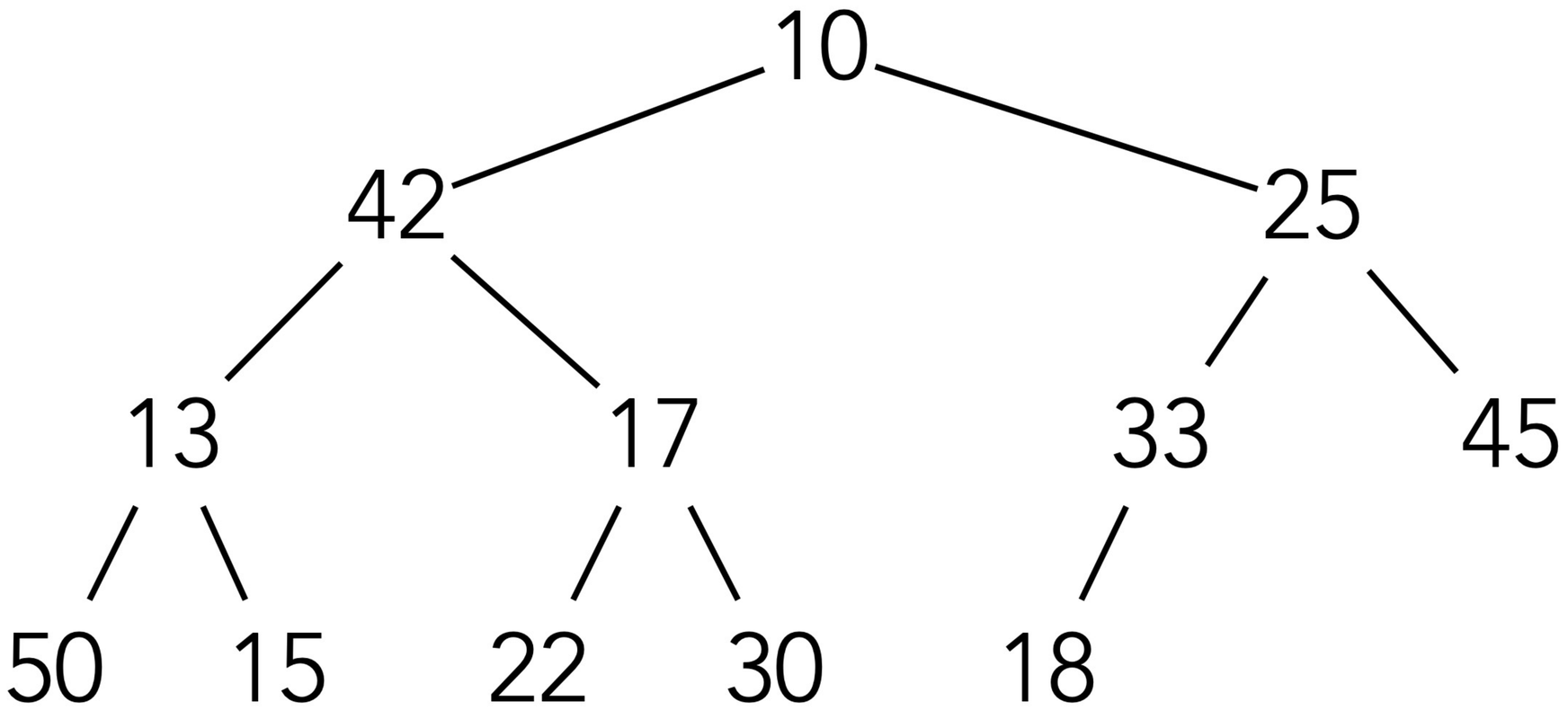
Perform **Down-Heap** on each internal node

from parent(n) to 1

keeps heap-order property

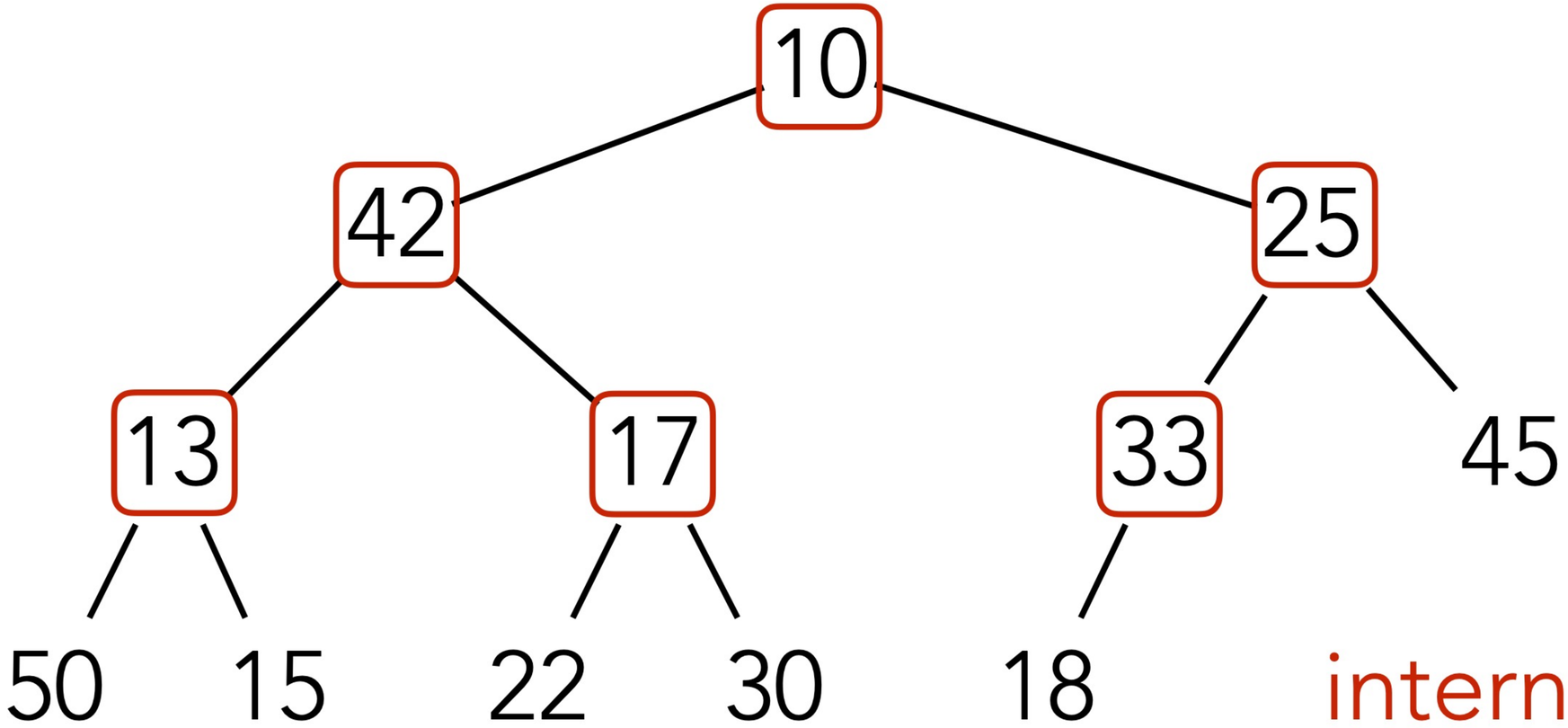
buildHeap example

input: 10 42 25 13 17 33 45 50 15 22 30 18



buildHeap example

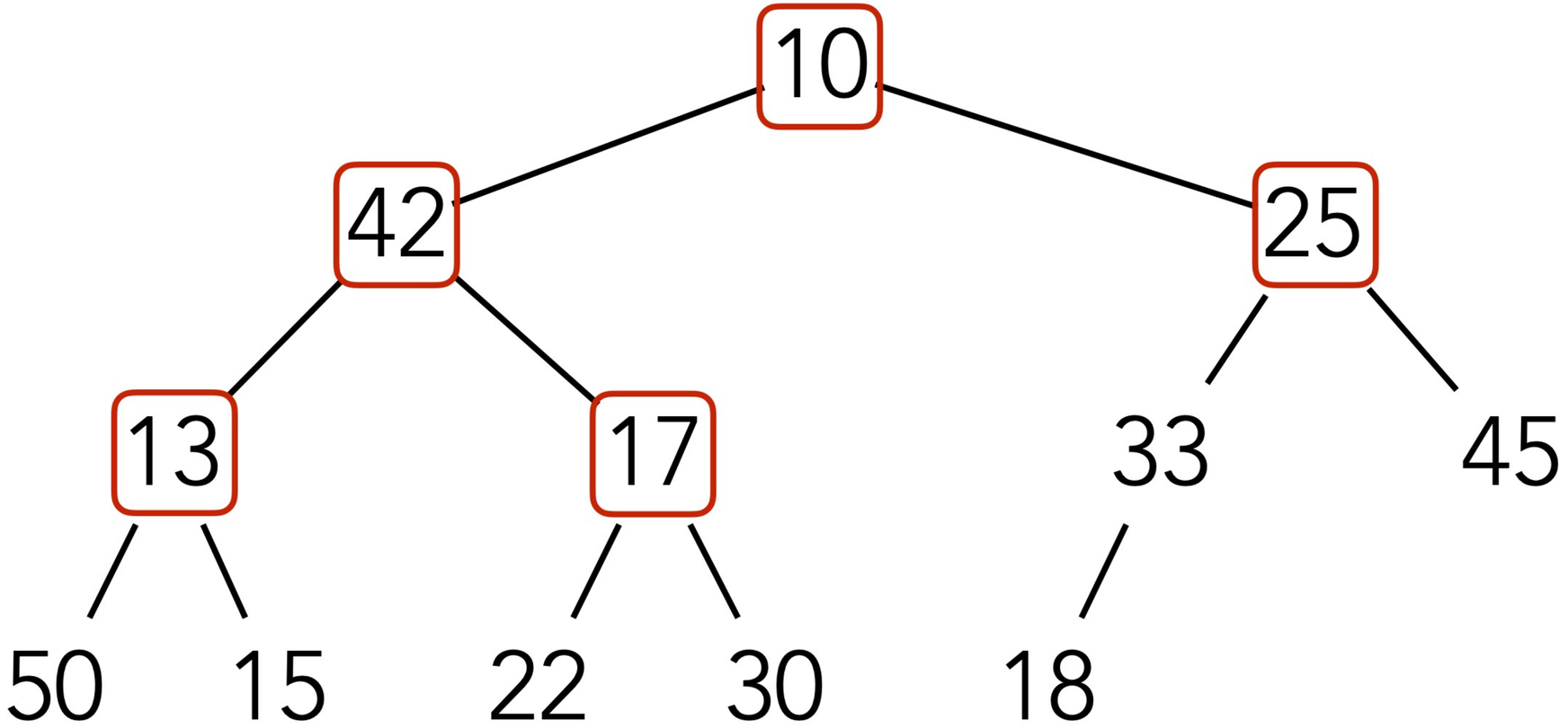
input: 10 42 25 13 17 33 45 50 15 22 30 18



internal nodes

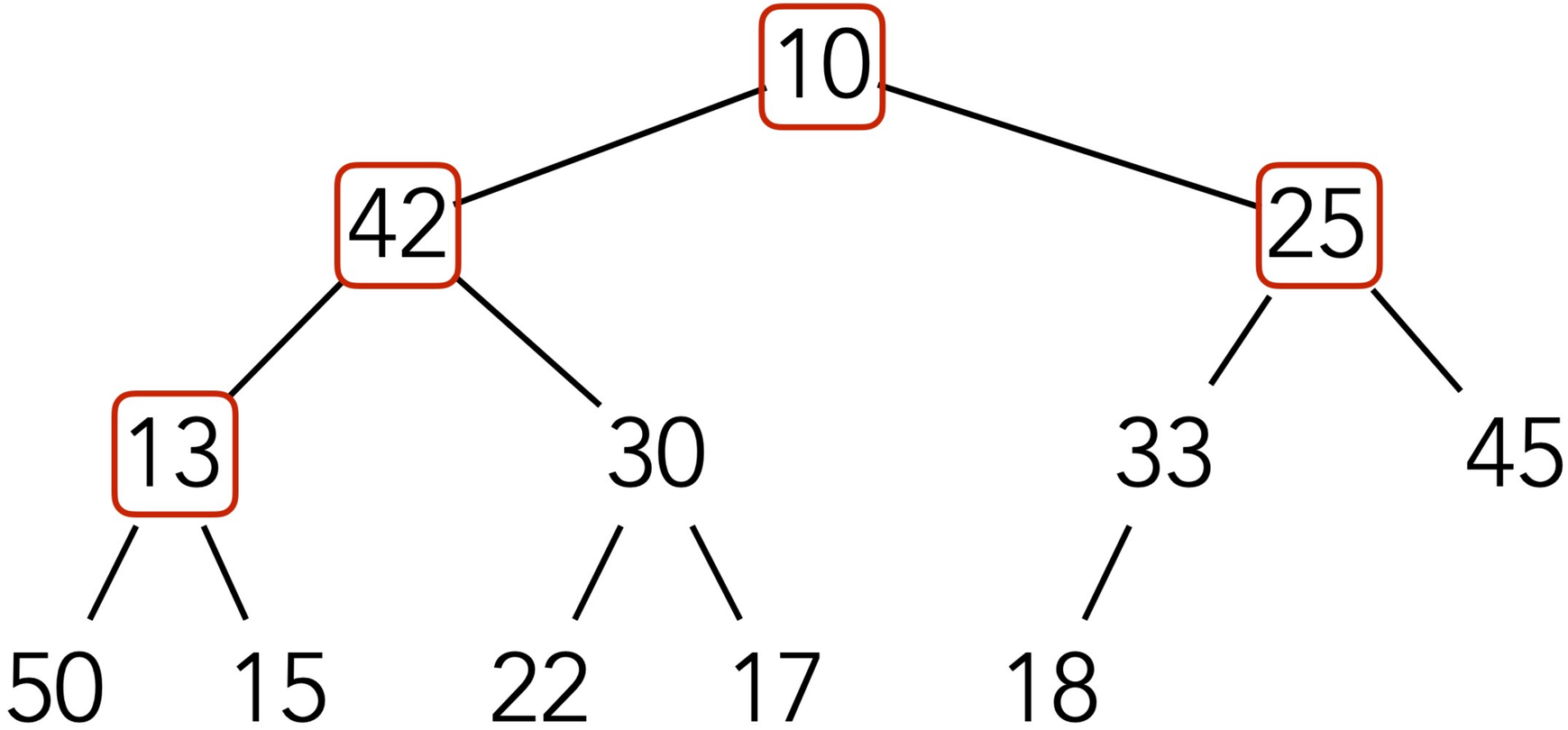
buildHeap example

input: 10 42 25 13 17 33 45 50 15 22 30 18



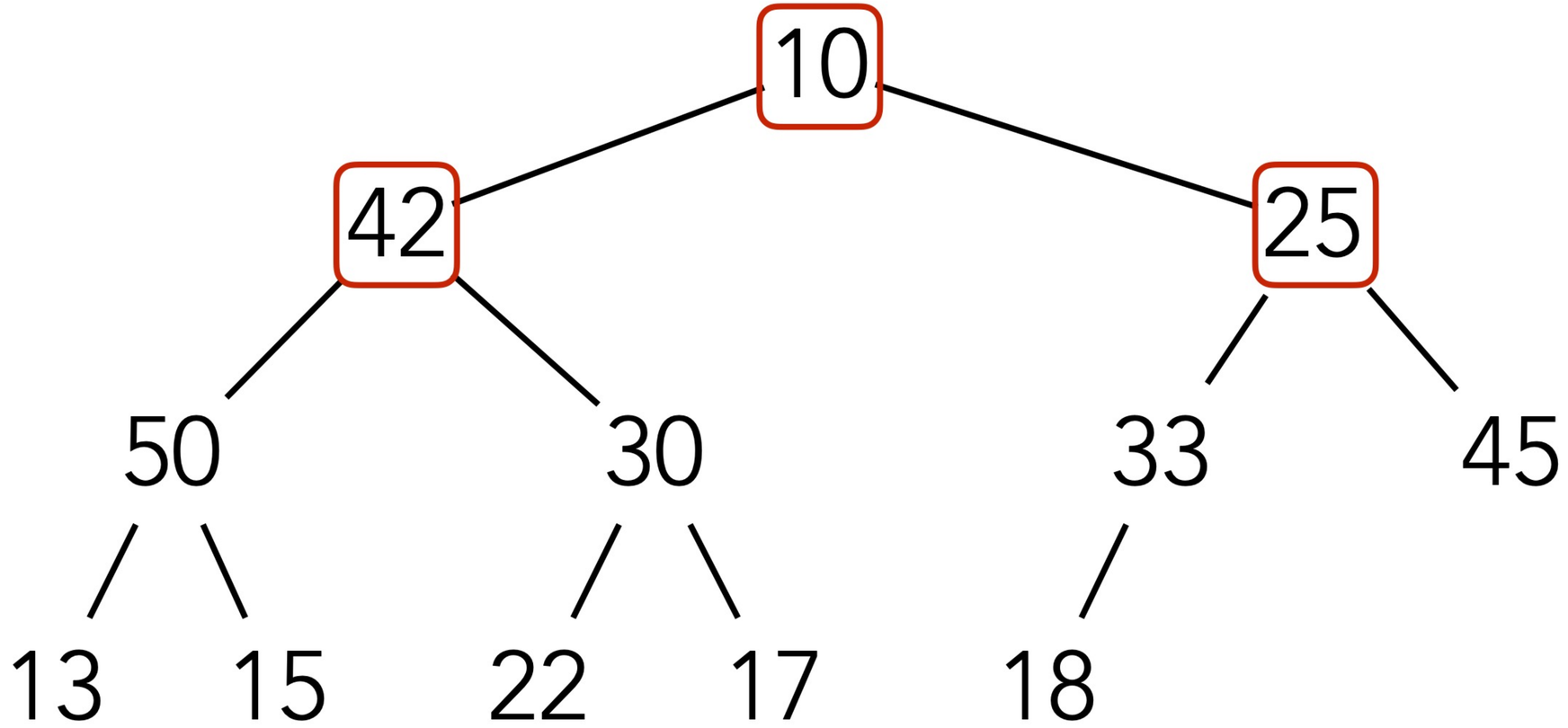
buildHeap example

input: 10 42 25 13 17 33 45 50 15 22 30 18



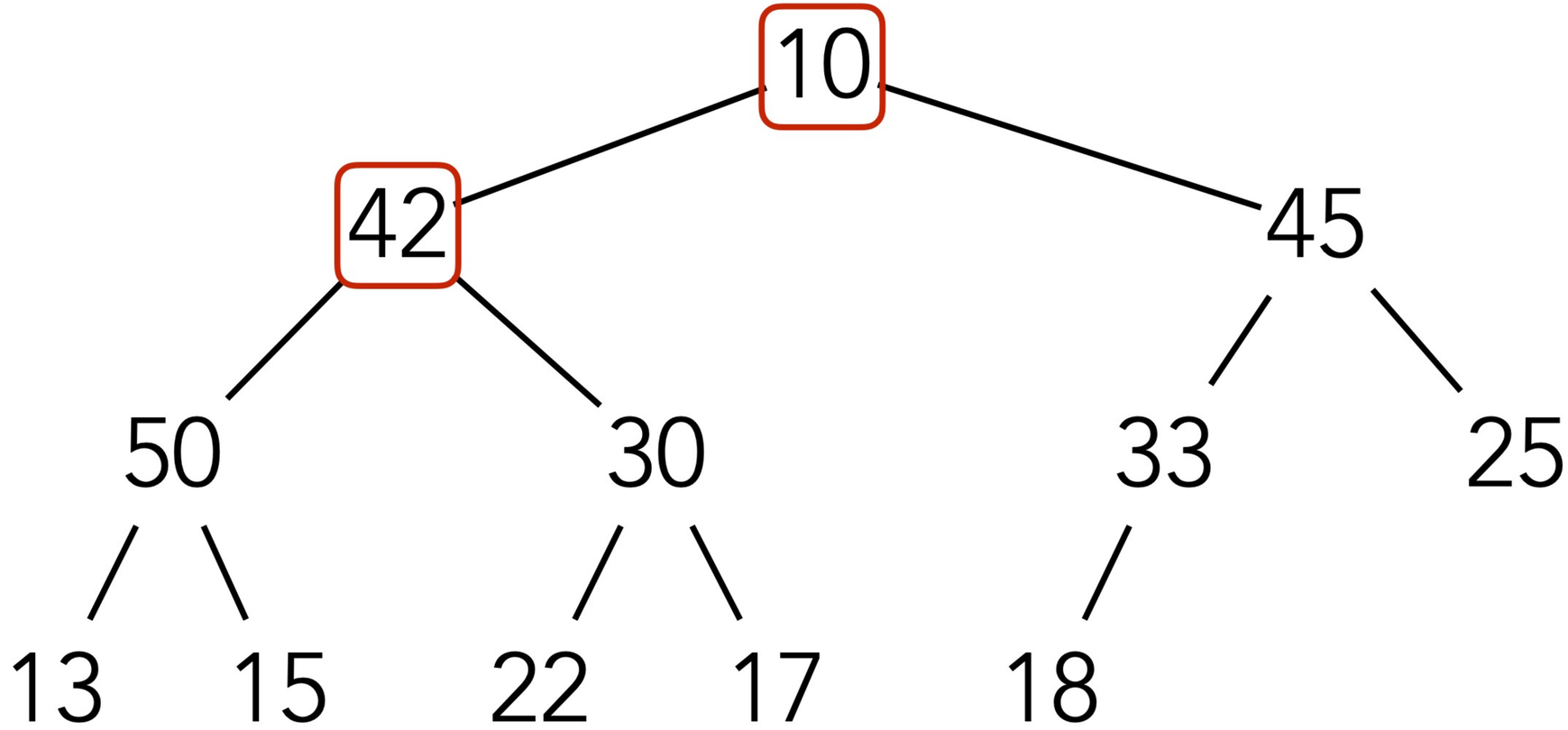
buildHeap example

input: 10 42 25 13 17 33 45 50 15 22 30 18



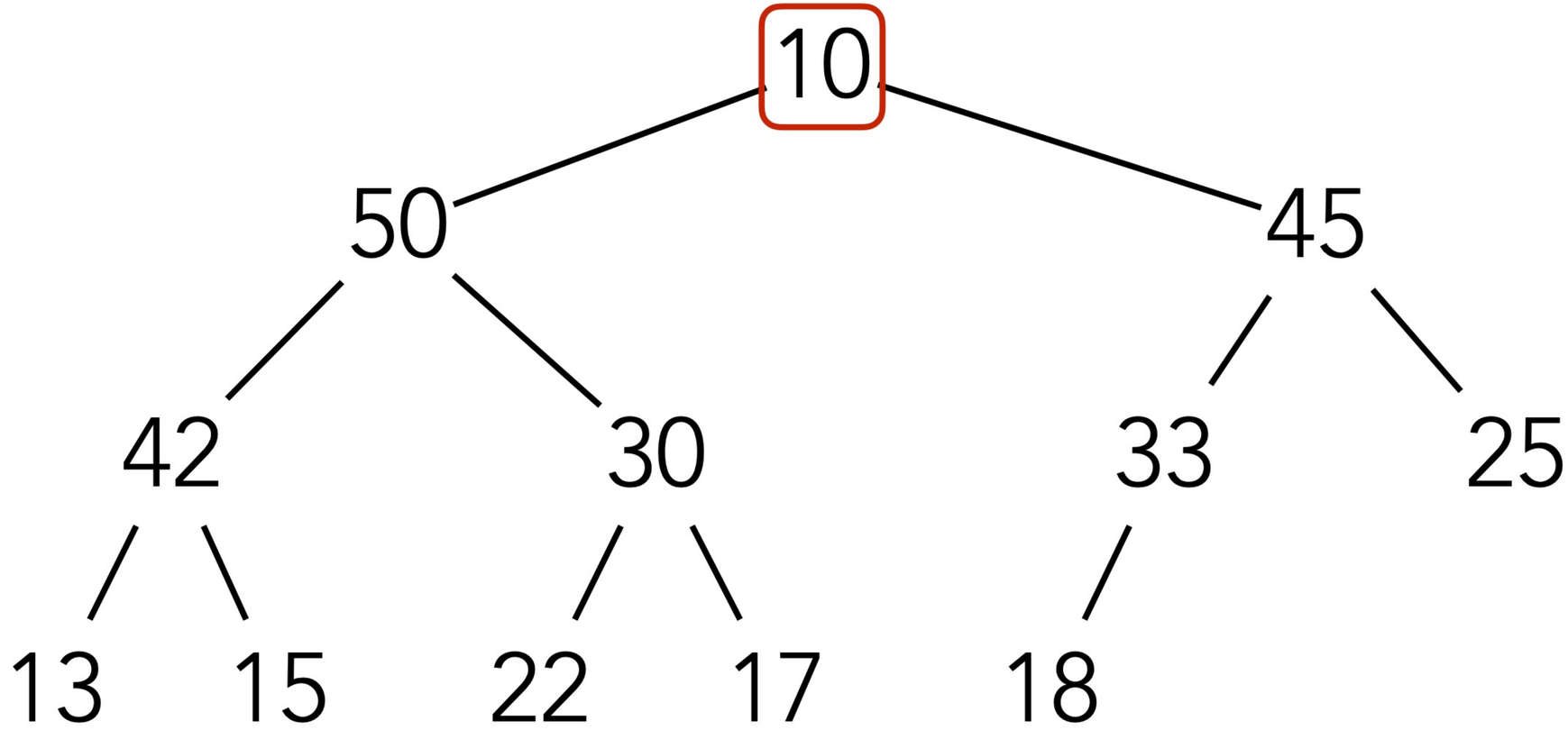
buildHeap example

input: 10 42 25 13 17 33 45 50 15 22 30 18



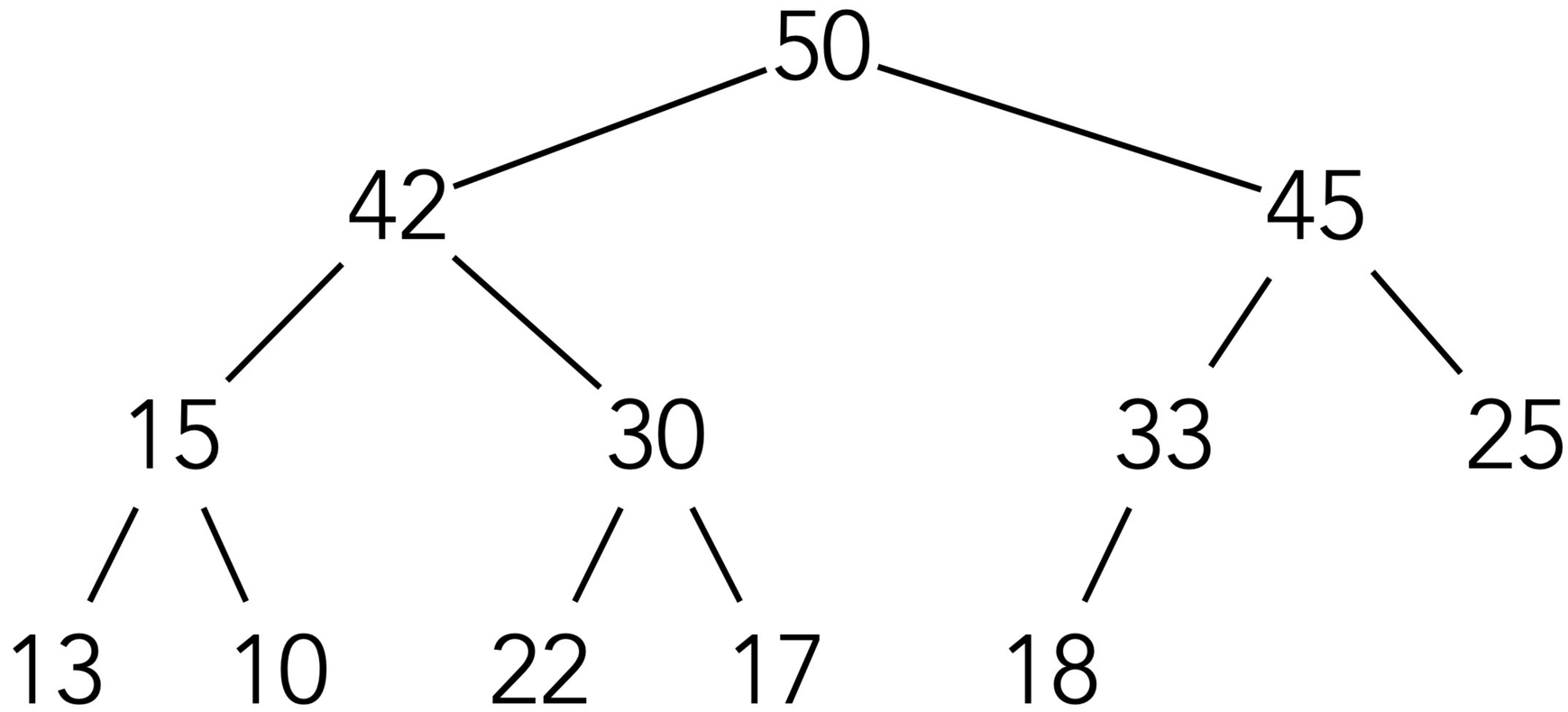
buildHeap example

input: 10 42 25 13 17 33 45 50 15 22 30 18



buildHeap example

input: 10 42 25 13 17 33 45 50 15 22 30 18



Analysis

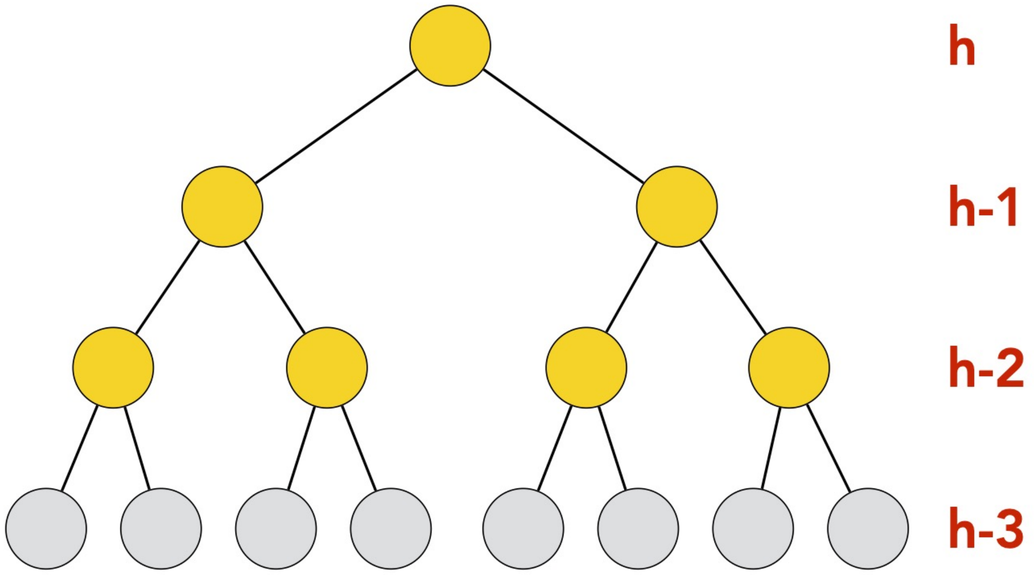
Cost is **sum of the heights** of all internal nodes

assume tree is full and complete, thus $\mathbf{n=2^{h+1}-1}$

Analysis

Cost is **sum of the heights** of all internal nodes

assume tree is full and complete, thus $\mathbf{n=2^{h+1}-1}$



Analysis

Cost is **sum of the heights** of all internal nodes

assume tree is full and complete, thus **$n=2^{h+1}-1$**

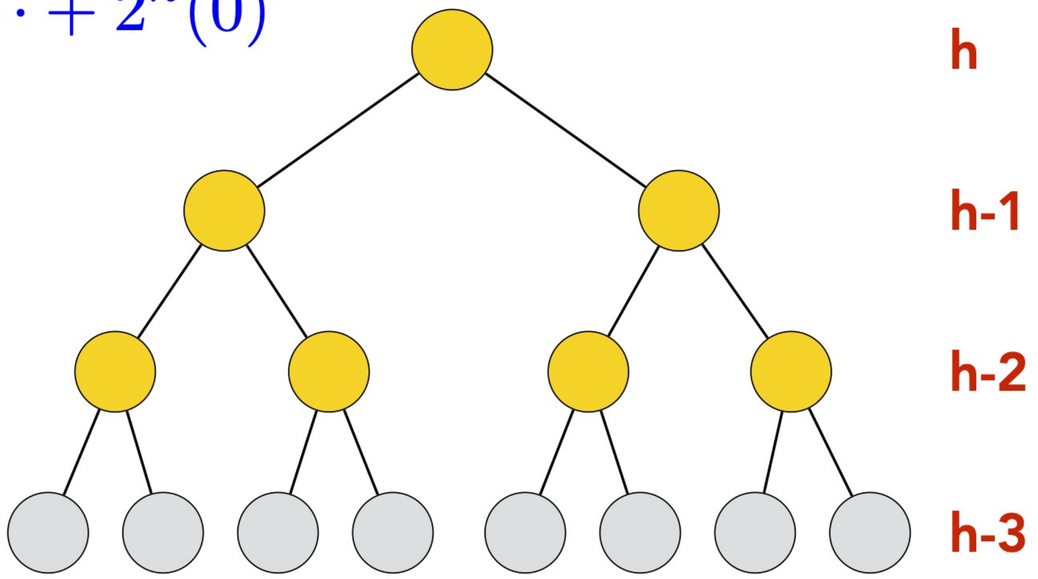
$$T(n) = h + 2(h-1) + 4(h-2) + 8(h-3) + \cdots + 2^h(0)$$

$$= \sum_{i=0}^{h} 2^i(h-i) = h\sum_{i=0}^{h} 2^i - \sum_{i=0}^{h} i2^i$$

$$= h\left[2^{h+1}-1\right] - \left[2 + (h+1-2)2^{h+1}\right]$$

$$= 2^{h+1} - h - 2 = n - (h+1)$$

$$= O(n)$$



Analysis

Cost is **sum of the heights** of all internal nodes

assume tree is full and complete, thus **$n=2^{h+1}-1$**

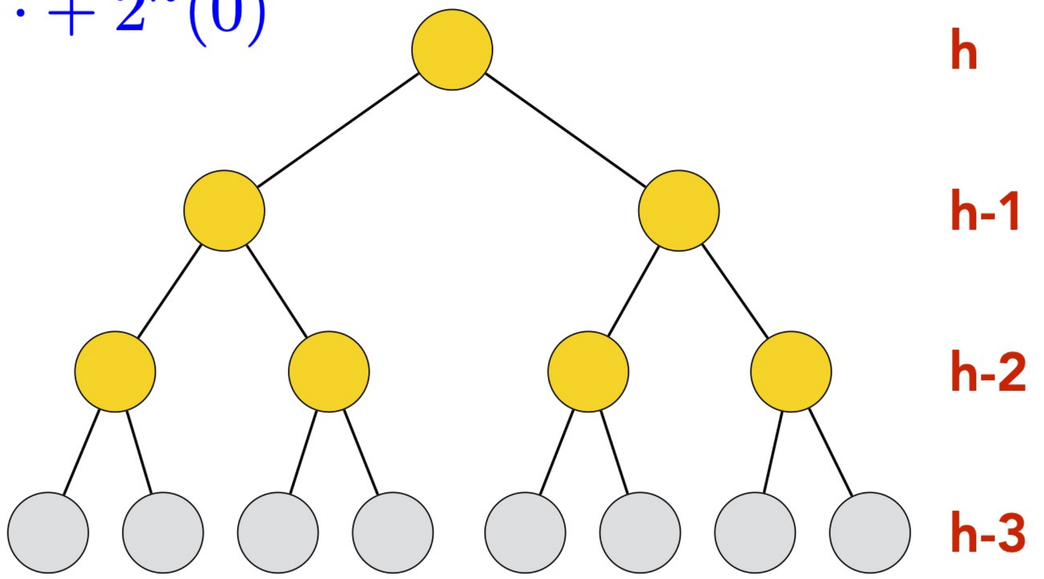
$$T(n) = h + 2(h-1) + 4(h-2) + 8(h-3) + \cdots + 2^h(0)$$

$$= \sum_{i=0}^{h} 2^i(h-i) = h\sum_{i=0}^{h} 2^i - \sum_{i=0}^{h} i2^i$$

$$= h\left[2^{h+1}-1\right] - \left[2 + (h+1-2)2^{h+1}\right]$$

$$= 2^{h+1} - h - 2 = n - (h+1)$$

$$= O(n)$$

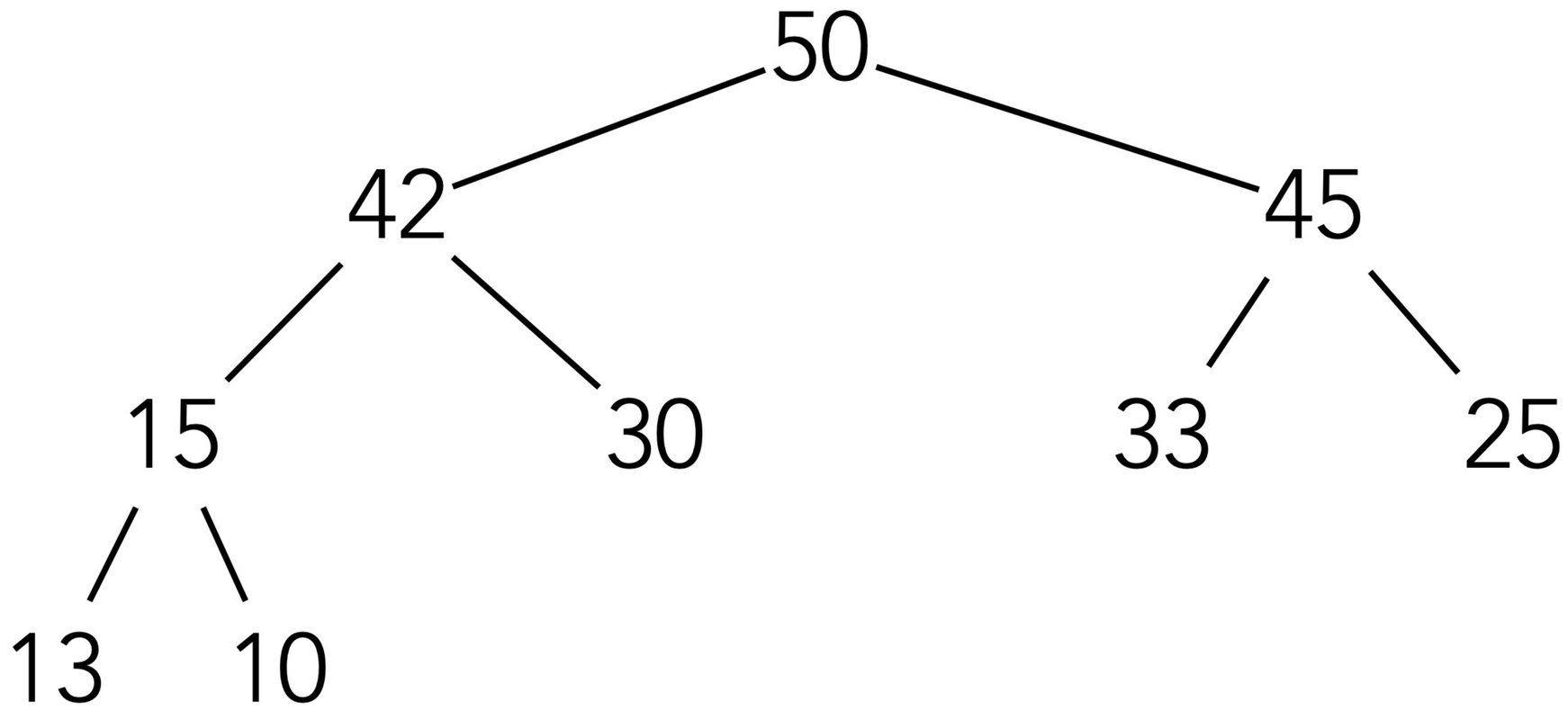


HeapSort Algorithm

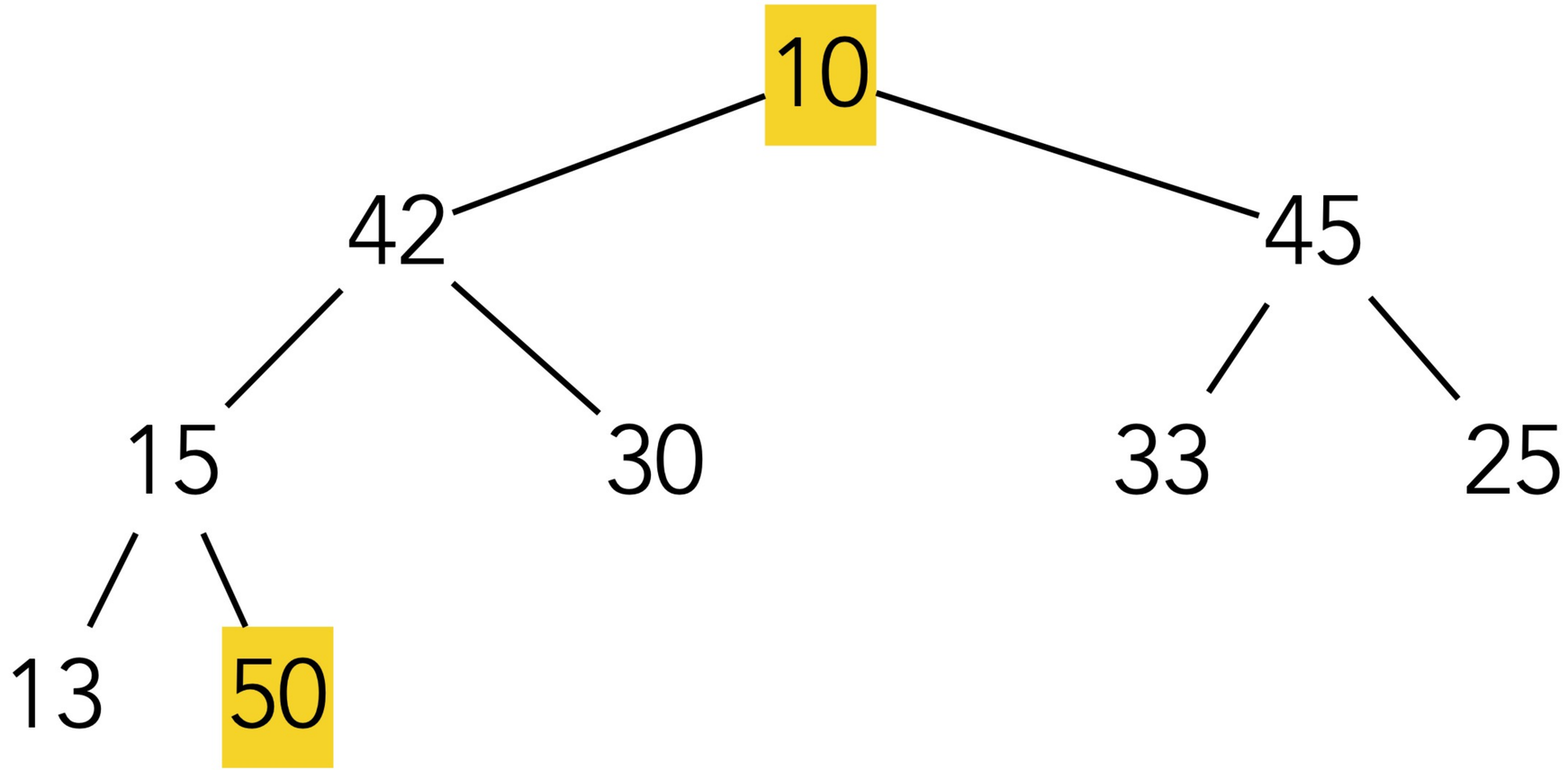
1. apply **buildHeap** to input array **A**
2. swap **first** with **last** element

   now the max element is at the end of **A**
3. decrement heap-size
4. apply **downHeap** to new **first** element
5. if heap is not empty go to step 2

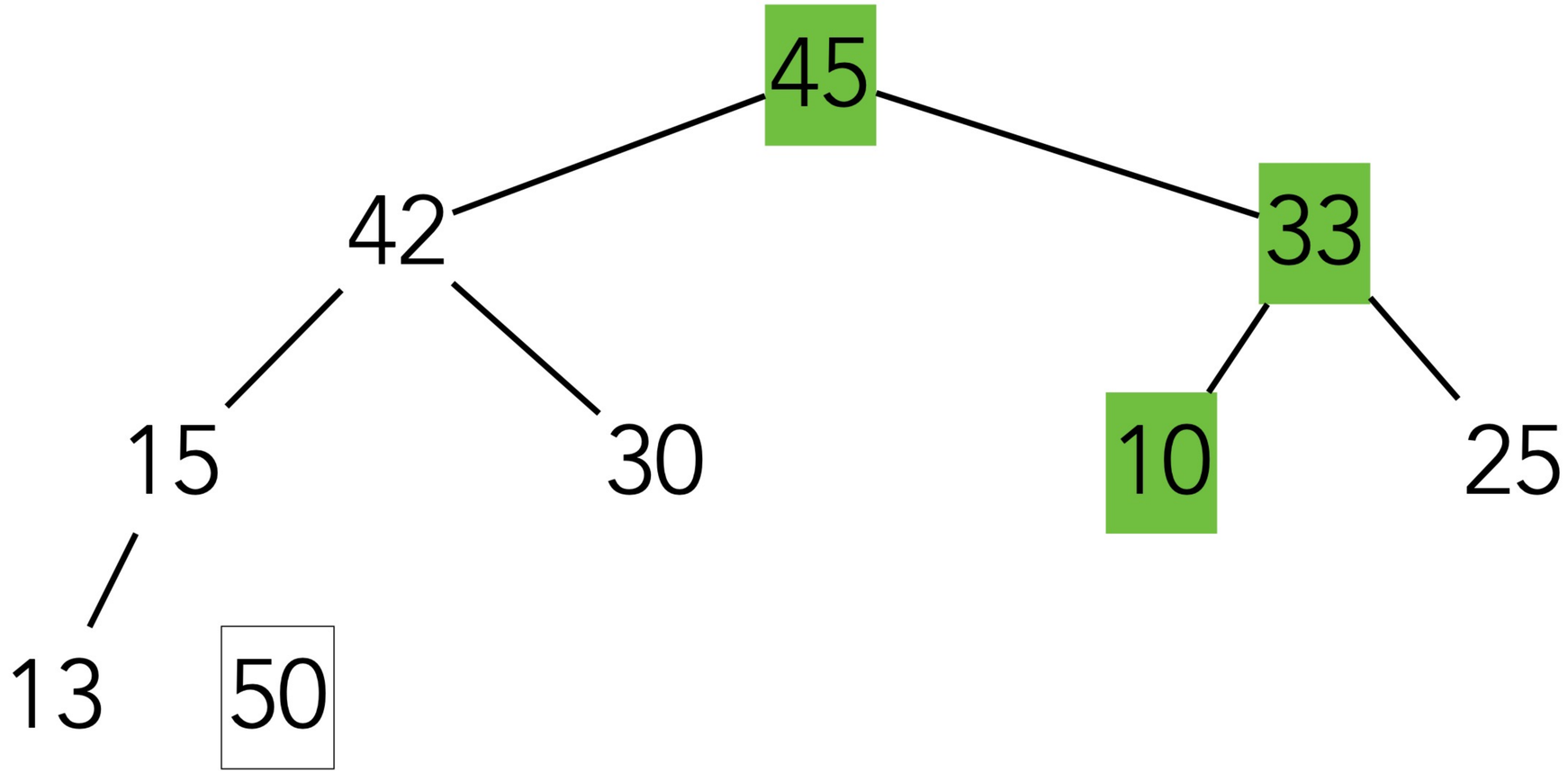
heapSort



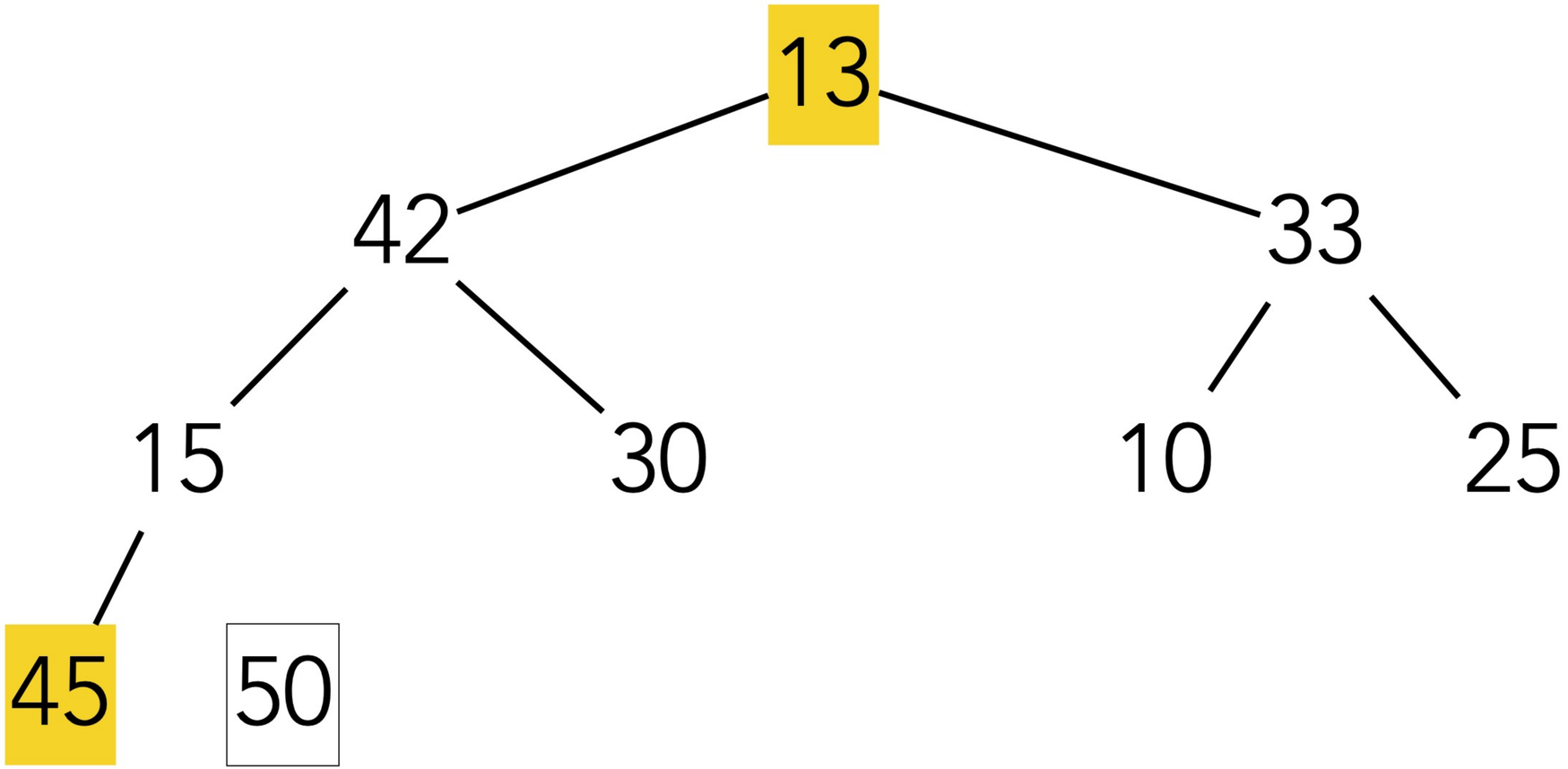
heapSort



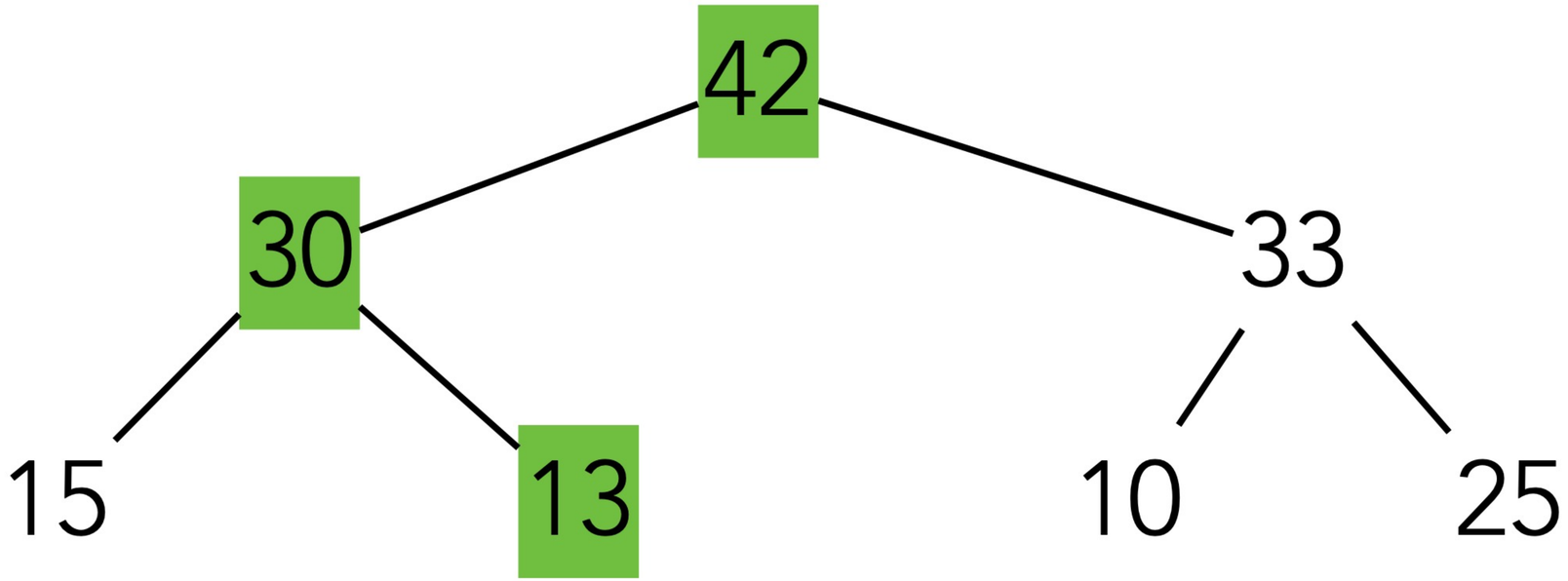
heapSort



heapSort

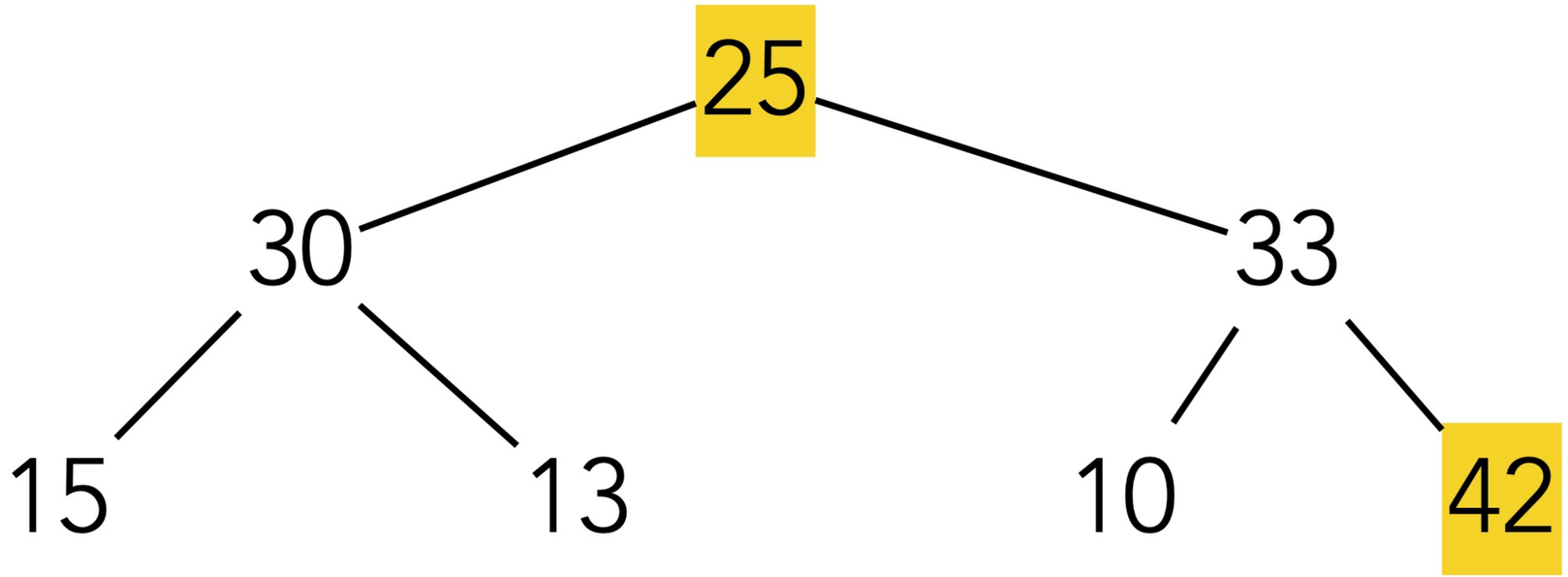


heapSort



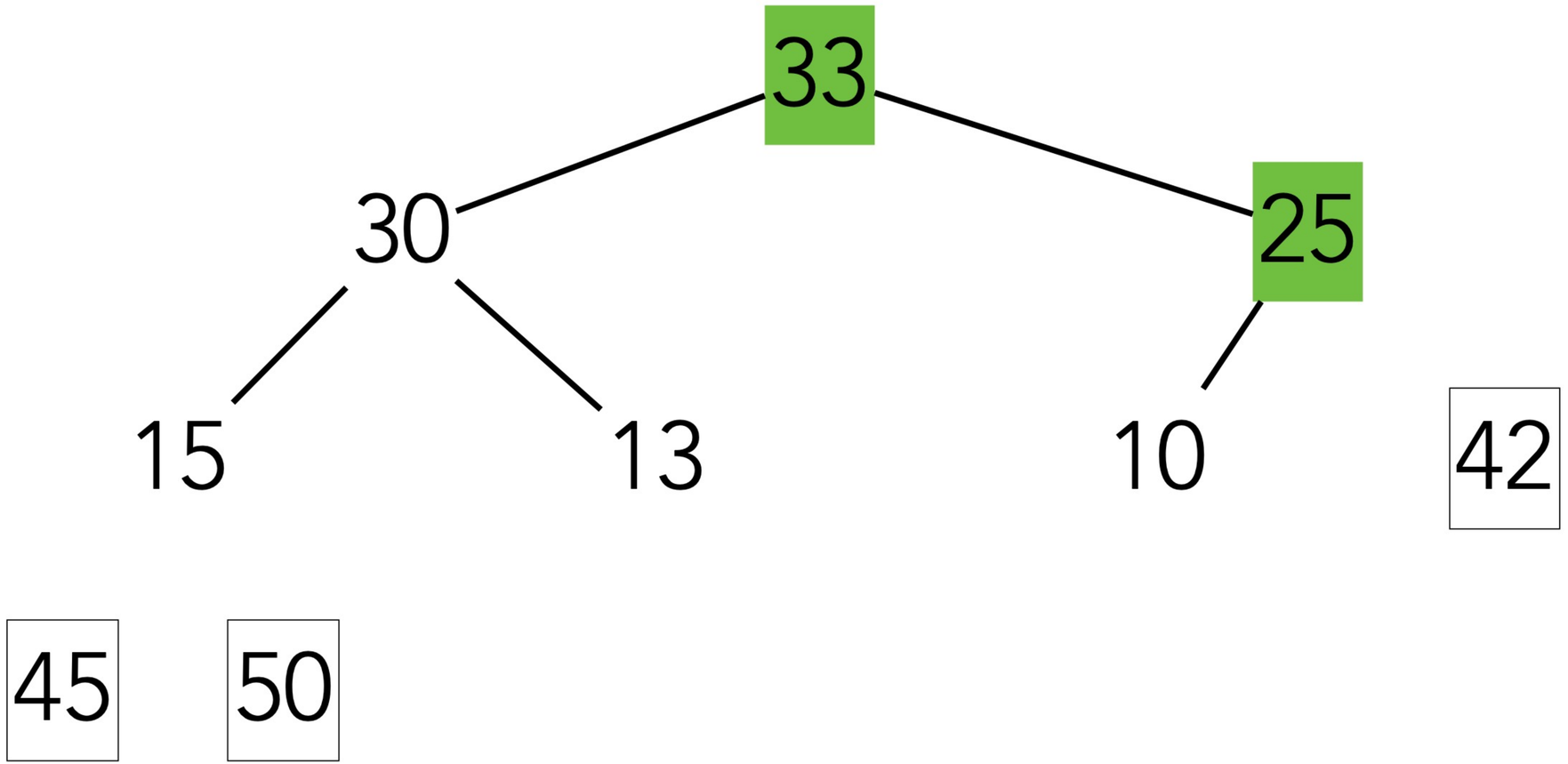
45 50

heapSort

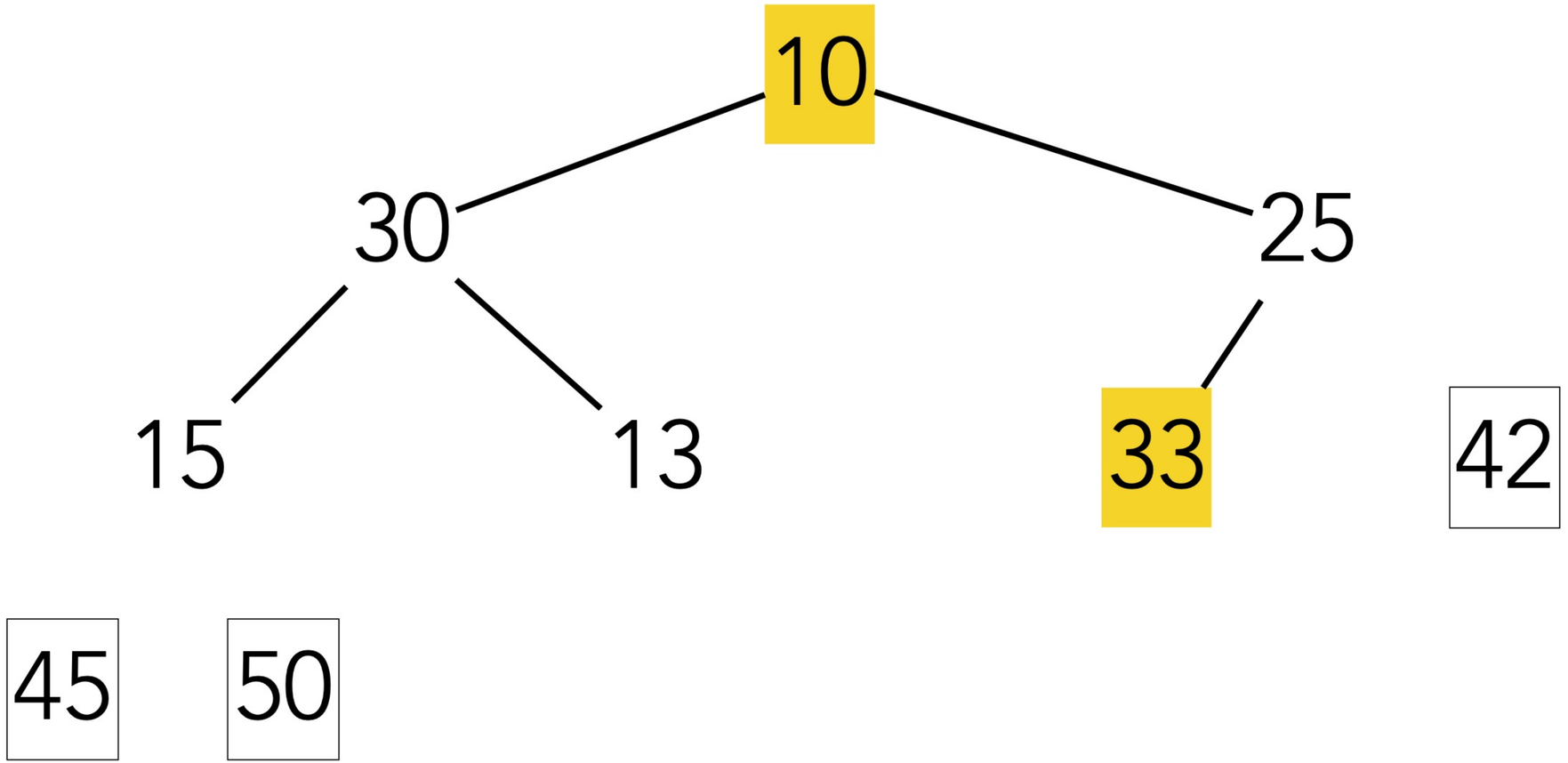


45 50

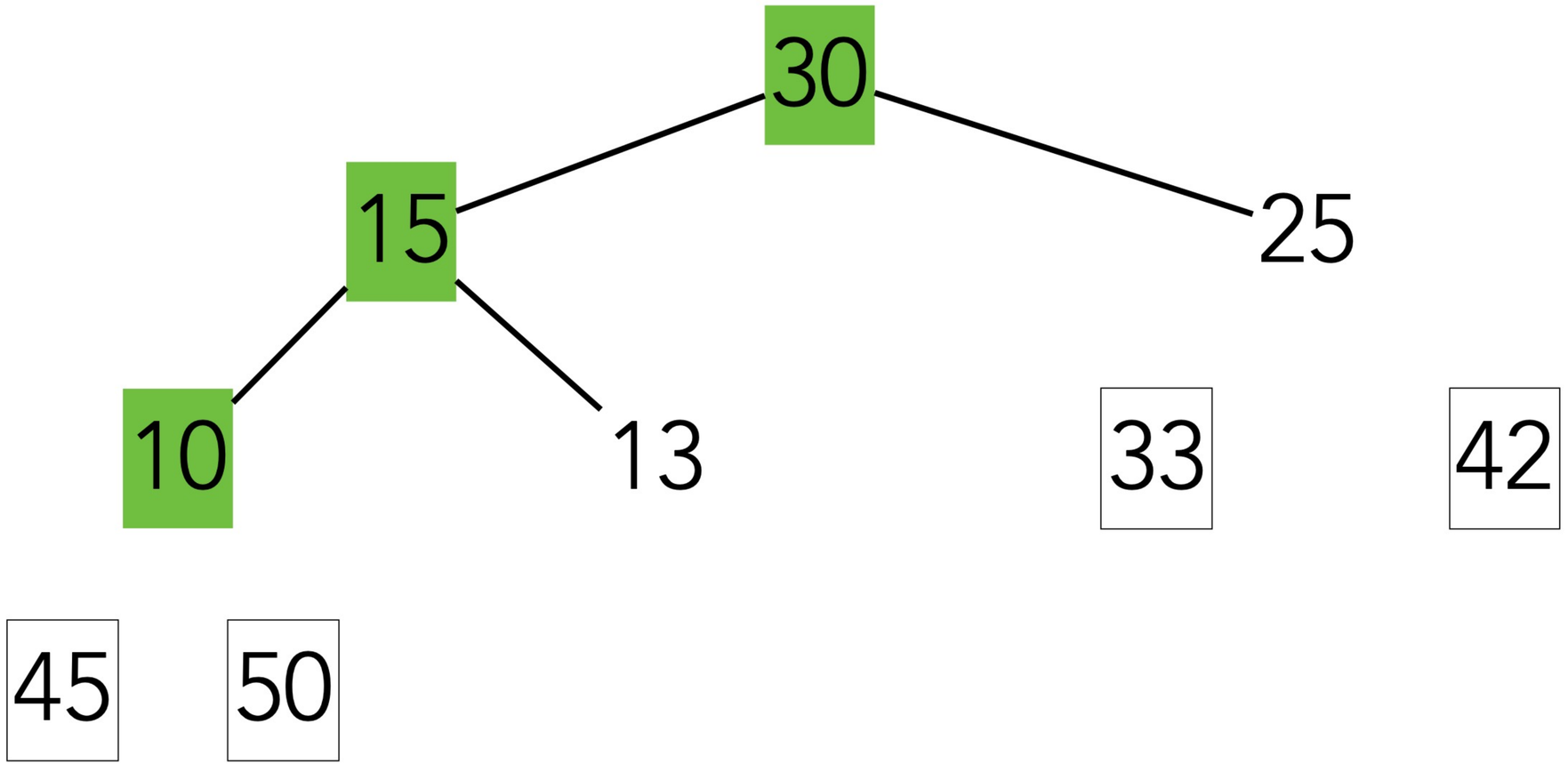
heapSort



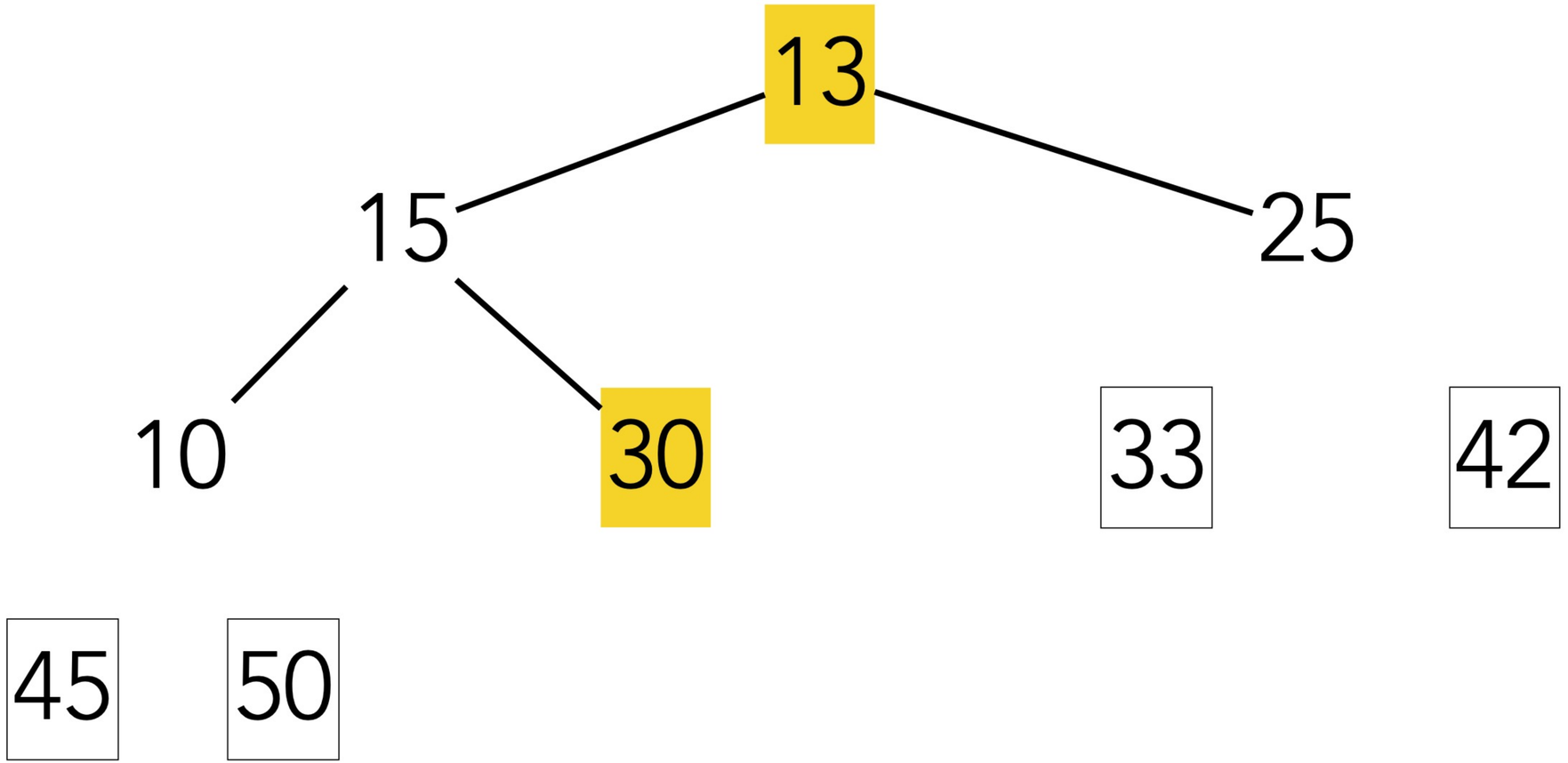
heapSort



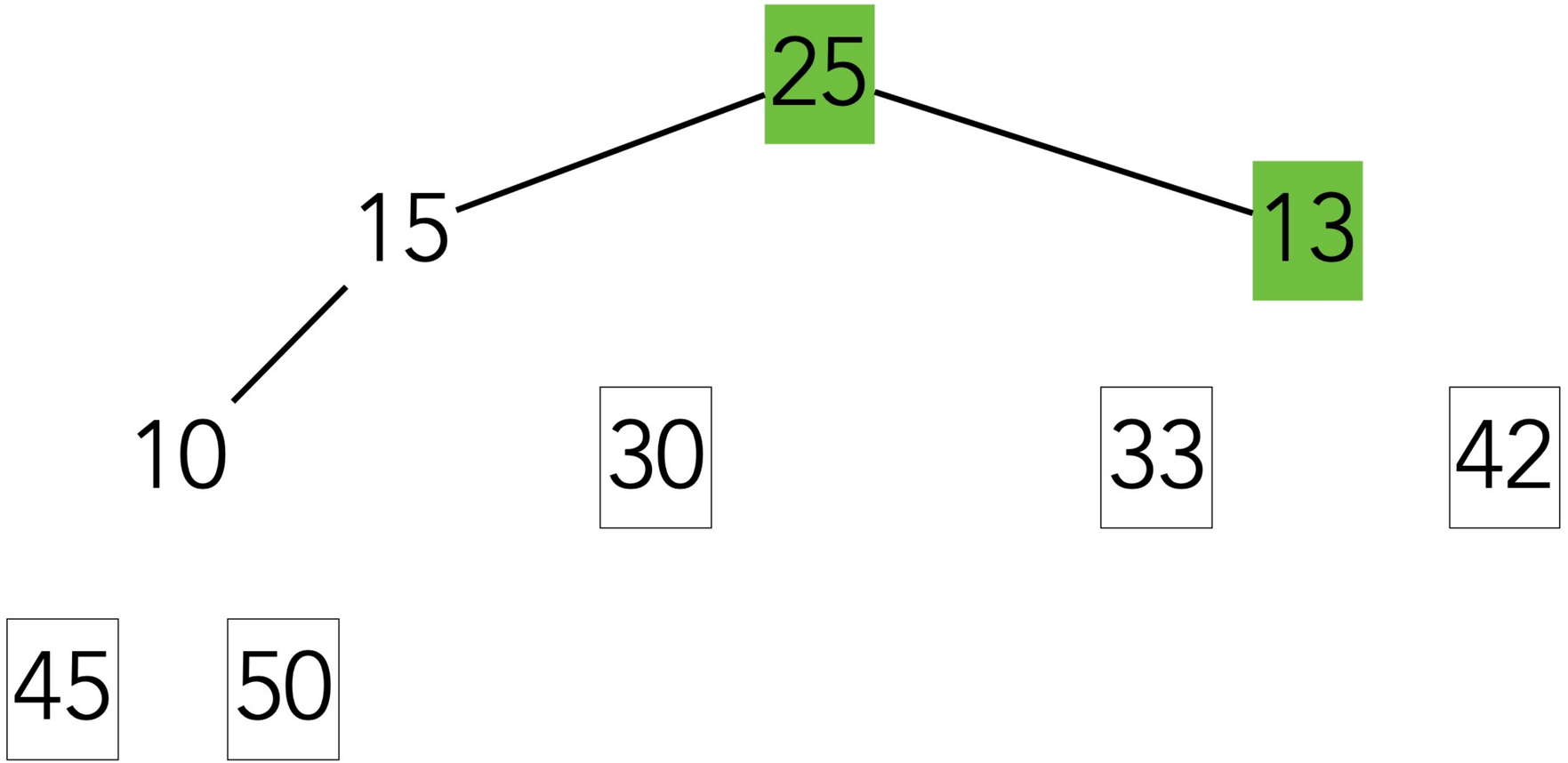
heapSort



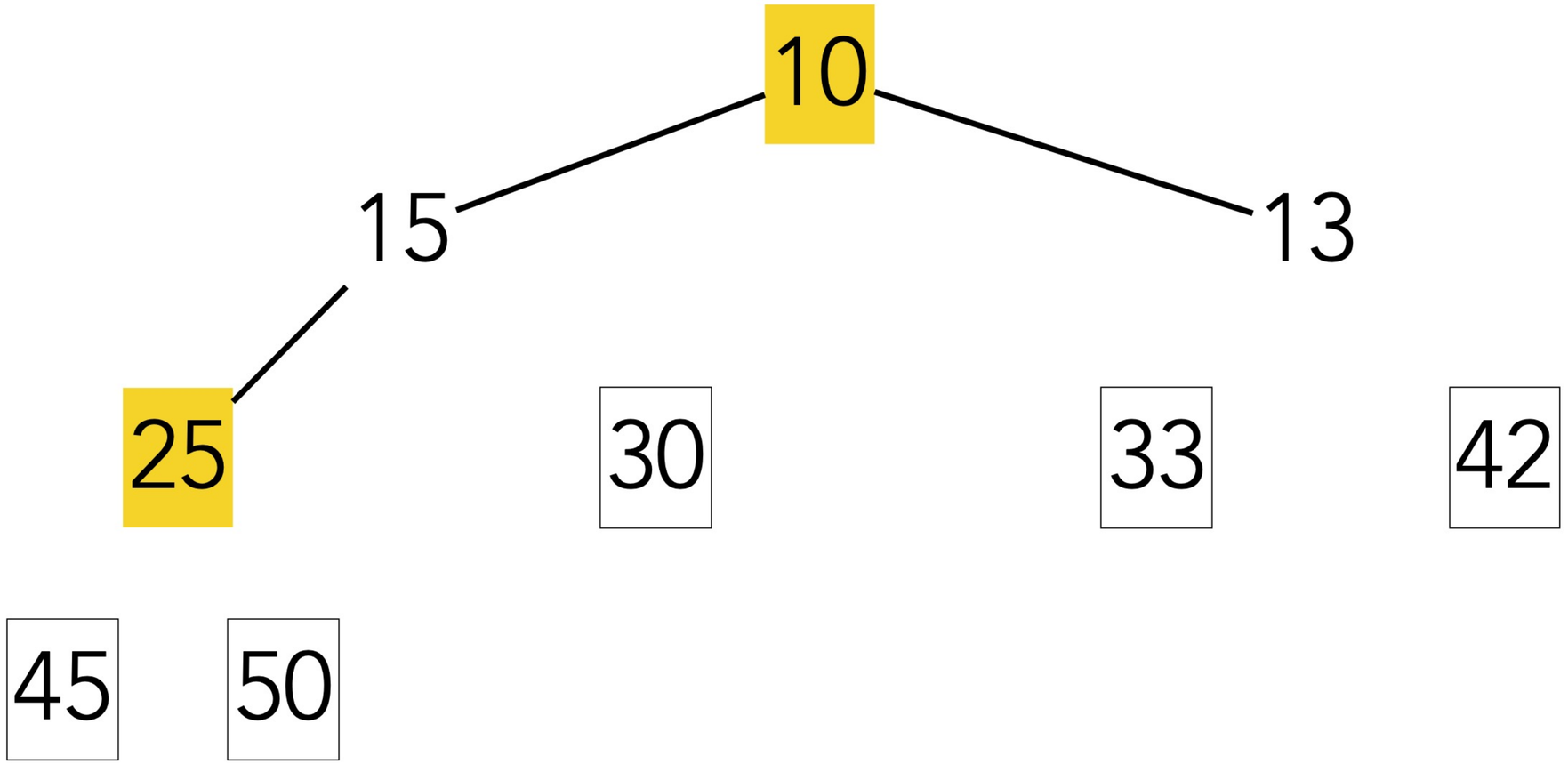
heapSort



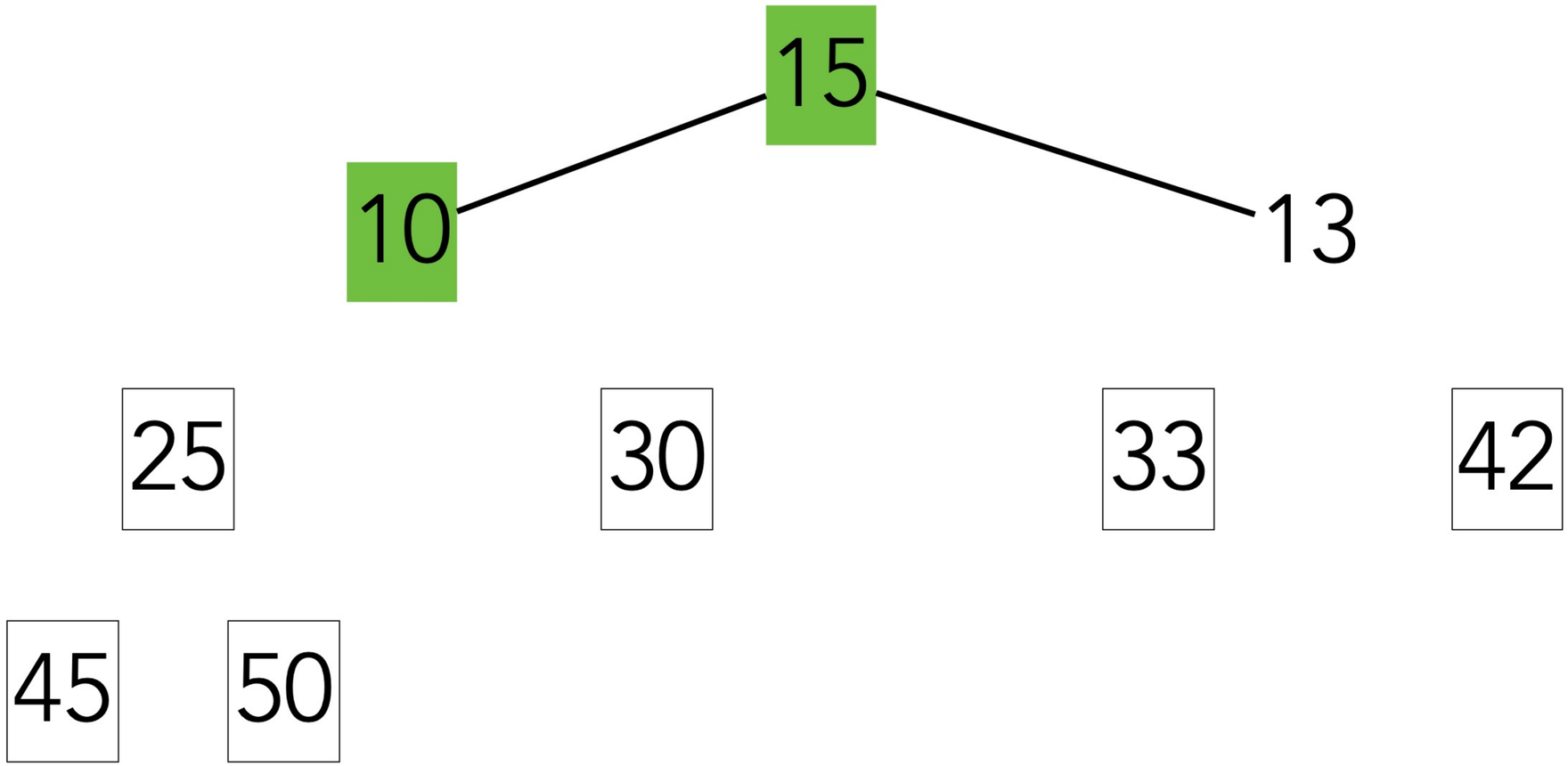
heapSort



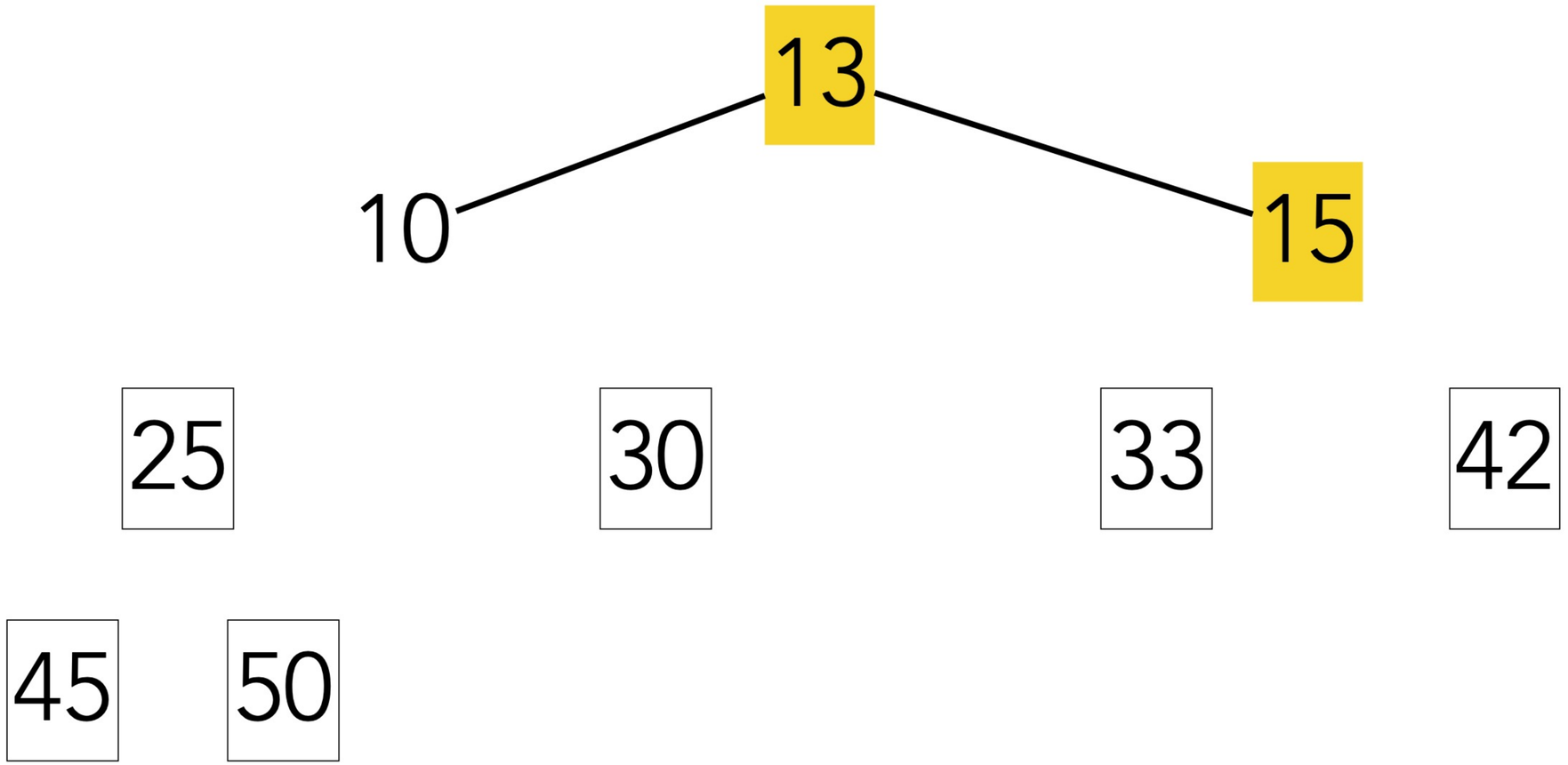
heapSort



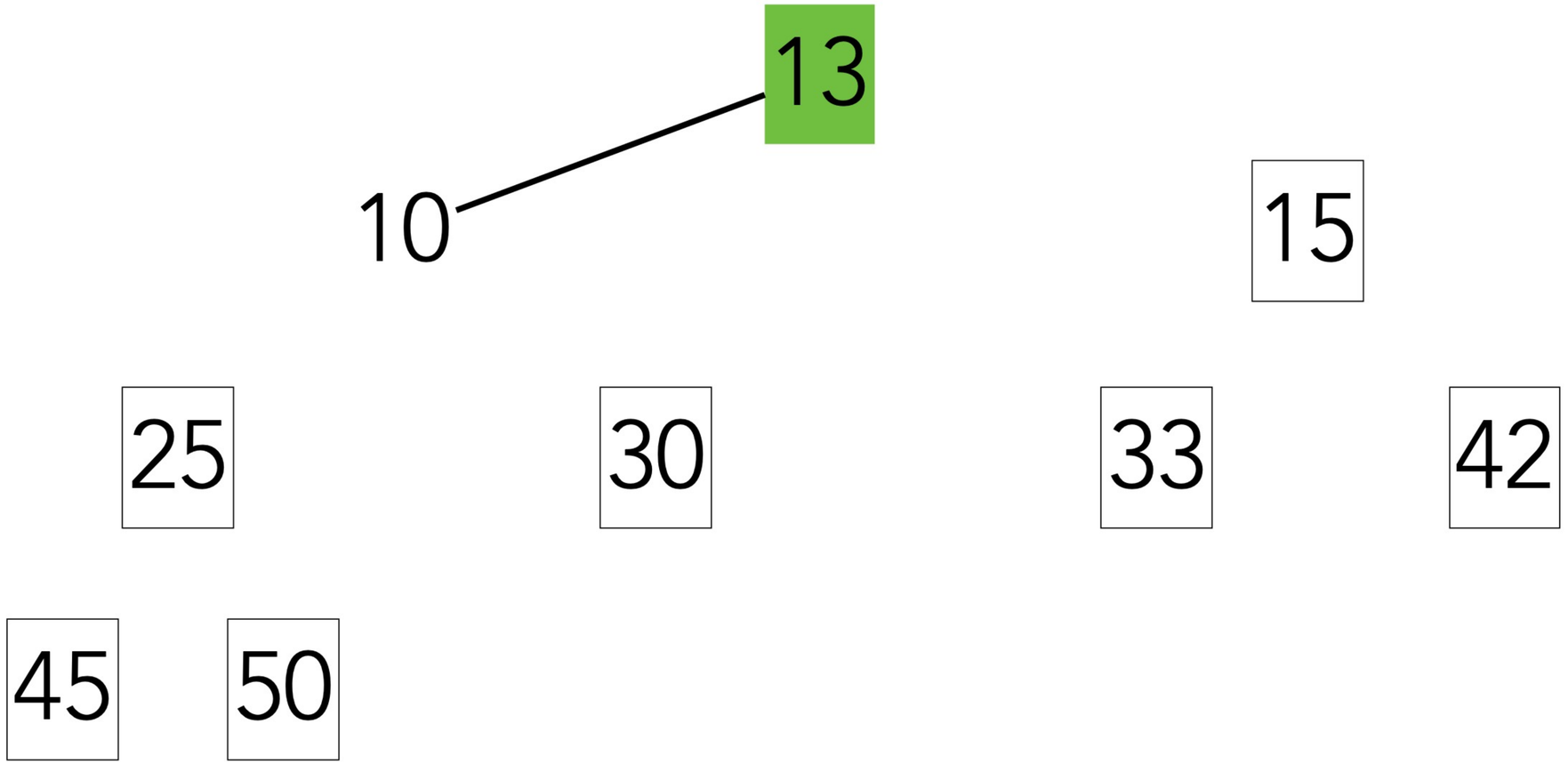
heapSort



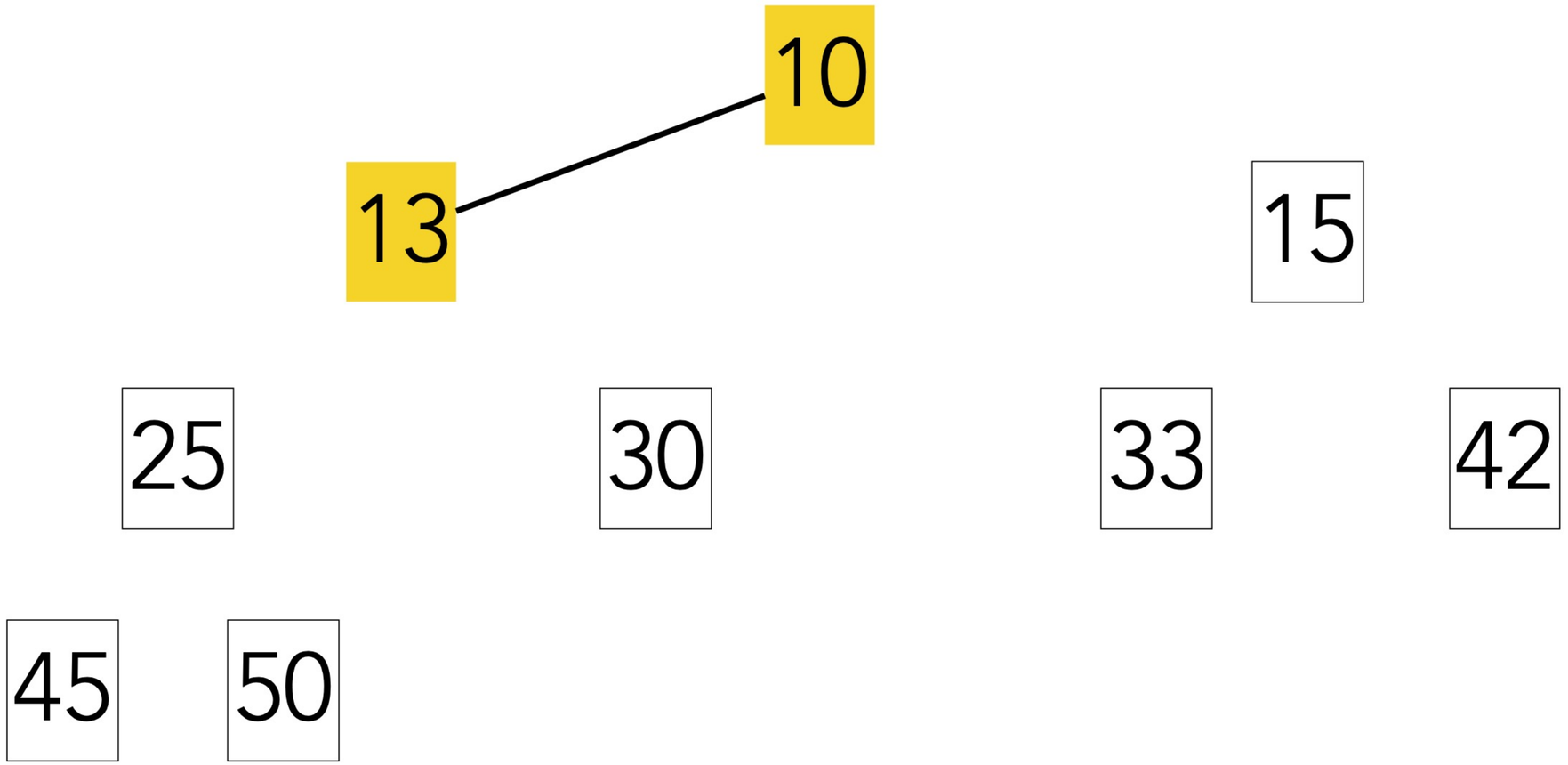
heapSort



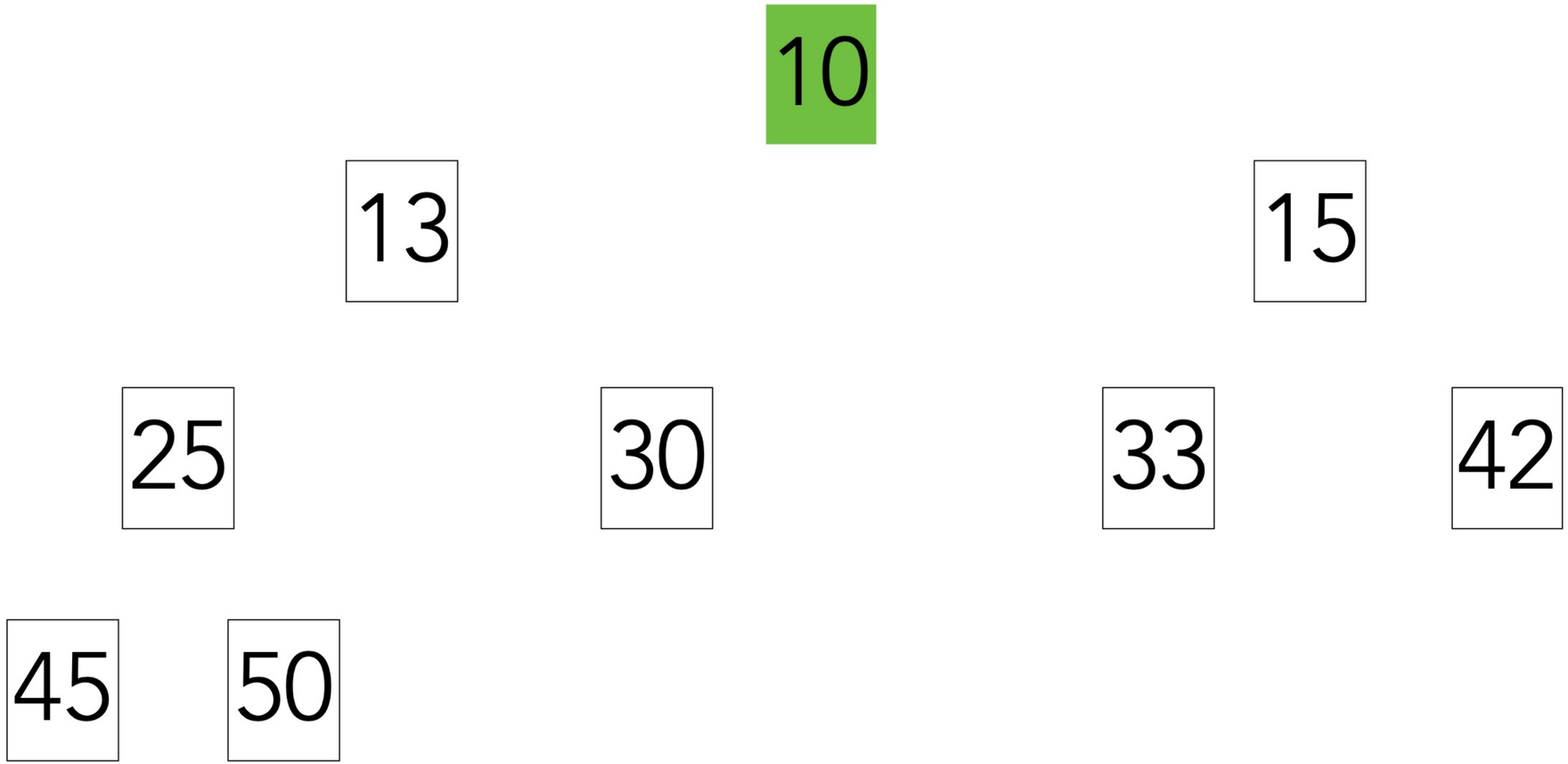
heapSort



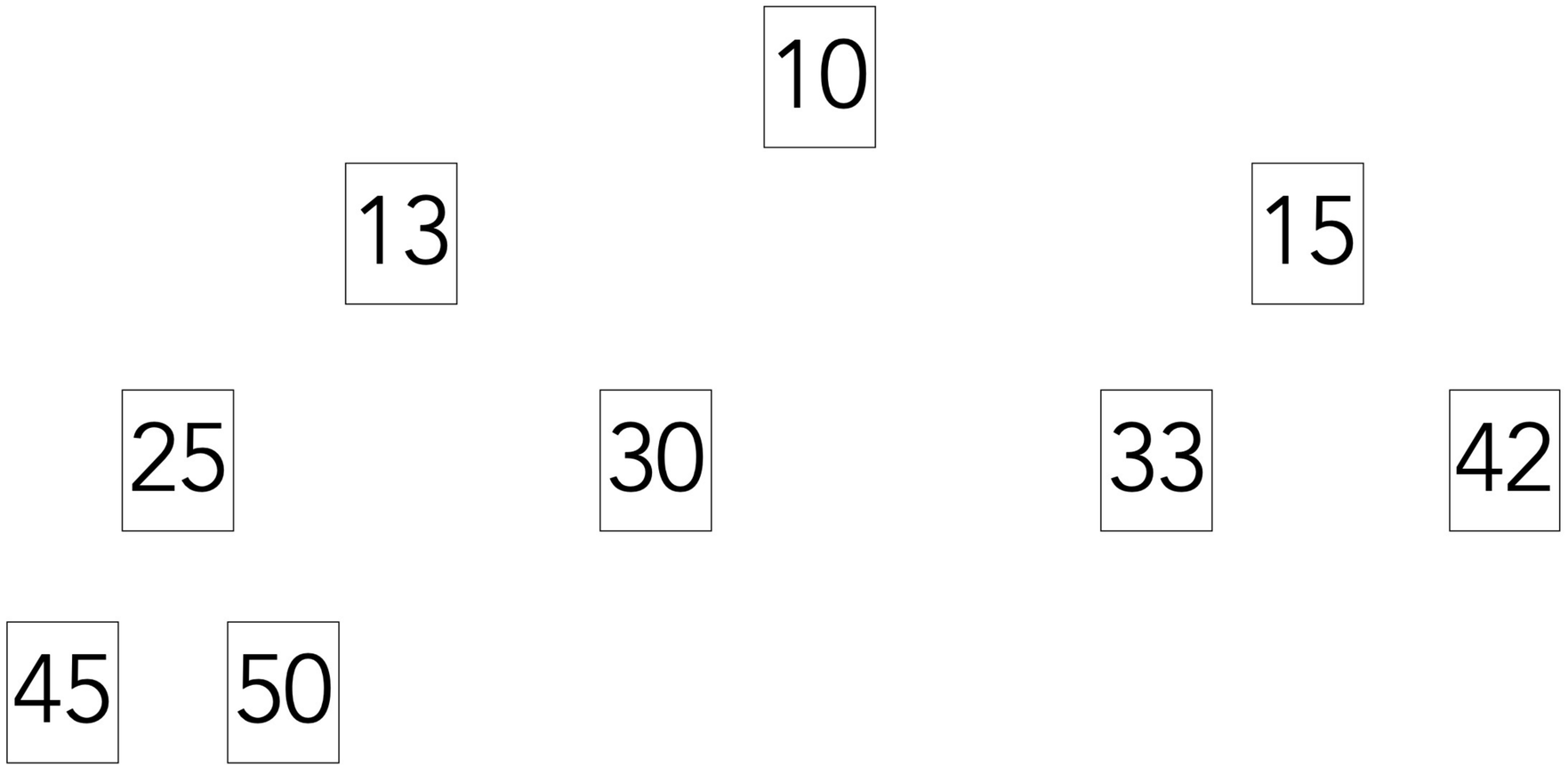
heapSort



heapSort



heapSort



Analysis?

buildHeap?

step 1

Analysis?

buildHeap?

step 1

$\Theta(n)$

Analysis?

buildHeap?

step 1

$\Theta(n)$

swap and downHeap?

steps 2-5

Analysis?

buildHeap?

step 1

$\Theta(n)$

swap and downHeap?

steps 2-5

$\Theta(n \log n)$