CHAPTER

11

Prospect Theory and the Brain

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OUTLINE

INTRODUCTION TO PROSPECT THEORY

Whether we like it or not, we face risk every day of our lives. From selecting a route home from work to selecting a mate, we rarely know in advance and with certainty what the outcome of our decisions will be. Thus, we are forced to make tradeoffs between the attractiveness (or unattractiveness) of potential outcomes and their likelihood of occurrence.

The lay conception of "risk" is associated with hazards that fill one with dread or are poorly understood (Slovic, 1987). Managers tend to see risk not as a gamble but as a "challenge to be overcome," and see risk as increasing with the magnitude of potential losses (e.g., March and Shapira, 1987). Decision theorists, in contrast, view risk as increasing with variance in the probability distribution of possible outcomes, regardless of whether a potential loss is involved. For example, a prospect that offers a 50–50 chance of paying $100 or nothing is more risky than a prospect that offers $50 for sure – even though the "risky" prospect entails no possibility of losing money.

Since Knight (1921), economists have distinguished decisions under *risk* from decisions under *uncertainty*. In decisions under risk, the decision maker knows with precision the probability distribution of possible outcomes, as when betting on the flip of a coin or entering a lottery with a known number of tickets.

In decisions under uncertainty, the decision maker is not provided such information but must assess the probabilities of potential outcomes with some degree of vagueness, as when betting on a victory by the home team or investing in the stock market.

In this chapter, we explore behavioral and neuroeconomic perspectives on decisions under risk. For simplicity we will confine most of our attention to how people evaluate simple prospects with a single non-zero outcome that occurs with known probability (e.g., a 50–50 chance of winning \$100 or nothing), though we will also mention extensions to multiple outcomes and to vague or unknown probabilities.

In the remainder of this section we provide a brief overview of economic models of decision making under risk, culminating in prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), the most influential descriptive account that has emerged to date. In subsequent sections, we provide an overview of various parameterizations of prospect theory's functions, and review methods for eliciting them. We then take stock of the early neuroeconomic studies of prospect theory, before providing some suggested directions for future research.

Historical Context

The origin of decision theory is traditionally traced to a correspondence between Pascal and Fermat in 1654 that laid the mathematical foundation of probability theory. Theorists asserted that decision makers ought to choose the option that offers the highest expected value (EV). Consider a prospect (x, p) that offers \$$x$ with probability p (and nothing otherwise):

$$\mathrm{EV} = px. \tag{11.1}$$

A decision maker is said to be "risk neutral" if he is indifferent between a gamble and its expected value; he is said to be "risk averse" if he prefers a sure payment to a risky prospect of equal or higher expected value; he is said to be "risk seeking" if he prefers a risky prospect to a sure payment of equal or higher expected value. Thus, expected value maximization assumes a neutral attitude toward risk. For instance, a decision maker who employs this rule will prefer receiving \$100 if a fair coin lands heads (and nothing otherwise) to a sure payment of \$49, because the expected value of the gamble (\$50 = .5 × \$100) is higher than the value of the sure thing (\$49).

Expected value maximization is problematic because it does not allow decision makers to exhibit risk aversion – it cannot explain, for example, why a person would prefer a sure \$49 over a 50–50 chance of receiving \$100 or nothing, or why anyone would purchase insurance. Swiss mathematician Daniel Bernoulli (1738) advanced a solution to this problem when he asserted that people do not evaluate options by their objective value but rather by their utility or "moral value." Bernoulli observed that a particular amount of money (say, \$1000) is valued more when a person is poor (wealth level W_1) than when he is wealthy (W_2) and therefore marginal utility decreases (from U_1 to U_2) as wealth increases (see Figure 11.1a). This gives rise to a utility function that is concave over states of wealth. In Bernoulli's model, decision makers choose the option with highest expected utility (EU):

$$EU = pu(x) \tag{11.2}$$

where $u(x)$ represents the utility of obtaining outcome x. For example, a concave utility function ($u''(x) < 0$) implies that the utility gained by receiving

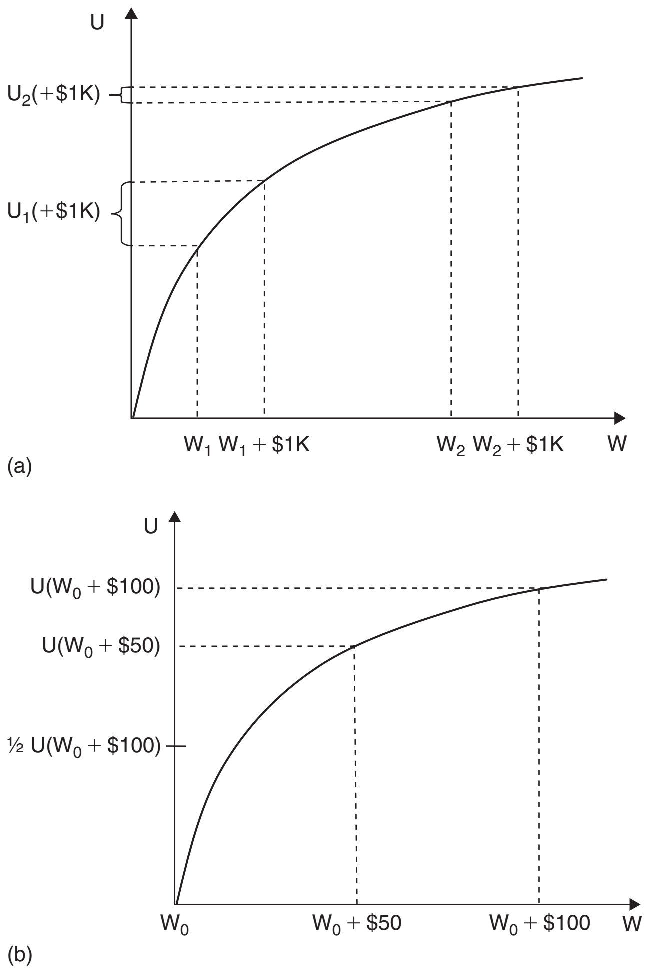

FIGURE 11.1 (a) A representative utility function over states of wealth illustrating the notion of diminishing marginal utility. (b) A representative utility function over states of wealth illustrating risk aversion for gains at an initial state of wealth W_0.

$50 is more than half the utility gained by receiving $100, and therefore a decision maker with such a utility function should prefer $50 for sure to a .5 probability of receiving $100 (see Figure 11.1b)

Axiomatization of Expected Utility

Expected utility became a central component of economic theory when von Neumann and Morgenstern (1947) articulated a set of axioms that are both necessary and sufficient for representing a decision-maker's choices by the maximization of expected utility (see also Jensen, 1967). Consider chance lotteries L_1 and L_2 that are known probability distributions over outcomes. For instance, L_1 might offer a .5 chance of $100 and a .5 chance of 0; L_2 might offer $30 for sure. Consider also a binary preference relation $\succeq$ over the set of all possible lotteries L; thus $L_1 \succeq L_2$ is interpreted as "L_1 is preferred or equivalent to L_2." Now consider the following axioms:

1. **Completeness**: People have preferences over all lotteries. Formally, for any two lotteries L_1 and L_2 in L, either $L_1 \succeq L_2$, $L_2 \succeq L_1$, or both.
2. **Transitivity**: People rank lotteries in a consistent manner. Formally, for any three lotteries L_1, L_2, and L_3, if $L_1 \succeq L_2$, and $L_2 \succeq L_3$, then $L_1 \succeq L_3$.
3. **Continuity**: For any three lotteries, some mixture of the best and worst lotteries is preferred to the intermediate lottery and *vice versa*. Formally, for any three lotteries $L_1 \succeq L_2 \succeq L_3$ there exist α, $\beta \in (0,1)$ such that $\alpha L_1 + (1-\alpha) L_3 \succeq L_2$, and $L_2 \succeq \beta L_1 + (1-\beta) L_3$.
4. **Substitution** (a.k.a. "independence"): If a person prefers one lottery to another, then this preference should not be affected by a mixture of both lotteries with a common third lottery. Formally, for any L_1, L_2, and L_3, and any $\alpha \in (0, 1)$, $L_1 \succeq L_2$ if and only if $\alpha L_1 + (1-\alpha) L_3 \succeq \alpha L_2 + (1-\alpha) L_3$.

Von Neumann and Morgenstern proved that these axioms are both necessary and sufficient to represent a decision-maker's decisions by the maximization of expected utility. That is,

$$L_1 \succeq L_2 \text{ if and only if } \sum_{i=1}^{n} p_i^1 u(x_i^1) \geq \sum_{j=1}^{m} p_j^2 u(x_j^2),$$

where superscripts indicate corresponding lottery numbers.

The completeness and transitivity axioms establish that decision makers can (weakly) order their preferences, which is necessary for using a unidimensional scale. The continuity axiom is necessary to establish a continuous tradeoff between probability and outcomes. The substitution axiom is necessary to establish that utilities of outcomes are weighted by their respective probabilities.

A more general formulation of expected utility theory that extended the model from risk to uncertainty (Savage, 1954) relies on a related axiom known as the *sure-thing principle*: If two options yield the same consequence when a particular event occurs, then a person's preferences among those options should not depend on the particular consequence (i.e., the "sure thing") or the particular event that they have in common. To illustrate, consider a game show in which a coin is flipped to determine where a person will be sent on vacation. Suppose the contestant would rather to go to Atlanta if the coin lands heads and Chicago if it lands tails (a, H; c, T) than go to Boston if the coin lands heads and Chicago if it lands tails (b, H; c, T). If this is the case, he should also prefer to go to Atlanta if the coin lands heads and Detroit (or any other city for that matter) if the coin lands tails (a, H; d, T), to Boston if it lands heads and Detroit if it lands tails (b, H; d, T).

Violations of Substitution and the Sure thing Principle

It was not long before the descriptive validity of expected utility theory and its axioms were called into question. One of the most powerful challenge has come to be known as the "Allais paradox" (Allais, 1953; Allais and Hagen, 1979). The following version was presented by Kahneman and Tversky (1979)[1].

Decision 1: Choose between (A) an 80% chance of $4000; (B) $3000 for sure.
Decision 2: Choose between (C) a 20% chance of $4000; (D) a 25% chance of $3000.

Most respondents chose (B) over (A) in the first decision and (C) over (D) in the second decision, which violates the substitution axiom. To see why, note that $C = 1/4\ A$ and $D = 1/4\ B$ (with a 3/4 chance of receiving 0 in both cases) so that according to the substitution axiom a decision maker should prefer C over D if and only if he prefers A to B. This systematic violation of substitution is known as the "common ratio effect."

A related demonstration from Allais was adapted by Kahneman and Tversky (1979) as follows:

Decision 3: Choose between (E) a 33% chance of $2500, a 66% chance of $2400, and a 1% chance of nothing; (F) $2400 for sure.
Decision 4: Choose between (G) a 33% chance of $2500; (H) a 34% chance of $2400.

[1] Kahneman & Tversky's version was originally denominated in Israeli Pounds.

TABLE 11.1 The Allais common consequence effect represented using a lottery with numbered tickets

Option	Ticket numbers 1–33	34	35–100
E	2500	0	2400
F	2400	2400	2400
G	2500	0	0
H	2400	2400	0

In this case most people prefer option (F) to option (E) in Decision 3, but they prefer option (G) to option (H) in Decision 4, which violates the sure-thing principle. To see why, consider options (E) through (H) as being payment schemes attached to different lottery tickets that are numbered consecutively from 1 to 100 (see Table 11.1). Note that one can transform options (E) and (F) into options (G) and (H), respectively, merely by replacing the common consequence (receive \$2400 if the ticket drawn is 35–100) with a new common consequence (receive \$0 if the ticket drawn is 35–100). Thus, according to the sure-thing principle, a person should favor option (G) over option (H) if and only if he prefers option (E) to option (F), and the dominant pattern of preferences violates this axiom. This violation of the sure-thing principle is known as the "common consequence effect."

Both the common ratio effect and common consequence effect resonate with the notion that people are more sensitive to differences in probability near impossibility and certainty than in the intermediate range of the probability scale. Thus, people typically explain their choice in Decision (1) as a preference for certainty over a slightly smaller prize that entails a possibility of receiving nothing; meanwhile, they explain their choice in Decision (2) as a preference for a higher possible prize given that the between a difference in probability of .20 and .25 is not very large. Likewise, people explain their choice in Decision (3) as a preference for certainty over a possibility of receiving nothing; meanwhile, they explain their choice in Decision (2) as a preference for a higher possible prize given that the difference between a probability of .33 and .34 seems trivial.

The Fourfold Pattern of Risk Attitudes

The Allais paradox is arguably the starkest and most celebrated violation of expected utility theory. In the years since it was articulated, numerous studies of decision under risk have shown that people often violate the principle of risk aversion that underlies much economic analysis. Table 11.2 illustrates a common pattern of risk aversion and risk seeking exhibited by participants in studies of Tversky and Kahneman (1992). Let $C(x, p)$ be the *certainty equivalent* of the prospect (x, p) that offers to pay \$$x$ with probability p (i.e., the sure payment that is deemed equally attractive to the risky prospect). The upper left-hand entry in Table 11.2 shows that the median participant was indifferent between receiving \$14 for sure and a 5% chance of gaining \$100. Because the expected value of the prospect is only \$5, this observation reflects risk seeking behavior.

TABLE 11.2 The fourfold pattern of risk attitudes (a); risk aversion for mixed (gain–loss) gambles (b) (both adapted from Tversky and Kahneman, 1992)

(a) $C(x, p)$ is the median certainty equivalent of the prospect that pays \$$x$ with probability p

	Gains	Losses
Low probability	C (\$100, .05) = \$14 *Risk seeking*	C (−\$100, .05) = − \$8 *Risk aversion*
High probability	C (\$100, .95) = \$78 *Risk aversion*	C (−\$100, .95) = − \$84 *Risk-seeking*

(b) Median gain amounts for which participants found 50–50 mixed gambles equally attractive to receiving nothing, listed fixed by loss amount

Gain	Loss	Ratio
61	25	2.44
101	50	2.02
202	100	2.02
280	150	1.87

Table 11.2a reveals a fourfold pattern of risk attitudes: risk seeking for low-probability gains and high-probability losses, coupled with risk aversion for high-probability gains and low-probability losses. Choices consistent with this fourfold pattern have been observed in several studies (Fishburn and Kochenberger, 1979; Kahneman and Tversky, 1979; Hershey and Schoemaker, 1980; Payne *et al.*, 1981). Risk seeking for low-probability gains may contribute to the attraction of gambling, whereas risk aversion for low-probability losses may contribute to the attraction of insurance. Risk aversion for high-probability gains may contribute to the preference for certainty, as in the Allais (1953) problem, whereas risk seeking for high-probability losses is consistent with the common tendency to undertake risk to avoid facing a sure loss.

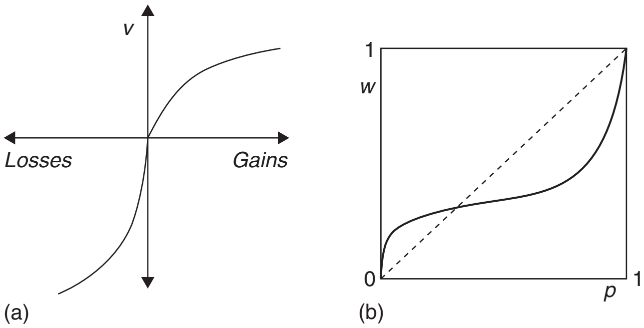

FIGURE 11.2 Representative value and weighting functions from prospect theory. (a) A hypothetical prospect theory value function illustrating concavity for gains, convexity for losses, and a steeper loss than gain limb. (b) A hypothetical prospect theory weighting function illustrating its characteristics inverse-S shape, the tendency to overweight low probabilities and underweight moderate to large probabilities, and the tendency for weights of complementary probabilities to sum to less than 1.

Prospect Theory

The Allais paradox and the fourfold pattern of risk attitudes are accommodated neatly by prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), the leading behavioral model of decision making under risk, and the major work for which psychologist Daniel Kahneman was awarded the 2002 Nobel Prize in economics.

According to prospect theory, the value V of a simple prospect that pays $\$x$ with probability p (and nothing otherwise) is given by:

$$V(x, p) = w(p)\, v(x) \tag{11.3}$$

where v measures the subjective value of the consequence x, and w measures the impact of probability p on the attractiveness of the prospect (see Figure 11.2).

Value Function

Prospect theory replaces the utility function $u(\cdot)$ over states of wealth with a value function $v(\cdot)$ over gains and losses relative to a reference point, with $v(0) = 0$. According to prospect theory, the value function $v(\cdot)$ exhibits the psychophysics of diminishing sensitivity. That is, the marginal impact of a change in value diminishes with the distance from a relevant reference point. For monetary outcomes, the *status quo* generally serves as the reference point distinguishing losses from gains, so that the function is concave for gains and convex for losses (see Figure 11.2a). Concavity for gains contributes to risk aversion for gains, as with the standard utility function (Figure 11.1). Convexity for losses, on the other hand, contributes to risk seeking for losses. For instance, the disvalue of losing \$50 is more than half the disvalue of losing \$100, which will contribute to a preference for the gamble over the sure loss. This tendency to be risk averse for moderate-probability gains and risk seeking for moderate-probability losses may contribute to the "disposition effect," in which investors have a greater tendency to sell stocks in their portfolios that have risen rather than fallen since purchase (Odean, 1998; but see also Barberis and Xiong, 2006).

The prospect theory value function is steeper for losses than gains – a property known as *loss aversion*. People typically require more compensation to give up a possession than they would have been willing to pay to obtain it in the first place (see, for example, Kahneman *et al*., 1990). In the context of decision under risk, loss aversion gives rise to risk aversion for mixed (gain–loss) gambles so that, for example, people typically reject a gamble that offers a .5 chance of gaining \$100 and a .5 chance of losing \$100, and require at least twice as much "upside" as "downside" to accept such gambles (see Table 11.2b). In fact, Rabin (2000) showed that a concave utility function over states of wealth cannot explain the normal range of risk aversion for mixed gambles, because this implies that a decision maker who is mildly risk averse for small-stakes gambles over a range of states of wealth must be unreasonably risk averse for large-stakes gambles. This tendency to be risk averse for mixed prospects has been used by Benartzi and Thaler (1995) to explain why investors require a large premium to invest in stocks rather than bonds (the "equity premium puzzle"): because of the higher volatility of stocks than bonds, investors who frequently check their returns are more likely to experience a loss in nominal value of their portfolios if they are invested in stocks than bonds (see also Barberis *et al*., 2001).

It is important to note that loss aversion, which gives rise to risk aversion for mixed (gain–loss) prospects (e.g., most people reject a 50–50 chance to gain \$100 or lose \$100) should be distinguished from convexity of the value function for losses, which gives rise to risk-seeking for pure loss prospects (e.g., most people prefer a 50–50 chance to lose \$100 or nothing, to losing \$50 for sure).

Weighting Function

In prospect theory, the value of an outcome is weighted not by its probability but instead by a decision weight, $w(\cdot)$, that represents the impact of the relevant probability on the valuation of the prospect (see equation 11.3). Decision weights are normalized so that $w(0) = 0$ and $w(1) = 1$. Note that w need not be interpreted as a measure of subjective belief – a person may believe that the probability of a fair coin landing heads is one-half, but afford this event a weight of less than one-half in the evaluation of a prospect.

Just as the value function captures diminishing sensitivity to changes in the number of dollars gained or lost, the weighting function captures diminishing sensitivity to changes in probability. For probability, there are two natural reference points: impossibility and certainty. Hence, diminishing sensitivity implies an inverse-S shaped weighting function that is concave near zero and convex near one, as depicted in Figure 11.2b. It can help explain the fourfold pattern of risk attitudes (Table 11.2a), because moderate to high probabilities are underweighted (which reinforces the pattern of risk aversion for gains and risk seeking for losses implied by the shape of the value function) and low probabilities are overweighted (which reverses the pattern implied by the value function and leads to risk seeking for gains and risk aversion for losses).

To appreciate the intuition underlying how the value- and weighting-functions contribute to the fourfold pattern, refer to Figure 11.2. Informally, the reason that most participants in Tversky and Kahneman's (1992) sample would rather have a .95 chance of $100 than $77 for sure is partly because they find receiving $77 nearly as appealing as receiving $100 (i.e., the slope of the value function decreases with dollars gained), and partly because a .95 chance "feels" like a lot less than a certainty (i.e., the slope of the weighting function is high near one). Likewise, most participants would rather face a .95 chance of losing $100 than pay $85 for sure is partly because paying $85 is almost as painful as paying $100, and partly because a .95 chance feels like it is much less than certain. On the other hand, the reason that most participants would rather have a .05 chance of $100 than $13 for sure is that a .05 chance "feels" like much more than no chance at all (i.e., the slope of the weighting function is steep near zero) – in fact it "feels" like more than its objective probability, and this distortion is more pronounced than the feeling that receiving $13 is more than 13% as attractive as receiving $100. Likewise, the reason most participants would rather lose $7 for sure than face a .05 chance of losing $100 is that the .05 chance of losing money looms larger than its respective probability, and this effect is more pronounced than the feeling that receiving $7 is more than 7% as attractive as receiving $100.

The inverse-S shaped weighting function also explains the Allais paradox because the ratio of weights of probabilities .8 and 1 is smaller than the ratio of weights of probabilities .20 and .25 (so that the difference between a .80 chance of a prize and a certainty of a prize in Decision 1 looms larger than the difference between a .20 and .25 chance of a prize in Decision 2); similarly, the difference in the weights of probabilities .99 and 1 is larger than the difference in the weights of probabilities .33 and .34 (so that the difference between a .99 chance and a certainty of receiving a prize in Decision 3 looms larger than the difference between a .33 chance and a .34 chance in Decision 4). This inverse S-shaped weighting function seems to be consistent with a range of empirical findings in laboratory studies (e.g., Camerer and Ho, 1994; Tversky and Fox, 1995; Wu and Gonzalez, 1996, 1998; Gonzalez and Wu, 1999; Wakker, 2001). Overweighting of low-probability gains can help explain why the attraction of lotteries tends to increase as the top prize increases even as the chances of winning decreases correspondingly (Cook and Clotfelter, 1993) and the attraction to long-shot bets over favorites in horse races. Overweighting of low-probability losses can also explain the attractiveness of insurance (Wakker *et al.*, 1997).

In sum, prospect theory explains attitudes toward risk via distortions in shape of the value and weighting functions. The data of Tversky and Kahneman (1992) suggest that the fourfold pattern of risk attitudes for simple prospects that offer a gain or a loss with low or high probability (Table 11.2a) is driven primarily by curvature of the weighting function, because the value function is not especially curved for the typical participant in those studies. Pronounced risk aversion for mixed prospects that offer an equal probability of a gain or loss (Table 11.2b) is driven almost entirely by loss aversion, because the curvature of the value function is typically similar for losses versus gains and decision weights are similar for gain versus loss components.

Framing and Editing

Expected utility theory and most normative models of decision making under risk assume *description invariance*: preferences among prospects should not be affected by how they are described. Decision makers should act as if they are assessing the impact of options on final states of wealth. Prospect theory, in contrast, explicitly acknowledges that choices are influenced by how prospects are cognitively represented in terms of losses and gains and their associated probabilities. There are two important manifestations of this principle.

First, this representation can be systematically influenced by the way in which options are described or "framed." Recall that the value function is applied to a reference point that distinguishes between losses and gains. A common default reference point is the *status quo*. However, by varying the description of options one can influence how they are perceived. For instance, decisions concerning medical treatments can differ depending on whether possible outcomes are described in terms of survival versus mortality rates (McNeil *et al.*, 1982); recall that people tend to be risk averse for moderate probability gains and risk seeking for moderate probability losses. Likewise, the weighting function is applied to probabilities of risky outcomes that a decision maker happens to identify. The description of gambles can influence whether probabilities are integrated or segregated, and therefore affect the decisions that people make (Tversky and Kahneman, 1986). For instance, people were more likely to favor a .25 chance of $32 over a .20 chance of $40 when this choice was described as a two-stage game in which there was a .25 chance of obtaining a choice between $32 for sure or a .80 chance of $40 (that is, the $32 outcome was more attractive when it was framed as a certainty). People may endogenously frame prospects in ways that are not apparent to observers, adopting aspirations as reference points (Heath *et al.*, 1999) or persisting in the adoption of old reference points, viewing recent winnings as "house money" (Thaler and Johnson, 1990).

Second, people may mentally transform or "edit" the description of prospects they have been presented. The original formulation of prospect theory (Kahneman and Tversky, 1979) suggested that decision makers edit prospects in forming their subjective representation. Consider prospects of the form ($\$x_1$, p_1; $\$x_2$, p_2; $\$x_3$, p_3) that offer $\$x_i$ with (disjoint) probability p_i (and nothing otherwise). In particular, decision makers are assumed to engage in the following mental transformations:

1. *Combination*. Decision makers tend to simplify prospects by combining common outcomes – for example, a prospect that offers ($10, .1; $10, .1) would be naturally represented as ($10, .2).
2. *Segregation*. Decision makers tend to segregate sure outcomes from the representation of a prospect – for instance, a prospect that offers ($20, .5; $30, .5) would be naturally represented as $20 for sure plus a ($10, .5).
3. *Cancellation*. Decision makers tend to cancel shared components of options that are offered together – for example, a choice between ($10, .1; $50, .1) or ($10, .1; $20, .2) would be naturally represented as a choice between a ($50, .1) or ($20, .2).
4. *Rounding*. Decision makers tend to simplify prospects by rounding uneven numbers or discarding extremely unlikely outcomes – for example, ($99, .51; $5, .0001) might be naturally represented as ($100, .5).
5. *Transparent dominance*. Decision makers tend to reject options without further evaluation if they are obviously dominated by other options – for instance, given a choice between ($18, .1; $19, .1; $20, .1) or ($20, .3), most people would naturally reject the first option because it is stochastically dominated by the second.

Applications to Riskless Choice

Although prospect theory was originally developed as an account of decision making under risk, many manifestations of this model in riskless choice have been identified in the literature.

Loss Aversion

Loss aversion implies that preferences among consumption goods will systematically vary with one's reference point (Kahneman and Tversky, 1991; see also Bateman *et al.*, 1997), which has several manifestations. First, the minimum amount of money a person is willing to accept (WTA) to part with an object generally exceeds the minimum amount of money that he is willing to pay (WTP) to obtain the same object. This pattern, robust in laboratory studies using student populations and ordinary consumer goods, is even more pronounced for non-market goods, non-student populations, and when incentives are included to encourage non-strategic responses (Horowitz and McConnell, 2002).

Likewise, people tend to value objects more highly after they come to feel that they own them – a phenomenon known as the *endowment effect* (Thaler, 1980). For instance, in one well-known study Kahneman *et al.* (1990) presented a coffee mug with a university logo to one group of participants ("sellers") and told them the mug was theirs to keep, then asked these participants whether they would sell the mug back to them at various prices. A second group of participants ("choosers") were told that they could have the option of receiving an identical mug or an amount of money, and asked which they preferred at various prices. Although both groups were placed in strategically identical situations (walk away with a mug or money), the sellers, who presumably framed the choice as a *loss* of a mug against a compensating gain of money, quoted a median price of $7.12, whereas the buyers, who presumably framed the choice as a *gain* of a mug against a gain of money, quoted a median price of $3.12.

Loss aversion is thought to contribute to the inertial tendency to stick with *status quo* options (Samuelson and Zeckhauser, 1988) and the reluctance to trade. For instance, in one study Knetsch (1989) provided students with a choice between a university mug and a bar of Swiss chocolate, and found that they had no significant preference for one over the other. However, when some students were assigned at random to receive the mug and given an opportunity to trade for the chocolate, 89% retained the mug; when other students were assigned at random to receive the chocolate and given an opportunity to trade for the mug, only 10% opted for the mug.

Loss aversion has been invoked to help explain a number of anomalous patterns in field data. Notably, loss aversion can partly account for the powerful attraction of defaults on behavior – for instance, why organ donation rates are much higher for European countries with an "opt-out" policy than those with an "opt-in" policy (Johnson and Goldstein, 2003), the tendency of consumer demand to be more sensitive to price increases than decreases (Hardie *et al*., 1993), and the tendency for taxi drivers to quit after they have met their daily income targets, even on busy days during which their hourly wages are higher (Camerer *et al*., 1997). In fact, Fehr and Gotte (2007) found a similar pattern among bicycle messengers in which only those who exhibited loss-averse preferences for mixed gambles tended to exert less effort per hour when their wage per completed job increased.

The stronger response to losses than foregone gains also manifests itself in evaluations of fairness. In particular, most people find it unfair for an employer or merchant to raise prices on consumers or to lower wages for workers unless the employer or merchant is defending against losses of their own, and this places a constraint on profit-seeking even when the market clearing price (wage) goes up (down) (Kahneman *et al*., 1986). For instance, people find it more fair to take away a rebate than to impose a price increase on customers; most people think it is unfair for a hardware store to exercise its economic power by raising the price of snow shovels after a snowstorm.

Loss aversion is also evident in riskless choice when consumers face tradeoffs of one product attribute against one another. For instance, Kahneman and Tversky (1991) asked participants to choose between two hypothetical jobs: Job x was characterized as "limited contact with others" and a 20-minute daily commute; Job y was characterized as "moderately sociable" with a 60-minute daily commute. Participants were much more likely to choose Job x if they had been told that their present job was socially isolated with a 10-minute commute than if they had been told it was very social but had an 80-minute commute, consistent with the notion that they are loss averse for relative advantages and disadvantages. Loss aversion when making tradeoffs may partially explain the ubiquity of brand loyalty in the marketplace.

Given the disparate manifestations of loss aversion, it is natural to ask to what extent there is any consistency in a person's degree of loss aversion across these different settings. Johnson *et al*. (2007) approached customers of a car manufacturer and, through a series of simple tasks, determined each customer's coefficient of loss aversion in a risky context, as well as a measure of the endowment effect that compares the minimum amount of money each participant was willing to accept to give up a model car and their maximum willingness to pay to acquire the model car. Remarkably, the Spearman correlation between the risky and riskless measures was .635, suggesting some consistency in the underlying trait of loss aversion.

Curvature of the Value Function

Not only does the difference in steepness of the value function for losses versus gains affect riskless choice, but so does the difference in curvature. Notably, Heath *et al*. (1999) asserted that goals can serve as reference points that inherit properties of the prospect theory value function. For instance, most people believe that a person who has completed 42 sit-ups would be willing to exert more effort to complete one last sit-up if he had set a goal of 40 than if he had set a goal of 30, because the value function is steeper (above the reference point) in the former than in the latter case. Conversely, most people believe that a person who has completed 28 sit-ups would be willing to exert more effort to complete one last sit-up if he had set a goal of 30 than if he had set a goal of 40, because value function is steeper (below the reference point) in the former case than in the latter case.

The cognitive activities that people use to frame and package gains and losses, known as "mental accounting" (Thaler, 1980, 1985, 1999), can influence the way in which riskless outcomes are experienced. In particular, due to the concavity of the value function for gains, people derive more enjoyment when gains are segregated (e.g., it's better to win two lotteries on two separate days); due to the convexity of the value function for losses, people find it less painful when losses are integrated (e.g., it's better to pay a parking ticket the same day I pay my taxes) – but see Linville and Fischer (1991).

Extensions of Prospect Theory

As mentioned earlier, decision theorists distinguish between decisions under risk, in which probabilities are known to the decision maker, and decisions under uncertainty, in which they are not. The original formulation of prospect theory (henceforth OPT; Kahneman and Tversky, 1979) applies to decisions under risk and involving at most two non-zero outcomes. *Cumulative prospect theory* (henceforth CPT; Tversky and Kahneman, 1992;

see also Luce and Fishburn, 1991; Wakker and Tversky, 1993) accommodates decisions under uncertainty and any finite number of possible outcomes. A thorough account of CPT is beyond the scope of this chapter, so we will only sketch out its distinctive features and refer the reader to the original paper for further detail.

Cumulative Prospect Theory

When considering simple chance prospects with at most two non-zero outcomes, two distinctive features of CPT are important.

First, cumulative prospect theory segregates value into gain portions and loss portions, with separate weighting functions for losses and gains (i.e., CPT decision weights are *sign-dependent*)[2].

Second, CPT applies decision weights to cumulative distribution functions rather than single events (i.e., CPT decision weights are *rank-dependent*)[3]. That is, each outcome x is weighted not by its probability but by the cumulated probabilities of obtaining an outcome at least as good as x if the outcome is positive, and at least as bad as x if the outcome is negative.

More formally, consider a chance prospect with two non-zero outcomes $(x, p; y, q)$ that offers \$$x$ with probability p and \$$y$ with probability q (otherwise nothing). Let $w^+(\cdot)$ and $w^-(\cdot)$ be the weighting function for gains and losses, respectively. The CPT valuation of the prospect is given by:

$$w^-(p)v(x) + w^+(q)v(y)$$

for mixed prospects, $x < 0 < y$

$$[w^+(p+q) - w^+(q)]v(x) + w^+(q)v(y)$$

for pure gain prospects, $0 \leq x < y$

$$[w^-(p+q) - w^-(q)]v(x) + w^-(q)v(y)$$

for pure loss prospects, $y < x \leq 0$.

The first equation illustrates sign dependence: a different weighting function is applied separately to the loss and gain portions of mixed prospects. The second and third equations illustrate rank dependence for gains and losses, respectively: extreme (y) outcomes are weighted by the impact of their respective probabilities, whereas intermediate outcomes (x) are weighted by the difference in impact of the probability of receiving an outcome at least as good as x and the impact of the probability of receiving an outcome that is strictly better than x. A more general characterization of CPT that applies to any finite number of outcomes and decisions under uncertainty is included in the Appendix to this chapter.

For decision under risk, the predictions of CPT coincide with OPT for all two-outcome risky prospects and all mixed (gain–loss) three-outcome prospects[4] when one outcome is zero, assuming $w^+ = w^-$. Because elicitation of prospect theory parameters (reviewed in the following section) usually requires the use of two-outcome prospects, we illustrate how they coincide for a two-outcome (pure gain) prospect below. Consider a prospect $(x, p; y)$ that offers \$$x$ with probability p and otherwise \$$y$, where $x > y$. According to CPT:

$$V(x, p; y) = [1 - w(p)]v(y) + w(p)\,v(x).$$

According to OPT, decision makers tend to invoke the editing operation of *segregation*, treating the smaller outcome y as a certainty, and reframing the prospect as a p chance of getting an additional $x - y$. Thus, we get:

$$V(x, p; y) = v(y) + w(p)[v(x) - v(y)]$$

which can be rearranged into the same expression as above. It is also easy to see that when $y = 0$, $V(x, p) = w(p)\,v(x)$ under both CPT and OPT.

Decision Weights Under Risk Versus Uncertainty: the Two-stage Model

As we have seen, the risky weighting function is assumed to exhibit greater sensitivity to changes in probability (i.e. higher slope) near the natural boundaries of 0 and 1 than in the midpoint of the scale. A characterization of the weighting function that generalizes

[2] Wu and Markle (2008) document systematic violations of gain–loss separability. Their results suggest different weighting function parameter values for mixed (gain–loss) prospects than for single domain (pure gain or pure loss) prospects.

[3] Rank-dependence is motivated in part by the concern that non-linear decision weights applied directly to multiple simple outcomes can give rise to violations of stochastic dominance. For instance, a prospect that offers a .01 chance of \$99 and a .01 chance of \$100 might be preferred to a prospect that offers a .02 chance of \$100 due to the overweighting of low probabilities, even though the latter prospect dominates the former prospect. OPT circumvents this problem for simple prospects by assuming that transparent violations of dominance are eliminated in the editing phase; CPT handles this problem through a rank-dependent decision weights that sum to one for pure gain or loss prospects. For further discussion of advantages of CPT over OPT when modeling preferences involving complex prospects, see Fennema and Wakker, 1997.

[4] Gonzalez and Wu (2003) estimated prospect theory weighting functions and value functions obtained from cash equivalents for two-outcome gambles, in which OPT and CPT coincide, and applied these estimates to predict cash equivalents for three-outcome gambles, in which they do not. Interestingly, they found systematic over-prediction for OPT and systematic under-prediction for CPT.

this observation from risk to uncertainty through the measure of "bounded subadditivity" is presented in Tversky and Fox (1995; see also Tversky and Wakker, 1995; Wu and Gonzalez, 1999). Informally, bounded subadditivity quantifies a decision-maker's diminished sensitivity to events when they are added to or subtracted from intermediate events compared to when they are added to impossibility or subtracted from certainty.

Several studies suggest that decisions under uncertainty accord well with a two-stage model in which participants first judge likelihood of events on which outcomes depend, then apply the inverse S-shaped weighting function to these probabilities, consistent with prospect theory (Tversky and Fox, 1995; Fox and Tversky, 1998; for a theoretical treatment, see Wakker, 2004). That is, the uncertain decision weight W of event E is given by

$$W(E) = w(P(E))$$

where $P(E)$ is the (non-additive) judged probability of event E and $w(\cdot)$ is the risky weighting function. For instance, consider the prospect "win \$100 if the Lakers beat the Celtics." A person's decision weight of "Lakers beat the Celtics" can be predicted well from his risky weighting function applied to his judged probability of the event "Lakers beat the Celtics." Judged probabilities are assumed to accord with support theory (Tversky and Koehler, 1994; Rottenstreich and Tversky, 1997), a behavioral model that conceives of judged probability as the proportion of support that a person associates with a focal hypothesis (e.g., that the Lakers will win) against its complement (the Celtics will win). Fox and Tversky (1998) review several studies that demonstrate the predictive validity of the two-stage model (see also Wu and Gonzalez, 1999; Fox and See, 2003; but see too Kilka and Weber, 2001).

Ambiguity Aversion and Source Preferences

Decisions under uncertainty can be further complicated by preferences to bet on particular sources of uncertainty. Ellsberg (1961) observed that people prefer to bet on events with known rather than unknown probabilities, a phenomenon known as ambiguity aversion (for a review, see Camerer and Weber, 1992; see also Fox and See, 2003). This phenomenon may partially explain, for example, the common preference to invest in the domestic stock market and under-diversify into foreign markets (French and Poterba, 1991). Ambiguity aversion appears to be driven by reluctance to act in situations in which a person feels comparatively ignorant of predicting outcomes (Heath and Tversky, 1991), and such preferences tend to diminish or disappear in the absence of a direct comparison between more and less familiar events or with more or less knowledgeable individuals (Fox and Tversky, 1995; Chow and Sarin, 2001; Fox and Weber, 2002). For a discussion of how source preferences can be incorporated into the two-stage model, see Fox and Tversky (1998).

Decisions from Experience

Finally, situations in which people learn relative frequencies of possible outcomes from experience (e.g., as in the Iowa Gambling Task or Balloon Analog Risk Task), learning can be complicated by sampling error. In particular, according to the binomial distribution very rare events are generally more likely to be under-sampled than over-sampled, and the opposite is true for very common events. For instance, imagine a situation in which a decision maker samples outcomes from two decks of cards: the first deck offers a .05 chance of \$100 (and nothing otherwise) while the second deck offers \$5 for sure. If decision makers sample a dozen cards from each deck, most will *never* sample \$100 from the first deck and therefore face an apparent choice between \$0 for sure and \$5 for sure, and therefore forego the 5% chance of \$100, contrary to the pattern observed in decision under risk. (For further discussion of these issues, see Hertwig *et al.*, 2004; Fox and Hadar, 2006). For further discussion of how the two-stage model can be extended to situations in which outcomes are learned from experience, see Hadar and Fox (2008).

PROSPECT THEORY MEASUREMENT

Several applications of prospect theory – from neuroeconomics to decision analysis to behavioral finance – require individual assessment of value and weighting functions. In order to measure the shape of the value and weighting functions exhibited by participants in the laboratory, we must first discuss how these functions can be formally modeled. We next discuss procedures for eliciting values and decision weights.

Parameterization

It is important to note that, in prospect theory, value and weighting functions are characterized by their qualitative properties rather than particular functional forms. It is often convenient, however, to fit data to equations that satisfy these qualitative properties. A survey of parameterizations of prospect theory's value and weighting functions can be found in Stott (2006). We review below the functional forms that have received the most attention in the literature to date.

Value Function

The value function is assumed to be concave for gains, convex for losses, and steeper for losses than for gains. By far the most popular parameterization, advanced by Kahneman and Tversky (1992) relies on a power function:

$$v(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -\lambda(-x)^{\beta} & x < 0 \end{cases} \tag{V1}$$

where α, $\beta > 0$ measure the curvature of the value function for gains and losses, respectively, and λ is the coefficient of loss aversion. Thus, the value function for gains (losses) is increasingly concave (convex) for smaller values of $\alpha(\beta) < 1$, and loss aversion is more pronounced for larger values of $\lambda > 1$. Tversky and Kahneman (1992) estimated median values of $\alpha = .88$, $\beta = .88$, and $\lambda = 2.25$ among their sample of college students. In prospect theory the power function is equivalent to *preference homotheticity*: as the stakes of a prospect (x, p) are multiplied by a constant k, then so is the certainty equivalent of that prospect, $C(x, p)$ so that $C(kx, p) = kC(x, p)$. (see, e.g., Tversky, 1967). Empirically this assumption tends to hold up only within an order of magnitude or so, and as the stakes of gambles increase by orders of magnitude, risk aversion tends to increase for gains – especially when the stakes are real (Holt and Laury, 2002); the evidence for losses is mixed (Fehr-Duda *et al.*, 2007). Thus, for example, a person who is indifferent between \$3 and (\$10, .5) will tend strictly to prefer \$30 over (\$100, .5). Nevertheless, most applications of prospect theory have assumed a power value function. Other common functional forms include the logarithmic function $v(x) = \ln(\alpha + x)$, originally proposed by Bernoulli (1738), which captures the notion that marginal utility is proportional to wealth, and quadratic $v(x) = \alpha x - x^2$, which can be reformulated in terms of a prospect's mean and variance, which is convenient in finance models. (For a discussion of additional forms including exponential and expo-power, see Abdellaoui *et al.*, 2007a.)

Surprisingly, there is no canonical definition or associated measure of loss aversion, though several have been proposed. First, in the original formulation of prospect theory (Kahneman and Tversky, 1979), loss aversion was defined as the tendency for the negative value of losses to be larger than the value of corresponding gains (i.e., $-v(-x) > v(x)$ for all $x > 0$) so that a coefficient of loss aversion might be defined, for example, by the mean or median value of $-v(-x)/v(x)$ over a particular range of x. Second, the aforementioned parameterization (V1) from Tversky and Kahneman (1992) that assumes a power value function implicitly defines the loss aversion as the ratio of value of losing a dollar to gaining a dollar (i.e., $-v(-\$1) > v(\$1)$) so that the coefficient is defined by $-v(-\$1)/v(\$1)$. Third, Wakker and Tversky (1993) defined loss aversion as the requirement that the slope of the value function for any amount lost is larger than the slope of the value function for the corresponding amount gained (i.e., $v'(-x) > v'(x)$) so that the coefficient can be defined by the mean or median value of $v'(-x)/v'(x)$. Note that if one assumes a simplified value function that is piecewise linear (as in, for example, Tom *et al.*, 2007), then all three of these definitions coincide. For a fuller discussion, see Abdellaoui *et al.* (2007b).

Weighting Function

In fitting their data, Tversky and Kahneman (1992) asserted a single-parameter weighting function:

$$w(p) = p^{\gamma}/(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}. \tag{W1}$$

This form is inverse-S shaped, with overweighting of low probabilities and underweighting of moderate to high probabilities for values of $\gamma < 1$. This function is plotted for various values of γ in Figure 11.3A.

Perhaps the most popular form of the weighting function, due to Lattimore *et al.* (1992; see also Goldstein and Einhorn, 1987) assumes that the relation between w and p is linear in a log-odds metric:

$$\ln\frac{w(p)}{1-w(p)} = \gamma \ln\frac{p}{1-p} + \ln\delta$$

which reduces to

$$w(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma} + (1-p)^{\gamma}} \tag{W2}$$

where $\delta > 0$ measures the elevation of the weighing function and $\gamma > 0$ measures its degree of curvature. The weighting function is more elevated (exhibiting less overall risk aversion for gains, more overall risk aversion for losses) as δ increases and more curved (exhibiting more rapidly diminishing sensitivity to probabilities around the boundaries of 0 and 1) as $\gamma < 1$ decreases (the function exhibits an S-shaped pattern that is more pronounced for larger values of $\gamma > 1$). Typically, the decision weights of complementary events sum to less than one ($w(p) + w(1 - p) < 1$), a property known as *subcertainty* (Kahneman and Tversky, 1979). This property is satisfied whenever $\delta < 1$. The Lattimore function is plotted for various values of the elevation parameter δ and curvature parameter γ in Figures 11.3b and 11.3c, respectively.

Prelec (1998; see also 2000) derived a functional form of the weighting function that accommodates three principles: (1) overweighting of low probabilities and underweighting of high probabilities; (2) subproportionality of decision weights (a condition that derives from the common ratio effect, decisions 1 and 2 above); and (3) sub-additivity of decision weights (a condition that derives from the common consequence effect, decisions 3 and 4 above). These three principles are all subsumed by a single axiom called *compound invariance*[5] which implies the following functional form of the weighting function:

$$w(p) = \exp[-\delta(-\ln p)^{\gamma}] \qquad \text{(W3A)}$$

where $\delta, \gamma > 0$. When $\delta = 1$, Prelec's function collapses to a single-parameter form:

$$w(p) = \exp[-(-\ln p)^{\gamma}] \qquad \text{(W3B)}$$

which implies a weighting function that crosses the identity at $1/e$. Prelec's two-parameter function is plotted for various values of the elevation parameter δ in Figure 11.3d, and the one-parameter function (i.e., $\delta = 1$) is plotted for various values of the curvature parameter γ in Figure 11.3e.

The prospect theory value and weighting function parameters can all be estimated for individuals using simple choice tasks on computer. Table 11.3 presents measured parameters for monetary gambles from several studies that have assumed a power value function and various weighting functions described above.

Although the typical measured values of these parameters suggest an S-shaped value function ($0 < \alpha, \beta < 1$) with loss aversion ($\lambda > 1$), and an inverse-S shaped weighting function that crosses the identity line below .5, there is considerable heterogeneity between individuals in these measured parameters. For instance, in a sample of 10 psychology graduate students evaluating gambles involving only the possibility of gains, Gonzalez and Wu (1999) obtained measures of α in the range from .23 to.68 (V1), δ in the range from .21 to 1.51, and γ in the range from .15 to .89 (W2).

As a practical matter, although the two-parameter functions (W2) and (W3) have different axiomatic implications, they are difficult to distinguish empirically in the normal range (i.e., .01 to .99) of probabilities (see Gonzalez and Wu, 1999). For the remainder of the chapter, we will refer to the parameters from the Lattimore *et al*. (1992) function (W2).

Interaction of v(·) and w(·)

As mentioned above, prospect theory value and weighting functions both contribute to observed risk attitudes: concavity (convexity) of the value function contributes to risk aversion (seeking) for pure gain (loss) prospects that is reinforced by underweighting of moderate to high probabilities and reversed by overweighting of low probabilities; loss aversion contributes to risk aversion for mixed prospects. To see more clearly how the value and weighting functions interact, consider the simple case of a prospect (x, p) that offers \$$x$ with probability p (and nothing otherwise). Let $c(x, p)$ be the certainty equivalent of (x, p). For instance, a decision maker for whom $c(100, .5) = 30$ is indifferent between receiving \$30 for sure or 50–50 chance of \$100 or nothing. Thus, this decision maker would strictly prefer the prospect to \$29 and would strictly prefer \$31 to the prospect. If we elicit certainty equivalents for a number of prospects in which we hold x constant and vary p, then we can derive a plot of *normalized certainty equivalents*, c/x as a function of probability. Such a plot can be instructive, because it indicates probabilities (of two-outcome gambles) for which the decision maker is risk seeking ($c/x > p$), risk neutral ($c/x = p$), and risk averse ($c/x < p$) by whether the curve lies above, on, or below the identity line, respectively.

To see how $w(\cdot)$ and $v(\cdot)$ jointly contribute to risk attitudes, note that, under prospect theory, $V(c) = V(x, p)$, so that $v(c) = w(p)v(x)$ or $w(p) = v(c)/v(x)$. Assuming the power value function (V1), we get $w(p) = (c/x)^{\alpha}$, or

$$c/x = w(p)^{1/\alpha}.$$

In the case of gains, normalized certainty equivalents will increase with the parameter α and, assuming a concave value function ($\alpha < 1$) that is correctly measured, they will be lower than corresponding decision weights. These observations give rise to two important implications. First, overweighting of low probabilities does not necessarily translate into risk-seeking for low-probability gains. To illustrate, consider the weighting function obtained from the median data of Gonzalez and

[5] Defined as: for any outcomes x, y, x', y', probabilities q, p, r, s, and the compounding integer $N \geq 1$, if $(x, p) \sim (y, q)$ and $(x, r) \sim (y, s)$ then $(x', p^N) \sim (y', q^N)$ implies $(x', r^N) \sim (y', s^N)$.

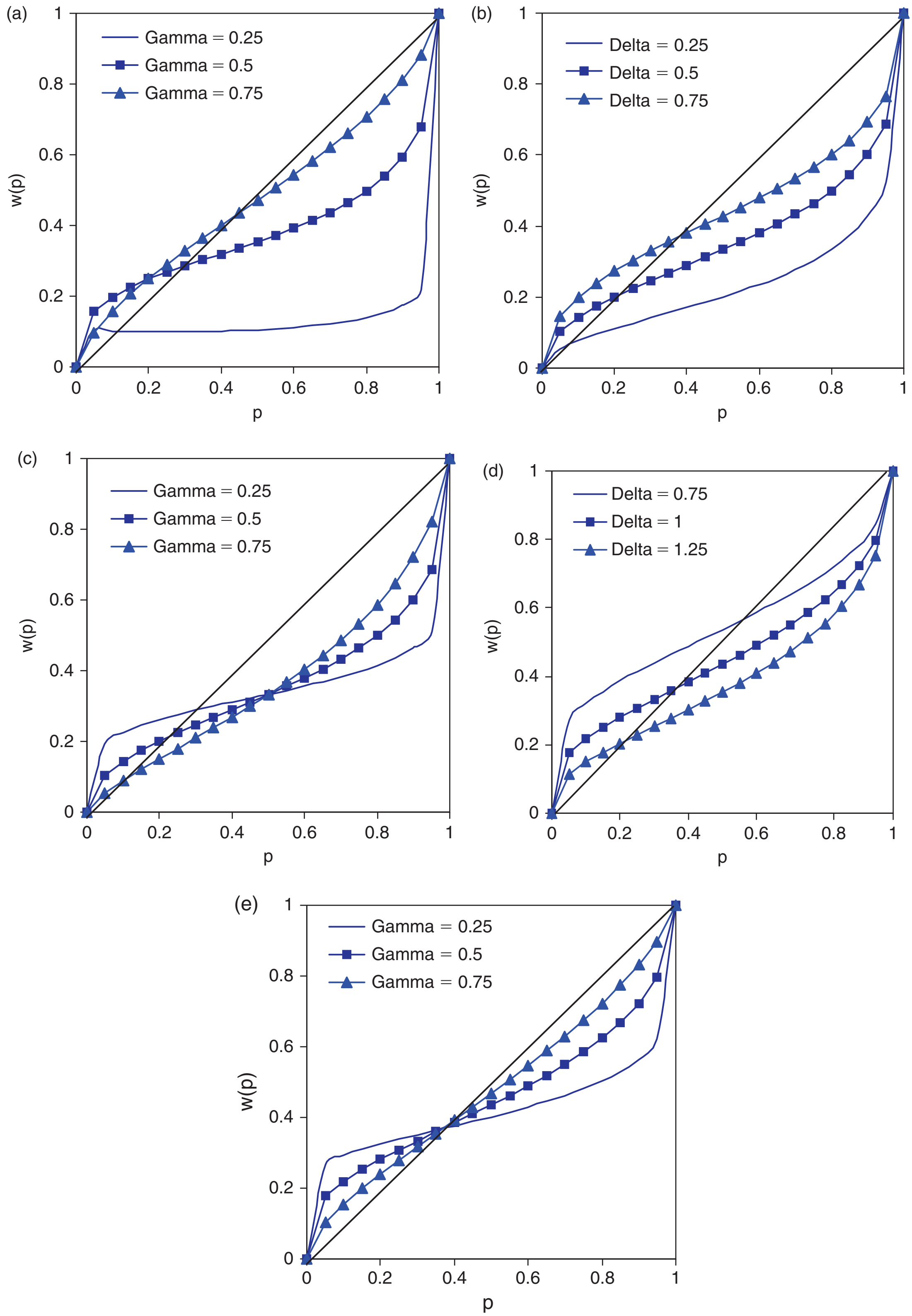

FIGURE 11.3 Most common parametric forms used for modeling the probability weighting function from prospect theory. (a) Tversky and Kahneman's (1992) function for various values of γ (W1). (b) Lattimore *et al.*'s (1992) function for various values of δ assuming $\gamma = .5$ (W2). (c) Lattimore *et al.*'s (1992) function for various values of γ assuming $\delta = .5$ (W2). (d) Prelec's (1998) function for various values of δ assuming $\gamma = .5$ (W3A). (e) Prelec's (1998) function for various values of γ assuming $\delta = 1$ (W3B).

TABLE 11.3 Measured value function parameters for money from several studies

Functional form	Study	Subject population	Parameter estimates
(a)			
(V1) $v(x) = \begin{cases} x^\alpha & x \geq 0 \\ -\lambda(-x)^\beta & x < 0 \end{cases}$	Tversky and Kahneman (1992)	n = 25 graduate students (median fitted parameters)	$\alpha = .88$ $\beta = .88$ $\lambda = 2.25$
	Camerer and Ho (1994)	Weighted average of nine studies reviewed	$\alpha = .23$
	Wu and Gonzalez (1996)	n = 420 undergraduates (fitted to binary choice data)	$\alpha = .49$
	Gonzalez and Wu (1999)	n = 10 psychology graduate students (median data)	$\alpha = .49$
	Abdellaoui (2000)	n = 46 economics students (median data)	$\alpha = .89$ $\beta = .92$
	Etchart-Vincent (2004)	n = 35 business students (median data)	$\beta = .97$
	Abdellaoui *et al*. (2005)	n = 41 business graduate students (median fitted parameters)	$\alpha = .91$ $\beta = .96$
	Stott (2006)	n = 96 university students (median fitted data)	$\alpha = .19$
	Abdellaoui *et al*. (2007b)	n = 48 economics students (median data)	$\alpha = .75$ $\beta = .74$
	Abdellaoui *et al*. (2007c)	n = 48 economics and math graduate students (median data)	$\alpha = .86$ $\beta = 1.06$ $\lambda = 2.61$
(b)			
(W1) $W(p) = p^\gamma/(p^\gamma + (1-p)^\gamma)^{1/\gamma}$	Tversky and Kahneman (1992)	n = 25 graduate students (median fitted parameters)	$\gamma^+ = .61$ $\gamma^- = .69$
	Camerer and Ho (1994)	Weighted average of nine studies reviewed	$\gamma^+ = .56$
	Wu and Gonzalez (1996)	n = 420 undergraduates (fitted to binary choice data)	$\gamma^+ = .71$
	Abdellaoui (2000)	n = 46 economics students (median data)	$\gamma^+ = .60$ $\gamma^- = .70$
	Stott (2006)	n = 96 university students (median fitted data)	$\gamma^+ = .96$
(W2) $w(p) = \dfrac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$	Tversky and Fox (1995)	n = 40 student football fans (median data, with $\alpha = .88$)	$\gamma^+ = .69$ $\delta^+ = .77$
	Wu and Gonzalez (1996)	n = 420 undergraduates (fitted to binary choice data)	$\gamma^+ = .68$ $\delta^+ = .84$
	Gonzalez and Wu (1999)	n = 10 psychology graduate students (median data)	$\gamma^+ = .44$ $\delta^+ = .77$
	Abdellaoui (2000)	n = 46 economics students (median data)	$\gamma^+ = .60$ $\delta^+ = .65$ $\gamma^- = .65$ $\delta^- = .84$
	Abdellaoui *et al*. (2005)	n = 41 business graduate students (median data)	$\gamma^+ = .83$ $\delta^+ = .98$ $\gamma^- = .84$ $\delta^- = 1.3$
	Stott (2006)	n = 96 university students (median fitted data)	$\gamma^+ = 1.4$ $\delta^+ = .96$
(W3A) $w(p) = \exp[-\delta(-\ln p)^\gamma]$	Stott (2006)	n = 96 university students (median fitted data)	$\gamma^+ = 1.0$ $\delta^+ = 1.0$
(W3B) $w(p) = \exp[-(-\ln p)^\gamma]$	Wu and Gonzalez (1996)	n = 420 undergraduates (fitted to binary choice data)	$\gamma^+ = .74$
	Stott (2006)	n = 96 university students (median fitted data)	$\gamma^+ = 1.0$

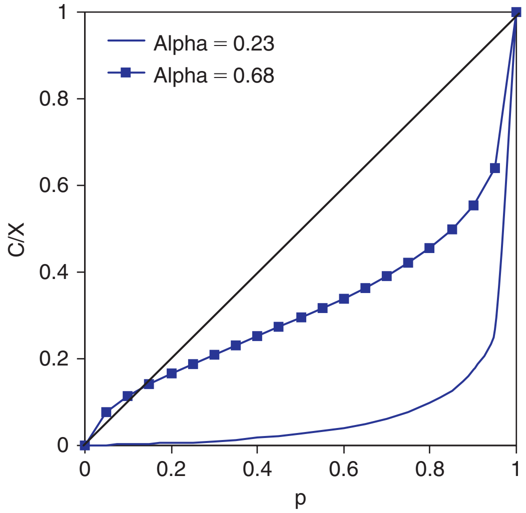

FIGURE 11.4 Normalized certainty equivalents as a function of probability assuming the Lattimore weighting function, with $\delta = .77$ and $\gamma = .44$ (median values from Gonzalez and Wu, 1999) and assuming a power value function, with $\alpha = .23$ and .68 (the range obtained from participants of Gonzalez and Wu, 1999). This figure illustrates the interaction of the value and weighting functions in determining risk attitudes.

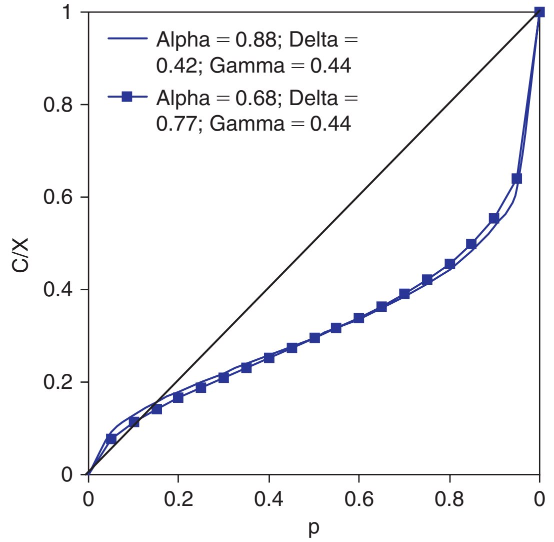

FIGURE 11.5 Normalized certainty equivalents as a function of probability assuming the Lattimore weighting function and power value function with $\alpha = .68$, $\delta = .77$, and $\gamma = .44$. versus $\alpha = .88$, $\delta = .42$, and $\gamma = .44$. This figure illustrates the difficulty empirically distinguishing between elevation of the weighting function and curvature of the value function.

Wu (1999), assuming the Lattimore *et al*. (1992) function (W2), with $\delta = .77$, $\gamma = .44$, which illustrates considerable overweighting of low probabilities; for example, $w(.05) = .17$. In that study, the authors obtained α in the range from .68 (moderate concavity) to .23 (extreme concavity) for their ten participants. Using these extreme values, we obtain wildly different c/x functions as depicted in Figure 11.4. For instance, given these values $c(100, .05) = 7.65$ and .05, respectively, indicating moderate risk-seeking and extreme risk aversion, respectively.

Second, the interaction of value- and weighting-functions makes it difficult empirically to distinguish variations in the measured elevation of the weighting function from variations in the measured curvature of the value function. For instance, as mentioned above, when $\alpha = .68$, $\delta = .77$, and $\gamma = .44$. we get $c(100, .05) = 7.65$. This same certainty equivalent follows assuming, for example, $\alpha = .88$, $\delta = .42$, and $\gamma = .44$. Both of these normalized certainty equivalent functions are illustrated in Figure 11.5. Thus, if one is concerned with parsing the contribution of subjective value versus probability weighting on observed risk attitudes, it is important to elicit the value and weighting functions with care. For instance, if one assumes a single parameter weighting function (e.g., (W1) or (W3B)) when "true" weighting functions vary in their elevation, incorrect measures may be obtained. A researcher may believe that a particular pattern of neural activity covaries with curvature of the value function, when in fact it covaries with elevation of the weighting function.

Elicitation

Several methods have been proposed for eliciting value and weighting function parameters. Broadly speaking, these methods fall into four categories:

1. Statistical methods that estimate $v(x_i)$ and $w(p_i)$ from a participant's cash equivalents for prospects that factorial combine each x_i, and p_i.
2. Non-parametric methods that separately assess values then assess decision weights, making no assumptions concerning the functional form of the value-and weighting-functions.
3. Semi-parametric methods that assume a functional form for the value- or weighting-function and assess the other function non-parametrically.
4. Parametric methods that assume a functional form of both the value and weighting functions.

We will review each of these methods in turn then evaluate their relative strengths and weaknesses.

Statistical Method: Gonzalez and Wu (1999)

Perhaps the most careful elicitation method of prospect theory value and weighting functions to date was advanced by Gonzalez and Wu (1999). Ten graduate students in Psychology from the University of Washington were paid \$50 plus an incentive-compatible payment (contingent on their choices) for their participation in four 1-hour sessions on computer[6]. Participants were presented with 15 two-outcome (non-negative) gambles crossed with 11 probabilities (=165 gambles), presented in a random order.

Certainty equivalents were assessed for each gamble through a series of choices. For instance, consider the prospect that offered a 50–50 chance of \$100 or nothing. A participant was asked if he preferred to receive the prospect or various sure amounts that ranged from \$100 to \$0 in increments of \$20. If a participant indicated that he preferred \$40 for sure over the prospect but preferred the prospect over \$20 for sure, then a second round of choices would be presented that spanned this narrower range (from \$40 to \$20). This process was repeated until certainty equivalents could be estimated to the nearest dollar. If, for example, a participant indicated a preference for a sure \$36 over the prospect but a preference for the prospect over a sure \$35, then the researchers estimated $c(100, .5) = 35.5$.

The estimation procedure used by Gonzalez and Wu (1999) was non-parametric in that it did not make any assumptions concerning the functional form of the $v(\cdot)$ or $w(\cdot)$. Their algorithm treated the value of each of the possible outcomes and the weight of each of the probabilities presented as a parameter to be estimated. These parameters were estimated using an alternating least squares procedure in which each step either held w constant and estimated v or held v constant and estimated w. The authors assert that this analysis converged on parameter estimates relatively quickly.

The statistical method of Gonzalez and Wu (1999) has several advantages over alternative methods. The elicitation is not very cognitively demanding, as participants are merely required to price two-outcome gambles. The procedure gives rise to estimates of values and decision weights that are not distorted by parametric misspecification. On the other hand, the procedure is demanding of participants' time as it requires pricing of a large number of gambles to get stable estimates (the original study required participants to assess 165 two-outcome gambles, each through a series of several choices). The procedure has not yet been applied to the domain of losses or mixed prospects, but such an extension would be straightforward.

Non-parametric Methods

Several other fully non-parametric methods have been advanced for analytically assessing $v(\cdot)$ and $w(\cdot)$. All of them rely on a two-stage process in which $v(\cdot)$ is assessed in a first phase, then applied to the measurement of $w(\cdot)$. The most popular approach to assessing values that makes no assumptions concerning the weighting of probabilities is the *tradeoff method* (Wakker and Deneffe, 1996). The tradeoff method requires participants to make choices between two two-outcome prospects $(x, p; y)$ that offer \$$x$ with probability p otherwise \$$y$, with one of the outcomes adjusted following each choice until indifference between the gambles can be established. Consider a pair of reference outcomes $R > r$, a pair of variable outcomes $x_1 > x_0$, and a fixed probability p. On each trial the values of R, r, x_0, and p are fixed, and x_1 is varied until the participant reveals that

$$(x_1, p; r) \sim (x_0, p; R).$$

For instance, a participant might be offered a choice between a 50–50 chance of \$100 or \$20 versus a 50–50 chance of \$70 or \$40. If the participant prefers the latter gamble, then the variable payoff of the first gamble (\$100) adjusts to a higher amount (say, \$110). The variable amount can be raised or lowered by decreasing increments until the participant confirms that both prospects are equally attractive. Once indifference is established for this first pair of prospects, the procedure is repeated for a second pair of prospects with the same probability and reference outcomes, but a new variable outcome $x_2 > x_1$, until it is established that:

$$(x_2, p; r) \sim (x_1, p; R).$$

According to CPT[7], the first indifference gives us

$$v(r)[1 - w(p)] + v(x_1)w(p) = v(R)[1 - w(p)] + v(x_0)w(p)$$

[6]An incentive-compatible payoff is a payment contingent on choice that encourages honest responses by participants. Experimental economists are generally skeptical of results of studies that do not include such incentives whereas experimental psychologists generally put more credence into responses to purely hypothetical choices. In practice, the addition of incentives tends to reduce noise in participant responses and may lead to decreased framing effects and greater risk aversion (for reviews, see Camerer and Hogarth, 1999; Hertwig and Ortmann, 2001).

[7]Assuming $x_0 > R$; this result can be relaxed without affecting the result of the elicitation.

so that

$$w(p)[v(x_1) - v(x_0)] = [1 - w(p)][v(R) - v(r)]$$

and the second indifference gives us

$$v(r)[1 - w(p)] + v(x_2)w(p) = v(R)[1 - w(p)] + v(x_1)w(p)$$

so that

$$w(p)[v(x_2) - v(x_1)] = [1 - w(p)][v(R) - v(r)].$$

Together these indifferences imply equal value intervals as follows:

$$v(x_1) - v(x_0) = v(x_2) - v(x_1).$$

Setting $x_0 = 0$ and $v(x_0) = 0$, we get $v(x_2) = 2v(x_1)$. By eliciting similar yoked indifferences to obtain x_3, x_4, etc., we can generate a standard sequence of outcomes that are spaced equally in subjective value space to construct a parameter-free value function for gains. A similar exercise can be repeated in the measurement of the value function for losses (for an example in the domain of losses, see Fennema and van Assen, 1999).

Once a measure of several values has been obtained from a participant, one can proceed to measure decision weights non-parametrically. Arguably the most popular method, advanced by Abdellaoui (2000), uses the standard sequence of outcomes x_0, ..., x_n to elicit a standard series of probabilities $p_1, \ldots, p_{n-1}$ that are equally spaced in terms of their decision weights. This is done by eliciting probabilities such that a mixture of the highest and lowest outcome in the standard sequence is equally attractive to each of the internal outcomes in that sequence. Thus, by establishing for each x_i ($i = 1, \ldots, n - 1$) the following indifference:

$$(x_n, p_i; x_0) \sim x_i.$$

CPT implies:

$$w(p_i) = \frac{v(x_i) - v(x_0)}{v(x_n) - v(x_0)}.$$

Because the values of x_i were constructed, using the tradeoff method, to be equally spaced in terms of their expected value, the above equation reduces to:

$$w(p_i) = i/n.$$

An analogous procedure can be followed for losses.

Bleichrodt and Pinto (2000) advanced a similar two-step procedure that first relies on the tradeoff method to elicit a standard sequence of outcomes, then elicits decision weights through a matching procedure. Instead of eliciting probabilities that lead to indifference between prospects, their method fixes probabilities and elicits outcomes that match pairs of two-outcome prospects[8]. Such a procedure was used to measure the weighting function for losses by Etchart-Vincent (2004). Another similar method has recently been proposed by van de Kuilen *et al.* (2006), though in an experiment this method yielded a weighting function for gains that was convex rather than the customary inverse-S shape (concave then convex).

The aforementioned non-parametric elicitations can be used to assess value- and weighting-functions separately for gains and losses. Because the value function is a ratio scale (unique to multiplication by a positive constant) a separate procedure using mixed (gain−loss) gambles is required to assess loss aversion. A parameter-free procedure has been advanced by Abdellaoui *et al.* (2007b). Details of the procedure are beyond the scope of this chapter, but the gist is as follows. The first step entails determining, through a series of indifferences between prospects, the probabilities p_g and p_l for which $w^+(p_g)$ and $w^-(p_l) = 1/2$. This allows determination, in a second stage, of outcome amounts that are midpoints in value space for losses. The third stage links value for losses and gains through a series of indifferences that determines a gain outcome that is the mirror image of a loss outcome in value space (i.e., has the same absolute value of utility/value). Finally, the fourth step repeats the second step by determining outcomes that are midpoints in value space for gains. The method of Abdellaoui *et al.* (2007b) is mathematically elegant and yielded clean results consistent with prospect theory in the analysis of aggregate data from a sample of 48 economics students. However, the task is cognitively demanding, as it involves choices between pairs of two-outcome gambles, and laborious, as it entails a complex four-step procedure.

Non-parametric methods tend to be less time consuming than statistical methods of elicitation. Also, unlike semi-parametric and fully parametric methods, they make no assumptions concerning the functional form of the value and weighting functions that might distort measurement, though functions can be fitted to the measured values and weights that are obtained.

[8]Note that because the new outcomes may not be included in the standard sequence this method requires an interpolation procedure and thus is not fully non-parametric.

Moreover, non-parametric methods preserve a direct link between specific choices and measured utilities so that specific inconsistencies can be traced to particular choices. Unfortunately, non-parametric methods are generally quite cognitively demanding, requiring choices between multiple two-outcome prospects (or even more complicated choices). Thus, these methods may not give utterly robust measurements, as participants may fall back on decision heuristics (such as expected value maximization) or respond in an inconsistent manner. Moreover, because these methods generally rely on elicitation of a standard sequence of values using the tradeoff method, there is the possibility that error in measuring the first step in the sequence will be propagated throughout the measurement of values and therefore lead to further error in the measurement of decision weights (however, studies that have investigated error propagation have thus far found no large effect; see Bleichrodt and Pinto, 2000; Abdellaoui *et al*., 2005). Note that only methods listed as allowing simultaneous measurement of both v^+ and v^- can also allow measurement of loss aversion.

Semi-Parametric Methods

Semi-parametric elicitation methods assume a parametric form of the value function in order to derive non-parametric estimates of decision weights. The simplest semi-parametric approach is to assume a power value function, $v(x) = x^\alpha$, as fitted to non-parametric measurement of value using the tradeoff method (or assuming representative parameters from previous studies of similar participant populations). Next, decision weights for various probabilities can be determined by eliciting certainty equivalents $c(x, p_i)$ for prospects that pay a fixed amount x with probabilities p_i. According to prospect theory, $c(x, p_i)^\alpha = w(p_i)x^\alpha$. Thus, each decision weight is given by:

$$w(p_i) = [c(x, p_i)/x]^\alpha.$$

Of course, this method depends on the accuracy of the first-stage measurement of utility.

A more elegant semi-parametric method was recently advanced by Abdellaoui *et al*. (2007c). This method entails three stages. In the first stage, the value function for gains is elicited and decision weights are measured parameters. This is done by eliciting certainty equivalents G_i for a series of prospects $(x_i, p_g; y_i)$ $(x_i > y_i \geq 0, i = 1, ..., k)$. According to CPT:

$$v(G_i) = v(y_i)[1 - w(p_g)] + v(x_i)w(p_g).$$

Define $w(p_g) \equiv \omega^+$ and assume a power value function $v(x) = x^\alpha$. We get:

$$G_i = (w^+(x_i^\alpha - y_i^\alpha) + y_i^\alpha)^{1/\alpha}.$$

Thus, by varying x_i and y_i and measuring cash equivalents G_i, the parameters ω^+ and α can be estimated using non-linear regression. An analogous method can be used for the domain of losses to measure ω^-, the decision weight of losing with probability $p_l = 1 - p_g$, and β, the power value coefficient for losses. Finally, a third stage links the value function for gains and losses by selecting a gain amount G^* within the range of value measured in step 1, then determining the loss amount L^* such that a participant finds the mixed prospect $(G^*, p_g; L^*)$ barely acceptable (i.e., is indifferent to playing the prospect or not). This implies that:

$$w^+v(G^*) + w^-\lambda v(L^*) = v(0) = 0$$

so one can easily solve for λ. Although the method of Abdellaoui *et al*. (2007c) is designed to elicit value function and loss aversion parameters, it also provides as a byproduct measurement of a decision weight. By repeating the procedure for various probabilities of gain and loss, several decision weights can be obtained for mapping more complete weighting functions.

Semi-parametric methods provide a compromise between accuracy of a non-parametric elicitation method and the efficiency of a parametric method. They tend to be less cognitively demanding and less time consuming than pure non-parametric methods and the statistical method.

Parametric Methods

The final method for eliciting prospect theory value- and weighting-functions is a purely parametric approach. Tversky and Kahneman (1992) elicited cash equivalents for a number of single- and two-outcome prospects entailing pure gains, pure losses, and mixed outcomes. These were entered into a non-linear regression assuming a power value function (V1) and single-parameter weighting function (W1).

A simpler procedure can be executed using Prelec's (1998) single-parameter weighting function (W3B) and a power value function. If we elicit a number of certainty equivalents c_{ij} for prospects that pay \$$x_i$ with probability p_j, then we get by prospect theory:

$$c_{ij}{}^\alpha = x_i{}^\alpha \exp[-(\ln p)^\gamma].$$

TABLE 11.4 Major elicitation methods

Method class	Reference	Prospect theory component(s)	Cognitive demands	Time required
Statistical	Gonzalez and Wu (1999)	All	Low	High
Non-parametric	Wakker and Deneffe (1996)	v^+ or v^-	High	Medium
	Abdellaoui *et al*. (2007b)	v^+ and v^-	High	Medium
	Abdellaoui (2000)	w^+ or w^-	High	Medium
	Bleichrodt and Pinto (2000)	w^+ or w^-	High	Medium
Semi-parametric	Abdellaoui *et al*. (2007c)	v^+ and v^-, limited w^+, w^-	Medium	Low
Parametric	Prelec (1998)	v^+, w^+ or v^-, w^-	Low	Medium

Collecting outcomes on the left side of the equation and taking the double log of both sides, we get:

$$-\ln[-\ln(c_{ij}/x_i)] = \ln(\alpha) + \gamma[-\ln(-\ln p_j)].$$

This equation lends itself to linear regression to determine the parameters α and γ.

Parametric estimation of value and weighting functions has several advantages over other methods. The task of pricing simple prospects is cognitively tractable, the time requirements are relatively small, and this method tends to yield relatively reliable measurement. On the other hand, this method is susceptible to parametric misspecification, particularly if one assumes a single parameter weighting function (as in the method of Prelec described above) so that it is difficult to distinguish the curvature of the value function from elevation of the weighting function.

Table 11.4 summarizes the major methods for prospect theory elicitation, listing strengths and weaknesses of each method. All entail tradeoffs, and the particular method used by researchers will be determined by the cognitive sophistication of participants, time constraints, and technical constraints of the study in question.

Determining Certainty Equivalents

Several elicitation methods discussed above require determination of certainty equivalents of various prospects. The most straightforward (but cognitively demanding) method is to elicit them directly by asking participants for the sure amount of money c that they find equally attractive to a prospect (x, p). Participants can be provided incentives for accuracy using the method described by Becker *et al*. (1964)[9]. Alternatively, one might ask participants for the probability p such that they find the prospect (x, p) equally attractive to the sure amount c. Empirically such elicitations tend to be quite noisy, but they are quick and convenient.

We caution researchers against such direct matching procedures. Prospect theory was originally articulated as a model of simple choice between prospects. Direct elicitation of sure amounts or probabilities to match prospects relies on the assumption of *procedure invariance*: two strategically equivalent methods of assessing preference should lead to the identical orderings between prospects. Unfortunately, this assumption is routinely violated. First, people generally afford more weight to outcomes relative to probabilities when they price prospects than when they choose between them. This can give rise to *preference reversal*, in which participants price a low-probability high-payoff bet (e.g., a 3/36 chance to win \$100) above a high-probability low-payoff bet (e.g., a 28/36 chance to win \$10) even though they prefer the latter to the former when facing a simple choice between them (see, for example, Tversky *et al*., 1990). Second, people tend to be more risk averse when matching prospects by varying probabilities than when matching prospects by varying outcomes (Hershey and Schoemaker, 1985). For instance, suppose that a participant is asked to report what p of receiving \$100 (or else nothing) is equally attractive to receiving \$35 for sure, and this participant reports a probability of .5. If that same participant is asked what certain amount is equally attractive to a .5 chance of \$100, he will generally report a value greater than \$35.

A popular alternative for overcoming limitations of direct matching procedures is to estimate cash equivalents from a series of choices. For instance, in pricing the prospect (100, .5) that offers a .5 chance of \$100, participants can be offered a series of choices between (100, .5) or \$100 for sure, (100, .5) or \$90 for sure, and so forth. For instance, if a participant

[9] This method is only incentive-compatible assuming the independence axiom, which of course is violated in prospect theory. For a further discussion see Karni and Safra, 1987.

chooses \$40 for sure over (100, .5) but also chooses (100, .5) over \$30 for sure, then by linear interpolation we can estimate his cash equivalent as approximately \$35. If a researcher tells participants that a randomly selected choice (from a randomly selected trial) will be honored for real money, then this method will be incentive-compatible (i.e., participants will have an economic incentive to respond honestly).

Sure amounts can be evenly spaced (e.g., Tversky and Fox, 1995) or logarithmically spaced (e.g., Tversky and Kahneman, 1992). If a researcher wishes to obtain higher-resolution estimates of cash equivalents, the sequential choice method cannot be readily accomplished in a single round. One approach is to use an iterated procedure in which a first-course evaluation is made followed by a more detailed series of choices etc. (e.g., Tversky and Kahneman, 1992; Tversky and Fox, 1995; Gonzalez and Wu, 1999). For instance, if a participant prefers \$40 to (100, .5) but \$30 to (100, .5) then four more choices might be presented between (100, .05) and \$28, \$26, \$24, \$22. Another, maximally efficient, approach is the "bisection method" in which each time a choice is made between two prospects (e.g. a risky and sure prospect) one of the outcomes is adjusted in smaller and smaller increments as preferences reverse. For instance, if a participant prefers \$50 to (100, .5) then he would be presented with a choice between \$25 and (100, .5). If he prefers the sure amount this time then he would be presented a choice between \$37.50 and (100, .5), and so forth. We note that, unlike single-round elicitations, the multi-round and bisection approaches to eliciting cash equivalents cannot easily be made incentive-compatible because if a randomly selected choice is honored for real money then participants can "game" the system so that a greater number of choices offer higher sure amounts (e.g., Harrison 1986). Pragmatically, however, this method remains popular, and there is no evidence that participants engage in such "gaming" (Peter Wakker, personal communication).

Empirical tests indicate that the bisection method performs much better than direct elicitation of cash equivalents (Bostic *et al*., 1990). Fischer *et al*. (1999) noted that elicitation of cash equivalents through a series of choices will suffer from some of the problems of direct elicitation when the goal of determining cash equivalents is transparent. This can be obscured by eliciting choices in a staggered order so that each successive choice entails measurement of the cash equivalent of a different prospect. The downside to this approach is that it is more time consuming than a more straightforward application of the bisection or sequential choice method that prices one prospect at a time.

Modeling choice variability

The elicitation methods described thus far have all assumed a deterministic model of decision under risk. Naturally, one would not expect a decision maker's choices in practice to be 100% consistent. At different moments in time, a participant may reverse preferences between prospects. Such reversals may be due to decision errors (i.e., carelessness or lapses in concentration) and/or transitory variations in the participant's genuine underlying preferences (e.g., due to emotional, motivational, and cognitive states that influence risk preference). Reversals in preference are more likely to occur when the participant has difficulty distinguishing between prospects or has only weak preferences between them – if a decision maker is indifferent between prospects g_1 and g_2, then one would expect a 50% chance of reversing preferences on a subsequent choice between the prospects; the more strongly g_1 is preferred to g_2 the more often we expect it to be chosen. Such response variability is typically substantial in studies of risky choice. For instance, in a survey of eight studies of risky choice, Stott (2006, Table 11.1) found a median 23% rate of reversal in preferences when participants chose between the same pair of prospects on separate occasions within or across sessions.

There are two distinct approaches to modeling choice variability. The first is to assume that preferences are consistent with prospect theory but allow preferences consistent with that theory to vary from moment to moment. The "random preference" approach assumes that choices reflect a random draw from a probability distribution over preferences that are consistent with an underlying core theory (see Becker *et al*., 1963, for an articulation of such a model under expected utility, and Loomes and Sugden, 1995, for a generalization). For instance, one could implement such a model using prospect theory value and weighting functions with variable parameters.

The second approach assumes a deterministic core theory but allows a specified error distribution to perturb the participant's response (see Becker *et al*., 1963, for an application to EU). Formally, let $f(g_1, g_2)$ be the relative frequency with which prospect g_1 is selected over prospect g_2 in a pairwise choice. Decisions are assumed to be stochastically independent from one another and symmetric, so that $f(g_1, g_2) = 1 - f(g_2, g_1)$. Let $V(g_i)$ be the prospect theory value of prospect g_i. Most response variability models assume that $f(g_1, g_2)$ increases monotonically with $V(g_1) - V(g_2)$, the difference in prospect theory value of prospects 1 and 2.

The choice function $f(\cdot)$ can take several forms (see Stott, 2006, Table 11.4). First, it can manifest itself as a *constant* error function in which there is a fixed probability of expressing one's true preference. Thus, $f(g_1, g_2) = \varepsilon$ whenever $V(g_1) < V(g_2)$, ½ whenever $V(g_1) = V(g_2)$, $1 - \varepsilon$ whenever $V(g_1) > V(g_2)$, where $0 \leq \varepsilon \leq ½$. Second, choice frequency might depend on the difference in prospect theory value between prospects, either following a *probit* transformation (e.g., Hey and Orme, 1994) or a *logit* transformation (e.g., Carbone and Hey, 1995). Thus, for the probit transformation,

$$f(g_1, g_2) = \Phi[(Vg_1) - V(g_2), 0, \sigma]$$

where $\Phi[x, \mu, \sigma]$ is the cumulative normal distribution with mean μ and SD σ at point x. Third, the choice function might follow a Luce (1959) choice rule, in which choice frequency depends on the ratio of prospect theory values of the prospects:

$$f(g_1, g_2) = \frac{V(g_1)^{\varepsilon}}{V(g_1)^{\varepsilon} + V(g_2)^{\varepsilon}}.$$

In an empirical test of several stochastic models assuming EU, Loomes and Sugden (1998) found that the random preference model tended to under-predict observed violations of dominance, and the error model assuming a probit transformation tended to over-predict such violations. The constant error form performed poorly.

The most comprehensive test to date of various choice functions and prospect theory value and weighting functional forms was reported by Stott (2006), who tested various combinations, including most of those described in this chapter. In his test, the model with the greatest explanatory power (adjusted for degrees of freedom) relied on a power value function (V1), a Prelec (1998) one-parameter weighting function (W3), and a logit function. However, for reasons already mentioned we recommend use of a two-parameter weighting function (W2) or (W3A).

The aforementioned models have been used to model preferences among pure gain or loss prospects. A stochastic method for measuring loss aversion was introduced by Tom *et al*. (2007). Their method required participants to make a series of choices as to whether or not to accept mixed prospects that offered a 50–50 chance of gaining $\$x$ or losing $\$y$ in which x and y were independently varied. These authors then assumed a piecewise linear value function, and also $w^{+}(.5) = w^{-}(.5)$[10]. They then determined the weight afforded the gain and loss portion of the gamble through logistic regression. This method has the advantage of allowing separate measurement of sensitivity to gains and losses (the regression coefficients), as well as response bias to accept or reject gambles (the intercept term).

[10]The former assumption is a customary and reasonable first approximation, and the latter assumption accords reasonably well with the data when it has been carefully tested (see Abdellaoui *et al*., 2007c).

NEUROSCIENTIFIC DATA

There has been substantial progress in understanding the neural correlates of prospect theory since we last reviewed the literature (Trepel *et al*., 2005). Below, we first outline some challenges to effective characterization of the relation between neural activity and theoretical quantities, and then review recent work that has characterized the brain systems involved in various components of prospect theory.

Paradigmatic Challenges

Integrating theories from behavioral decision-making research with neuroscientific evidence has posed a number of challenges to researchers in both fields.

Developing Clean Comparisons

A neuroimaging study is only as good as its task design. In particular, in the context of behavioral decision theory it is critical that tasks cleanly manipulate particular theoretical quantities or components. For example, a study designed to examine the nature of probability weighting must ensure that the manipulation of probability does not also affect value. Because it is often impossible cleanly to isolate quantities in this way using any specific task, another alterative is to vary multiple quantities simultaneously and then model these manipulations parametrically. This allows the response to each quantity to be separately estimated. For example, Preuschoff *et al*. (2006) manipulated both expected reward and risk in a gambling task, and were able to demonstrate different regions showing parametric responses to each variable.

Isolating Task Components

One of the most difficult challenges of fMRI is the development of task paradigms and analytic approaches that allow isolation of specific task components. For example, in tasks where participants make a decision and then receive an outcome, it is desirable to be able separately to estimate the evoked response to the decision and to the outcome. Because the fMRI signal provides a delayed and smeared representation of the underlying neuronal activity, the evoked response lags the mental event by several seconds. A number of earlier studies used an approach where specific timepoints following a particular component are assigned to that component; however, this approach is not a reliable way to isolate trial components, as it will provide at best a weighted average of nearby events (Zarahn, 2000). It is possible to model the individual components using the general linear model, but the regressors that model the different components are often highly correlated, resulting in inflated variance. One solution to this problem involves the use of random-length intervals between trial components; this serves to decorrelate the model regressors for each task component and allow more robust estimation of these responses (see, for example, Aron *et al*., 2004).

Inferring Mental States from Neural Data

It is very common in the neuroeconomics literature to infer the engagement of particular mental states from neuroimaging data. For example, Greene *et al*. (2001) found that moral decision making for "personal" moral dilemmas was associated with greater activity in a number of regions associated with emotion (e.g., medial frontal gyrus) compared to "impersonal" moral dilemmas. On the basis of these results, they concluded that the difference between these tasks lies in the engagement of emotion when reasoning about the personal dilemmas. Poldrack (2006) referred to this approach as "reverse inference," and showed that its usefulness is limited by the selectivity of the activation in question. That is, if the specific regions in question only activate for the cognitive process of interest, then reverse inference may be relatively powerful; however, there is little evidence for strong selectivity in current neuroimaging studies, and this strategy should thus be used with caution. For example, ventral striatal activity is often taken to imply that the participant is experiencing reward, but activity in this region has also been found for aversive stimuli (Becerra *et al*., 2001) and novel non-rewarding stimuli (Berns *et al*. 1997), suggesting that this reverse inference is not well founded.

Reference-dependence and Framing Effects

The neural correlates of reference-dependence in decision making have been examined in two studies. De Martino *et al*. (2006) manipulated framing in a decision task in which participants chose between a sure outcome and a gamble after receiving an initial endowment on each trial; gambles were not resolved during scanning. Framing was manipulated by offering participants a choice between a sure loss and a gamble (e.g., lose £30 vs gamble) or a sure win and a gamble (e.g., keep £20 vs gamble). Participants showed the standard behavioral pattern of risk seeking in the loss frame and risk aversion in the gain frame, with substantial individual variability. Amygdala activity was associated with the dominant choices, with increased activity for sure choices in the gain frame and risky choices in the loss frame; the dorsal anterior cingulate cortex (ACC) showed an opposite pattern across conditions. Individual differences in behavioral framing-related bias were correlated with framing-related activation in orbitofrontal and medial prefrontal cortex; that is, participants who showed less framing bias (and thus "behaved more rationally") showed more activity for sure choices in the gain frame and risky choices in the loss frame compared to the other two conditions. Thus, whereas amygdala showed the framing-related pattern across all participants on average, in the orbitofrontal cortex (OFC) this pattern was seen increasingly for participants who showed less of a behavioral framing effect. Although amygdala activation is often associated with negative outcomes, it has also been associated with positive outcomes (e.g., Ghahremani and Poldrack, unpublished work; Weller *et al*., 2007), and the correlation of amygdala activity with choice in the de Martino study is consistent with coding of value in the amygdala.

Windmann *et al*. (2006) compared two versions of the Iowa Gambling Task (IGT): a "standard" version (in which participants must learn to choose smaller constant rewards in order to avoid large punishments) and an "inverted" version (in which participants must choose large constant punishments in order to obtain large rewards). This is similar to an inverted version of the IGT examined by Bechara *et al*. (2000), who found that patients with ventromedial prefrontal cortex (PFC) lesions were equally impaired on the standard and inverted versions of the task. Windmann *et al*. (2006) found that the inverted IGT was associated with a greater neural response to rewards compared to punishments in the lateral and ventromedial OFC when contrasted with the standard task. Interestingly, it appeared that some of the same lateral OFC regions activated for punishments vs rewards in the standard task were also activated for rewards vs punishments in the inverted task. These results suggest that the OFC response to outcomes is strongly modulated by the framing of outcomes. However, it is difficult to interpret results strongly from the IGT because of its conflation of risk and ambiguity. Because participants begin the task with no knowledge about the relevant probabilities and must learn them over time, it is not possible to know whether activation in the task reflects differences in the learning process or differences in the representation of value and/or probability.

Together, these studies provide initial evidence for the neural basis of framing effects, but much more work is needed. In particular, because neuroimaging methods are correlational, it is difficult to determine whether these results reflect the neural causes or neural effects of reference-dependence. Further work with lesion patients should provide greater clarity on this issue.

Value Function

Before reviewing papers that purport to examine neurophysiological correlates of the prospect theory value function, we pause to distinguish different varieties of utility. Traditionally, the utility construct in neoclassical economics refers to a hypothetical function that cannot be directly observed mapping states of wealth to numbers; a decision maker whose choices adhere to the four axioms reviewed in the first section of this chapter can be represented as maximizing expected utility. Thus, utility is a mathematical construct that may or may not reflect the mental states of decision makers.

Although prospect theory also has an axiomatic foundation (Wakker and Tversky, 1993), the model is motivated by behavioral phenomena, such as the psychophysics of diminishing sensitivity, that are assumed to correspond to mental states of decision makers. However, it is important to distinguish different varieties of utility when using tools of neuroscience to interpret mental states of decision makers. In particular, "utility" in the context of making a decision may not be the same thing as "utility" in the context of experiencing or anticipating the receipt of an outcome. Economists have focused primarily on a measure of what Kahneman *et al*. (1997) call *decision utility*, which is the weight of potential outcomes in decisions. However, as these authors point out, the original concepts of utility from Bentham and others focused on the immediate experience of pleasure and pain, which they refer to as *experienced utility*. Others have highlighted the importance of the utility related to anticipating a positive or negative outcome (e.g., Loewenstein, 1987), referred to as *anticipation utility*. Of particular interest is the fact that these different forms of utility can be dissociated; for example, individuals sometimes make decisions that serve to decrease their experienced or anticipation utility. In order to be able to interpret clearly the results of neuroimaging studies, it is critical to distinguish between these different forms of utility.

The distinction between different forms of utility in behavioral decision theory parallels the distinction between "wanting" and "liking" that has developed in the animal literature (Berridge, 2007). A large body of work has shown that the neural systems involved in motivating aspects of reward ("wanting") can be dissociated from those involved in the hedonic aspects of reward ("liking"). This work has largely focused on neurochemical dissociations. Whereas dopamine is often thought to be involved with pleasurable aspects of reward, a large body of work in rodents has shown that disruption of the dopamine system impairs animals' motivation to obtain rewards (particularly when effort is required), but does not impair their hedonic experience (as measured using conserved behavioral signals of pleasure such as tongue protrusion and paw licking; Pecina *et al*., 2006). The hedonic aspects of reward appear to be mediated by opioid systems in the ventral striatum and pallidum. Although the mapping of neurochemical systems to functional neuroimaging results is tricky (Knutson and Cooper, 2005), these results provide further suggestion that "utility" is not a unitary concept.

Because it is most directly relevant to the prospect theory value function, we focus here on decision utility. This is the value signal that is most directly involved in making choices, particularly when there is no immediate outcome of the decision, as in purchasing a stock or lottery ticket. It has received relatively little interest in the neuroeconomics literature compared to experienced and anticipation utility, but several recent studies have examined the neural basis of decision utility using fMRI. Tom *et al*. (2007) imaged participants during a gamble acceptability paradigm, in which participants decided whether to accept or reject mixed gambles offering a 50% chance of gain and 50% chance of loss. The size of the gain and loss were varied parametrically across trials, with gains ranging from \$10 to \$40 (in \$2 increments) and losses from \$5 to \$20 (in \$1 increments). Participants received an endowment in a separate session 1 week before scanning, in order to encourage integration of the endowment into their assets and prevent the risk-seeking associated with "house money" effects (Thaler and Johnson, 1990). Participants exhibited loss-averse decision behavior, with a median loss aversion parameter $\lambda = 1.93$ (range: 0.99 to 6.75). Parametric analyses examined activation in relation to gain and loss magnitude. A network of regions (including ventral and dorsal striatum, ventromedial and ventrolateral

PFC, ACC, and dopaminergic midbrain regions) showed increasing activity as potential gain increased. Strikingly, no regions showed increasing activity as potential loss increased (even using weak thresholds in targeted regions including amygdala and insula). Instead, a number of regions showed *decreasing* activation as losses increased, and these regions overlapped with the regions whose activity increased for increasing gains.

The Tom *et al*. (2007) study further characterized the neural basis of loss aversion by first showing that a number of regions (including ventral striatum) showed "neural loss aversion," meaning that the decrease in activity for losses was steeper than the increase in activity for gains. Using whole-brain maps of these neural loss aversion parameters, they found that behavioral loss aversion was highly correlated across individuals with neural loss aversion in a number of regions including ventral striatum and ventrolateral PFC. These data are strongly consistent with prospect theory's proposal of a value function with a steeper slope for losses than for gains.

Decision utility was examined by Plassmann *et al*. (2007) using a "willingness-to-pay" (WTP) paradigm in which participants placed bids for a number of ordinary food items in a Becker–DeGroot–Marschak (BDM) auction, which ensures that participants' choices are an accurate reflection of their preferences. "Free bid" trials, in which participants decided how much to bid on the item, were compared with "forced bid" trials, in which participants were told how much to bid. Activity in ventromedial and dorsolateral PFC was correlated with WTP in the free bid trials but not the forced bid trials, suggesting that these regions are particularly involved in coding for decision utility.

The neural correlates of purchasing decisions were also examined by Knutson *et al*. (2007). Participants were presented at each trial with a product, and then given a price for that product and asked to indicate whether they would purchase the product for that price. Participants also provided WTP ratings after scanning was completed. Activity in ventral striatum and ventromedial PFC was greater for items that were purchased, whereas activity in anterior insula was lower for items that were purchased. A logistic regression analysis examined whether decisions could be better predicted by self report data or brain activity; although self-report data were much more predictive of purchasing decisions, a small (~1% of variance) increase in predictability was obtained when self-report and fMRI data were combined.

Because of the oft-noted association of the amygdala with negative emotions, it might be suspected that it would be involved in loss aversion in decision making. However, only one study has found amygdala activity in relation to loss aversion. Weber *et al*. (2007) examined reference-dependence using a design in which participants either bought or sold MP3 songs in a BDM auction. Comparison of selling trials versus buying trials showed greater activity in both amygdala and dorsal striatum, whereas comparison of buying versus selling trials showed greater activity in the parahippocampal gyrus. Given the association of amygdala with both positive and negative outcomes, it is unclear whether the effect for selling versus buying reflects the disutility of losing a good, the utility of gaining money, or some other factor. Further, a recent study by Weller *et al*. (2007) shows that patients with amygdala damage are actually impaired in making decisions about potential gains, whereas they are unimpaired in decisions about potential losses. These findings highlight the complexity of the amygdala's role in decision making, potentially suggesting that there are underlying factors modulating amygdala activity that have yet to be discovered.

Together, these results begin to characterize a system for decision utility, with the ventromedial PFC appearing as the most consistent region associated with decision utility. These results are consistent with other data from neurophysiology in non-human primates suggesting a representation of the value of goods such as foods (Padoa-Schioppa and Assad, 2006). However, the results also raise a number of questions. First, they cast some doubt over a simple two-system model with separate regions processing potential gains and losses. It is clear that the neural activity evoked by potential gains and losses is only partially overlapping with that evoked by actual gains and losses, but further work is needed to better characterize exactly how the nature of the task (such as the participants' anticipation of outcomes) changes neural activity. Second, they cast doubt over the common inference that amygdala activity is related to negative emotion, as it is clear that positive outcomes can also activate the amygdala. Further work is necessary to better understand the amygdala's role in decision making. Third, they leave unexplained how neural activity relates to the characteristic S-shaped curvature of the value function that contributes a tendency toward risk aversion for gains and risk seeking for losses.

Probability Weighting Distortions

A number of recent studies have attempted to identify neural correlates of distortions in probability weighting. Paulus and Frank (2006) used a certainty equivalent paradigm in which participants chose between a gamble and a sure outcome on each trial; the gamble was altered in successive trials to estimate the certainty equivalent. Non-linearity of the probability weighting function was estimated using the Prelec (1998) weighting function. Regression of activation for high- versus low-probability prospects showed that activity in the ACC was correlated with the non-linearity parameter, such that participants with more ACC activity for high versus low prospects were associated with more linear weighting of probabilities.

Non-linearities in probability weighting were also examined by Hsu *et al*. (2008). Participants chose between pairs of simple gambles, which varied in outcome magnitude and probability; on each trial, each gamble was first presented individually, then they were presented together and the participant chose between them. Weighting function non-linearity was estimated using the Prelec (1998) one-parameter weighting function (W3B). In order to isolate regions exhibiting non-linear responses with probability, separate regressors were created which modeled a linear response with p and a deflection from that linear function which represents non-linear effects. Significant correlations with both linear and non-linear regressors were found in several regions, including the dorsal striatum. Further analysis of individual differences showed a significant correlation between behavioral non-linearity and non-linearity of striatal response across participants.

Probability weighting distortion for aversive outcomes was examined by Berns *et al*. (2007). In a first phase, participants passively viewed prospects which specified the magnitude and probability of an electric shock. In a second phase, participants chose between pairs of lotteries. A quantity was estimated ("neurological probability response ratio," NPRR) which indexed the response to a lottery with probability less than one to a lottery with a probability of one (normalized by respect to the response to probability 1/3, which is the sampled point nearest to the likely intersection of the non-linear weighting function and linear weighting function – see Figure 11.3e). For the passive phase, NPRR was significantly non-linear for most regions examined, including regions in the dorsal striatum, prefrontal cortex, insula, and ACC. Activity from the passive phase was also used to predict choices during the choice phase; the fMRI signals provided significant predictive power, particularly for lotteries that were near the indifference point. Thus, there appears to be fairly wide-scale overweighting of low-probability aversive events in a number of brain regions.

Although the results of these studies are preliminary and not completely consistent, they suggest that it should be possible to identify the neural correlates of probability weighting distortions. It will be important to determine which regions are causally involved in these distortions (as opposed to simply reflecting the distortions) by testing participants with brain lesions or disorders. If non-linearities are the product of a specific brain system, then it should be possible to find participants whose choices are rendered linear with probability following specific lesions, similar to findings that VMPFC lesions result in more advantageous behavior in risky choice (Shiv *et al*., 2005).

CONCLUSIONS AND FUTURE DIRECTIONS

The field of neuroeconomics is providing a rapidly increasing amount of data regarding the phenomena that lie at the heart of prospect theory, such as framing effects and loss aversion. But we might ask: what have these data told us about prospect theory? It is clear that the demonstrations of neural correlates of several of the fundamental behavioral phenomena underlying prospect theory (loss aversion, framing effects, and probability weighting distortions) provide strong evidence to even the most entrenched rational choice theorists that these "anomalies" are real. The data have also started to provide more direct evidence regarding specific claims of the theory.

Our review of behavioral and neuroscience work on prospect theory and the neuroscience of behavioral decision making suggests a number of points of caution for future studies of decision making in the brain:

1. It is critical to distinguish between the different varieties of utility in designing and interpreting neuroscience studies. Studies in which participants make a decision and then receive an immediate outcome may be unable to disentangle the complex combination of decision, anticipation, and experienced utilities that are likely to be in play in such a task.
2. Under prospect theory, risk attitudes toward different kinds of prospects are interpreted in different ways. Risk aversion for mixed gambles is attributed to loss aversion; the fourfold pattern of risk attitudes for pure gain or loss gambles is attributed to diminishing sensitivity both to money (as reflected by curvature of the value function) and probability (as reflected by the inverse S-shaped weighting function). It is easy to conflate these factors empirically; for instance, if one assumes a single-parameter weighting function that only allows variation in curvature

but not elevation, then variations in observed risk attitudes across all probability levels may be misattributed to curvature of the value function.
3. Reverse inference (i.e., the inference of mental states from brain-imaging data) should be used with extreme care. As a means for generating hypotheses it can be very useful, but its severe limitations should be recognized.

Challenges for the Future

As neuroeconomics charges forward, we see a number of important challenges for our understanding of the neurobiology of prospect theory. First, it is critical that neuroimaging studies are integrated with studies of neuropsychological patients in order to determine not just which regions are correlated with particular theoretical phenomena, but also whether those regions are necessary for the presence of the phenomena. A nice example of this combined approach was seen in the study of ambiguity aversion by Hsu *et al*. (2005). It is likely that many of the regions whose activity is correlated with theoretical quantities (e.g., curvature of weighting function) may be effects rather than causes of the behavioral phenomena.

Another challenge comes in understanding the function of complex neural structures, such as the ventral striatum and amygdala, in decision making. Each of these regions is physiologically heterogeneous, but the resolution of current imaging techniques leads them to be treated as singular entities. In the amygdala, the heterogeneous nuclei are large enough that they could potentially be differentiated using currently available neuroimaging methods (e.g., Etkin *et al*., 2004). The neurobiological heterogeneity of the ventral striatum is more difficult to address using current neuroimaging methods; there are both structural features that are not currently visible to human neuroimaging (e.g., accumbens core vs. shell) as well as substantial cellular heterogeneity (e.g., striosomes vs. matrix, direct vs. indirect pathway) at an even finer grain. Finally, there is still substantial controversy over the degree to which imaging signals in the ventral striatum reflect dopamine release as opposed to excitatory inputs or interneuron activity. It is clear that imaging signals in the ventral striatum often exhibit activity that parallels the known patterns of dopamine neuron firing (in particular, prediction error signals), and dopamine has strong vascular as well as neuronal effects, so it is likely that it exerts powerful effects on imaging signals, but it is not currently known how to disentangle these effects from local neuronal effects.

Finally, one critical extension of present work will be to relate it to other work in the domain of cognitive control. The role of frontal and basal ganglia regions in the control of cognitive processes (including inhibition, selection, and interference resolution) is becoming increasingly well specified, but how these processes relate to decision making remains unknown. Given the availability of the prefrontal cortex to both neuroimaging and disruption by transcranial magnetic stimulation (TMS), there is hope that an understanding of the relation between cognitive control and decision making will be relatively tractable in comparison to subcortical regions.

APPENDIX

Formal Presentation of Cumulative Prospect Theory (adapted from Tversky and Kahneman, 1992)

Let S be the set whose elements are interpreted as states of the world, with subsets of S called *events*. Thus, S is the certain event and ϕ is the null event. A weighting function W (on S), also called a *capacity*, is a mapping that assigns to each event in S a number between 0 and 1 such that $W(\phi) = 0$, $W(S) = 1$, and $W(A) \geq W(B)$ if and only if $A \supseteq B$.

Let X be a set of consequences, also called *outcomes*, that also includes a neutral outcome 0. An uncertain prospect f is a function from S into X that assigns to each event A_i a consequence x_i. Assume that the consequences are ordered by magnitude so that $x_i > x_j$ if $i > j$. Cumulative prospect theory separates prospects into a positive part, f^+, that includes all $x_i > 0$, and a negative part, f^-, that includes all $x_i < 0$. CPT assumes a strictly increasing value function $v(x)$ satisfying $v(x_0) = v(0) = 0$.

CPT assigns to each prospect f a number $V(f)$ such that $f \succeq g$ if and only if $V(f) \geq V(g)$. Consider a prospect $f = (x_i, A_i)$, $-m \leq i \leq n$, in which positive (negative) subscripts refer to positive (negative) outcomes and decision weights $\pi^+(f^+) = (\pi_0^+, \ldots, \pi_n^+)$ and $\pi^-(f^-) = (\pi_{-m}^-, \ldots, \pi_0^-)$ for gains and losses, respectively. The value V of the prospect is given by

$$V(f) = V(f^+) + V(f^-)$$

where

$$V(f^+) = \sum_{i=1}^{n} \pi_i^+ v(x_i), \quad \text{and} \quad V(f^-) = \sum_{i=-m}^{0} \pi_i^- v(x_i)$$

where π^+ and π^- are defined as follows:

$$\pi_n^+ = W^+(A_n), \quad \pi_{-m}^- = W^-(A_{-m})$$

$$\pi_i^+ = W^+(A_i \cup \ldots \cup A_n) - W^+(A_{i+1} \cup \ldots \cup A_n), \quad \text{for } 0 \leq i \leq n-1$$

$$\pi_i^- = W^-(A_{-m} \cup \ldots \cup A_i) - W^-(A_{-m} \cup \ldots \cup A_{i-1}), \quad \text{for } 1-m \leq i \leq 0.$$

Acknowledgments

We thank Mohammed Abdellauoi, Han Bleichrodt, Paul Glimcher and Peter Wakker for useful feedback on earlier versions of this chapter and Liat Hadar for helpful assistance.

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