Electric Propulsion Systems

Physics, Engineering, and System Integration

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November 24, 2025

Lecture Outline

Solid Chemical Propulsion Overview

Solid Chemical Propulsion Systems:

Solid chemical propulsion offers high volumetric energy density, long-term storability, and instant readiness. Their scale ranges from micro-scale thrusters for attitude control to large boosters for heavy-lift launch vehicles.

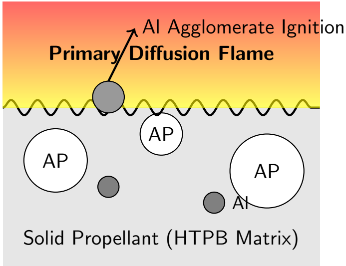
Critical Physics:

- Saint-Robert's Law ($r_b = aP_c^n$) – burning rate dependence on chamber pressure.
- Heterogeneous combustion physics – with multiple reacting phases.
- Erosive burning regimes in high L/D motors.

Example Composite Propellant (NH_4ClO_4/HTPB/Al)

Constituents & Roles:

- **Oxidizer:** NH_4ClO_4 (AP)
- **Binder:** HTPB. Structure + fuel source via pyrolysis.
- **Fuel:** Aluminum. $T_{flame} \approx 3400K$; acoustic damper.



Sequential Combustion Mechanism:

1. AP phase change (orthorhombic to cubic)
2. HTPB binder pyrolysis → gaseous HC fuels + solid char
3. AP Decomposition: $NH_4ClO_4 \longrightarrow NH_3 + HClO_4 \longrightarrow Cl_2 + O_2 + H_2O + N_2$
4. Surface diffusion flame
5. Al Melting & Combustion (far from surface).

Combustion Chamber Control Volume (V_c) Analysis

- Gaseous Mass Conservation:

$$\frac{d}{dt}(\rho_g V_c) = \dot{m}_{gen} - \dot{m}_{out}$$
$$V_c \frac{d\rho_g}{dt} + \rho_g \frac{dV_c}{dt} = \dot{m}_{gen} - \dot{m}_{out}$$

- V_c increases as the grain burns:

$$\frac{dV_c}{dt} = A_b r_b$$

- A_b is the burning surface area, and r_b is the burn rate.
- Mass generation rate from propellant burning: $\dot{m}_{gen} = \rho_p A_b r_b$
- Mass outflow rate through nozzle: $\dot{m}_{out} = \frac{P_c A_t}{c^*}$

Unsteady Pressure Equation

Goal: Derive an expression for dP_c/dt in terms of chamber properties and burn rate.

$$V_c \frac{d\rho_g}{dt} + \rho_g \frac{dV_c}{dt} = \dot{m}_{gen} - \dot{m}_{out}$$

$$\frac{V_c}{RT_c}\left[\frac{dP_c}{dt} - \underbrace{\frac{P_c}{T_c}\frac{dT_c}{dt}}_{\text{Assume } \frac{dT_c}{dt}=0}\right] = \rho_p A_b r_b - \frac{P_c A_t}{c^*} - \rho_g \frac{dV_c}{dt}$$

$$\frac{dP_c}{dt} = \frac{RT_c}{V_c}\left[\rho_p A_b r_b \left(1 - \frac{\rho_g}{\rho_p}\right) - \frac{P_c A_t}{c^*}\right]$$

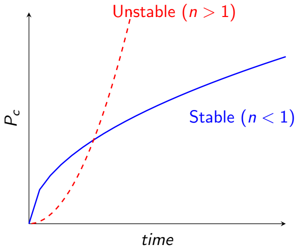
Saint-Robert's Law

Saint-Robert's Law: $r_b = aP_c^n$

- a = temperature dependent coefficient (hotter grains burn faster).
- n = pressure exponent

Stability Criterion ($\dot{m}_{gen} \propto P_c^n$, $\dot{m}_{out} \propto P_c$)

$$\begin{cases} n < 1 & \text{Stable} : \uparrow P_c \Rightarrow |\Delta \dot{m}_{gen}| < |\Delta \dot{m}_{out}| \\ n > 1 & \text{Unstable} : \uparrow P_c \Rightarrow |\Delta \dot{m}_{gen}| > |\Delta \dot{m}_{out}| \end{cases}$$



Steady State Chamber Pressure

- **Assume Steady State:**

$$\frac{dP_c}{dt} = 0$$

$$\rho_p A_b r_b \left(1 - \underbrace{\frac{\rho_g}{\rho_p}}_{neglect} \right) = \frac{P_c A_t}{c^*}$$

- **Substitute Saint-Robert's Law ($r_b = aP_c^n$):**

$$\rho_p A_b a P_c^n = \frac{P_c A_t}{c^*}$$

- **Equilibrium Pressure:**

$$P_c^{eq} \approx \left[\rho_p a c^* K_n\right]^{\frac{1}{1-n}}$$

- Note sensitivity to $K_n = A_b/A_t$. If $n = 0.5$, $P \propto K_n^2$.

Burn Area (A_b) Geometric Evolution

Evolution of Grain Geometry ($A_b(t)$) Controls Thrust Profile:

- **Progressive:** A_b increases $\Rightarrow$ T increases.
- **Neutral:** A_b constant $\Rightarrow$ T constant.
- **Regressive:** A_b decreases $\Rightarrow$ T decreases.

Common Grain Geometries:

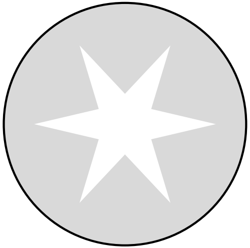
- **Progressive:**Circular Perforation or Center-Perforated Cylinder
- **Neutral:** Star or Wagon Wheel
- **Regressive:** Outer-burning grains or multi-propellant stacks

Advanced Grain Geometries: Star and Finocyl

Star:

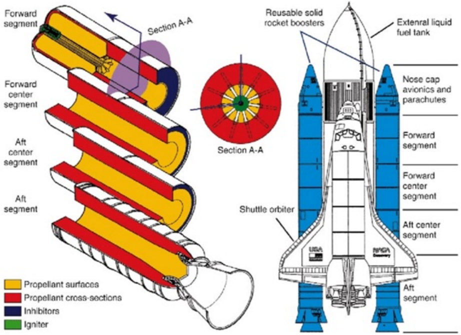
- $A_b \approx$ constant (neutral burn).
- Sliver formation an efficiency challenge.

Star Grain



Finocyl (Fin-On-Cylinder):

- Provides flexibility in thrust profile design.
- Shuttle SRM: High thrust after launch, lower thrust during max-Q.



Burnback Analysis & Optimization

Goal: Optimize $A_b(t = 0)$ to provide the $A_b(t)$ that enables the target thrust profile.

- **Analytical Methods:** Simplified geometries (cylinders, stars) allow closed-form $A_b(t)$ expressions.
- **Level Set Methods::** Implicit surface tracking via signed distance functions.

$$\phi(x, y, t) = \begin{cases} > 0 & \text{in solid} \\ = 0 & \text{on surface} \\ < 0 & \text{in gas} \end{cases}$$

$$\frac{\partial \phi}{\partial t} + r_b |\nabla \phi| = 0$$

- **Optimization:** Iterate on initial geometry to achieve desired $A_b(t)$ and thrust profile.

Erosive Burning & Velocity Coupling

Phenomenon:

- Stagnant chamber assumption fails in high L/D motors.
- High velocity (u) enhances heat transfer to the propellant, increasing burn rate beyond Saint-Robert's prediction.

Lenoir-Robillard Model

$$r_{total} = r_{base} + r_{erosive}$$

$$r_{base} = aP_c^n$$

$$r_{erosive} = \alpha G^{0.8} L^{-0.2} e^{-\beta r_{total} \rho_p / G}$$

where $G = \rho_g u$ is the mass flux; α, β are empirical constants.

Implications of Erosive Burning

System Implications:

Erosive burning typically occurs in high L/D motors immediately after ignition when u is highest. As the grain burns, the port area (A_{port}) opens, reducing u and thus erosive effects.

Negative Erosion:

In some geometries, high velocity can cool the surface and slow the burning rate.

Acoustic Instability: Hart-McClure Criterion

Combustion Instability

Acoustic eigenmodes in the chamber can couple with unsteady heat release from the burning propellant, leading to large pressure oscillations or DC pressure increases that can damage or destroy the motor.

Stability: Determined by the net growth rate α_{net}.

$$\alpha_{net} = \underbrace{\alpha_{pressure} + \alpha_{velocity}}_{\text{Gains (Driving)}} - \underbrace{(\alpha_{nozzle} + \alpha_{particle} + \alpha_{wall})}_{\text{Losses (Damping)}}$$

$$\alpha_{net} < 0 \Rightarrow \text{Stable}; \quad \alpha_{net} > 0 \Rightarrow \text{Unstable}$$

- **Pressure Coupling:** Increased surface pressures can accelerate burning.
- **Particle Damping:** Al_2O_3 droplets lag gas oscil. & dissipate energy via viscous drag.
- **Design Trade-off:** "Smokeless" missiles (no Al) lose particle damping (stability risk).

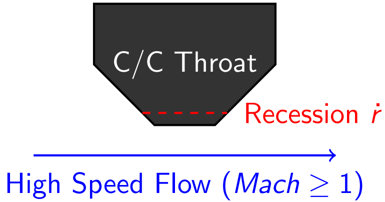
Ablative Nozzle Throat Erosion & Cooling

Nozzle Cooling Challenge

- Combustion gas temperature ($3400K$) exceeds melting point of all metals.
- Active cooling impractical due to solid grain geometry.

Solution: Carbon-Carbon (C/C) Ablation

- Mechanism is chemical erosion, not melting.
- Diffusion-controlled oxidation by H_2O and CO_2.
- $C_{(s)} + H_2O_{(g)} \rightarrow CO_{(g)} + H_2$



Bartz Correlation Scaling: $\dot{r}_{erosion} \propto P_c^{0.8} D_t^{-0.2}$

Solid Propulsion Summary

Takeaways

1. **Chemistry is Destiny:** APCP formulation dictates energy density and fundamental burn rate.
2. **Geometry is Control:** 3D grain design (Finocyl/Star) determines the thrust profile ($P \propto K_n^{1/(1-n)}$).
3. **Stability is Critical:** Acoustic eigenmodes must be damped by particles or mechanical resonators to prevent catastrophic failure.
4. **Materials Limit Performance:** Nozzle erosion imposes limits on pressure and burn time.

Lecture Outline

Chemical to Electric Propulsion

Chemical Propulsion:

- Energy source – reacting fuel and oxidizer – is the propellant.
- Max $v_e \approx 4.5$ km/s (H_2/O_2).

Electric Propulsion:

- Energy source decoupled from propellant.
- $v_e > 30$ km/s achievable.
- At fixed power, I_{sp} and thrust are inversely related.

The Power-Limited Rocket Equation

Thrust (T) is the momentum flux of the beam:

$$T = \dot{m} v_e \tag{1}$$

The Jet Power (P_{jet}) is the kinetic energy flux of the beam:

$$P_{jet} = \frac{1}{2}\dot{m} v_e^2 = \frac{1}{2} T v_e \tag{2}$$

The overall efficiency is given by $\eta_T = P_{jet}/P_{in}$:

$$T = \frac{2\eta_T P_{in}}{v_e} = \frac{2\eta_T P_{in}}{g_0 I_{sp}} \tag{3}$$

- Thrust (T) is *inversely proportional* to Specific Impulse (I_{sp}) @ fixed power (P_{in}).
- **Design Trade:** Minimize propellant mass (high I_{sp}) vs. Minimize trip time (high Thrust).

Efficiency Drivers

Diagnosing performance requires breaking down η_T:

$$\eta_T = \eta_e \cdot \eta_m \cdot \eta_b \cdot \eta_v \cdot \alpha \tag{4}$$

- **Electrical Efficiency (η_e):** Beam power vs. Total power (losses in magnets, heater, PPU).
- **Mass Utilization (η_m):** Fraction of propellant ionized ($\dot{m}_{ion}/\dot{m}_{total}$).
- **Beam Divergence (η_b or η_{div}):** Loss due to plume spread ($\langle\cos\theta\rangle^2$).
- **Voltage Utilization (η_v):** Effective beam voltage vs. Discharge voltage (V_b/V_d).
- **Doubles Correction (α):** Thrust loss due to multiply charged ions (Xe^{++}).

Comparative Analysis of Electric Propulsion Systems

Technology	I_{sp} (s)	Thrust	η_T	Power	TRL	Key Pros	Key Cons
Resistojet	280–350	10mN–0.5N	65–80%	<1 kW	9	Simple; Low cost; Shared propellant	Material thermal limits; Low I_{sp}
Arcjet	450–600	0.1N–2N	30–45%	0.5–2 kW	9	Robust; Higher I_{sp} than resistojet	Electrode erosion; Frozen flow losses
Gridded Ion	2500–4000+	20–250mN	60–80%	0.5–7 kW	9	Highest Efficiency; Longest Life	Low thrust density (Grid limits); Complex PPU
Hall Effect	1500–3000	40mN–1N+	50–65%	0.2–20 kW	9	High Thrust-to-Power; Compact	Beam divergence; Channel erosion (if unshielded)
Electrospray	1000–6000	μN–mN	$>70\%$	mW–50W	6–9	Precision control; Scalable arrays	Very low thrust; High voltage; Clogging risks
PPT	800–1500	μN-s	5–15%	<100 W	9	Solid fuel (Teflon); Simple storage	Very low efficiency; Carbon contamination
MPD	2000–10000	1N–100N	30–60%	MW Class	4–5	High thrust density at	Requires MW power;

Designer's Guide to Thruster Selection (1 of 2)

1. **Is Power the Primary Constraint?**
   - **Selection:** Hall Effect Thruster
   - *Rationale:* Offers higher Thrust-to-Power ratio (60–80 mN/kW) than Ion engines. Reduces maneuver time for power-limited commercial satellites.
2. **Is Propellant Mass the Primary Constraint?**
   - **Selection:** Gridded Ion Thruster
   - *Rationale:* Maximizes Specific Impulse ($I_{sp} > 3000$ s). Essential for high-Δv deep space missions (e.g., Dawn, BepiColombo) to minimize launch mass.

Designer's Guide to Thruster Selection (2 of 2)

3. **Is Cost or Volume Constrained?**
   - **Selection:** Argon Hall (Cost) or Iodine Hall (Volume)
   - *Rationale:* Argon is abundant and cheap for constellations (Starlink). Iodine stores as a dense solid, eliminating high-pressure tanks for CubeSats.
4. **Is Precision Pointing Required?**
   - **Selection:** FEEP / Electrospray
   - *Rationale:* Provides μN thrust resolution with no moving parts (valves). Critical for drag-free science missions (e.g., LISA).

Resistojets & Arcjets

Resistojets:

- Mechanism: Gas flows over electrically heated element (Re, W).
- Limit: Material melting point.
- Performance: $I_{sp} \approx 300$ s (Hydrazine).
- Application: Station-keeping, attitude control.

Arcjets:

- Mechanism: High-current arc heats gas core $> 10{,}000$ K.
- Physics: Cool boundary layer protects nozzle walls.
- Performance: $I_{sp} \approx 500 - 600$ s.
- Limitation: Electrode erosion, high power density required.

Arcjet Thruster

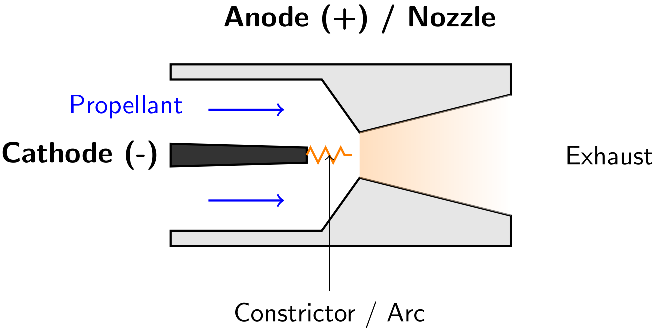


Figure: Arcjet Thruster Schematic

Gridded Ion Thruster Architecture

Core Concept:

- Decoupled ionization and acceleration.
- Highest efficiency EP device.

Ionization Generation:

- **DC (Kaufman):** Hollow cathode + Ring-cusp magnetic confinement.
- **RF (RIT):** Inductive coil, electrodeless discharge (longer life).

Acceleration:

- Multi-grid assembly (Screen + Accel).
- Electrostatic field extracts ions.

Gridded Ion Thruster Schematic

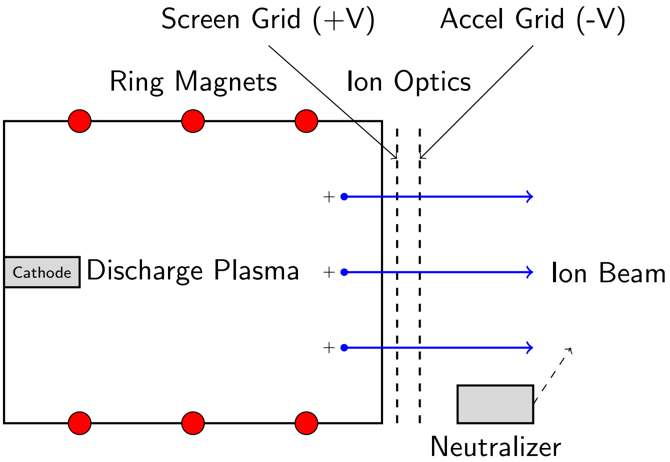


Figure: Electrostatic: Gridded Ion Thruster

The Child-Langmuir Limit

Thrust density is limited by space charge shielding between grids:

$$J_{CL} = \frac{4\epsilon_0}{9}\sqrt{\frac{2q}{m_i}}\frac{V_T^{3/2}}{d^2} \tag{5}$$

- J: Current density (A/m^2)
- V_T: Total voltage $(V_{beam} + |V_{accel}|)$
- d: Grid gap

Implication: To increase thrust, you must increase voltage or decrease grid gap (manufacturing limit). Ion thrusters are "large" for their thrust.

Optics Lifetime: Erosion Physics

Primary Failure Mode: Accelerator Grid Erosion

- Mechanism: Charge Exchange (CEX) Collisions.
- Fast Ion + Slow Neutral $\rightarrow$ Fast Neutral + Slow Ion.
- Slow ions form in the gap/plume and accelerate into the negative Accel Grid.
- Result: "Pits and Grooves" erosion.

Perveance Matching:

- **Over-perveance:** Sheath bulges, direct impingement.
- **Under-perveance:** Beam focuses too sharply, CEX crossover.

Hall Thruster Operating Principle

Cross-Field ($E \times B$) Discharge:

- Radial Magnetic Field (B_r) + Axial Electric Field (E_z).
- Electrons are magnetized ($\Omega_e \gg 1$), Ions are un-magnetized.

$$\Omega_e = \frac{eB}{m_e \nu_{coll}} \tag{6}$$

Magnetic Insulation:

- High impedance to electron flow allows supporting large potential drop ($V_d \approx 300 - 600$ V) in a quasi-neutral plasma.
- **Advantage:** No Space Charge Limit! Higher thrust density than Ion Thrusters.

Hall Effect Thuster Schematic

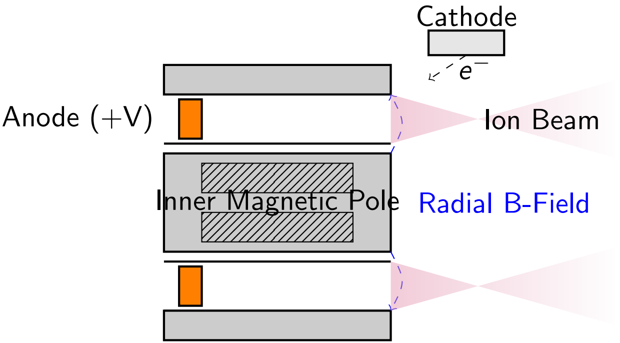


Figure: Electrostatic: Hall Effect Thruster (Cross Section)

Anomalous Transport & Instabilities

Anomalous Transport:

- Classical collision theory predicts current $100\times$ lower than observed.
- Real transport driven by:
  - Plasma turbulence/waves (Bohm diffusion).
  - Near-Wall Conductivity (Secondary electron emission).

Breathing Mode Instability (10-30 kHz):

- "Predator-Prey" cycle between Neutrals and Ions.
- Ionization depletes neutrals $\rightarrow$ Current drops $\rightarrow$ Neutrals refill $\rightarrow$ Current spikes.
- Requires active filtering in PPU.

The Revolution: Magnetic Shielding

The Problem (Pre-2010):

- Field lines intersected ceramic walls. High energy ions eroded channel.
- Lifetime limit: $\approx 5,000$ hours.

The Solution (Magnetic Shielding):

- Topology shaping: Field lines parallel to walls, grazing the anode.
- Creates equipotential zone near wall (T_e is low).
- Potential drop ¡ Sputtering threshold.
- **Result:** Erosion reduced $100 - 1000\times$. Lifetimes $>$ 50,000 hours (e.g., AEPS for Lunar Gateway).

Electromagnetic Propulsion

Lorentz Force: $\vec{F} = \vec{J} \times \vec{B}$

- **MPD (Magnetoplasmadynamic):**
  - Multi-MW power handling.
  - Self-field or Applied-field.
  - Issue: Cathode erosion at kA currents.
- **PPT (Pulsed Plasma Thruster):**
  - Solid Teflon ablation. Simple, robust.
  - Low efficiency, used for CubeSats/Attitude control.
- **VASIMR:**
  - RF Ionization (Helicon) + RF Heating (ICRH).
  - Variable I_{sp} at constant power.

Pulsed Plasma Thruster Schematic

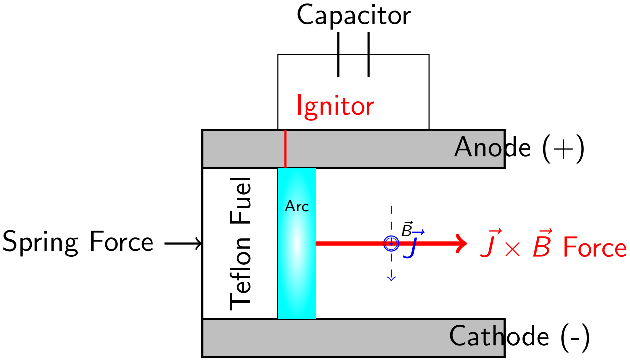


Figure: Electromagnetic: Pulsed Plasma Thruster (PPT)

Propellant Economics: The Shift from Xenon

Propellant	Mass (amu)	Cost	Use Case
Xenon	131.3	High (~$3k/kg)	Science / GEO
Krypton	83.8	Moderate	Starlink V1
Argon	39.9	Low (~$10/kg)	Starlink V2
Iodine	126.9	Moderate	CubeSats (Solid)

Starlink V2 (Argon):

- 4.2 kW, $I_{sp} \approx 2500$ s.
- $\eta_T \approx 50\%$ (Lower than Xe, but economically viable).
- High ionization energy of Ar (15.8 eV) challenges efficiency.

Micro-Propulsion: Electrospray

Physics:

- Electrostatic extraction from liquid meniscus.
- **Taylor Cone** formation balance (Surface tension vs. Electric stress).

$$V_{start} \approx \sqrt{\frac{\gamma r_c}{\epsilon_0}} \ln\left(\frac{4d}{r_c}\right) \tag{7}$$

Modes:

- **Cone-Jet (Colloid):** Charged droplets.
- **Ionic (FEEP/ILIS):** Pure ion evaporation ($I_{sp} > 6000$ s).

Scaling: MEMS arrays (thousands of emitters) required for useful thrust.

Electrospray Thruster Schematic

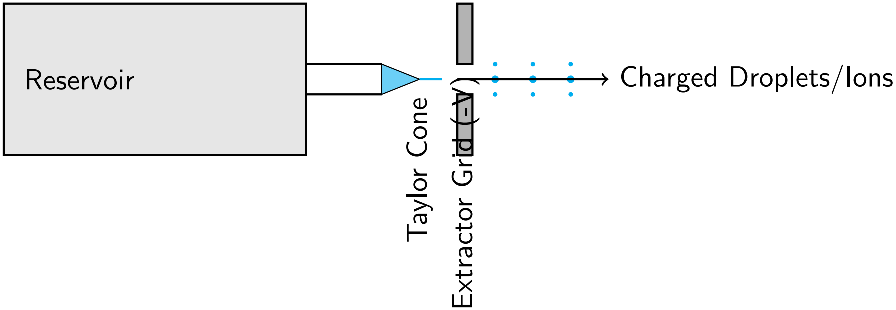


Figure: Micro-Propulsion: Electrospray Thruster

Conclusion

- **Physics:** Power-limited rocket equation drives the design.
- **Technology Bifurcation:**
  - **High Power/Life:** Magnetic Shielding enables Gateway/Mars (AEPS).
  - **Low Cost/High Volume:** Argon Hall thrusters enable Mega-constellations (Starlink).
- **Systems View:** PPU and Thermal management are dominant mass/volume drivers.
- **Future:** Higher power (100 kW+), alternative propellants (Iodine), and micro-scale integration.