Solid Chemical & Electric Propulsion Systems

Physics, Engineering, and System Integration

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Lecture Outline

Solid Chemical Propulsion Overview

Energy dense, storable, and instantly available

Solid Chemical Propulsion Systems:

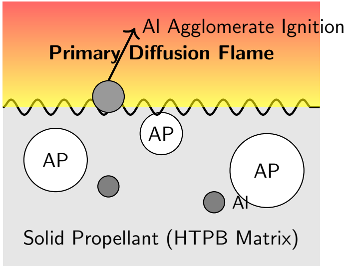
Solid chemical propulsion offers high volumetric energy density, long-term storability, and instant readiness. Their scale ranges from micro-scale thrusters for attitude control to large boosters for heavy-lift launch vehicles.

Critical Physics:

- Saint-Robert's Law ($r_b = aP_c^n$) – burning rate dependence on chamber pressure.
- Heterogeneous combustion physics – with multiple reacting phases.
- Erosive burning regimes in high L/D motors.

Example Composite Propellant (NH_4ClO_4/HTPB/Al)

Ammonium Perchlorate, Hydroxyl-Terminated Polybutadiene, & Aluminum



Constituents & Roles:

- **Oxidizer:** NH_4ClO_4 (AP)
- **Binder:** HTPB. Structure + fuel source via pyrolysis.
- **Fuel:** Aluminum. $T_{flame} \approx 3400K$; acoustic damper.

Sequential Combustion Mechanism:

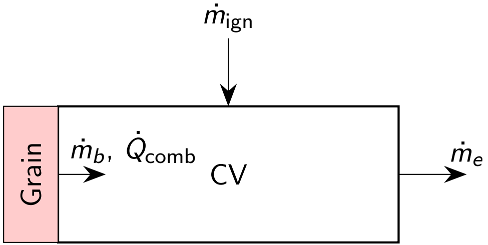
1. AP phase change (orthorhombic to cubic)
2. HTPB binder pyrolysis $\rightarrow$ gaseous HC fuels + solid char
3. AP Decomposition: $NH_4ClO_4 \longrightarrow NH_3 + HClO_4 \longrightarrow Cl_2 + O_2 + H_2O + N_2$
4. Surface diffusion flame
5. Al Melting & Combustion (far from surface).

Combustion Chamber Control Volume (V_c) Analysis

Predict 0-D chamber conditions, thrust & I_{sp} versus time

- Control volume $\Rightarrow$ gas inside the combustion chamber.
- Gas assumed perfectly mixed and spatially uniform.
- Ideal gas: $pV = mRT$.

- Sources:
  - Propellant–generated mass $\dot{m}_b$
  - Igniter mass $\dot{m}_{\text{ign}}$
  - Combustion heat release $\dot{Q}_{\text{comb}}$
- Sinks:
  - Nozzle mass flow $\dot{m}_e$
  - Wall heat losses $\dot{Q}_{\text{wall}}$



Mass Balance

Unsteady mass conservation with propellant burn & nozzle outflow

Unsteady mass conservation:	$\dfrac{dm}{dt} = \dot{m}_b + \dot{m}_{\mathrm{ign}} - \dot{m}_e$
Propellant mass generation:	$\dot{m}_b = \rho_p \, r_b \, A_b(s)$
Saint–Robert burn law:	$r_b = f_{\mathrm{ign}}(t) \, a \, p^n$
Ignition ramp:	$f_{\mathrm{ign}}(t) = 1 - \exp\left(-\dfrac{t - t_{\mathrm{ign,start}}}{\tau_{\mathrm{ign}}}\right)$
Choked nozzle mass flow:	$\dot{m}_e = C_d A_t p \sqrt{\dfrac{\gamma}{RT}} \left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$

Energy Balance with Heat Release

Unsteady energy conservation with combustion heat release & wall losses

Internal energy:

$$U = mc_v T$$

Full unsteady first law:

$$\frac{d}{dt}(mc_v T) = \dot{m}_b h_b + \dot{m}_{\text{ign}} h_{\text{ign}} - \dot{m}_e h_e - p\frac{dV}{dt} + \dot{Q}_{\text{wall}} + \dot{Q}_{\text{comb}}$$

Combustion heat release:

$$\dot{Q}_{\text{comb}} = \dot{m}_b \Delta h_c$$

Total heat term:

$$\dot{Q} = \dot{Q}_{\text{wall}} + \dot{Q}_{\text{comb}} + \dot{Q}_{\text{ign}}.$$

Deriving $\frac{dp}{dt}$ (Step 1)

Use the mass and energy equations to derive an expression for the evolution of chamber pressure with time.

Start with:

$$U = mc_v T$$

$$\frac{d}{dt}(mc_v T) = \dot{m}_b h_b + \dot{m}_{\text{ign}} h_{\text{ign}} - \dot{m}_e h_e - p\frac{dV}{dt} + \dot{Q}$$

Assume:

$$h_b \approx h_e \approx c_p T, \qquad h_{\text{ign}} \approx c_p T_{\text{ign}}$$

Left side:

$$\frac{d}{dt}(mc_v T) = c_v \left(m\frac{dT}{dt} + T\frac{dm}{dt} \right)$$

Right side becomes:

$$c_p T \frac{dm}{dt} - p\frac{dV}{dt} + \dot{Q}$$

Use mass balance:

$$\frac{dm}{dt} = \dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e$$

Deriving $\frac{dp}{dt}$ (Step 2)

Rearrange:

$$c_v m \frac{dT}{dt} = (c_p - c_v) T \frac{dm}{dt} - p \frac{dV}{dt} + \dot{Q}$$

Use $c_p - c_v = R$:

$$\frac{dT}{dt} = \frac{RT \frac{dm}{dt} - p \frac{dV}{dt} + \dot{Q}}{c_v m}$$

Differentiate ideal gas:

$$pV = mRT$$

$$\frac{dp}{dt} V + p \frac{dV}{dt} = R \left(T \frac{dm}{dt} + m \frac{dT}{dt} \right)$$

Substitute above dT/dt expression.

Deriving $\frac{dp}{dt}$ (Step 3)

Compute:

$$R\left(T\frac{dm}{dt} + m\frac{dT}{dt}\right) = RT\frac{dm}{dt} + \frac{R}{c_v}\left(RT\frac{dm}{dt} - p\frac{dV}{dt} + \dot{Q}\right)$$

Use $\gamma = \frac{c_p}{c_v}$ and $\frac{R}{c_v} = \gamma - 1$:

$$= \gamma RT\frac{dm}{dt} - (\gamma - 1)p\frac{dV}{dt} + (\gamma - 1)\dot{Q}$$

Insert into:

$$\frac{dp}{dt}V + p\frac{dV}{dt} = R\left(T\frac{dm}{dt} + m\frac{dT}{dt}\right)$$

Thus:

$$\frac{dp}{dt}V = \gamma RT\frac{dm}{dt} - \gamma p\frac{dV}{dt} + (\gamma - 1)\dot{Q}$$

Deriving $\frac{dp}{dt}$ (Final Form)

Divide by V:

$$\frac{dp}{dt} = \frac{\gamma RT}{V}\frac{dm}{dt} - \frac{\gamma p}{V}\frac{dV}{dt} + \frac{\gamma - 1}{V}\dot{Q}$$

Use ideal gas $pV = mRT$:

$$\frac{RT}{V} = \frac{p}{m}$$

Therefore:

$$\boxed{\frac{dp}{dt} = \gamma p\left[\frac{1}{m}\frac{dm}{dt} - \frac{1}{V}\frac{dV}{dt}\right] + \frac{\gamma - 1}{V}\dot{Q}}$$

Volume Evolution and Web Regression

Chamber volume and propellant burn area calculated from propellant geometry model

Web regression:

$$\frac{ds}{dt} = r_b = f_{\text{ign}}(t)\, a\, p^n$$

Grain geometry tables:

$$A_b(s), \qquad V(s), \qquad \frac{dV}{ds}(s)$$

Chamber volume evolution:

$$\frac{dV}{dt} = \frac{dV}{ds}(s)\,\frac{ds}{dt}$$

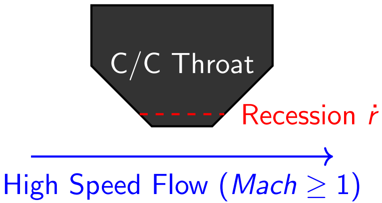
Ablative Nozzle Throat Erosion & Cooling

Nozzle Cooling Challenge

- Combustion gas temperature ($> 3000K$) exceeds melting point of all metals.
- Active cooling impractical due to solid grain geometry.

Solution: Carbon-Carbon (C/C) Ablation

- Mechanism is chemical erosion, not melting.
- Diffusion-controlled oxidation by H_2O and CO_2.
- $C_{(s)} + H_2O_{(g)} \rightarrow CO_{(g)} + H_2$



Bartz Correlation Scaling: $\dot{r}_{erosion} \propto P_c^{0.8} D_t^{-0.2}$

Simple Alternative: $\dot{r}_{erosion} = k_{\text{eros}} |\dot{m}_e|$

Final Pressure Equation (SRM Form)

With the SRM source terms:

$$\frac{dm}{dt} = \dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e, \qquad \frac{dV}{dt} = \frac{dV}{ds}(s)\frac{ds}{dt},$$

the chamber pressure ODE is:

$$\boxed{\frac{dp}{dt} = \gamma p \left[\frac{\dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e}{m} - \frac{1}{V}\frac{dV}{dt}\right] + \frac{\gamma - 1}{V}\dot{Q}}$$

Final ODE System

System of four ordinary differential equations

State vector:

$$y(t) = \begin{bmatrix} p(t) \\ m(t) \\ s(t) \\ r_t(t) \end{bmatrix}$$

ODEs:

$$\frac{dp}{dt} = \gamma p \left[\frac{\dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e}{m} - \frac{1}{V}\frac{dV}{dt} \right] + \frac{\gamma - 1}{V}\dot{Q}$$

$$\frac{dm}{dt} = \dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e$$

$$\frac{ds}{dt} = f_{\text{ign}}(t)\, a\, p^n$$

$$\frac{dr_t}{dt} = k_{\text{eros}} |\dot{m}_e|$$

Assumptions & Limitations of the 0-D SRM Model

Major assumptions:

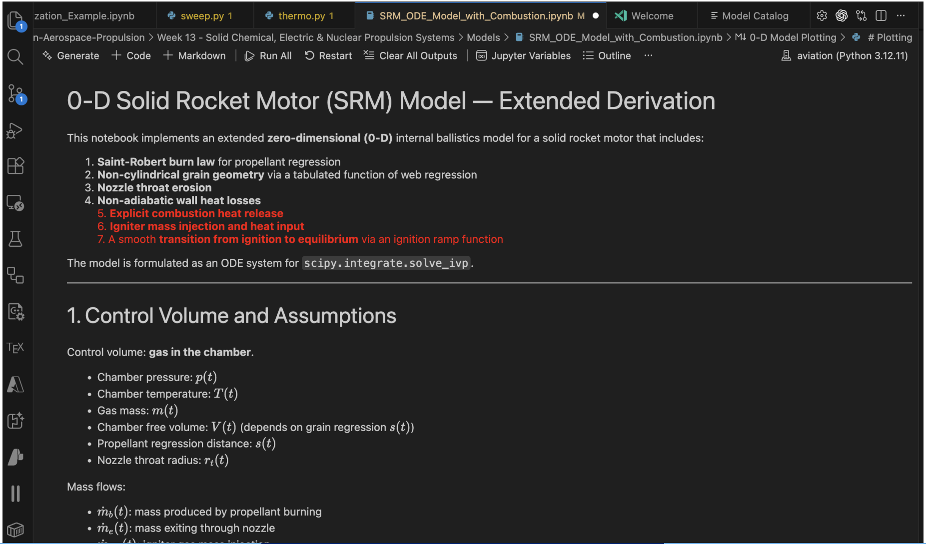
- Chamber gas is **perfectly mixed** (no spatial gradients).
- Propellant gases instantaneously reach chamber temperature.
- Ideal-gas thermodynamics: $pV = mRT$.
- Quasi-steady Saint–Robert burn law: $r_b = ap^n$.
- Choked nozzle flow with fixed C_d.
- Lumped heat loss: $\dot{Q}_{\text{wall}} = -h_w A_w (T - T_w)$.
- Grain geometry encoded through tables $A_b(s)$ and $V(s)$.

Limitations:

- Cannot predict combustion instabilities or pressure oscillations.
- No axial or transverse wave dynamics (1-D/3-D neglected).
- No local flame chemistry or finite-rate kinetics.
- No particle dynamics (Al droplets, slag accumulation).
- No two-phase flow in nozzle.

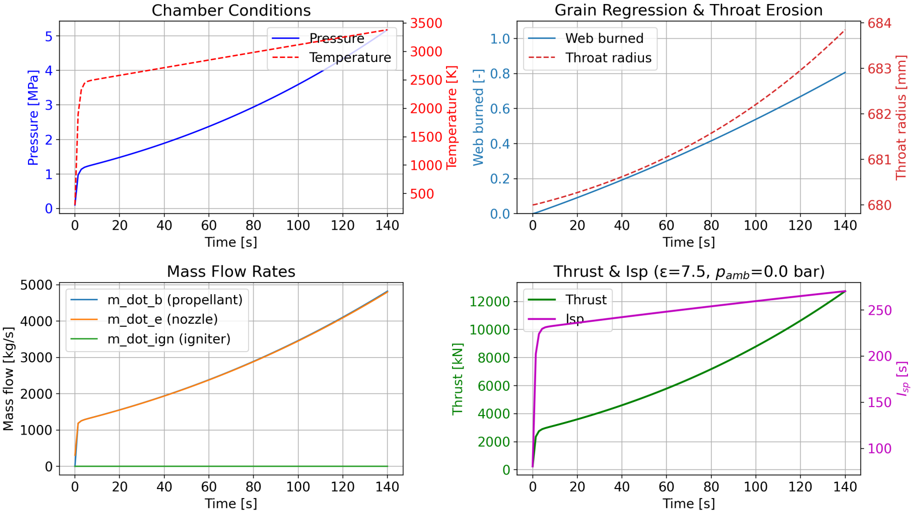
SRM Model Implementation

Uploaded to Course Github Repository



Example SRM Simulation Results

Intended to be an approximate representation of the Space Shuttle's SRB

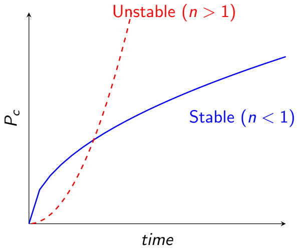


Saint-Robert's Law: $r_b = aP_c^n$

- a = temperature dependent coefficient (hotter grains burn faster).
- n = pressure exponent

Stability Criterion ($\dot{m}_{gen} \propto P_c^n$, $\dot{m}_{out} \propto P_c$)

$$\begin{cases} n < 1 & \text{Stable} :\uparrow P_c \Rightarrow |\Delta \dot{m}_{gen}| < |\Delta \dot{m}_{out}| \\ n > 1 & \text{Unstable} :\uparrow P_c \Rightarrow |\Delta \dot{m}_{gen}| > |\Delta \dot{m}_{out}| \end{cases}$$



Burn Area (A_b) Geometric Evolution

Evolution of Grain Geometry ($A_b(t)$) Controls Thrust Profile:

- **Progressive:** A_b increases $\Rightarrow$ T increases.
- **Neutral:** A_b constant $\Rightarrow$ T constant.
- **Regressive:** A_b decreases $\Rightarrow$ T decreases.

Common Grain Geometries:

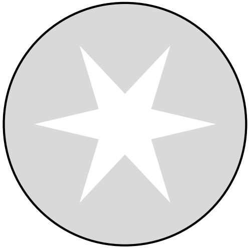
- **Progressive:** Circular Perforation or Center-Perforated Cylinder
- **Neutral:** Star or Wagon Wheel
- **Regressive:** Outer-burning grains or multi-propellant stacks

Advanced Grain Geometries: Star and Finocyl

Star:

- $A_b \approx$ constant (neutral burn).
- Sliver formation an efficiency challenge.

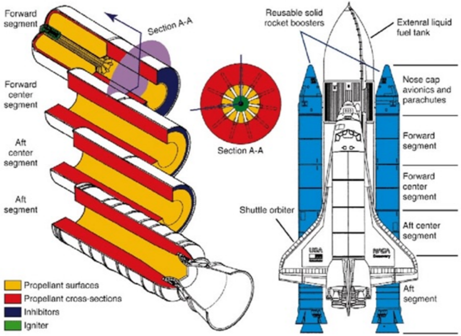
Star Grain



Finocyl (Fin-On-Cylinder):

- Provides flexibility in thrust profile design.
- Shuttle SRM: High thrust after launch, lower thrust during max-Q.

https://wikis.mit.edu/confluence/display/RocketTeam/Commercial+Grain+Geometries



Burnback Analysis & Optimization

Goal: Optimize $A_b(t = 0)$ to provide the $A_b(t)$ that enables the target thrust profile.

- **Analytical Methods:** Simplified geometries (cylinders, stars) allow closed-form $A_b(t)$ expressions.
- **Level Set Methods:**: Implicit surface tracking via signed distance functions.

$$\phi(x, y, t) = \begin{cases} > 0 & \text{in solid} \\ = 0 & \text{on surface} \\ < 0 & \text{in gas} \end{cases}$$

$$\frac{\partial \phi}{\partial t} + r_b |\nabla \phi| = 0$$

- **Optimization:** Iterate on initial geometry to achieve desired $A_b(t)$ and thrust profile.

Erosive Burning & Velocity Coupling

Phenomenon:

- Stagnant chamber assumption fails in high L/D motors.
- High velocity (u) enhances heat transfer to the propellant, increasing burn rate beyond Saint-Robert's prediction.

Lenoir-Robillard Model

$$r_{total} = r_{base} + r_{erosive}$$

$$r_{base} = aP_c^n$$

$$r_{erosive} = \alpha G^{0.8} L^{-0.2} e^{-\beta r_{total} \rho_p / G}$$

where $G = \rho_g u$ is the mass flux; α, β are empirical constants.

Implications of Erosive Burning

System Implications:

Erosive burning typically occurs in high L/D motors immediately after ignition when u is highest. As the grain burns, the port area (A_{port}) opens, reducing u and thus erosive effects.

Negative Erosion:

In some geometries, high velocity can cool the surface and slow the burning rate.

Acoustic Instability: Hart-McClure Criterion

Combustion Instability

Acoustic eigenmodes in the chamber can couple with unsteady heat release from the burning propellant, leading to large pressure oscillations or DC pressure increases that can damage or destroy the motor.

Stability: Determined by the net growth rate α_{net}.

$$\alpha_{net} = \underbrace{\alpha_{pressure} + \alpha_{velocity}}_{\text{Gains (Driving)}} - \underbrace{(\alpha_{nozzle} + \alpha_{particle} + \alpha_{wall})}_{\text{Losses (Damping)}}$$

$$\alpha_{net} < 0 \Rightarrow \text{Stable}; \quad \alpha_{net} > 0 \Rightarrow \text{Unstable}$$

- **Pressure Coupling:** Increased surface pressures can accelerate burning.
- **Particle Damping:** Al_2O_3 droplets lag gas oscil. & dissipates energy via viscous drag.
- **Design Trade-off:** "Smokeless" missiles (no Al) lose particle damping (stability risk).

Solid Propulsion Summary

Takeaways

1. **Chemistry is Destiny:** Chemistry dictates energy density and fundamental burn rate.
2. **Geometry is Control:** Grain geometry determines the thrust profile ($P \propto K_n^{1/(1-n)}$).
3. **Stability is Critical:** Acoustic eigenmodes must be damped to prevent failure.
4. **Materials Limit Performance:** Nozzle erosion imposes limits on pressure and burn time.

Solid vs. Liquid Chemical Propulsion

Aspect	Solid Propulsion	Liquid Propulsion
Energy Density	High (volumetric)	Moderate (mass-based)
Thrust Control	Limited (grain geometry)	Precise (throttleable)
Ignition	Instantaneous	Requires ignition system
Storability	Long-term stable	Cryogenic or limited life
Complexity	Simple, few moving parts	Complex (pumps/valves)
Reliability	High (few failure modes)	Moderate (more components)
Applications	Boosters, missiles	Upper stages, deep space

Exhaust Velocity & Specific Impulse: Solid vs. Liquid

Ideal Exhaust Velocity & Specific Impulse

$$V_e = \sqrt{\frac{2\gamma R_u}{\gamma - 1}\left(\frac{\mathbf{T_c}}{\mathfrak{M}}\right)\left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$I_{sp} = \frac{V_e}{g_0} \propto \sqrt{\frac{T_c}{\mathfrak{M}}}$$

where γ = specific heat ratio, R_u = universal gas constant, T_c = chamber temperature, $\mathfrak{M}$ = molecular weight of exhaust, P_c = chamber pressure, P_e = exit pressure.

I_{sp} **Drivers:**

- **Chamber Temperature (T_c):** Both solids and liquids achieve $3000K+$.
- **Molecular Weight ($\mathfrak{M}$):** $\mathfrak{M}_{liquid} < \mathfrak{M}_{solid}$ Lower $\mathfrak{M}$ yields higher velocity.

Exhaust Molecular Weight ($\mathfrak{M}$) Differerences

Solid propellants yield heavy metal oxides and Cl species; liquid H_2 and CH_4 yield light H_2O and CO_2.

$H_2O + CO_2$ versus Metal Oxides + Cl

Liquid H_2 (and CH_4) yield low molecular weight exhaust streams, while solids are burdened by heavy metal oxides and chlorine species.

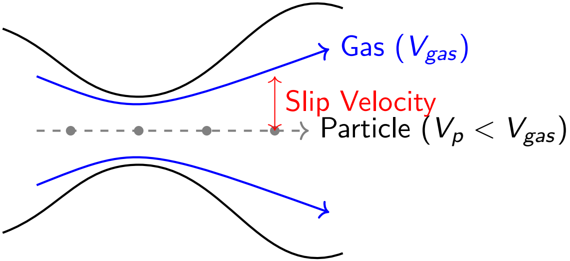
Property	**LOX/LH2**	**AP/Al/HTPB**
Products	$H_2O + H_2$ (Fuel Rich)	CO_2, HCl, Al_2O_3
Avg. $\mathfrak{M}$	9–12 g/mol	28–32 g/mol
I_{sp}	≈ 450 s	≈ 265 s

Two Phase Flow Loss Differences

Solid propellants use Al, yielding condensed-phase Al_2O_3 particles

Two Phase Flow Loss Drivers & Implication

- Al_2O_3 particles (liquid/solid) do not expand to do pressure work.
- **Velocity Lag:** Particles accelerate slower than gas.
- **Thermal Lag:** Particles stay hot, failing to transfer heat to kinetic energy.
- **Result:** 1–3% I_{sp} penalty.

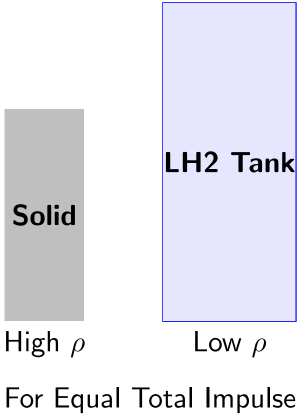


The Trade-off: Density Impulse

Solids lose on Specific Impulse (I_{sp}) but win on Density Impulse ($I_{sp} \times \rho$).

Strategic Implications:

- **Liquid Hydrogen:** $\rho \approx 0.07$ g/cm^3. Requires massive tank volume. High drag, high structural mass.
- **Solid Propellant:** $\rho \approx 1.75$ g/cm^3. Compact storage.
- **Application:** Solids are ideal for first stages (Boosters) where thrust-to-volume is critical. Liquids are ideal for upper stages where mass efficiency (I_{sp}) dominates.



Solid vs Liquid Comparision Summary

Liquids achieve higher I_{sp} due to lightweight fuels, while solids suffer from heavier fuels and two-phase flow losses.

Parameter	Solids (APCP)	Liquids (LOX/LH2)
Combustion Temp (T_c)	≈ 3400 K	≈ 3600 K
Exhaust MW ($\mathfrak{M}$)	High (~ 30)	Low (~ 12)
Exhaust Phase	Gas + Solids	Pure Gas
Loss Mechanisms	Two-Phase Lag	Dissociation
Typical I_{sp} (Vac)	~ 280 s	~ 450 s

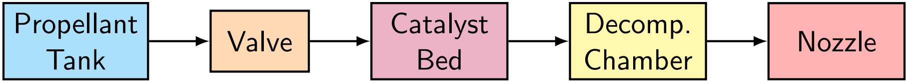
Lecture Outline

Liquid Monopropellant Thrusters

Leverages a single propellant that exothermically decomposes over a catalyst

Monopropellant thrusters:

- Use a **single fluid** (e.g., N_2H_4 [hydrazine] or H_2O_2 [peroxide])
- Decompose the fluid **catalytically** to form a hot gas mixture
- Expands mixture through a **C–D nozzle** to generate thrust
- Avoids mixture ratio control, igniters, and complex plumbing
- Used for attitude control, station keeping, "small" Δv maneuvers



Monopropellant Comparison

Hydrazine vs High Test Hydrogen Peroxide

Aspect	Hydrazine (N_2H_4)	High-Test Peroxide (H_2O_2)
Primary Reaction	$3\,N_2H_4 \rightarrow 4\,NH_3 + N_2$	$2\,H_2O_2 \rightarrow 2\,H_2O + O_2 + \text{heat}$
Secondary Step	$4\,NH_3 \rightarrow 2\,N_2 + 6\,H_2$ (cracking)	–
Catalyst	Ir/Al_2O_3 bed	Silver or Pt mesh
Products	N_2, H_2, NH_3 (some uncracked NH_3)	Steam (H_2O) + O_2
Nominal T_c	~ 1000 K	850–1150 K (conc. dependent)
Typical γ	≈ 1.30	1.25–1.30
Gas Constant R	~ 400 J/kg-K	300–350 J/kg-K
Vacuum I_{sp}	220–235 s	150–180 s

Lecture Outline

Chemical to Electric Propulsion

Chemical Propulsion:

- Energy source – reacting fuel and oxidizer – is the propellant.
- Max $v_e \approx 4.5$ km/s (H_2/O_2).

Electric Propulsion:

- Energy source (e.g., solar, nuclear) decoupled from propellant.
- $v_e > 30$ km/s achievable.
- At fixed power, I_{sp} and thrust are inversely related.

Power-Limited Rocket Equation

Thrust (T) is inversely proportional to Specific Impulse (I_{sp}) at fixed power (P_{in})

Thrust (T) is the momentum flux of the beam:

$$T = \dot{m} v_e \tag{1}$$

Jet Power (P_{jet}) is the kinetic energy flux of the beam:

$$P_{jet} = \frac{1}{2}\dot{m} v_e^2 = \frac{1}{2} T v_e \tag{2}$$

Overall efficiency is given by $\eta_T = P_{jet}/P_{in}$:

$$T = \frac{2\eta_T P_{in}}{v_e} = \frac{2\eta_T P_{in}}{g_0 I_{sp}} \tag{3}$$

Design Trade: Minimize propellant mass (high I_{sp}) vs. Minimize trip time (high Thrust).

Efficiency Drivers

Electrical conversion, ionization fraction, beam divergence, voltage utilization, doubles correction

Thruster overall efficiency (η_T)

$$\eta_T = \eta_e \cdot \eta_m \cdot \eta_b \cdot \eta_v \cdot \alpha$$

- **Electrical Efficiency (η_e):** Beam / Input Electric power (magnets, heat, PPU losses).
- **Mass Utilization (η_m):** Fraction of propellant ionized ($\dot{m}_{ion}/\dot{m}_{total}$).
- **Beam Divergence (η_b or η_{div}):** Loss due to plume spread ($\langle\cos\theta\rangle^2$).
- **Voltage Utilization (η_v):** Effective beam voltage vs. Discharge voltage (V_b/V_d).
- **Doubles Correction (α):** Thrust loss due to multiply charged ions (Xe^{++}).

Comparative Analysis of Electric Propulsion Systems

Technology	I_{sp} (s)	Thrust	η_T	Power	TRL	Key Pros	Key Cons
Resistojet	280–350	10mN–0.5N	65–80%	< 1 kW	9	Simple; Low cost; Shared propellant	Material thermal limits; Low I_{sp}
Arcjet	450–600	0.1N–2N	30–45%	0.5–2 kW	9	Robust; Higher I_{sp} than resistojet	Electrode erosion; Frozen flow losses
Gridded Ion	2500–4000+	20–250mN	60–80%	0.5–7 kW	9	Highest Efficiency; Longest Life	Low thrust density (Grid limits); Complex PPU
Hall Effect	1500–3000	40mN–1N+	50–65%	0.2–20 kW	9	High Thrust-to-Power; Compact	Beam divergence; Channel erosion (if unshielded)
Electrospray	1000–6000	μN–mN	$> 70\%$	mW–50W	6–9	Precision control; Scalable arrays	Very low thrust; High voltage; Clogging risks
PPT	800–1500	μN-s	5–15%	< 100 W	9	Solid fuel (Teflon); Simple storage	Very low efficiency; Carbon contamination

Designer's Guide to Thruster Selection (1 of 2)

1. **Is Power the Primary Constraint?**
   - **Selection:** Hall Effect Thruster
   - *Rationale:* Offers higher Thrust-to-Power ratio (60–80 mN/kW) than Ion engines. Reduces maneuver time for power-limited commercial satellites.
2. **Is Propellant Mass the Primary Constraint?**
   - **Selection:** Gridded Ion Thruster
   - *Rationale:* Maximizes Specific Impulse ($I_{sp} > 3000$ s). Essential for high-Δv deep space missions (e.g., Dawn, BepiColombo) to minimize launch mass.

Designer's Guide to Thruster Selection (2 of 2)

3. Is Cost or Volume Constrained?

- **Selection:** Argon Hall (Cost) or Iodine Hall (Volume)
- *Rationale:* Argon is abundant and cheap for constellations (Starlink). Iodine stores as a dense solid, eliminating high-pressure tanks for CubeSats.

4. Is Precision Pointing Required?

- **Selection:** FEEP / Electrospray
- *Rationale:* Provides μN thrust resolution with no moving parts (valves).

Electrothermal: Resistojets & Arcjets

Propellant energization & acceleration via electric heating.

Resistojets:

- Mechanism: Gas flows over electrically heated element (Re, W).
- Limit: Material melting point.
- Performance: $I_{sp} \approx 300$ s (Hydrazine).
- Application: Station-keeping, attitude control.

Arcjets:

- Mechanism: High-current arc heats gas core $> 10{,}000$ K.
- Physics: Cool boundary layer protects nozzle walls.
- Performance: $I_{sp} \approx 500 - 600$ s.
- Limitation: Electrode erosion, high power density required.

Electrothermal: Arcjet Thruster

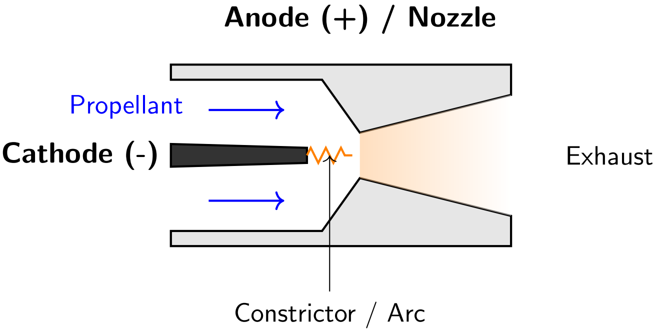


Figure: Arcjet Thruster Schematic

Electrostatic: Gridded Ion Thruster (GIT)

Ionized *propellant acceleration via electric fields*

Core Concept:

- Decoupled ionization and acceleration.
- Highest efficiency electric propulsion device.

Ionization Generation:

- **DC:** Hollow cathode + Ring-cusp magnetic confinement.
- **RF:** Inductive coil, electrodeless discharge (longer life).

Acceleration:

- Multi-grid assembly (Screen + Accel).
- Electrostatic field extracts ions.

Electrostatic: GIT Schematic

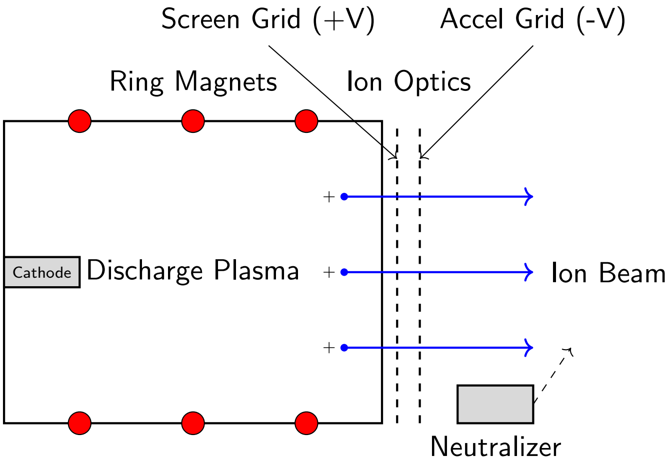


Figure: Electrostatic: Gridded Ion Thruster

Child-Langmuir Limit

Current/Thrust density is limited by space charge shielding between grid: Child–Langmuir Law

Child–Langmuir Law for Ion Thrusters:

$$J_{CL} = \frac{4\epsilon_0}{9}\sqrt{\frac{2q}{m_i}}\frac{V_{gap}^{3/2}}{d^2}$$

Where:

- J = Current density (A/m^2)
- V_{gap} = Screen gap voltage $(V_{screen} - V_{accel})$
- d = Grid gap

Implications:

- To increase J (and thrust density), increase voltage and decrease grid spacing.
- Heavier ions (larger m_i) reduce J_{CL}.
- Screen–accelerator gap behaves like a planar diode.

Child-Langmuir Law Derivation (1 of 2)

1D Planar diode with voltage V across gap d. Ions of mass m and charge q starting from rest at $x = 0$.

- Poisson's equation for the electric potential ϕ ($\vec{E} = -\nabla\phi$):

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\varepsilon_0} = -\frac{qn}{\varepsilon_0}$$

q = ion charge, n = ion density, ϵ_0 = permittivity of free space.

- Continuity and energy conservation equations:

$$J = qnv = \text{const}, \qquad \frac{1}{2}mv^2 = q\phi$$

- Solve continuity and energy conservation equations for the charge density n:

$$n = \frac{J}{qv} = \frac{J}{q}\sqrt{\frac{m}{2q}}\,\phi^{-1/2}$$

- Substitute the expresssion for n into Poisson's equation:

$$\frac{d^2\phi}{dx^2} = \frac{J}{\varepsilon_0}\sqrt{\frac{m}{2q}}\,\phi^{-1/2}$$

Child-Langmuir Law Derivation (2 of 2)

Manipulate & Integrate (twice) to yield J_{CL} expression mentioned earlier.

- Let $y = d\phi/dx$:

$$\frac{d^2\phi}{dx^2} = \frac{dy}{dx} = \frac{dy}{d\phi}\frac{d\phi}{dx} = y\frac{dy}{d\phi}$$

- So Poisson's equation becomes:

$$y\frac{dy}{d\phi} = \frac{J}{\varepsilon_0}\sqrt{\frac{m}{2q}}\,\phi^{-1/2}$$

- Integrate with $\phi(0) = 0$ at the emitting surface:

$$\frac{1}{2}y^2 = \frac{2J}{\varepsilon_0}\sqrt{\frac{m}{2q}}\,\phi^{1/2}$$

- So

$$\frac{d\phi}{dx} = C\,\phi^{1/4}, \quad C = \left(\frac{4J}{\varepsilon_0}\sqrt{\frac{m}{2q}}\right)^{1/2}$$

- Integrate from 0 to d, $\phi : 0 \to V$:

$$\int_0^V \phi^{-1/4}d\phi = C\int_0^d dx \Rightarrow \frac{4}{3}V^{3/4} = Cd$$

- Solve for J:

$$J = \frac{4}{9}\,\varepsilon_0\sqrt{\frac{2q}{m}}\,\frac{V^{3/2}}{d^2}$$

Numerical Example: Xenon Gridded Ion Thruster

- Xe^+, $q = e$, $m \approx 2.18 \times 10^{-25}\,\mathrm{kg}$.
- Grid spacing: $d = 1mm$.
- Beam voltage: $V = 1100V$.
- Child–Langmuir current density:

$$J_{\mathrm{CL}} \approx 1.7 \times 10^2 A/m^2 \approx 0.17 A/cm^2$$

- For a $15cm$ diameter active area:

$$A \approx 1.8 \times 10^{-2} m^2, \quad I \approx JA \approx 3A$$

- Thrust:

$$T = I\sqrt{\frac{2mV}{q}} \approx 0.17N$$

- Very close to real NEXT / NSTAR thruster operating points.

Electrostatic: Hall Thruster Operating Principle

Ionizing magnetic electrons trapped by radial $\vec{B}$ field. Non-magnetic (e.g., Xe^+) ions accelerated by axial $\vec{E}$ field.

Cross-Field ($E \times B$) Discharge:

- Radial Magnetic Field (B_r) + Axial Electric Field (E_z).
- Light electrons are magnetized. Heavy ions are un-magnetized.
- Electrons undergo an azimuthal $\mathbf{E} \times \mathbf{B}$ Hall drift.
- Ions accelerated axially by E_z and exit at $\sim$10–20 $\mathrm{km/s}$.
- Quasi-neutral plasma throughout; no grids $\Rightarrow$ No space charge limit!

Magnetic Insulation:

- Electrons are trapped by $\vec{B}$ field, limiting axial current.
- High discharge voltage ($V_d \approx$ 300–600 V) achievable in quasi-neutral plasma.
- High V_d + No Space Charge Limit $\Rightarrow$ Higher thrust density than Ion Thrusters.

Electrostatic: Hall Effect Thruster Cross-Section

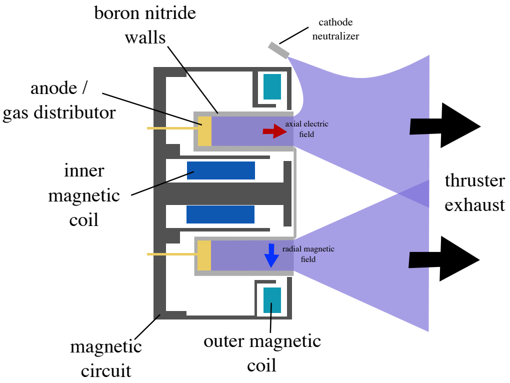


Figure: Source: Wikipedia

Propellant Economics: The Shift from Xenon

Propellant	Mass (amu)	Cost	Use Case
Xenon	131.3	High ($\sim$\$3k/kg)	Science / GEO
Krypton	83.8	Moderate	Starlink V1
Argon	39.9	Low ($\sim$\$10/kg)	Starlink V2
Iodine	126.9	Moderate	CubeSats (Solid!)

Starlink V2 (Argon):

- 4.2 kW, $I_{sp} \approx 2500$ s.
- $\eta_T \approx 50\%$ (Lower than Xe, but economically viable).
- High ionization energy of Ar (15.8 eV) challenges efficiency.

Electromagnetic Propulsion

EM acceleration with potential for high thrust & power density with MW-scale potential.

Lorentz Force: $\vec{F} = \vec{J} \times \vec{B}$

- **MPD (Magnetoplasmadynamic):**
  - Central cathode + annular anode.
  - High J_r discharge current arc ionizes propellant & induces strong B_θ field.
  - $F = J \times B$ acceleration of plasma in axial direction.
  - Efficient at high power (> 100 kW).
- **PPT (Pulsed Plasma Thruster):**
  - Oldest form of electric propulsion (1960s).
  - Capacitor discharge arc across face of Teflon fuel bar between coaxial electrodes.
  - Ablated plasma accelerated via $J \times B$ force.
  - Low efficiency ($< 15\%$), used for CubeSats/Attitude control.
- **VASIMR - Variable Specific Impulse Magnetoplasma Rocket:**
  - RF Ionization (Helicon) + RF Heating (ICRH).
  - Variable I_{sp} at constant power.

Micro-Propulsion: Electrospray

CubeSat-scale propulsion

Physics:

- Electrostatic charged particle extraction from conductive liquid surface.
- **Taylor Cone** formation balance (Surface tension vs. Electric stress).

$$V_{start} \approx \sqrt{\frac{\gamma r_c}{\epsilon_0}} \ln\left(\frac{4d}{r_c}\right) \tag{4}$$

Modes:

- **Cone-Jet (Colloid):** Charged droplets.
- **Ionic (FEEP/ILIS):** Pure ion evaporation ($I_{sp} > 6000$ s).

Scaling: MEMS arrays (thousands of emitters) required for useful thrust.

Electric Propulsion: Key Takeaways

- **Performance:** High I_{sp} & low thrust density compared to chemical rockets.
- **System Types:**
  - Electrothermal (Resistojet, Arcjet).
  - Electrostatic (Ion, Hall).
  - Electromagnetic (PPT, MPD).
  - Micro-propulsion (Electrospray, FEEP).
- **Systems View:** PPU and Thermal management are dominant mass/volume drivers.
- **Future:** Higher power (100 kW+), alternative propellants (Iodine), and micro-scale integration.

Lecture Outline