

# Solid Chemical & Electric Propulsion Systems

## Physics, Engineering, and System Integration

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# Lecture Outline

- 1 Solid Chemical Propulsion
  - Internal Ballistics – 0-D SRM Model
  - Grain Design
  - Flow Physics & Erosion
  - Combustion Instability
  - Thermal Protection & Nozzle Erosion
  - Summary
- 2 Liquid Monopropellant Propulsion
- 3 Electric Propulsion
  - Introduction to Electric Propulsion
  - Electrothermal Propulsion
  - Electrostatic I: Gridded Ion Thrusters
  - Electrostatic II: Hall Effect Thrusters
  - Electromagnetic Systems & Propellant Economics
  - Summary

# Solid Chemical Propulsion Overview

*Energy dense, storable, and instantly available*

## Solid Chemical Propulsion Systems:

Solid chemical propulsion offers high volumetric energy density, long-term storability, and instant readiness. Their scale ranges from micro-scale thrusters for attitude control to large boosters for heavy-lift launch vehicles.

## Critical Physics:

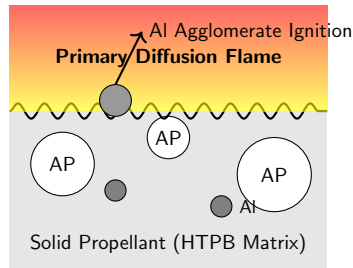
- Saint-Robert's Law ( $r_b = aP_c^n$ ) – burning rate dependence on chamber pressure.
- Heterogeneous combustion physics – with multiple reacting phases.
- Erosive burning regimes in high  $L/D$  motors.

# Example Composite Propellant ( $NH_4ClO_4$ /HTPB/Al)

*Ammonium Perchlorate, Hydroxyl-Terminated Polybutadiene, & Aluminum*

## Constituents & Roles:

- **Oxidizer:**  $NH_4ClO_4$  (AP)
- **Binder:** HTPB. Structure + fuel source via pyrolysis.
- **Fuel:** Aluminum.  $T_{flame} \approx 3400K$ ; acoustic damper.



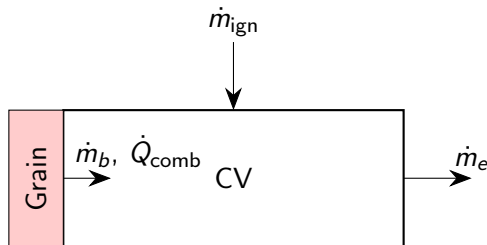
## Sequential Combustion Mechanism:

- 1 AP phase change (orthorhombic to cubic)
- 2 HTPB binder pyrolysis  $\rightarrow$  gaseous HC fuels + solid char
- 3 AP Decomposition:  $NH_4ClO_4 \rightarrow NH_3 + HClO_4 \rightarrow Cl_2 + O_2 + H_2O + N_2$
- 4 Surface diffusion flame
- 5 Al Melting & Combustion (far from surface).

# Combustion Chamber Control Volume ( $V_c$ ) Analysis

*Predict 0-D chamber conditions, thrust &  $I_{sp}$  versus time*

- Control volume  $\Rightarrow$  gas inside the combustion chamber.
- Gas assumed perfectly mixed and spatially uniform.
- Ideal gas:  $pV = mRT$ .
- Sources:
  - Propellant-generated mass  $\dot{m}_b$
  - Igniter mass  $\dot{m}_{ign}$
  - Combustion heat release  $\dot{Q}_{comb}$
- Sinks:
  - Nozzle mass flow  $\dot{m}_e$
  - Wall heat losses  $\dot{Q}_{wall}$



# Mass Balance

*Unsteady mass conservation with propellant burn & nozzle outflow*

**Unsteady mass conservation:**  $\frac{dm}{dt} = \dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e$

Propellant mass generation:  $\dot{m}_b = \rho_p r_b A_b(s)$

Saint–Robert burn law:  $r_b = f_{\text{ign}}(t) a p^n$

Ignition ramp:  $f_{\text{ign}}(t) = 1 - \exp\left(-\frac{t - t_{\text{ign,start}}}{\tau_{\text{ign}}}\right)$

Choked nozzle mass flow:  $\dot{m}_e = C_d A_t p \sqrt{\frac{\gamma}{RT}} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$

# Energy Balance with Heat Release

*Unsteady energy conservation with combustion heat release & wall losses*

Internal energy:

$$U = mc_v T$$

Full unsteady first law:

$$\frac{d}{dt}(mc_v T) = \dot{m}_b h_b + \dot{m}_{\text{ign}} h_{\text{ign}} - \dot{m}_e h_e - p \frac{dV}{dt} + \dot{Q}_{\text{wall}} + \dot{Q}_{\text{comb}}$$

Combustion heat release:

$$\dot{Q}_{\text{comb}} = \dot{m}_b \Delta h_c$$

Total heat term:

$$\dot{Q} = \dot{Q}_{\text{wall}} + \dot{Q}_{\text{comb}} + \dot{Q}_{\text{ign}}.$$

# Deriving $\frac{dp}{dt}$ (Step 1)

Use the mass and energy equations to derive an expression for the evolution of chamber pressure with time.

Start with:

$$U = mc_v T$$

$$\frac{d}{dt}(mc_v T) = \dot{m}_b h_b + \dot{m}_{\text{ign}} h_{\text{ign}} - \dot{m}_e h_e - p \frac{dV}{dt} + \dot{Q}$$

Assume:

$$h_b \approx h_e \approx c_p T, \quad h_{\text{ign}} \approx c_p T_{\text{ign}}$$

Left side:

$$\frac{d}{dt}(mc_v T) = c_v \left( m \frac{dT}{dt} + T \frac{dm}{dt} \right)$$

Right side becomes:

$$c_p T \frac{dm}{dt} - p \frac{dV}{dt} + \dot{Q}$$

Use mass balance:

$$\frac{dm}{dt} = \dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e$$



## Deriving $\frac{dp}{dt}$ (Step 2)

Rearrange:

$$c_v m \frac{dT}{dt} = (c_p - c_v) T \frac{dm}{dt} - p \frac{dV}{dt} + \dot{Q}$$

Use  $c_p - c_v = R$ :

$$\frac{dT}{dt} = \frac{RT \frac{dm}{dt} - p \frac{dV}{dt} + \dot{Q}}{c_v m}$$

Differentiate ideal gas:

$$pV = mRT$$

$$\frac{dp}{dt} V + p \frac{dV}{dt} = R \left( T \frac{dm}{dt} + m \frac{dT}{dt} \right)$$

Substitute above  $dT/dt$  expression.

## Deriving $\frac{dp}{dt}$ (Step 3)

Compute:

$$R \left( T \frac{dm}{dt} + m \frac{dT}{dt} \right) = RT \frac{dm}{dt} + \frac{R}{c_v} \left( RT \frac{dm}{dt} - p \frac{dV}{dt} + \dot{Q} \right)$$

Use  $\gamma = \frac{c_p}{c_v}$  and  $\frac{R}{c_v} = \gamma - 1$ :

$$= \gamma RT \frac{dm}{dt} - (\gamma - 1)p \frac{dV}{dt} + (\gamma - 1)\dot{Q}$$

Insert into:

$$\frac{dp}{dt} V + p \frac{dV}{dt} = R \left( T \frac{dm}{dt} + m \frac{dT}{dt} \right)$$

Thus:

$$\frac{dp}{dt} V = \gamma RT \frac{dm}{dt} - \gamma p \frac{dV}{dt} + (\gamma - 1)\dot{Q}$$

# Deriving $\frac{dp}{dt}$ (Final Form)

Divide by  $V$ :

$$\frac{dp}{dt} = \frac{\gamma RT}{V} \frac{dm}{dt} - \frac{\gamma p}{V} \frac{dV}{dt} + \frac{\gamma - 1}{V} \dot{Q}$$

Use ideal gas  $pV = mRT$ :

$$\frac{RT}{V} = \frac{p}{m}$$

Therefore:

$$\frac{dp}{dt} = \gamma p \left[ \frac{1}{m} \frac{dm}{dt} - \frac{1}{V} \frac{dV}{dt} \right] + \frac{\gamma - 1}{V} \dot{Q}$$

# Volume Evolution and Web Regression

Web regression:

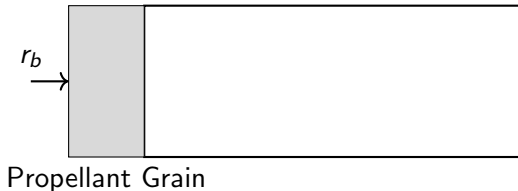
$$\frac{ds}{dt} = r_b = f_{\text{ign}}(t) a p^n$$

Grain geometry tables:

$$A_b(s), \quad V(s), \quad \frac{dV}{ds}(s)$$

Chamber volume evolution:

$$\frac{dV}{dt} = \frac{dV}{ds}(s) \frac{ds}{dt}$$



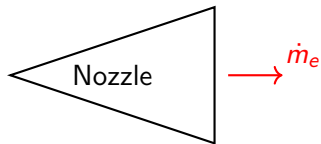
# Nozzle Throat Erosion

Throat area:

$$A_t = \pi r_t^2$$

Erosion model:

$$\frac{dr_t}{dt} = k_{\text{eros}} |\dot{m}_e|$$



# Final Pressure Equation (SRM Form)

With the SRM source terms:

$$\frac{dm}{dt} = \dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e, \quad \frac{dV}{dt} = \frac{dV}{ds}(s) \frac{ds}{dt},$$

the chamber pressure ODE is:

$$\boxed{\frac{dp}{dt} = \gamma p \left[ \frac{\dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e}{m} - \frac{1}{V} \frac{dV}{dt} \right] + \frac{\gamma - 1}{V} \dot{Q}}$$

# Final ODE System

State vector:

$$y(t) = \begin{bmatrix} p(t) \\ m(t) \\ s(t) \\ r_t(t) \end{bmatrix}$$

ODEs:

$$\frac{dp}{dt} = \gamma p \left[ \frac{\dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e}{m} - \frac{1}{V} \frac{dV}{dt} \right] + \frac{\gamma - 1}{V} \dot{Q}$$

$$\frac{dm}{dt} = \dot{m}_b + \dot{m}_{\text{ign}} - \dot{m}_e$$

$$\frac{ds}{dt} = f_{\text{ign}}(t) a p^n$$

$$\frac{dr_t}{dt} = k_{\text{eros}} |\dot{m}_e|$$

# Assumptions & Limitations of the 0-D SRM Model

## Major assumptions:

- Chamber gas is **perfectly mixed** (no spatial gradients).
- Propellant gases instantaneously reach chamber temperature.
- Ideal-gas thermodynamics:  $pV = mRT$ .
- Quasi-steady Saint–Robert burn law:  $r_b = ap^n$ .
- Choked nozzle flow with fixed  $C_d$ .
- Lumped heat loss:  $\dot{Q}_{\text{wall}} = -h_w A_w (T - T_w)$ .
- Grain geometry encoded through tables  $A_b(s)$  and  $V(s)$ .

## Limitations:

- Cannot predict combustion instabilities or pressure oscillations.
- No axial or transverse wave dynamics (1-D/3-D neglected).
- No local flame chemistry or finite-rate kinetics.
- No particle dynamics (Al droplets, slag accumulation).
- No two-phase flow in nozzle.



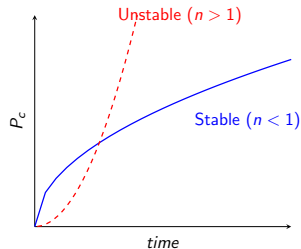
# Saint-Robert's Law

**Saint-Robert's Law:**  $r_b = aP_c^n$

- $a$  = temperature dependent coefficient (hotter grains burn faster).
- $n$  = pressure exponent

Stability Criterion ( $\dot{m}_{gen} \propto P_c^n$ ,  $\dot{m}_{out} \propto P_c$ )

$$\begin{cases} n < 1 & \text{Stable : } \uparrow P_c \Rightarrow |\Delta \dot{m}_{gen}| < |\Delta \dot{m}_{out}| \\ n > 1 & \text{Unstable : } \uparrow P_c \Rightarrow |\Delta \dot{m}_{gen}| > |\Delta \dot{m}_{out}| \end{cases}$$



# Steady State Chamber Pressure

- **Assume Steady State:**

$$\frac{dP_c}{dt} = 0$$
$$\rho_p A_b r_b \left( 1 - \underbrace{\frac{\rho_g}{\rho_p}}_{neglect} \right) = \frac{P_c A_t}{c^*}$$

- **Substitute Saint-Robert's Law ( $r_b = aP_c^n$ ):**

$$\rho_p A_b a P_c^n = \frac{P_c A_t}{c^*}$$

- **Equilibrium Pressure:**

$$P_c^{eq} \approx [\rho_p a c^* K_n]^{\frac{1}{1-n}}$$

- Note sensitivity to  $K_n = A_b/A_t$ . If  $n = 0.5$ ,  $P \propto K_n^2$ .

# Burn Area ( $A_b$ ) Geometric Evolution

## Evolution of Grain Geometry ( $A_b(t)$ ) Controls Thrust Profile:

- **Progressive:**  $A_b$  increases  $\Rightarrow T$  increases.
- **Neutral:**  $A_b$  constant  $\Rightarrow T$  constant.
- **Regressive:**  $A_b$  decreases  $\Rightarrow T$  decreases.

## Common Grain Geometries:

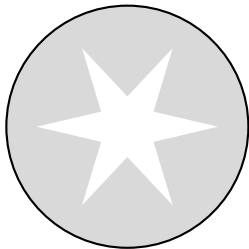
- **Progressive:** Circular Perforation or Center-Perforated Cylinder
- **Neutral:** Star or Wagon Wheel
- **Regressive:** Outer-burning grains or multi-propellant stacks

# Advanced Grain Geometries: Star and Finocyl

## Star:

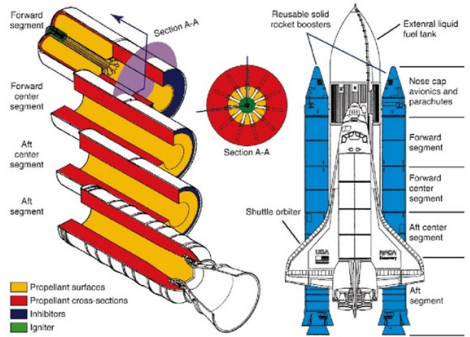
- $A_b \approx \text{constant}$  (neutral burn).
- Sliver formation an efficiency challenge.

Star Grain



## Finocyl (Fin-On-Cylinder):

- Provides flexibility in thrust profile design.
- Shuttle SRM: High thrust after launch, lower thrust during max-Q.



# Burnback Analysis & Optimization

**Goal:** Optimize  $A_b(t = 0)$  to provide the  $A_b(t)$  that enables the target thrust profile.

- **Analytical Methods:** Simplified geometries (cylinders, stars) allow closed-form  $A_b(t)$  expressions.
- **Level Set Methods:** Implicit surface tracking via signed distance functions.

$$\phi(x, y, t) = \begin{cases} > 0 & \text{in solid} \\ = 0 & \text{on surface} \\ < 0 & \text{in gas} \end{cases}$$

$$\frac{\partial \phi}{\partial t} + r_b |\nabla \phi| = 0$$

- **Optimization:** Iterate on initial geometry to achieve desired  $A_b(t)$  and thrust profile.

# Erosive Burning & Velocity Coupling

## Phenomenon:

- Stagnant chamber assumption fails in high  $L/D$  motors.
- High velocity ( $u$ ) enhances heat transfer to the propellant, increasing burn rate beyond Saint-Robert's prediction.

## Lenoir-Robillard Model

$$r_{total} = r_{base} + r_{erosive}$$

$$r_{base} = aP_c^n$$

$$r_{erosive} = \alpha G^{0.8} L^{-0.2} e^{-\beta r_{total} \rho_p / G}$$

where  $G = \rho_g u$  is the mass flux;  $\alpha, \beta$  are empirical constants.

# Implications of Erosive Burning

## System Implications:

Erosive burning typically occurs in high  $L/D$  motors immediately after ignition when  $u$  is highest. As the grain burns, the port area ( $A_{port}$ ) opens, reducing  $u$  and thus erosive effects.

## Negative Erosion:

In some geometries, high velocity can cool the surface and slow the burning rate.

# Acoustic Instability: Hart-McClure Criterion

## Combustion Instability

Acoustic eigenmodes in the chamber can couple with unsteady heat release from the burning propellant, leading to large pressure oscillations or DC pressure increases that can damage or destroy the motor.

**Stability:** Determined by the net growth rate  $\alpha_{net}$ .

$$\alpha_{net} = \underbrace{\alpha_{pressure} + \alpha_{velocity}}_{\text{Gains (Driving)}} - \underbrace{(\alpha_{nozzle} + \alpha_{particle} + \alpha_{wall})}_{\text{Losses (Damping)}}$$

$$\alpha_{net} < 0 \Rightarrow \text{Stable}; \quad \alpha_{net} > 0 \Rightarrow \text{Unstable}$$

- **Pressure Coupling:** Increased surface pressures can accelerate burning.
- **Particle Damping:**  $Al_2O_3$  droplets lag gas oscil. & dissipate energy via viscous drag.
- **Design Trade-off:** "Smokeless" missiles (no Al) lose particle damping (stability risk).



# Ablative Nozzle Throat Erosion & Cooling

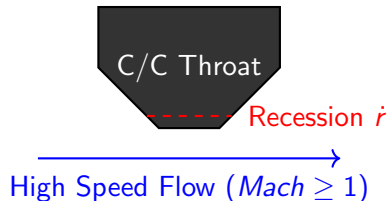
## Nozzle Cooling Challenge

- Combustion gas temperature ( $3400K$ ) exceeds melting point of all metals.
- Active cooling impractical due to solid grain geometry.

## Solution: Carbon-Carbon (C/C) Ablation

- Mechanism is chemical erosion, not melting.
- Diffusion-controlled oxidation by  $H_2O$  and  $CO_2$ .
- $C_{(s)} + H_2O_{(g)} \rightarrow CO_{(g)} + H_2$

**Bartz Correlation Scaling:**  $\dot{r}_{erosion} \propto P_c^{0.8} D_t^{-0.2}$



## Takeaways

- ① **Chemistry is Destiny:** Chemistry dictates energy density and fundamental burn rate.
- ② **Geometry is Control:** Grain geometry determines the thrust profile ( $P \propto K_n^{1/(1-n)}$ ).
- ③ **Stability is Critical:** Acoustic eigenmodes must be damped to prevent failure.
- ④ **Materials Limit Performance:** Nozzle erosion imposes limits on pressure and burn time.

# Solid vs. Liquid Chemical Propulsion

Aspect	Solid Propulsion	Liquid Propulsion
Energy Density	High (volumetric)	Moderate (mass-based)
Thrust Control	Limited (grain geometry)	Precise (throttleable)
Ignition	Instantaneous	Requires ignition system
Storability	Long-term stable	Cryogenic or limited life
Complexity	Simple, few moving parts	Complex (pumps/valves)
Reliability	High (few failure modes)	Moderate (more components)
Applications	Boosters, missiles	Upper stages, deep space

# Exhaust Velocity & Specific Impulse: Solid vs. Liquid

## Ideal Exhaust Velocity & Specific Impulse

$$V_e = \sqrt{\frac{2\gamma R_u}{\gamma - 1} \left( \frac{T_c}{\mathfrak{M}} \right) \left[ 1 - \left( \frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

$$I_{sp} = \frac{V_e}{g_0} \propto \sqrt{\frac{T_c}{\mathfrak{M}}}$$

where  $\gamma$  = specific heat ratio,  $R_u$  = universal gas constant,  $T_c$  = chamber temperature,  $\mathfrak{M}$  = molecular weight of exhaust,  $P_c$  = chamber pressure,  $P_e$  = exit pressure.

### $I_{sp}$ Drivers:

- **Chamber Temperature ( $T_c$ ):** Both solids and liquids achieve 3000K+.
- **Molecular Weight ( $\mathfrak{M}$ ):**  $\mathfrak{M}_{liquid} < \mathfrak{M}_{solid}$  Lower  $\mathfrak{M}$  yields higher velocity.

# Exhaust Molecular Weight ( $\mathfrak{M}$ ) Differences

$H_2O + CO_2$  versus Metal Oxides +  $Cl$

Liquid  $H_2$  (and  $CH_4$ ) yield low molecular weight exhaust streams, while solids are burdened by heavy metal oxides and chlorine species.

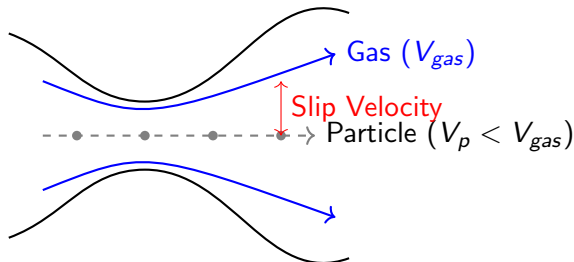
Property	LOX/LH2	AP/Al/HTPB
Products	$H_2O + H_2$ (Fuel Rich)	$CO_2, HCl, Al_2O_3$
Avg. $\mathfrak{M}$	9–12 g/mol	28–32 g/mol
$I_{sp}$	$\approx 450$ s	$\approx 265$ s

# Two Phase Flow Loss Differences

*Solid propellants use Al, yielding condensed-phase  $Al_2O_3$  particles*

## Two Phase Flow Loss Drivers & Implication

- $Al_2O_3$  particles (liquid/solid) do not expand to do pressure work.
- **Velocity Lag:** Particles accelerate slower than gas.
- **Thermal Lag:** Particles stay hot, failing to transfer heat to kinetic energy.
- **Result:** 1–3%  $I_{sp}$  penalty.

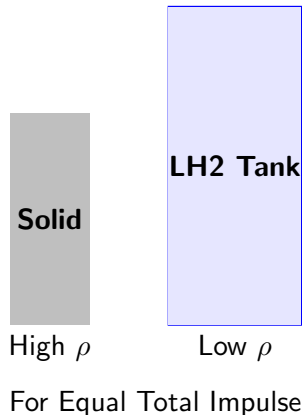


# The Trade-off: Density Impulse

*Solids lose on Specific Impulse ( $I_{sp}$ ) but win on Density Impulse ( $I_{sp} \times \rho$ ).*

## Strategic Implications:

- **Liquid Hydrogen:**  $\rho \approx 0.07 \text{ g/cm}^3$ . Requires massive tank volume. High drag, high structural mass.
- **Solid Propellant:**  $\rho \approx 1.75 \text{ g/cm}^3$ . Compact storage.
- **Application:** Solids are ideal for first stages (Boosters) where thrust-to-volume is critical. Liquids are ideal for upper stages where mass efficiency ( $I_{sp}$ ) dominates.



# Solid vs Liquid Comparison Summary

*Liquids achieve higher  $I_{sp}$  due to lightweight fuels, while solids suffer from heavier fuels and two-phase flow losses.*

Parameter	Solids (APCP)	Liquids (LOX/LH2)
Combustion Temp ( $T_c$ )	$\approx 3400$ K	$\approx 3600$ K
Exhaust MW ( $\mathcal{M}$ )	High ( $\sim 30$ )	Low ( $\sim 12$ )
Exhaust Phase	Gas + Solids	Pure Gas
Loss Mechanisms	Two-Phase Lag	Dissociation
Typical $I_{sp}$ (Vac)	$\sim 280$ s	$\sim 450$ s



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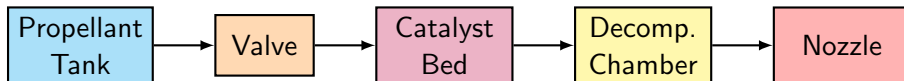
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# Liquid Monopropellant Thrusters

*Leverages a single propellant that exothermically decomposes over a catalyst*

## Monopropellant thrusters:

- Use a **single fluid** (e.g.,  $N_2H_4$  [hydrazine] or  $H_2O_2$  [peroxide])
- Decompose the fluid **catalytically** to form a hot gas mixture
- Expands mixture through a **C-D nozzle** to generate thrust
- Avoids mixture ratio control, igniters, and complex plumbing
- Used for attitude control, station keeping, "small"  $\Delta v$  maneuvers



# Monopropellant Comparison

## *Hydrazine vs High Test Hydrogen Peroxide*

Aspect	Hydrazine ( $\text{N}_2\text{H}_4$ )	High-Test Peroxide ( $\text{H}_2\text{O}_2$ )
Primary Reaction	$3 \text{N}_2\text{H}_4 \rightarrow 4 \text{NH}_3 + \text{N}_2$	$2 \text{H}_2\text{O}_2 \rightarrow 2 \text{H}_2\text{O} + \text{O}_2 + \text{heat}$
Secondary Step	$4 \text{NH}_3 \rightarrow 2 \text{N}_2 + 6 \text{H}_2$ (cracking)	–
Catalyst	Ir/ $\text{Al}_2\text{O}_3$ bed	Silver or Pt mesh
Products	$\text{N}_2$ , $\text{H}_2$ , $\text{NH}_3$ (some uncracked $\text{NH}_3$ )	Steam ( $\text{H}_2\text{O}$ ) + $\text{O}_2$
Nominal $T_c$	$\sim 1000 \text{ K}$	850–1150 K (conc. dependent)
Typical $\gamma$	$\approx 1.30$	1.25–1.30
Gas Constant $R$	$\sim 400 \text{ J/kg-K}$	300–350 J/kg-K
Vacuum $I_{sp}$	220–235 s	150–180 s

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# Chemical to Electric Propulsion

## Chemical Propulsion:

- Energy source – reacting fuel and oxidizer – is the propellant.
- Max  $v_e \approx 4.5 \text{ km/s}$  ( $H_2/O_2$ ).

## Electric Propulsion:

- Energy source (e.g., solar, nuclear) decoupled from propellant.
- $v_e > 30 \text{ km/s}$  achievable.
- At fixed power,  $I_{sp}$  and thrust are inversely related.

# The Power-Limited Rocket Equation

Thrust ( $T$ ) is the momentum flux of the beam:

$$T = \dot{m}v_e \quad (1)$$

The Jet Power ( $P_{jet}$ ) is the kinetic energy flux of the beam:

$$P_{jet} = \frac{1}{2}\dot{m}v_e^2 = \frac{1}{2}Tv_e \quad (2)$$

The overall efficiency is given by  $\eta_T = P_{jet}/P_{in}$ :

$$T = \frac{2\eta_T P_{in}}{v_e} = \frac{2\eta_T P_{in}}{g_0 I_{sp}} \quad (3)$$

- Thrust ( $T$ ) is *inversely proportional* to Specific Impulse ( $I_{sp}$ ) @ fixed power ( $P_{in}$ ).
- **Design Trade:** Minimize propellant mass (high  $I_{sp}$ ) vs. Minimize trip time (high Thrust).

Diagnosing performance requires breaking down  $\eta_T$ :

$$\eta_T = \eta_e \cdot \eta_m \cdot \eta_b \cdot \eta_v \cdot \alpha \quad (4)$$

- **Electrical Efficiency ( $\eta_e$ ):** Beam / Input Electric power (losses in magnets, heater, PPU).
- **Mass Utilization ( $\eta_m$ ):** Fraction of propellant ionized ( $\dot{m}_{ion}/\dot{m}_{total}$ ).
- **Beam Divergence ( $\eta_b$  or  $\eta_{div}$ ):** Loss due to plume spread ( $\langle \cos \theta \rangle^2$ ).
- **Voltage Utilization ( $\eta_v$ ):** Effective beam voltage vs. Discharge voltage ( $V_b/V_d$ ).
- **Doubles Correction ( $\alpha$ ):** Thrust loss due to multiply charged ions ( $Xe^{++}$ ).

# Comparative Analysis of Electric Propulsion Systems

Technology	$I_{sp}$ (s)	Thrust	$\eta_T$	Power	TRL	Key Pros	Key Cons
Resistojet	280–350	10mN–0.5N	65–80%	< 1 kW	9	Simple; Low cost; Shared propellant	Material thermal limits; Low $I_{sp}$
Arcjet	450–600	0.1N–2N	30–45%	0.5–2 kW	9	Robust; Higher $I_{sp}$ than resistojet	Electrode erosion; Frozen flow losses
Gridded Ion	2500–4000+	20–250mN	60–80%	0.5–7 kW	9	Highest Efficiency; Longest Life	Low thrust density (Grid limits); Complex PPU
Hall Effect	1500–3000	40mN–1N+	50–65%	0.2–20 kW	9	High Thrust-to-Power; Compact	Beam divergence; Channel erosion (if unshielded)
Electrospray	1000–6000	$\mu$ N–mN	> 70%	mW–50W	6–9	Precision control; Scalable arrays	Very low thrust; High voltage; Clogging risks
PPT	800–1500	$\mu$ N-s	5–15%	< 100 W	9	Solid fuel (Teflon); Simple storage	Very low efficiency; Carbon contamination



## 1. Is Power the Primary Constraint?

- **Selection:** Hall Effect Thruster
- *Rationale:* Offers higher Thrust-to-Power ratio (60–80 mN/kW) than Ion engines. Reduces maneuver time for power-limited commercial satellites.

## 2. Is Propellant Mass the Primary Constraint?

- **Selection:** Gridded Ion Thruster
- *Rationale:* Maximizes Specific Impulse ( $I_{sp} > 3000$  s). Essential for high- $\Delta v$  deep space missions (e.g., Dawn, BepiColombo) to minimize launch mass.

## 3. Is Cost or Volume Constrained?

- **Selection:** Argon Hall (Cost) or Iodine Hall (Volume)
- *Rationale:* Argon is abundant and cheap for constellations (Starlink). Iodine stores as a dense solid, eliminating high-pressure tanks for CubeSats.

## 4. Is Precision Pointing Required?

- **Selection:** FEPP / Electrospray
- *Rationale:* Provides  $\mu\text{N}$  thrust resolution with no moving parts (valves). Critical for drag-free science missions (e.g., LISA).

# Resistojets & Arcjets

## Resistojets:

- Mechanism: Gas flows over electrically heated element (Re, W).
- Limit: Material melting point.
- Performance:  $I_{sp} \approx 300$  s (Hydrazine).
- Application: Station-keeping, attitude control.

## Arcjets:

- Mechanism: High-current arc heats gas core  $> 10,000$  K.
- Physics: Cool boundary layer protects nozzle walls.
- Performance:  $I_{sp} \approx 500 - 600$  s.
- Limitation: Electrode erosion, high power density required.

# Arcjet Thruster

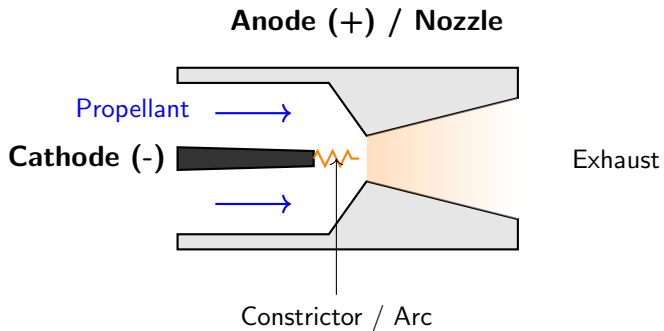


Figure: Arcjet Thruster Schematic

# Gridded Ion Thruster Architecture

## Core Concept:

- Decoupled ionization and acceleration.
- Highest efficiency EP device.

## Ionization Generation:

- **DC (Kaufman):** Hollow cathode + Ring-cusp magnetic confinement.
- **RF (RIT):** Inductive coil, electrodeless discharge (longer life).

## Acceleration:

- Multi-grid assembly (Screen + Accel).
- Electrostatic field extracts ions.

# Gridded Ion Thruster Schematic

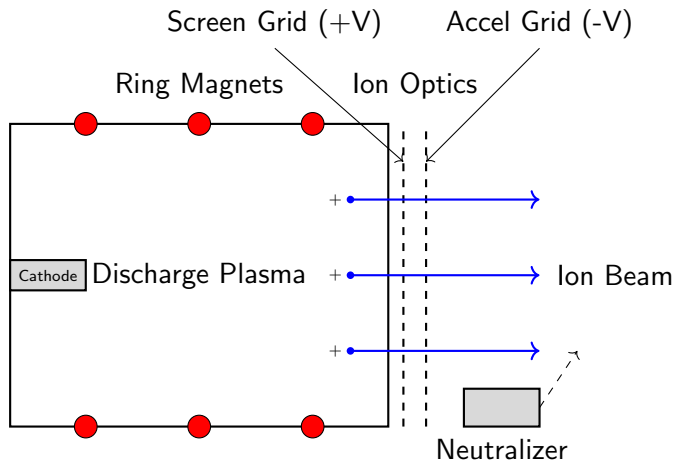


Figure: Electrostatic: Gridded Ion Thruster

# The Child-Langmuir Limit

Thrust density is limited by space charge shielding between grids:

$$J_{CL} = \frac{4\epsilon_0}{9} \sqrt{\frac{2q}{m_i}} \frac{V_T^{3/2}}{d^2} \quad (5)$$

- $J$ : Current density ( $A/m^2$ )
- $V_T$ : Total voltage ( $V_{beam} + |V_{accel}|$ )
- $d$ : Grid gap

**Implication:** To increase thrust, you must increase voltage or decrease grid gap (manufacturing limit). Ion thrusters are "large" for their thrust.

## Primary Failure Mode: Accelerator Grid Erosion

- Mechanism: Charge Exchange (CEX) Collisions.
- Fast Ion + Slow Neutral  $\rightarrow$  Fast Neutral + Slow Ion.
- Slow ions form in the gap/plume and accelerate into the negative Accel Grid.
- Result: "Pits and Grooves" erosion.

## Perveance Matching:

- **Over-perveance:** Sheath bulges, direct impingement.
- **Under-perveance:** Beam focuses too sharply, CEX crossover.



# Hall Thruster Operating Principle

## Cross-Field ( $E \times B$ ) Discharge:

- Radial Magnetic Field ( $B_r$ ) + Axial Electric Field ( $E_z$ ).
- Electrons are magnetized ( $\Omega_e \gg 1$ ), Ions are un-magnetized.

$$\Omega_e = \frac{eB}{m_e \nu_{coll}} \quad (6)$$

## Magnetic Insulation:

- High impedance to electron flow allows supporting large potential drop ( $V_d \approx 300 - 600$  V) in a quasi-neutral plasma.
- **Advantage:** No Space Charge Limit! Higher thrust density than Ion Thrusters.

# Hall Effect Thruster Schematic

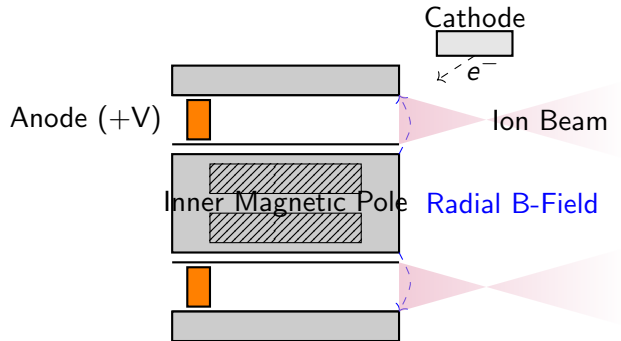


Figure: Electrostatic: Hall Effect Thruster (Cross Section)

# Anomalous Transport & Instabilities

## Anomalous Transport:

- Classical collision theory predicts current  $100\times$  lower than observed.
- Real transport driven by:
  - Plasma turbulence/waves (Bohm diffusion).
  - Near-Wall Conductivity (Secondary electron emission).

## Breathing Mode Instability (10-30 kHz):

- "Predator-Prey" cycle between Neutrals and Ions.
- Ionization depletes neutrals  $\rightarrow$  Current drops  $\rightarrow$  Neutrals refill  $\rightarrow$  Current spikes.
- Requires active filtering in PPU.

# The Revolution: Magnetic Shielding

## The Problem (Pre-2010):

- Field lines intersected ceramic walls. High energy ions eroded channel.
- Lifetime limit:  $\approx 5,000$  hours.

## The Solution (Magnetic Shielding):

- Topology shaping: Field lines parallel to walls, grazing the anode.
- Creates equipotential zone near wall ( $T_e$  is low).
- Potential drop  $\downarrow$  Sputtering threshold.
- **Result:** Erosion reduced  $100 - 1000\times$ . Lifetimes  $> 50,000$  hours (e.g., AEPS for Lunar Gateway).

**Lorentz Force:**  $\vec{F} = \vec{J} \times \vec{B}$

- **MPD (Magnetoplasdynamic):**

- Multi-MW power handling.
- Self-field or Applied-field.
- Issue: Cathode erosion at kA currents.

- **PPT (Pulsed Plasma Thruster):**

- Solid Teflon ablation. Simple, robust.
- Low efficiency, used for CubeSats/Attitude control.

- **VASIMR:**

- RF Ionization (Helicon) + RF Heating (ICRH).
- Variable  $I_{sp}$  at constant power.

# Pulsed Plasma Thruster Schematic

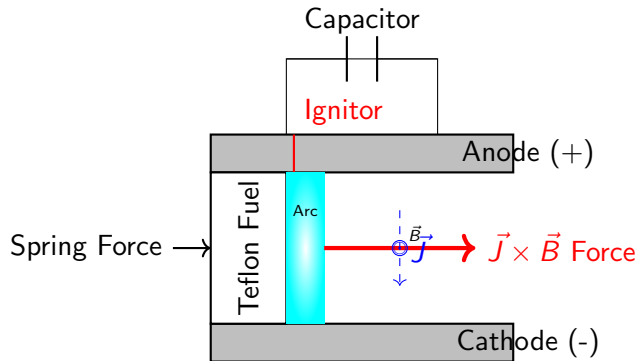


Figure: Electromagnetic: Pulsed Plasma Thruster (PPT)

# Propellant Economics: The Shift from Xenon

Propellant	Mass (amu)	Cost	Use Case
Xenon	131.3	High ( $\sim \$3\text{k/kg}$ )	Science / GEO
Krypton	83.8	Moderate	Starlink V1
Argon	39.9	Low ( $\sim \$10\text{/kg}$ )	Starlink V2
Iodine	126.9	Moderate	CubeSats (Solid)

## Starlink V2 (Argon):

- 4.2 kW,  $I_{sp} \approx 2500$  s.
- $\eta_T \approx 50\%$  (Lower than Xe, but economically viable).
- High ionization energy of Ar (15.8 eV) challenges efficiency.

## Physics:

- Electrostatic extraction from liquid meniscus.
- **Taylor Cone** formation balance (Surface tension vs. Electric stress).

$$V_{start} \approx \sqrt{\frac{\gamma r_c}{\epsilon_0}} \ln \left( \frac{4d}{r_c} \right) \quad (7)$$

## Modes:

- **Cone-Jet (Colloid)**: Charged droplets.
- **Ionic (FEEP/ILIS)**: Pure ion evaporation ( $I_{sp} > 6000$  s).

**Scaling:** MEMS arrays (thousands of emitters) required for useful thrust.



# Electrospray Thruster Schematic

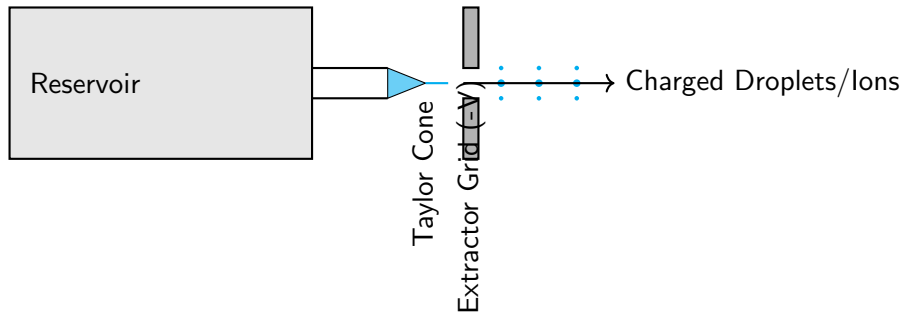


Figure: Micro-Propulsion: Electrospray Thruster

- **Physics:** Power-limited rocket equation drives the design.
- **Technology Bifurcation:**
  - **High Power/Life:** Magnetic Shielding enables Gateway/Mars (AEPS).
  - **Low Cost/High Volume:** Argon Hall thrusters enable Mega-constellations (Starlink).
- **Systems View:** PPU and Thermal management are dominant mass/volume drivers.
- **Future:** Higher power (100 kW+), alternative propellants (Iodine), and micro-scale integration.