

# Regenerative Nozzle Cooling & Full-Flow Staged Combustion Cycles

## Modeling, Analysis, and Optimization

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# Lecture Overview

## Topics Covered

- ① Regenerative nozzle cooling fundamentals
- ② Full-flow staged combustion (FFSC) cycle architecture
- ③ Governing equations for 1D cooling model
- ④ Turbopump power balance
- ⑤ Python implementation and optimization
- ⑥ Example:  $CH_4/LOX$  engine design

**Goal:** Understand coupled thermal-fluid-cycle analysis of liquid rocket engines.

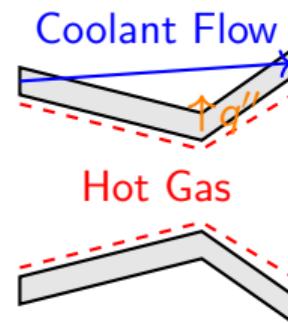
# Why Regenerative Cooling?

## The Challenge:

- Combustion gas temperature:  $T_0 \approx 3600$  K ( $CH_4/O_2$ )
- Nozzle wall materials melt at  $\sim 2000$  K
- Film/ablative cooling inefficient for reusability

## The Solution: Regenerative Cooling

- Route cryogenic propellant through jacket around nozzle
- Convective heat transfer cools walls
- Heated propellant feeds combustion chamber
- Enables reusable, high-performance engines



# Full-Flow Staged Combustion (FFSC) Cycle

## Key Features:

- All propellant flows through preburners
- Two preburners: fuel-rich and ox-rich
- Turbines driven by preburner exhaust
- High chamber pressure capability
- Highest theoretical  $I_{sp}$  of any cycle

## Advantages:

- No wasted propellant (vs. gas generator)
- Lower turbine temperatures than ox-rich SC
- Excellent for methane/LOX

# FFSC Flow Split Strategy

## Propellant Distribution:

- **Fuel side:**
  - Large fraction ( $\sim 95\%$ ) routed through cooling jacket
  - Remainder to fuel-rich preburner with small ox injection
  - Preburner operates at  $\phi \approx 1.3$  (fuel-rich)
- **Oxidizer side:**
  - Majority to ox-rich preburner with small fuel injection
  - Preburner operates at  $\phi \approx 0.7$  (ox-rich)
  - Remainder injected directly to main chamber

## Design Constraint:

$$P_{\text{turb,avail}} \geq \frac{P_{\text{pump,req}}}{\eta_{\text{mech}}} \quad (\text{on each side})$$

# 1D Nozzle Flow Model

Isentropic core flow with Bartz heat transfer

## Isentropic Relations:

$$T(x) = \frac{T_0}{1 + \frac{\gamma-1}{2} M(x)^2}$$
$$p(x) = p_0 \left( \frac{T(x)}{T_0} \right)^{\gamma/(\gamma-1)}$$

## Bartz Gas-Side Heat Transfer Coefficient:

$$h_g = \frac{0.026}{D_t^{0.2}} \left( \frac{\mu^{0.2} C_p}{Pr^{0.6}} \right)_0 \left( \frac{P_c}{c^*} \right)^{0.8} \left( \frac{A_t}{A} \right)^{0.9} \sigma$$

$h_g$  highest at the throat due to high pressure and temperature.

# Coolant-Side Heat Transfer

Turbulent internal flow with property variations

## Reynolds and Prandtl Numbers:

$$\text{Re} = \frac{GD_h}{\mu}, \quad \text{Pr} = \frac{c_p \mu}{k}, \quad D_h = \frac{4A_c}{P_{\text{wet}}}$$

## Gnielinski Correlation (Re > 2300):

$$\text{Nu} = \frac{(f/8)(\text{Re} - 1000) \text{Pr}}{1 + 12.7 \sqrt{f/8} (\text{Pr}^{2/3} - 1)} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

## Coolant Heat Transfer Coefficient:

$$h_c = \frac{\text{Nu} \cdot k}{D_h}$$

Sieder-Tate correction accounts for viscosity variation near wall.

# Conjugate Heat Transfer

Series thermal resistances

**Total Thermal Resistance:**

$$R_{\text{tot}} = \frac{1}{h_g} + \frac{t_{\text{wall}}}{k_{\text{wall}}} + \frac{1}{h_c}$$

**Heat Flux:**

$$q''(x) = \frac{T_g(x) - T_{\text{cool}}(x)}{R_{\text{tot}}(x)}$$

**Inner Wall Temperature:**

$$T_{\text{wall,inner}}(x) = T_g(x) - \frac{q''(x)}{h_g(x)}$$

**Design Limit:**  $T_{\text{wall,inner}} < T_{\text{melt}}$  or structural limit ( $\sim 900$  K for Inconel/Cu alloys)

# Coolant Energy Balance

Axial integration of coolant heating

## Energy Equation (Steady 1D):

$$\dot{m}_{\text{cool}} c_{p,c} \frac{dT_{\text{cool}}}{dx} = q''(x) P_{\text{inner}}(x)$$

## Discrete Form:

$$T_{\text{cool},i+1} = T_{\text{cool},i} + \frac{q_i'' P_{\text{inner},i} \Delta x}{\dot{m}_{\text{cool}} c_{p,c,i}}$$

## Coolant Pressure Drop (Darcy-Weisbach):

$$\Delta p_{\text{cool}} = f \frac{L}{D_h} \frac{\rho v^2}{2}$$

Integrated along nozzle to ensure  $p_{\text{cool}} > p_0$  at all stations.

# Turbopump Power Balance

Ensure turbines can drive pumps

## Pump Power Required:

$$P_{\text{pump}} = \frac{\dot{m}\Delta p}{\rho\eta_{\text{pump}}}$$

## Turbine Power Available:

$$P_{\text{turb}} = \dot{m}_{\text{turb}} c_p (T_{\text{in}} - T_{\text{out}}) \eta_{\text{turb}}$$

## Feasibility Criterion (Each Side):

$$P_{\text{turb,avail}} \geq \frac{P_{\text{pump,req}}}{\eta_{\text{mech}}}$$

Typical values:  $\eta_{\text{pump}} \approx 0.75$ ,  $\eta_{\text{turb}} \approx 0.90$ ,  $\eta_{\text{mech}} \approx 0.98$

# Python Model Implementation

## Modular Structure:

- thermo: Cantera equilibrium chemistry for chamber state
- regen: 1D regenerative cooling along nozzle contour
- cycle: Full FFSC power balance and feasibility check
- sweep: Parameter sweeps and optimization routines

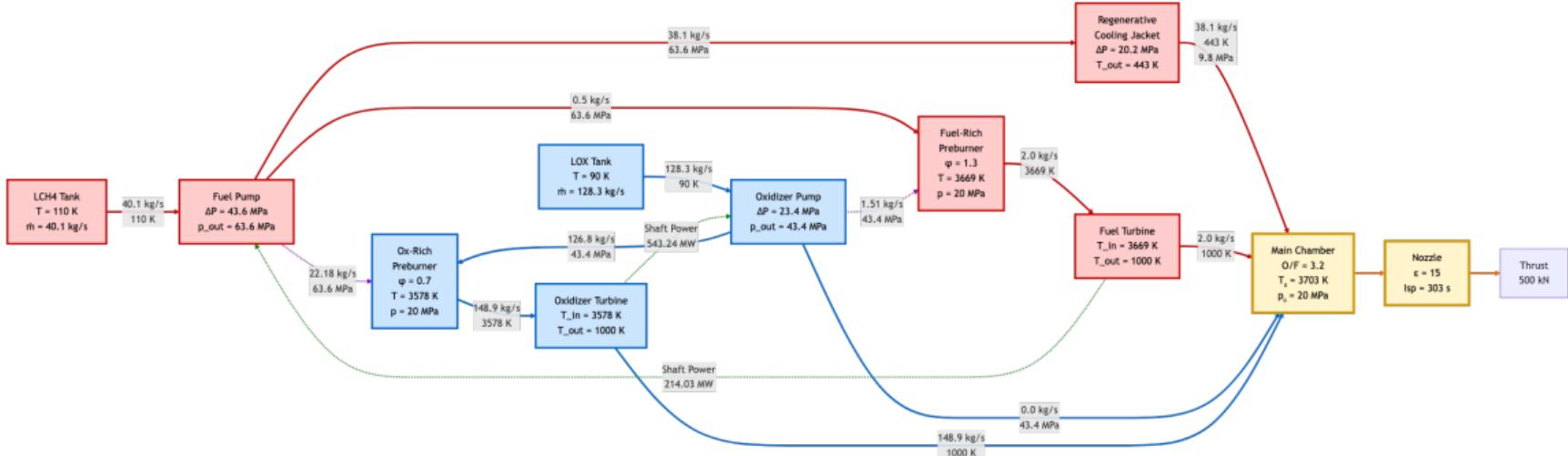
## Key Inputs:

- Thrust  $F_{vac}$ , chamber pressure  $p_0$ , O/F ratio
- Nozzle geometry: throat radius  $r_t$ , expansion ratio  $\varepsilon$
- Cooling parameters: channel count, dimensions, coolant fraction
- Turbomachinery efficiencies and preburner temperatures

## Outputs:

- Wall temperature profile, coolant heating and pressure drop
- Pump/turbine power requirements and margins
- FFSC cycle feasibility (fuel and ox sides)
- Mermaid flow diagram generation for visualization

# FFSC "Mermaid" Schematic Diagram



Schematic of FFSC cycle showing propellant flow paths and turbopump arrangement

# Example: 500 kN $CH_4/LOX$ Engine

Baseline design parameters

## Design Parameters:

- $F_{vac} = 500 \text{ kN}$
- $p_0 = 20 \text{ MPa}$
- O/F = 3.2
- $r_t = 0.10 \text{ m}$
- $\varepsilon = 15$
- $L_{noz} = 1.2 \text{ m}$
- $T_{fuel,tank} = 110 \text{ K}$
- $T_{ox,tank} = 90 \text{ K}$

## Cooling Configuration:

- 300 channels
- Channel width: 3 mm
- Wall thickness: 1.5 mm
- $p_{cool,in} = 30 \text{ MPa}$
- Coolant fraction: 95% of fuel

## Turbomachinery:

- $T_{turb,out} = 1000 \text{ K}$
- $\eta_{pump} = 0.75$
- $\eta_{turb} = 0.92$

# Nozzle Cooling Results

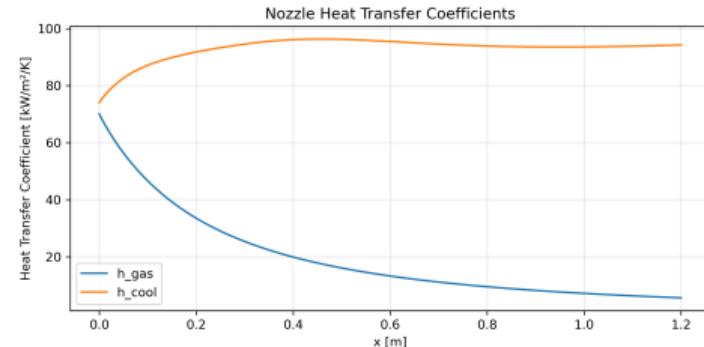
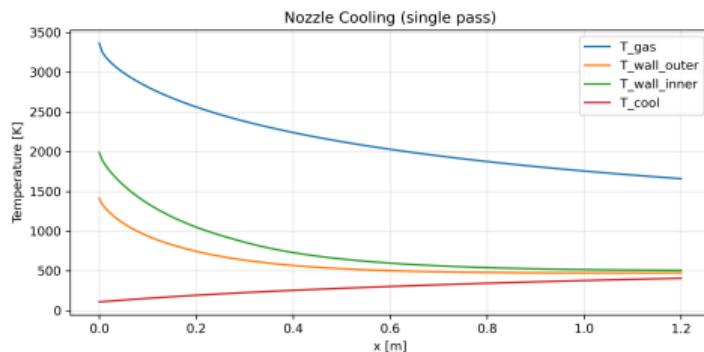
Temperature and heat transfer coefficient profiles

## Key Results:

- $T_{\text{wall,max}} = 2000 \text{ K}$
- $\Delta p_{\text{cool}} = 2.5 \text{ MPa}$
- $T_{\text{cool,out}} = 350 \text{ K}$
- Peak heat flux at throat:  $TBD \text{ MW/m}^2$

## Observations:

- Wall temp peaks near throat
- Coolant heats rapidly in throat region
- Gas-side HTC dominates thermal resistance
- Pressure drop manageable (<10% of inlet)



# FFSC Cycle Performance

Turbopump power balance and feasibility

## Performance:

- $I_{sp,\text{vac,eff}} = 360 \text{ s}$
- $I_{sp,\text{ideal}} = 379 \text{ s}$
- Efficiency: 95%
- $\dot{m}_{\text{total}} = 142 \text{ kg/s}$

## Chamber State:

- $T_0 = 3580 \text{ K}$
- $\gamma = 1.18$

## Pump Power (MW):

- Fuel: 1.12 MW
- Ox: 1.85 MW

## Turbine Power (MW):

- Fuel-side avail: 1.25 MW
- Ox-side avail: 2.05 MW

## Margins:

- Fuel side: +11%
- Ox side: +10%

**FFSC cycle is FEASIBLE — both turbopump sides balanced!**

# Preburner Operating Points

Fuel-rich and ox-rich preburner conditions

## Fuel-Rich Preburner:

- Equivalence ratio:  $\phi = 1.3$
- Temperature:  $T_{pb} = 1200 \text{ K}$
- Drives fuel-side turbine
- Exhaust rich in  $CH_4, H_2$
- Lower temperature protects turbine

## Ox-Rich Preburner:

- Equivalence ratio:  $\phi = 0.7$
- Temperature:  $T_{pb} = 1100 \text{ K}$
- Drives ox-side turbine
- Exhaust rich in  $O_2, H_2O$
- Materials challenge (oxidizing)

## Trade-off

Lower preburner temperatures reduce turbine stress but require larger turbine area and higher mass flow fractions through preburners.

# Design Optimization

Maximize  $I_{sp}$  subject to feasibility & thermal constraints

## Optimization Problem:

$$\begin{aligned} & \text{maximize} && I_{sp,\text{eff}}(p_0, O/F, \varepsilon) \\ & \text{subject to} && P_{\text{turb,avail}} \geq P_{\text{pump,req}} \quad (\text{fuel \& ox sides}) \\ & && T_{\text{wall,max}} \leq 900 \text{ K} \\ & && p_{\text{cool}} > p_0 \quad (\text{all axial stations}) \\ & && 10 \leq p_0 \leq 30 \text{ MPa} \\ & && 2.5 \leq O/F \leq 4.0 \\ & && 10 \leq \varepsilon \leq 30 \end{aligned}$$

## Result for 500 kN Thrust:

- Optimal  $I_{sp} = 365 \text{ s}$
- $p_0^* = 22 \text{ MPa}$ ,  $O/F^* = 3.15$ ,  $\varepsilon^* = 18$

# Key Design Trades

- **Chamber Pressure:**
  - Higher  $p_0 \rightarrow$  higher  $I_{sp}$ , but increases pump power
  - Must balance turbine power available vs. pump power required
- **O/F Ratio:**
  - Near stoichiometric ( $O/F \approx 4$ ) maximizes  $T_0$  and  $I_{sp}$
  - Off-stoichiometric O/F in preburners controls turbine temperature
- **Coolant Mass Flow:**
  - More coolant  $\rightarrow$  lower wall temps, but reduces cooling  $\Delta T$
  - Trade: thermal margin vs. turbopump power (higher  $\dot{m}$  = higher  $\Delta p$ )
- **Expansion Ratio:**
  - Higher  $\varepsilon \rightarrow$  higher  $I_{sp}$ , but longer/heavier nozzle
  - Cooling becomes more challenging with larger nozzle area

# Summary

## Key Takeaways:

- ① Regenerative cooling enables high-performance reusable engines by leveraging cryogenic propellants.
- ② FFSC cycles route all propellant through preburners, maximizing  $I_{sp}$  while controlling turbine temperatures.
- ③ 1D coupled thermal-fluid-cycle model captures essential physics:
  - Bartz correlation for gas-side heat transfer
  - Gnielinski/Sieder-Tate for coolant-side
  - Turbopump power balance for feasibility
- ④ Python implementation with Cantera/CoolProp enables rapid iteration and optimization.
- ⑤ Visualization includes combined temperature and HTC profiles for comprehensive analysis.
- ⑥ Example 500 kN  $CH_4/LOX$  engine demonstrates feasible FFSC design with  $I_{sp} \approx 360$  s.
- ⑦ Trade studies reveal optimal operating points in  $(p_0, O/F, \varepsilon)$  space.

**Real-world application:** SpaceX Raptor ( $CH_4/LOX$  FFSC,  $p_0 \sim 300$  bar,  $I_{sp,vac} \sim 380$  s)

# Further Reading & Extensions

## Model Extensions:

- 2D/3D CFD for detailed wall temperature distribution
- Transient startup/shutdown analysis
- Structural analysis (thermal stresses, creep)
- Multi-objective optimization (mass, cost, reliability)

## References:

- Huzel & Huang, *Modern Engineering for Design of Liquid-Propellant Rocket Engines* (1992)
- Sutton & Biblarz, *Rocket Propulsion Elements*, 9th ed. (2017)
- Bartz, D. R., "A Simple Equation for Rapid Estimation of Rocket Nozzle Convective Heat Transfer Coefficients," *Jet Propulsion* (1957)

## Visualization Tools:

- Mermaid diagrams for dynamic flow schematic generation
- Matplotlib for temperature, pressure, and HTC profiles