STA 255 Tutorial 6

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Agenda

1. Kahoot
2. Change of Variables (pg 221 Devore & Berk)
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4. Chg of Variables Example
5. Chg of Variables Example
6. Joint Probability Example (Exercise 5.1.1 Devore & Berk)

Kahoot

Change of Variables (pg 221 Devore & Berk)

Where $y = g(x)$ and $x = h(y) = g^{-1}(y)$

$$F_Y(y) = P(Y \leq y) = P[g(X) \leq y] = P[X \leq h(y)] = F_x[h(y)]$$

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Now differentiate both sides!

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Now differentiate both sides!

$$\frac{d}{dy}F_Y(y) = \frac{d}{dy}F_x[h(y)]$$

$$f_Y(y) = \left[\frac{d}{dx}F_X(x)\right]\left|\frac{d}{dy}h(y)\right|$$

$$f_Y(y) = f_X(x)|h'(y)|$$

$$f_Y(y) = f_X[h(y)]|h'(y)|$$

Chg of Variables Example

Let X be a random variable, where $0 \leq x \leq 1$ with pdf $f_X(x) = 2x$. Let $Y = 3x + 3$. What is the pdf of Y? Draw it and compare it to X's pdf.

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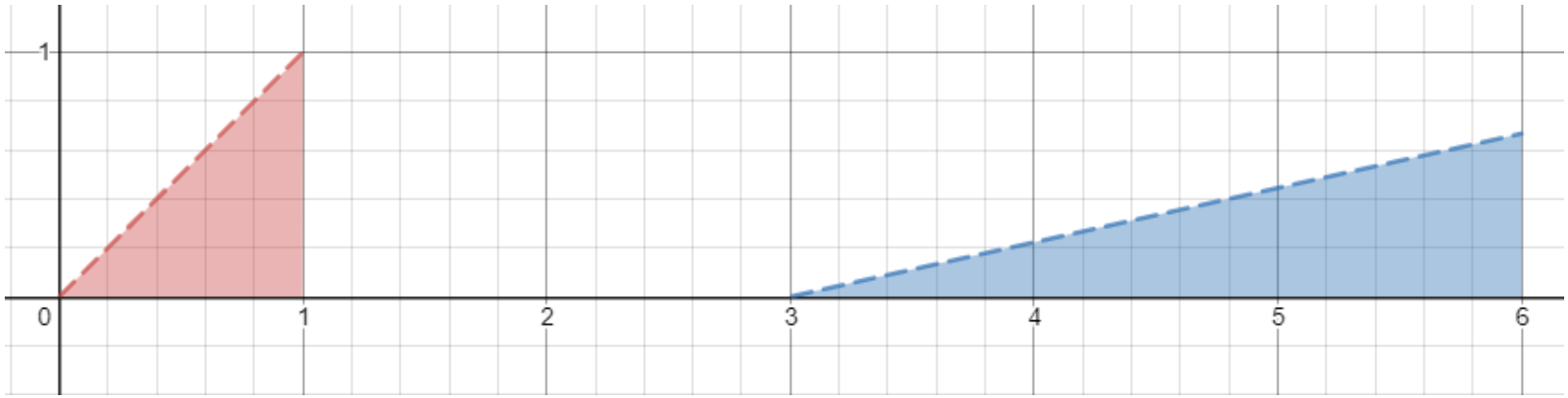

Joint Probability Example (Exercise 5.1.1 Devore & Berk)

A service station has both self-service and full service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint pmf of X and Y appears in the accompanying tabulation.

$p(x, y)$		y 0	1	2
	0	.10	.04	.02
x	1	.08	.20	.06
	2	.06	.14	.30

1. Find $P(X = 1 \cap Y = 1), P(X \leq 1 \cap Y \leq 1), P(X \neq 0 \cap Y \neq 0)$
2. Compute the marginal pmf of X and Y