STA 255 Tutorial 5

David Veitch

University of Toronto

daveveitch.github.io

July 25, 2019

Agenda

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Joint Distribution Question (Devore & Berk Exercise 3.76)

An airport limousine can accommodate up to four passengers on any one trip. The company will accept a maximum of six reservations for a trip, and a passenger must have a reservation. From previous records, 20% of all those making reservations do not appear for the trip. In the following questions, assume independence, but explain why there could be dependence. Suppose the probability distribution of the number of reservations made is given in the accompanying table. Let X denote the number of passengers on a randomly selected trip. Obtain the probability mass function of X.

Number of reservations	3	4	5	6
Probability	.1	.2	.3	.4

Midterm Q1

1. A professional organization (for statisticians, of course) sells term life insurance and major medical insurance. Of those who have just life insurance, 70% will renew next year. Of those with only a major medical policy, 80% will renew next year. However, 90% of policyholders who have both types of policy will renew at least one of the insurances next year. Suppose we know that, among all the policy holders, 75% have term life insurance, 45% have major medical insurance, and 20% have both insurances.

   (a) (*5 points*) What is the chance that a randomly selected policyholder only have a major medical policy?

   (b) (*5 points*) What percentage of policyholders that will renew at least one policy next year?

   (c) (*5 points*) If a randomly selected policy holder renews at least one insurance next year, what is the probability that he or she has both life and major medical insurances?

Midterm Q2

2. Suppose Lin is going to host a potato-lotto. She puts 10 potatoes each labelled a unique letter

$$A, B, C, D, E, F, G, H, I, J$$

in a lotto-box. Anyone can play the lotto by paying 2 dollars. Each participant writes down 5 unique letters as his/her lotto ticket. Lin will randomly draw 5 potatoes **without replacement** from the lotto box. If a ticket matches all the 5 letters that Lin draws (**order doesn't matter**), then it is considered a winning ticket and Lin will pay 4,000 dollars to every winner.

(a) (*5 points*) What is the probability that a participant wins a prize?

(b) (*8 points*) Suppose 200 people participate in this potato-lotto, and each of them writes down their letters independently (they cannot affect each other's decision). What is the probability that more than one participant wins a prize?

(c) (*12 points*) What is Lin's expected income from this potato-lotto?

Midterm Q3

3. Let X be a continuous random variable with probability density function (pdf)

$$f(x) = \begin{cases} c - \frac{x}{8} & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) (*5 points*) What is the value of c?

(b) (*10 points*) Find the cumulative density function of X.

(c) (*10 points*) Find the variance of X.

Midterm Q4

4. Suppose the diameter at breast height (in inches) of trees of a certain type is normally distribution with mean 9 and variance 9.

   (a) (*10 points*) What is the probability that a randomly selected tree will be at least 6.75 inches? (Hint: the R output may be useful.)

(b) (*10 points*) What is the 75th percentile of all the diameter at breast height in inches? (Hint: the R output may be useful.)

Midterm Q5

5. Liam and Alex decided to play Ludo on a boardgame night. At the beginning of the game, each player's four tokens are out of play and staged in the player's yard. The players will enter their tokens one per time when they roll a six on a **fair dice**. Liam moved one of his tokens out of his yard every time he rolled a six until he moved all of his tokens out of his yard.

   (a) (*7 points*) What is the chance that Liam moved his first token on his 6th roll of the fair dice?

   (b) (*8 points*) What is the chance that Liam moved all his four tokens on his 20th roll of the fair dice?