Notes on gauge dependence of backreaction

March 29, 2021
Dave Verweg
s3438015

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1 GR as gauge theory

We follow [1, p. 306]. In order to formulate GR as a gauge theory, one considers the frame bundle $L(M) = \big(LM, M, \pi, GL(4,\mathbb{R})\big)$, where $GL(4,R)$ is the structure group consisting of all real 4×4 invertible matrices and is the group of gauge transformations of the first kind. The group $\mathcal{G} \cong \mathrm{Diff}(M)$ of gauge transformations of the second kind is a subgroup of $\mathrm{Aut}\big(L(M)\big)$ leaving M pointwise fixed.

1.1 Gauge transformations: first kind

Any element $g \in GM(4,\mathbb{R})$ is a local Lorentz transformation, i.e. change of coordinate system at every point or equivalently a rule which rotates Lorentz frames.

1.2 Gauge transformations: second kind

According to [2, p. 222], the *principle of gauge theory* is that "the laws of physics should be differential equations such that if the section s if a solution, so is gs for any $g \in \mathcal{G}$. Such differential equations are said to be *gauge invariant*."

We consider $\mathcal{G}$, which are automorphisms $\phi : LM \to LM$ of the frame bundle $L(M)$ which cover the identity, i.e. $\pi \circ \phi = \mathrm{id}_M \circ \pi$ making that each point in M is preserved, or equivalently each fibre is mapped to itself. It turns out that $\mathcal{G} \cong \mathrm{Diff}(M)$; hence we want the laws of relativity to be formulated up to diffeomorphisms of the spacetime manifold.

1.3 Why introduce spinors?

The spin connection is a principal connection and a 1-form with values in the Lie algebra of the corresponding Lorentz group, while the Levi-Civita connection is not a principal connection nor a 1-form. Therefore, the theory of exterior differentiation and differential forms can only be applied to the spin connection of the principal bundle of orthonormal frames [3].

References

[1] A. Trautman, "Fiber bundles, gauge fields, and gravitation," *General relativity and gravitation*, vol. 1, pp. 287–308, 1980.

[2] J. C. Baez and J. P. Muniain, *Gauge fields, knots and gravity*, vol. 4. World Scientific Publishing Company, 1994.

[3] K. Krasnov, *Formulations of General Relativity: Gravity, Spinors and Differential Forms*. Cambridge University Press, 2020.