Notes on gauge dependence of backreaction

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Contents

1 Relativistic gauge theory

1.1 Einstein's field equations

The Einstein field equations,

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}, \tag{1}$$

express the relation between the curvature of spacetime (represented by the metric tensor g_{ab}) and the distribution of energy and mass (represented by the energy-momentum tensor T_{ab}). Since $1 \leq a, b \leq 4$, (1) describes 16 equations. As the metric, the energy-momentum and the Ricci[1] tensor are symmetric, we see that the Einstein equations describe a system of 10 distinct equations [2, p. 74].

Remark 1.1 (Solving for the metric). Important to note is that if one wants to solve (1), one should solve for g_{ij} and T_{ij} simultaneously [1, Sec. 2.7]. However, given a smooth metric g_{ab}, there exists a smooth symmetric field T_{ab} satisfying the Einstein equations, where one has the possibility to define T_{ab} as the right hand-side of (1).

1.2 What is a gauge?

We describe what a gauge is in general relativity. We do so with the help of an analogy.

Definition 1.1 (Coordinate system)**.** A *coordinate system* is a quantity satisfying the following three conditions:

1. Locally, it measures distances.
2. It transforms diffeomorphically under a *coordinate transformation*.
3. One can derive all previously derived physical quantities after a *coordinate transformation*.

Similarly, we define a gauge.

Definition 1.2 (Gauge)**.** A *gauge* is a quantity satisfying the following three conditions:

1. Locally, it measures distances.
2. It transforms diffeomorphically under a *gauge transformation*.
3. One can derive all previously derived physical quantities after a *gauge transformation*.

[1] Symmetry of the Ricci tensor R_{ab} is shown in e.g. [1, p. 84].

One can already verify that the metric tensor is a potential candidate as it measures infinitesimal distances and it transforms diffeomorphically under a gauge transformation [2, p. 74]. Furthermore, this is in line with the principle of General Covariance as the metric is the only quantity pertaining to space that can appear in physical laws [3].

We are left to prove that a coordinate system is in some sense a special case of a gauge.

References

[1] D. Malament, *Topics in the foundations of general relativity and Newtonian gravitation theory*. University of Chicago Press, 2012.

[2] S. Hawking and G. Ellis, *The large scale structure of space-time*, vol. 1. Cambridge university press, 1973.

[3] R. M. Wald, *General relativity*. University of Chicago press, 1984.