Assignment 5 Part 1

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5.2 - 9 Prove for all integers $n \geq 3$, $4^3 + 4^4 + 4^5 \ldots 4^n = \frac{4(4^n - 16)}{3}$

Let $P(n)$ be the statement $4^3 + 4^4 + 4^5 \ldots 4^n = \frac{4(4^n - 16)}{3}$

$P(3) = 4^3 = 4(4^3 - 16)/3$

$64 = 4(64 - 16)/3$

$4(48)/3$

$192/3$

64

Thus $P(3)$ is true.

Let k be a positive integer with $k \geq 3$, and suppose $P(k)$ is true.

$4^3 + 4^4 + 4^5 \ldots 4^k = \frac{4(4^k + 16)}{3}$

We must show that $P(k+1)$ is true

$4^3 + 4^4 + 4^5 \ldots 4^k + 4^{k+1} = \frac{4(4^{k+1} - 16)}{3}$

$\frac{4(4^k - 16)}{3} + 4^{k+1} = \frac{4(4^{k+1} - 16)}{3}$

$\frac{4(4^k - 16) + 4^{k+1}(3)}{3}$

$\frac{4^{k+1} - 4^2(4) + 4^{k+1}(3)}{3}$

$4^{k+1} - \frac{4^2(4) + 4^{k+1}}{3} + 4^{k+1} + 4^{k+1} = \frac{4(4^{k+1}) - 4(4^2)}{3} = \frac{4(4^{k+1} - 16)}{3}$

5.2 - 27 $5^3 + 5^4 + 5^5 + \ldots + 5^k$ where k is an integer $k \geq 3$

$5^3 + 5^4 + 5^5 + \ldots + 5^k = 5^3(1 + 5 + 5^2 + \ldots + 5^{k-3})$

$= 5^3\left(\frac{5^{k-3} - 1}{4}\right)$

5.2 - 35 For any integer $n \geq 1$

$\sum_{i=1}^{n} i(i!) = (n+1)! - 1$

Prove by mathematical induction

$P(n)$ be $\sum_{i=n} i(i!) = (n+1)! - 1$

Show $P(1)$ is true when $n = 1$

$\sum_{i=1} i(i!) = (1+1)! - 1$

so $1(1!) = 2! - 1$

and $1 = 1$

Thus $P(1)$ is true

This proof is invalid since we are manipulating both sides simultaneously. The correct answer is to transform the left side, then the right until both sides can be seen as equal

$i(i!)$

$P(1) = 1(1!) = 1$

the right side will be

$P(1) = (1+1)! - 1$

$= 2! - 1$

$= 1$

Since both sides are equal $P(1)$ is true

Lets also assume k where $k \geq 1$. we must show $k+1$ to be true

$\sum_{i=k}^{k} k(k!) + (k+1)(k+1)!$

$(k+1)! - 1 + (k+1)(k+1)! = (k^{+1} + 1)! - 1$

$(k+1)!(-1 + (k+1))$

$(k^{+1} + 1) - 1 = (k^{+1} + 1)! - 1$

5.3 - 10 Prove by mathematical induction

$n^3 - 7n + 3$ is divisible by 3 for each integer where $n \geq 0$

$P(n)$ $n^3 - 7n + 3$ is divisible by 3

$P(0)$ $0^3 - 7(0) + 3$ is divisible by 3

$P(0) = 3$ is divisible by 3

$P(0)$ is true since 3 is divisible by itself

lets assume $k \geq 0$ and $k^3 - 7k + 3$ is divisible by 3

I will show that $P(k+1)$ is also divisible by 3

$(k+1)^3 - 7(k+1) + 3$

$(k+1)(k^2 + 2k + 1) - (7k + 1) + 3$

$k^3 + 2k^2 + k + k^2 + 2k + 1 - 7k - 1 + 3$

$k^3 + 3k^2 + 3k + 1 - 7k - 1 + 3$

$3k^2 + 3k + 1 - 1 + k^3 - 7k + 3$

$3k^2 + 3k + \underbrace{k^3 - 7k + 3}_{\text{divisible by } 3}$

since $3k^2 + 3k = 3(k^2 + k)$ is also divisible by 3. We can claim that $P(k+1)$ is also divisible by 3. Thus the proposition is true.

5.3 - 18 Prove by mathematical induction

$5^n + 9 < 6^n$ for all integer $n \geq 2$

$P(n)$ $5^n + 9 \leq 6^n$ where $n \geq 2$

$P(2)$ $5^2 + 9 \leq 6^n$

$25 + 9 = 34$

$6^2 = 36$

$34 \leq 36$

$P(2)$ is true.

Suppose that k is any integer ≥ 2 such that

$5^k + 9 \leq 6^k$

We must prove that $P(k+1)$ is true

$5^{k+1} + 9 \leq 6^{k+1}$

$5(5^k) + 9 \leq 6(6^k)$

$\frac{5^k + \frac{9}{5}}{6} \leq \frac{6^k}{5}$???

5.3 - 23b $n! \geq n^2$ for all integer $n \geq 4$

$P(n) = n! > n^2$ where $n \geq 4$

$P(4) = 4! > 4^2$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$4^2 = 16$

$24 > 16$

suppose there is a k such - - $k \geq 4$

$k! > k^2$

we must show that $(k+1)! > (k+1)^2$

$(k+1)(k!) > (k+1)^2$

$k! \geq k+1$

since we know that $k! \geq k^2$

we must know that $k! \geq k+1$ thus the proposition is true.