Assignment 3 Part 1

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4.6 – 12 If a and b are rational numbers, $b \neq 0$, and r is an irrational number, then a+br is irrational

Suppose not, That is, suppose that a+br is irrational when r is rational.

lets assume $a = \frac{d}{e}$ where $e \neq 0$

$b = \frac{f}{g}$ where $g \neq 0$

$r = \frac{h}{i}$ where $i \neq 0$

$a + br$ is irrational?

$\frac{d}{e} + \left(\frac{f}{g}\right)\left(\frac{h}{i}\right)$

$\frac{d}{e} + \frac{fh}{gi}$

$\left(\frac{gi}{gi}\right)\frac{d}{e} + \left(\frac{e}{e}\right)\frac{fh}{gi}$

$\frac{gid + efh}{egi}$ because $egi \neq 0$ a+br must be rational

the supposition is false and the proposed theorem is true

4.6 – 16 For all odd integer a, b, and c, if z is a solution of $ax^2+bx+c=0$ then z is irrational.

Suppose not. $ax^2+bx+c=0$ where z is a solution and is rational

$z = \frac{d}{e}$ where $e \neq 0$

$a\left(\frac{d}{e}\right)^2 + b\left(\frac{d}{e}\right) + c = 0$

$\frac{ad^2}{e^2} + \frac{bd}{e} + c = 0$

$e^2\left(\frac{ad^2}{e^2}\right) + e^2\left(\frac{bd}{e}\right) + e^2(c) = e^2(0)$

$ad^2 + ebd + e^2c = 0$ if $d = 2m$ = even

$a4m^2 + eb2m + e^2c = 0$ $e = 2n$ = even

$a4m^2 + b(2n)(2m) + 4n^2c = 0$

$a4m^2 + 4bnm + 4n^2c = 0$

$4(am^2 + bnm + n^2c) = 0$ the only way for this equation to equal 0 is if a, b, and c are 0

the supposition is false and the proposed theorem is true.

4.6 - 18 Suppose n is any integer such that $5|n$ [we must show that $5|n^2$]. By definition of divisibility, $n = 5k$ for some integer k. By substitution, $n^2 = (5k)^2 = 5(5k^2)$. $5k^2$ is an integer because it was the product of integers. Hence $n^2 = 5\cdot$(an integer), and also $5|n^2$ [as was to be shown]