

S.9 - 6 Define a set S recursively as follows

I - BASE $a \in S$

II - RECURSION: if $s \in S$ then a) $sa \in S$
b) $sb \in S$

III - RESTRICTION: Nothing is in S other than objects defined in I and II

Use structural induction to prove every string in S begins with an a

The only object in the base case is " a "

The recursion for S consists of two rules a: $sa \in S$ and b: $sb \in S$.

Suppose s is a string in S that begins with a . In the case where $sa \in S$ is applied to s , the resulting string is sa , which begins with a .

When we look at $sb \in S$ the resulting string is ab which also begins with a . When the rules of recursion is applied in string S the string will begin with a .

S.9 - 10 Define a set S recursively as follows.

I - BASE $0 \in S, 5 \in S$

II - RECURSIVE If $s \in S$ then $t \in S$
a) $s+t \in S$ b) $s-t \in S$

III RESTRICTION: Nothing is in S other than objects defined in I and II

Use structural induction to prove every integer S is divisible by 5

The two base cases for this set is 0 and 5. Both are divisible by 5

The recursion function consists of two rules $s+t \in S$ and $s-t \in S$.

Suppose s and t are numbers that are divisible by 5. Since both $s \in S$ and $t \in S$ then the sum of the two numbers must be divisible by 5.

With the rule of recursion applied $s-t$ must also be divisible by 5

S.9 - 13b Consider the set P of parenthesis structures defined in S.9.4. Give derivations showing that each of the following is in P .

↳ $(())()$

(1) by I $()$ is in P

(2) by (1) and II(a), $(())$ is in P

(3) by (2) and II(b), $(())()$ is in P

S.9 - 16 Give a recursive definition for the set of all strings of 0's and 1's for which all of the 0's precede the 1's

Let S be the set of all strings of 0's and 1's for which all of the 0's precede the 1's.

I. BASE $0 \in S$.

II. RECURSION a) $01 \in S$ b) $01 \in S$ c) $01 \in S$

III RESTRICTION There are no elements S other than those obtained by II