Assignment 3 Part 2

Daria Yan

Thursday, January 21, 2016 6:55 PM

6.1-3 Let sets R, S, and T be defined as follows

$R = \{x \in Z \mid x \text{ is divisible by } 2\}$

$S = \{y \in Z \mid y \text{ is divisible by } 3\}$

$T = \{z = Z \mid z \text{ is divisible by } 6\}$

a) Is $R \subseteq T$? Explain

No since there are numbers in R that are not in T. numbers like 2, 4, 8, 10... are divisible by 2 but not divisible by 6

b) Is $T \subseteq R$? Explain

Yes every number that is divisible by 6 (an even number) must be divisible by 2

c) Is $T \subseteq S$? Explain

Yes every number that is divisible by 6 must be divisible by it's factor 3.

If the set notation was the other way $S \subseteq T$ then it would be false (eg 9, 15, 21)

6.1-7 Let $A = \{x \in Z \mid x = 6a + 4 \text{ for some integer } a\}$

$B = \{y \in Z \mid y = 18b - 2 \text{ for some integer } b\}$

$C = \{z \in Z \mid z = 18c + 16 \text{ for some integer } c\}$

a) $A \subseteq B$

Suppose x is a particular but arbitrary element of A.

If $A \subseteq B$ then $x = 18(\text{some integer}) - 2$

$6a + 4 = 18(\text{some integer}) - 2$

$\frac{6a + 6}{18} = \text{some integer}$ $\qquad \frac{a + 1}{3} = \text{some integer}$

since not all values of a is an integer $\qquad A \not\subseteq B$

b) $B \subseteq A$

Suppose x is a particular but arbitrary element of B

If $B \subseteq A$ then $x = 6(\text{some integer}) + 4$

$18b - 2 = 6(\text{some integer}) + 4$

$18b - 6 = 6(\text{some integer})$

$3b - 1 = \text{some integer}$

$B \subseteq A$ since $3b - 1$ is an integer

c) $B = C$

Suppose x is a particular but arbitrary element of C

If $B = C$ then $B \subseteq C$ and $C \subseteq B$

$18b - 2 = 18(\text{some int}) + 16$ and $18c + 16 = 18(\text{some int}) - 2$

$18b - 18 = 18(\text{some int})$ $\qquad 18c + 18 = 18(\text{some int})$

$b - 1 = \text{some int}$ $\qquad c + 1 = \text{some int}$

Both equations are integers $\therefore B \subseteq C$ and $C \subseteq B$

$\therefore B = C$

6.1-13 Indicate which of the following relationships are true and which are false

Z = int

Q = rational

R = real

a) $Z^+ \subseteq Q$ True all ints are rational

b) $R^- \subseteq Q$ False Some negative real numbers are not rational

c) $Q \subseteq Z$ False not all rational numbers can be expressed as integers

d) $Z^- \cup Z^+ = Z$ False 0 does not exist in either Z^- or Z^+

e) $Z^- \cap Z^+ = \emptyset$ True both Z^- and Z^+ do not have a common element

f) $Q \cap R = Q$ True since all rational numbers are real. the intersect should only contain rational #

g) $Q \cup Z = Q$ True the union would contain both rational and integers

h) $\mathbb{Z}^+ \cap \mathbb{R} = \mathbb{Z}^+$ True since $\mathbb{Z}^+$ is contained within $\mathbb{R}$
i) $\mathbb{Z} \cup \mathbb{Q} = \mathbb{Z}$ False since the union would also contain rational numbers that are not integers

6.1 - 18

a) Is the number 0 in $\emptyset$? No because a null set does not contain any elements
b) Is $\emptyset = \{\emptyset\}$? No $\emptyset$ contains no elements. The second is a set that contains the "element" $\emptyset$
c) Is $\emptyset \in \{\emptyset\}$? Yes $\emptyset$ is an element in the set $\{\emptyset\}$
d) Is $\emptyset \in \emptyset$? No there is no set to see if $\emptyset$ belongs to it.

6.1 - 33

a) $P(\emptyset) = \{\emptyset\}$
b) $P(P(\emptyset)) = \{\emptyset, \{\emptyset\}\}$
c) $P(P(P\emptyset)) = \{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

6.1 - 34 Let $A_1 = \{1,2,3\}$, $A_2 \{u, v\}$ and $A_3 = \{m, n\}$

a) $A_1 \times (A_2 \times A_3) =$

$\{(1,(u,m)), (2,(u,m)), (3,(u,m))$
$(1,(u,n)), (2,(u,n)), (3,(u,n))$
$(1,(v,m)), (2,(v,m)), (3,(v,m))$
$(1,(v,n)), (2,(v,n)), (3,(v,n))\}$

b) $(A_1 \times A_2) \times A_3 = \{((1,u),m), ((1,u),n)$
$((1,v),m), ((1,v),n)$
$((2,u),m), ((2,u),n)$
$((2,v),m), ((2,v),n)$
$((3,u),m), ((3,u),n)$
$((3,v),m), ((3,v),n)\}$

c) $A_1 \times A_2 \times A_3 \{(1,u,m) \ (2,u,m) \ (3,u,m)$
$(1,u,n) \ (2,u,n) \ (3,u,n)$
$(1,v,m) \ (2,v,m) \ (3,v,m)$
$(1,v,n) \ (2,v,n) \ (3,v,n)\}$