

# Assignment 4 Part 1

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Daria Yan

6.1-12 Let the universal set be the set  $K$  for all real numbers and let

$$\begin{aligned} A &= \{x \in K \mid -3 \leq x \leq 0\} & \{-3, -2, -1, 0\} \\ B &= \{x \in K \mid -1 < x < 2\} & \{0, 1\} \\ C &= \{x \in K \mid 6 < x \leq 8\} & \{7, 8\} \end{aligned}$$

$$\begin{aligned} a) A \cup B &= \{-3, -2, -1, 0, 1\} \\ b) A \cap B &= \{0\} \\ c) A^c &= \{x \in K \mid 0 < x < 9\} \\ d) A \cup C &= \{-3, -2, -1, 0, 1, 7, 8\} \\ e) A \cap C &= \emptyset \\ f) B^c &= \{x \in K \mid -1 \leq x \leq 2\} \\ g) A^c \cap B^c &= \{x \in K \mid -1 < x < 9\} \\ h) A^c \cup B^c &= \{x \in K \mid -4 < x < 2\} \\ i) (A \cap B)^c &= \{0\}^c = \{x \in K \mid -1 < x < 1\} \\ j) (A \cup B)^c &= \{x \in K \mid -4 < x < 2\} \end{aligned}$$

6.1-16 Let  $A = \{a, b, c\}$   $B = \{b, c, d\}$   $C = \{b, c, e\}$

a)  $(A \cup (B \cap C))$ ,  $(A \cup B) \cap C$  and  $(A \cup B) \cap (A \cup C)$  which of these sets are equal

$\rightarrow$  distributive property  
 $(A \cup B) \cap (A \cup C)$ ,  $(A \cap C) \cup (B \cap C)$  and  $(A \cup B) \cap (A \cup C)$   
 $\rightarrow$  the first and third are equal  
 $\rightarrow \{a, b, c, d\} \cap \{a, b, c, e\} = \{a, b, c\}$   
 $\rightarrow \{b, c\} \cup \{b, c\} = \{b, c\}$

b)  $A \cap (B \cup C)$ ,  $(A \cap B) \cup C$  and  $(A \cap B) \cup (A \cap C)$

$\rightarrow$  distributive property  
 $(A \cap B) \cup (A \cap C)$ ,  $(A \cup C) \cap (B \cup C)$  and  $(A \cap C) \cup (A \cap C)$   
 $\rightarrow$  the first and third are equal  
 $\rightarrow$  same as the question before  
 $\rightarrow \{a, b, c\}$

c)  $(A - B) - C$  and  $A - (B - C)$

$\rightarrow \{a\} - \{b, c, e\} = \{a\}$   
 $\rightarrow \{a, b, c\} - \{d\} = \{a, b, c\}$   
 $\rightarrow \{a\} \neq \{a, b, c\}$   
 $\rightarrow$  they are not equal

$\rightarrow \{x \in U \mid x \in A \text{ or } x \in B\}$

6.2-4 The following is a proof that for all sets  $A$  and  $B$  if  $A \subseteq B$  then  $A \cup B \subseteq B$

Proof Suppose  $A$  and  $B$  are any sets and  $A \subseteq B$  [We must show that  $A \cup B \subseteq B$ ]  
 Let  $x \in K$  [We must show that  $A \cup B \subseteq B$ ] By definition of Union  $x \in K \mid x \in A$   
 $x \in B$ . In case  $x \in A \cup B$ , then since  $A \subseteq B$ ,  $x \in A$  In case  $x \in B$ , then clearly  $x \in B$ . So in either case  $x \in B$  [as was to be shown]

6.2-10

$$(A - B) \cap (C - B) = (A \cap C) - B$$

We must show that  $(A - B) \cap (C - B) \subseteq (A \cap C) - B$  and  $(A \cap C) - B \subseteq (A - B) \cap (C - B)$   
 based on the distributive property  
 $(A - B) \cap (C - B) \subseteq (A - B) \cap (C - B)$

6.2-14 For all sets  $A, B$ , and  $C$ , if  $A \subseteq B$  then  $A \cap C \subseteq B \cap C$

if  $A \subseteq B$  then we know that  $x \in B$  and  $x \in A$

$$\begin{aligned} A \cap C &= \{x \in A \mid x \in C\} \\ B \cap C &= (\{x \in B \text{ and } x \in A\}) \cap \{x \in C\} \end{aligned}$$

6.3 - 12  $A \cap (B - C) = (A \cap B) - (A \cap C)$

$x \in A \cap (B - C) \iff x \in A \quad x \in B \quad x \notin C$

$x \in (A \cap B) - (A \cap C)$

$(x \in A \text{ and } x \in B) \quad (x \in A - x \in C)$

$x \in A \quad x \in B \quad x \notin C$

6.3 - 37 For all sets A and B

$B^c \cup (B^c - A)^c = B$

$B^c \cup (B^c \cap A^c)^c = B$  set difference law

$B^c \cup (B \cup A) = B$  de Morgan's law

$(B^c \cup B) \cup A = B$  associative law

$U \cup A = B$  complement law

$U$  universal bond law

I believe 37 is wrong

$(B^c - B) + (B^c - A)^c$

$(U - B) + ((U - B) - A)^c$

$U - B + A + B$

$U \cup A$

6.3 - 42  $(A - (A \cap B)) \cap (B - (A \cap B))$

$(A \cap (A \cap B)^c) \cap (B \cap (A \cap B)^c)$  De Morgan's law

$A \cap (A \cup B^c) \cap (B \cap (A \cup B^c))$  set difference

$(A \cap A^c) \cup (A \cap B^c) \cap (B \cap A^c) \cup (B \cap B^c)$  distributive law

$(\emptyset) \cup (A \cap B^c) \cap (B \cap A^c) \cup (\emptyset)$  complement law

$A \cap B^c \cap B \cap A^c$  identity law

$(B^c \cap B) \cap (A \cap A^c)$  associative law

$\emptyset \cap \emptyset$

$\emptyset$