Assignment 2 Part 2

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4.1 : 32 If a is an odd integer and b is an even integer, then $2a+3b$ is even

$a = 2k+1$ odd $\forall x \in D,\ P(x) \to Q(x)$

$b = 2m$ even

$2(2k+1) + 3(2m)$ $2x+2n$

$4k+2+6m$ $2(x+n)$

$2(2k+1+3m)$ $\square$

4.1 : 61 Suppose that integers m and n are perfect squares. Then $m+n+2\sqrt{mn}$ is also a perfect square. Why?

$m = \sqrt{m}\cdot\sqrt{m}$ let $\sqrt{m} = x$ $k^2 = \sqrt{m}^2 + \sqrt{n}^2 + 2\sqrt{m}\sqrt{n}$

$n = \sqrt{n}\cdot\sqrt{n}$ let $\sqrt{n} = y$ $k^2 = x^2 + y^2 + 2xy$

let k^2 = perfect square $k^2 = (x+y)(x+y)$

$k^2 = (x+y)^2$

$k = (x+y)$

4.2 : 20 Given any two rational numbers r and s, with $r<s$, there is another rational number between r and s.

let $r = \frac{a}{b}$

$\frac{r+s}{2}$ $s = \frac{c}{d}$ $\frac{\frac{a}{b}+\frac{c}{d}}{2} = \frac{(ad+bc)/bd}{2}$ $\frac{ad+bc}{2bd}$ rational. products and sum of integers $ad+bc$ are both integers. the denominator $2bd \neq 0$

Since we prove that $\frac{r+s}{2}$ is a rational number we must prove there is a rational number between them

if $a<b$ then $2a < a+b$ $\therefore a < \frac{a+b}{2} < b$

assuming a and b is an integer (which is also rational Theorem 4.2.1) we can say that there is a rational number between two rational numbers r and s.

4.2 : 25 If r is any rational number, then $3r^2-2r+4$ is rational.

Theorem 4.2.2

the sum of any two rational number is rational

$r+s = \frac{a}{b}+\frac{c}{d} = \frac{ad+bc}{bd}$ $p = ad+bc$, $q = bd$ $= r+s = \frac{p}{q}$ $q \neq 0$

let $a = 3r^2$ $a+b+c$ is rational

$b = -2r$

$c = 4$

4.6 : 28 For all integers m and n, if mn is even then m is even or n is even

$\forall x$ in D, if $P(x)$ then $Q(x)$

$\forall x$ in D if $Q(x)$ is false then $P(x)$ is false.

$P(x)$ = m is even or n is even

$Q(x)$ = mn is even

$\sim Q(x) \to \sim P(x)$ if m is not even and n is not even, then mn is not even

if m is odd and n is odd, then mn is odd

$m = 2a+1$

$n = 2b+1$

$mn = (2a+1)(2b+1) = 4ab + 2a + 2b + 2 = 2(2ab + a + b + 1)$

mn is even.