Assignment 8 Part 1

David Yan

Monday, February 22, 2016 4:22 PM

9.2-32c How many ways can letters of the word ALGORITHM be arranged in a row if the letters GOR must remain together in a row

"GOR" = 1!

"ALITHM" = 6!

$(1!)(6!) = 6! = 720$

9.2-33 Six people attend the theater together and sit in a row with exactly 6 seats.

a. How many ways can they be seated together in a row?

$6! = 720$

b. Suppose one of the doctors must sit on the aisle incase they get paged. How many ways can the group sit together with the doctor in the aisle seat?

Two cases. Doctor in the first seat or doctor in the 6^{th} seat

$(2!)(5!) = 2 \cdot 120 = 240$

c. Suppose the six people consisted of mixed couples and each want to sit together with the husbands on the left. How many ways can the 6 be seated together in a row?

With the assumptions that they wanted to sit together in a single row

there are 3 possible positions for a couple $= 3! = 6$

9.2-36 Write all 3-permutations of $\{s,t,u,v\}$ $= P(4,3) = \frac{4!}{4-3!} \quad \frac{4!}{1!} = 24$

stu, sut, suv, svu, stv, svt,

tsu, tus, tsv, tvs, tuv, tvu,

ust, uts, usv, uvs, utv, uvt,

vst, vts, vsu, vus, vtu, vut,

9.2-39b How many ways can 6 of the letters of ALGORITHM be selected and written in a row?

ALGORITHM = 9! $\quad P(9,6) = \frac{9!}{(9-6)!} = 9 \cdot 8 \cdot 7 = 504$

9.2-39d How many ways can six of the letters of the word ALGORITHM be selected and written in a row if the first two letters must be OR?

"OR" = 1! $\quad = \frac{7!}{4!} = 7 \cdot 6 \cdot 5 = 210$

ALGRITHM = 7!

9.5 - 7b A computer programming team has 13 members

Suppose 7 are women and 6 are men

i How many groups of seven can be chosen that contain 4 women and 3 men?

women $= \binom{7}{4}$ men $= \binom{6}{3}$ $= \left(\frac{7!}{4!3!}\right)\left(\frac{6!}{3!3!}\right) = \left(\frac{7\cdot6\cdot5}{\cancel{3\cdot2}}\right)\left(\frac{6\cdot5\cdot4}{\cancel{3\cdot2}}\right) = 700$

ii How many groups of 7 can be chosen that contain at least 1 man?

total number of teams $= \binom{13}{7} = \frac{13!}{7!6!} = \frac{13\cdot\cancel{12}\cdot11\cdot\cancel{10}\cdot9\cdot8}{6\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}$ $\frac{13\cdot11\cdot9\cdot8}{6} = 1716$

number of teams without men $\binom{7}{7} = 1$

1715

iii How many groups of 7 can be chosen with at most 3 women

teams with 3 women $\binom{6}{4}\binom{7}{3}$ $\frac{6!}{4!2!}$ $\frac{7!}{3!4!}$ $\frac{6\cdot\cancel{5\cdot4\cdot3\cdot2}}{\cancel{4\cdot3\cdot2}\ 2}$ $\frac{30}{2}$ $\frac{7\cdot6\cdot5\cdot\cancel{4\cdot3\cdot2}}{3\cdot2\ \ \cancel{4\cdot3\cdot2}}$ $\frac{35}{1} = 525$

teams with 2 women $\binom{6}{5}\binom{7}{2}$ $\frac{6}{5!1!}$ $\frac{7!}{2!5!}$ $\frac{6\cdot\cancel{5\cdot4\cdot3\cdot2}}{\cancel{5\cdot4\cdot3\cdot2}}$ $\frac{6}{1}$ $\frac{7\cdot6\cdot\cancel{5\cdot4\cdot3\cdot2}}{2\cdot\cancel{5\cdot4\cdot3\cdot2}}$ $\frac{42}{2} = 126$

teams with 1 women $\binom{6}{6}\binom{7}{1}$ $\frac{6!}{6!}$ $\frac{7!}{1!6!}$ 1 $\frac{7\cdot\cancel{6\cdot5\cdot4\cdot3\cdot2}}{\cancel{6\cdot5\cdot4\cdot3\cdot2}}$ $\frac{7}{1} = 7$ $= 658$

teams with 0 women : impossible since there is only 6 men

9.5-14 a How many 16-bit strings contains exactly seven 1's

$\binom{16}{7} = \frac{16!}{7!9!} = \frac{16\cdot\cancel{15}\cdot14\cdot13\cdot\cancel{12}\cdot11\cdot\cancel{10}}{7\cdot6\cdot\cancel{5}\cdot4\cdot\cancel{3}\cdot2} = \frac{480480}{35} = 13728$

b How many 16 bit strings contain at least thirteen 1's

$\binom{16}{13} + \binom{16}{14} + \binom{16}{15} + \binom{16}{16}$

$\frac{16!}{13!3!} + \frac{16!}{14!2!} + \frac{16!}{15!1!} + 1$

$\frac{16\cdot15\cdot14}{3\cdot2} + \frac{16\cdot15}{2} + \frac{16}{1} + 1$

$\frac{3360}{6}$ $\frac{240}{2} + 16 + 1$

$560 + 120 + 16 + 1 = 697$

c How many 16 bit strings contain at least one 1

$2^{16} - \binom{16}{0} = 65535$

d How many 16 bit strings contain at most one 1

$\binom{16}{0} + \binom{16}{1} = 1 + 16 = 17$

9.5-20 a How many distinguishable ways can the letters MILLIMICRON be arranged in order?

M $= \binom{11}{2}$ I $= \binom{9}{3}$ L $= \binom{6}{2}$ C $= \binom{4}{1}$ R $= \binom{3}{1}$ O $= \binom{2}{1}$ N $= \binom{1}{1}$

$\frac{11!}{2!\cancel{9!}} \cdot \frac{\cancel{9!}}{3!\cancel{6!}} \cdot \frac{\cancel{6!}}{2!\cancel{4!}} \cdot \frac{\cancel{4!}}{1!\cancel{3!}} \cdot \frac{\cancel{3!}}{1!\cancel{2!}} \cdot \frac{\cancel{2!}}{1!\cancel{1!}} \cdot \cancel{1!} =$

$\frac{11!}{2!3!2!1!} = \frac{11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot\cancel{5\cdot4\cdot3\cdot2}}{\cancel{2\cdot3\cdot2\cdot2}\cdot1} = 332640$

b How many distinguishable orders of the letters MILLIMICRON be arranged that begin with a M and end with an N

$M = \binom{10}{1}$ $I = \binom{9}{3}$ $L = \binom{6}{2}$ $C = \binom{4}{1}$ $R = \binom{3}{1}$ $O = \binom{2}{1}$ $N = \binom{1}{1}$

$\frac{10!}{2!3!2!1!}$ $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30240$

c How many distinguishable orders of the letters MILLIMICRON with the letters CR and ON be arranged next to each other

$CR = \binom{9}{1}$ $ON = \binom{8}{1}$ $M = \binom{7}{2}$ $I = \binom{5}{3}$ $L = \binom{2}{2}$

$\frac{9!}{1!\cancel{8!}}$ $\frac{\cancel{8!}}{1!\cancel{7!}}$ $\frac{\cancel{7!}}{2!5!}$ $\frac{\cancel{5!}}{3!\cancel{2!}} = \frac{\cancel{2!}}{2!}$

$\frac{9!}{2!3!2!} = \frac{9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2}}{2 \cdot \cancel{3 \cdot 2} \cdot \cancel{2}} = 5040$