

Assignment 5 Part 2

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5.4 -2 Suppose b_1, b_2, b_3 is a sequence defined as such

$$b_1 = 4 \quad b_2 = 12 \quad b_k = b_{k-2} + b_{k-1} \quad \text{for all integers } k \geq 3$$

Prove b_n is divisible by 4 for all integers $n \geq 1$

$P(n)$ b_n is divisible by 4

note that both $b_1 = 4$ and $b_2 = 12$ and both 4 and 12 are divisible by 4

Therefore $P(1)$ and $P(2)$ are both true.

Let k be any integer with $k \geq 4$ and suppose

i is divisible by 4

i is from 4 to k

we must show that $k+1$ is also divisible by 4

$$b_{k+1} = b_{(k+1)-2} + b_{(k+1)-1}$$

$$b_{k+1} = b_{(k-1)} + b_k \quad \text{Since we know } b_k = b_{k-2} + b_{k-1}$$

$$b_{k+1} = b_{(k-1)} + b_{k-2} + b_{k-1}$$

5.4 -10 Let $P(n)$ be " n ¢ can be obtained using a combination of 3¢ and 5¢. Use strong mathematical induction to prove that $P(n)$ is true for all integers $n \geq 8$

$P(14)$ can be made from (3) 3¢ + 5¢

$P(15)$ can be made from (3) 5¢

$P(16)$ can be made from (2) 5¢ + (2) 3¢

Let there be an integer k where $k \geq 8$ and $P(i)$ is true for all integers i from 2 through k . Then I will show that $P(k+1)$ is also true

$$P(k+1) = [(k+1) - 3] + 3 \quad \text{if } k \geq 16 \text{ then } (k+1) - 3 \geq 14$$

8	3	5
9	3	5
10		5
11	3	5
12	3	3
13	5	5
14	3	3
15		5
16	3	5

5.4 -29 It is a fact that every integer $n \geq 1$ can be written in the form

$$c_r \cdot 2^r + c_{r-1} \cdot 2^{r-1} + \dots + c_2 \cdot 2^2 + c_1 \cdot 2^1 + c_0$$

where $c_r = 1$ or 2 and $c_i = 0, 1, \text{ or } 2$ for all integers $s, i = 0, 1, 2, \dots, r-1$

sketch a proof of this fact

Let the property $P(n)$ be the equation $c_r \cdot 2^r + c_{r-1} \cdot 2^{r-1} + \dots + c_2 \cdot 2^2 + c_1 \cdot 2^1 + c_0$

$P(1)$ Let $r = 0$ and $c_0 = 1$. We can then express $c_r \cdot 2^r = 1$

If we want to prove that all int ≥ 1 can be expressed this way we must show that both odd and even number ($2k$) and ($2k+1$) can be expressed as such.

if $k+1/2$ is an integer since $1 \leq (k+1)/2 \leq k$ then

$$\frac{k+1}{2} = c_r \cdot 2^r + c_{r-1} \cdot 2^{r-1} + \dots + c_2 \cdot 2^2 + c_1 \cdot 2^1 + c_0$$