

Assignment 6 Part 1

David Yan

Thursday, February 11, 2016 2:58 PM

$$\begin{aligned} 5.6-2 \quad b_k &= b_{k-1} + 3k \quad \text{for all int } k \geq 2 \\ b_1 &= 1 \\ b_2 &= b_1 + 3(2) = 1 + 6 = 7 \\ b_3 &= b_2 + 3(3) = 7 + 9 = 16 \\ b_4 &= b_3 + 3(4) = 16 + 12 = 28 \end{aligned}$$

$$\begin{aligned} 5.7-4 \quad d_k &= k(d_{k-1})^2 \quad \text{for all int } k \geq 1 \\ d_0 &= 3 \\ d_1 &= 1(d_0)^2 = 1(3)^2 = 9 \\ d_2 &= 2(d_1)^2 = 2(9)^2 = 162 \\ d_3 &= 3(d_2)^2 = 3(162)^2 = 78732 \end{aligned}$$

$$\begin{aligned} 5.7-4 \quad b_k &= \frac{b_{k-1}}{1+b_{k-1}} \quad \text{for all int } k \geq 1 \\ b_0 &= 1 \\ b_1 &= \frac{b_0}{1+b_0} = \frac{1}{1+1} = \frac{1}{2} \\ b_2 &= \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3} \\ b_3 &= \frac{\frac{1}{3}}{\frac{4}{3}} = \frac{1}{3} \cdot \frac{3}{4} = \frac{1}{4} \\ b_n &= \frac{1}{n} \end{aligned}$$

$$\begin{aligned} 5.7-6 \quad d_k &= 2d_{k-1} + 3 \quad \text{for all int } k \geq 2 \\ d_1 &= 2 \\ d_2 &= 2(d_1) + 3 = 2 \cdot 2 + 3 \\ d_3 &= 2(d_2) + 3 = 2(2 \cdot 2 + 3) + 3 = 2 \cdot 2 \cdot 2 + 2 \cdot 3 + 3 \\ d_4 &= 2(d_3) + 3 = 2(2 \cdot 2 \cdot 2 + 2 \cdot 3 + 3) + 3 = 2 \cdot 2 \cdot 2 \cdot 2 + 2 \cdot 2 \cdot 3 + 2 \cdot 3 + 3 \\ d_n &= 2^n + 2^{n-2}(3) + 2^{n-3}(3) + \dots + 2^2(3) + 2(3) + 3 \\ &= 2^n + 2^1(3)(1 + 2 + 2^2 + 2^3 + \dots + 2^{n-2}) \\ &= 2^n + 2(3) \frac{2^{n-1} - 1}{2 - 1} \\ &= 2^n + 6(2^{n-1} - 1) \end{aligned}$$

$$\begin{aligned} 5.7-7 \quad e_k &= 4e_{k-1} + 5 \quad \text{for all integers } k \geq 1 \\ e_0 &= 2 \\ e_1 &= 4(e_0) + 5 = 4(2) + 5 \\ e_2 &= 4(4(2) + 5) + 5 = 4 \cdot 4 \cdot 2 + 4 \cdot 5 + 5 \\ e_3 &= 4(4 \cdot 4 \cdot 2 + 4 \cdot 5 + 5) + 5 = 4 \cdot 4 \cdot 4 \cdot 2 + 4 \cdot 4 \cdot 5 + 4 \cdot 5 + 5 \\ e_n &= 4^n(2) + 4^{n-1}(5) + 4^{n-2}(5) + \dots + 4^2(5) + 4(5) + 5 \\ &= 4^n(2) + 45 \left(\frac{4^{n-1} - 1}{4 - 1} \right) \\ &= 4^n(2) + 20 \left(\frac{4^{n-1} - 1}{3} \right) \end{aligned}$$