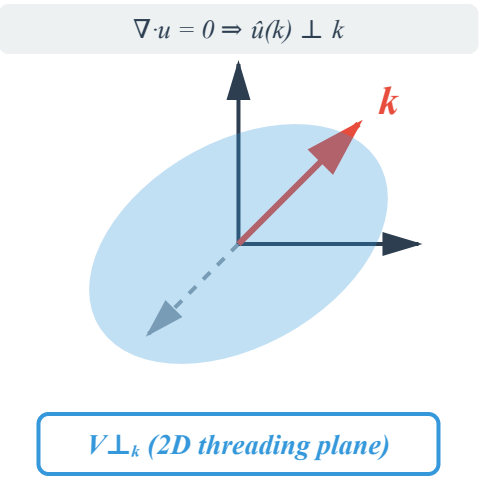# 3D Navier-Stokes Global Regularity Proof
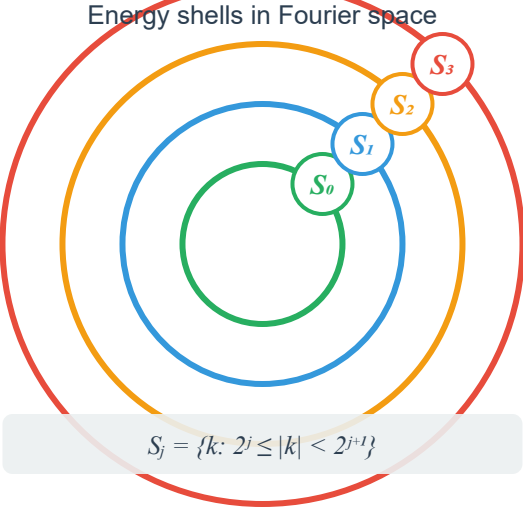
Operational Geometry Framework | Clay Millennium Problem
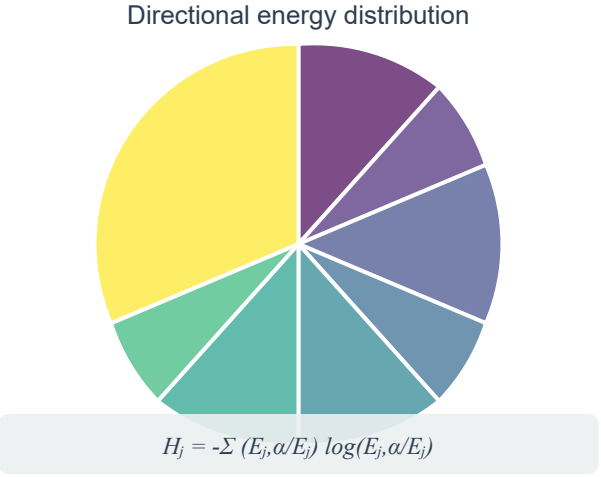
Threading Subspace Constraint

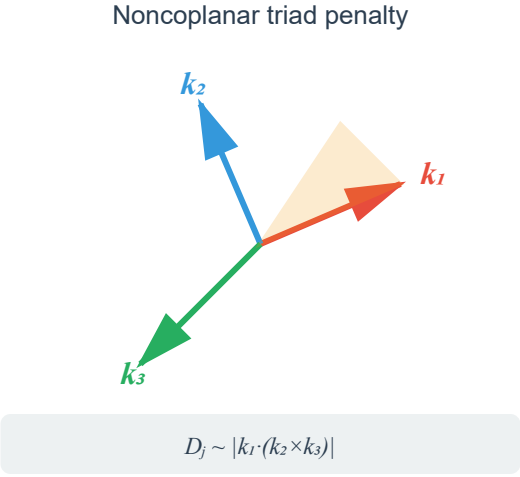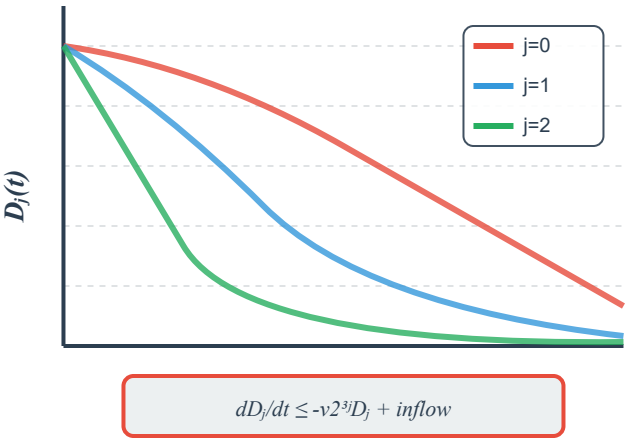$$\nabla \cdot u = 0 \Rightarrow \hat{u}(k) \perp k$$

k

$V \perp_k$ (2D threading plane)

Dyadic Shell Structure

Energy shells in Fourier space

S_3
S_2
S_1
S_0

$$S_j = \{k: 2^j \le |k| < 2^{j+1}\}$$

Angular Entropy H_j

Directional energy distribution

$$H_j = -\Sigma\, (E_{j,\alpha}/E_j)\, \log(E_{j,\alpha}/E_j)$$

Threading Deficit D_j

Noncoplanar triad penalty

k_2
k_1
k_3

$$D_j \sim |k_1 \cdot (k_2 \times k_3)|$$

Exponential Deficit Decay

$D_j(t)$

- j=0
- j=1
- j=2

$$dD_j/dt \le -\nu 2^{3j} D_j + inflow$$

Dissipation vs Inflow

Rate

- Dissipation $\sim \nu 2^{3j}$
- Inflow $\sim 2^{2j}$

Shell j

Separation: $2^{5j} \gg 1$
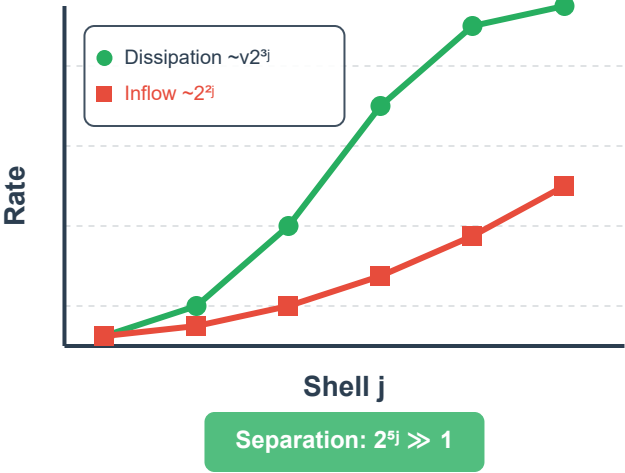
Bounded Angular Entropy

$H_j(t)$

- $H_j(0)$
- $H_j(t)$ bound

$$H_j(t) \le H_j(0) + C/(\nu 2^{4j})$$

Proof Chain

Incompressibility
$\nabla \cdot u = 0$

2D Threading Constraint

Deficit Decay
$D_j \sim e^{-\nu 2^{3j}t}$

Entropy Bound
$\Sigma_j\, 2^{-4j} < \infty$

Global Regularity
✓ QED
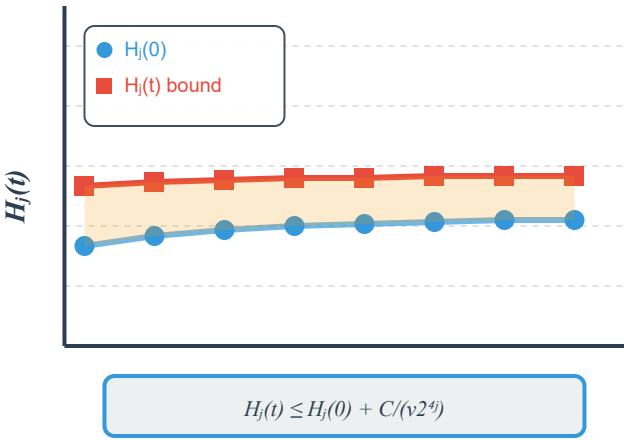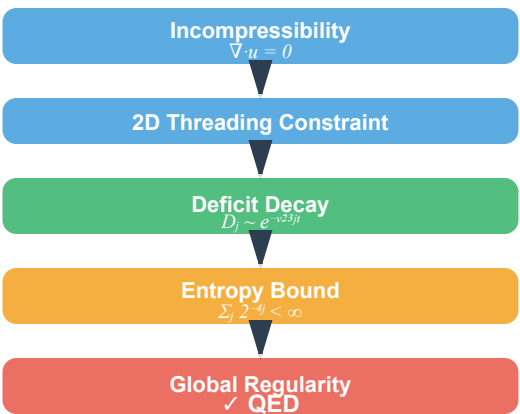
Key Insight: Incompressibility forces 2D threading → Angular entropy bounded → No blowup