BPA-SAS : individual work 3.

Task no.1

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1) continuous LTI system: $5\dfrac{dy(t)}{dt} + y(t) = 10u(t)$

- Transfer function $(H(s))$

↳ find using Laplace transform: $5[s\,y(s)] + y(s) = 10\,u(s)$

$$y(s)(5s+1) = 10\,u(s)$$

$$H(s) = \frac{y(s)}{u(s)} = \frac{10}{5s+1}$$

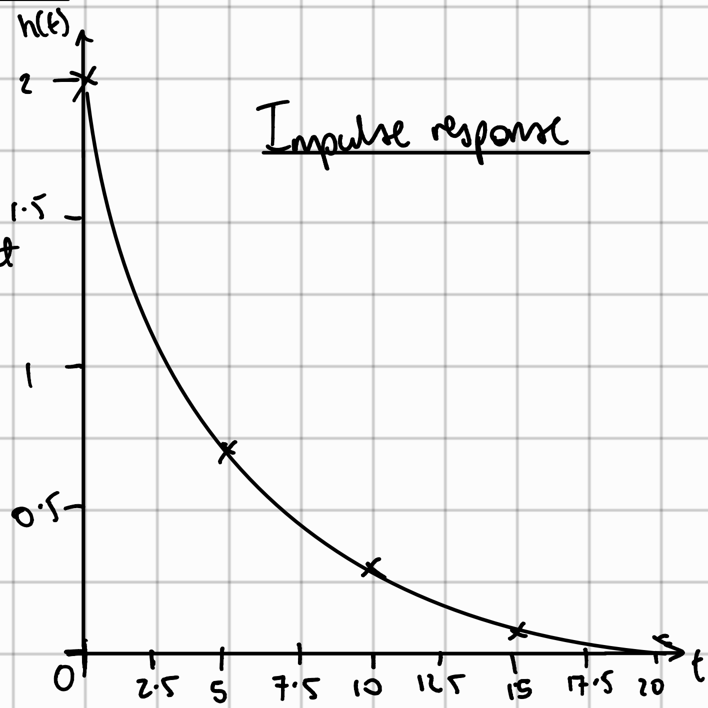
- Impulse response $(h(t))$

↳ inverse laplace transform

from table, closest is: $\dfrac{1}{s+a} \Rightarrow \mathcal{L}^{-1}\{H(s)\} = e^{-at}$

Rewrite $H(s)$: $\dfrac{2}{s+\frac{1}{5}}$

$$\mathcal{L}^{-1}\{H(s)\} = 2e^{-\frac{t}{5}}u(t) = h(t)$$



- Step response $(y_{step}(t))$

↳ transfer function $\times \frac{1}{s}$: $\dfrac{10}{5s+1}\cdot\dfrac{1}{s} \Rightarrow \dfrac{10}{s(5s+1)} = \dfrac{A}{s} + \dfrac{B}{5s+1}$

↓ $\mathcal{L}^{-1}$ of result

$$\Rightarrow 10 = A(5s+1) + B(s)$$

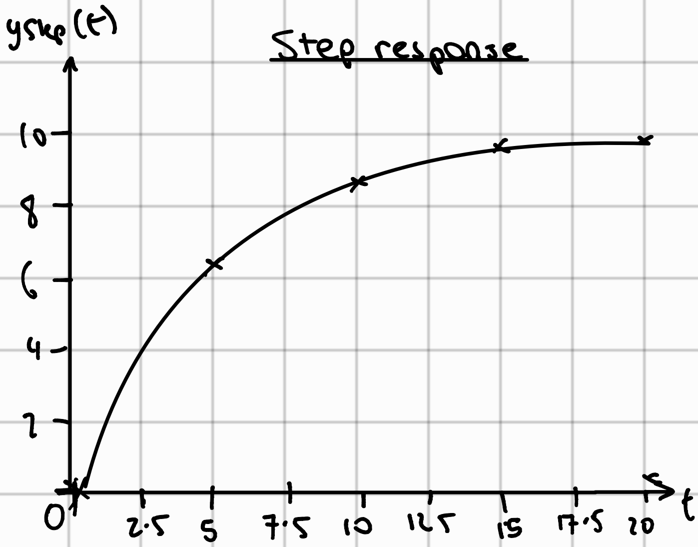
- let $s = 0$: $10 = A(1) \rightarrow A = 10$
- let $s = -\frac{1}{5}$: $10 = B(-\frac{1}{5}) \rightarrow B = -50$

↓

$$\frac{10}{s} - \frac{\cancel{50}^{10}}{\cancel{5}^{1}(s+\frac{1}{5})} = 10\left(\frac{1}{s} - \frac{1}{s+\frac{1}{5}}\right)$$

$\mathcal{L}^{-1}\{\frac{1}{s}\} = 1$ $\mathcal{L}^{-1}\{\frac{1}{s+\frac{1}{5}}\} = e^{-\frac{t}{5}}$

$$y_{step}(t) = 10(1-e^{-\frac{t}{5}})u(t)$$



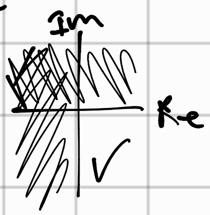
- Frequency transfer function ($H(j\omega)$)
  ↳ replace s of H(s) with $j\omega$

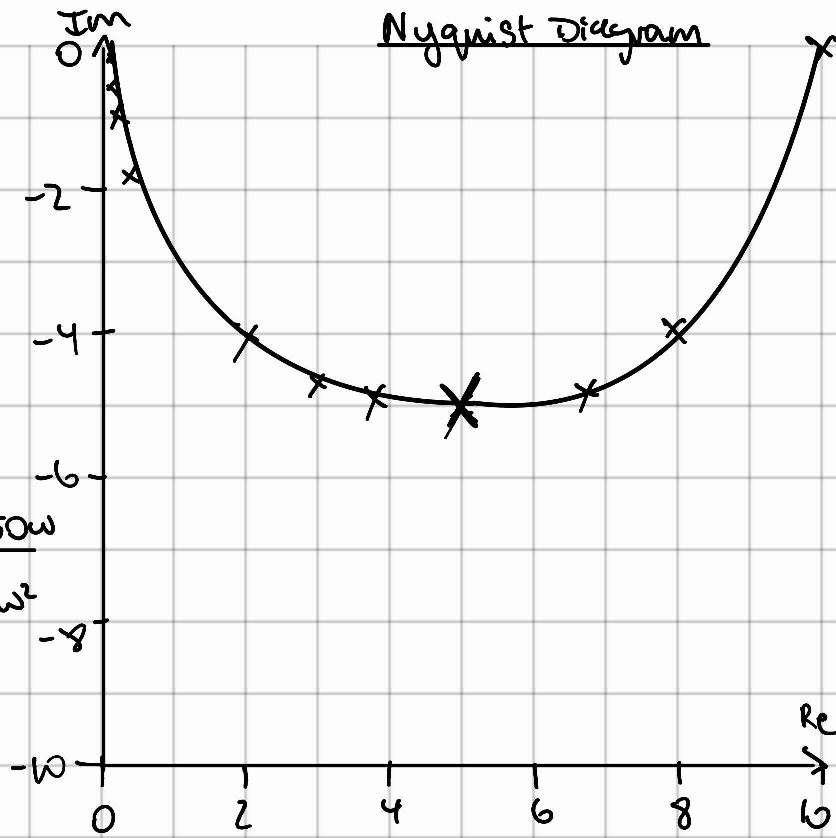
$$H(s) = \frac{10}{5s+1} \rightarrow \frac{10}{1+j5\omega} = H(j\omega)$$

$$\frac{10}{1+j5\omega} \cdot \frac{(1-j5\omega)}{(1-j5\omega)} = \frac{10-j50\omega}{1-j^2 25\omega^2} = \frac{10-j50\omega}{1+25\omega^2}$$

$$Re = \frac{10}{1+25\omega^2} \qquad Im = \frac{-50\omega}{1+25\omega^2}$$

$Re > 0$ $\qquad$ $Im < 0$





Plotting:

	Re	Im
$w=0$	10	0
$w=1$	0.38	-1.92
$w=2$	0.1	-1
$w=3$	0.04	-0.66
$w=4$	0.02	-0.5
$w=\infty$	$\rightarrow 0$	$\rightarrow 0$

	Re	Im
$W=0.1$	8	-4
$w=0.2$	5	-5
$W=0.3$	3.1	-4.6
$W=0.4$	2	-4
$W=\infty$	0	0

	Re	Im
$W=0.15$	6.4	-4.8
$W=0.25$	3.9	-4.9

- Amplitude of harmonic output signal

> applied input signal = $u(t) = \sin(0.2)t$

$w = 0.2 \Rightarrow im = -5, re = 5.$ $\quad$ Amplitude $= \sqrt{5^2 + (-5)^2} = \sqrt{50} = 7.07$

↓ from plot.

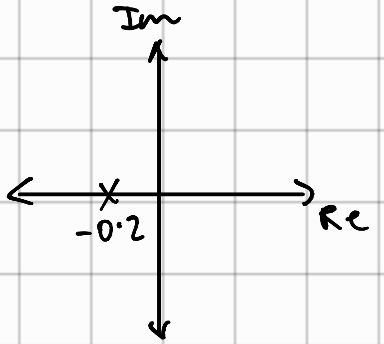
- Phase shift of harmonic output signal

$H(j\omega) = \arctan\left(\frac{-5}{5}\right) = -45^\circ =$ phase.

• **Poles and zeros** $(H(s)) = \frac{10}{5s+1}$

zero if numerator $=0$. since numerator is constant, no finite zeros.

poles if denominator $=0$. denom $= 5s+1$.



$5s+1=0$

$5s=-1$

$s=\frac{-1}{5}$ → one pole at $s=-0.2$.

$s=-0.2+0j$

left hand plane = stable ⇒ since all poles lie on left-half plane ⇒ stable

right hand plane = unstable

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3) continuous LTI system: $5\frac{dy(t)}{dt} = 10u(t)$

• **Transfer function** $(H(s))$

$$5sy(s) = 10u(s)$$

$$H(s) = \frac{y(s)}{u(s)} = \frac{10}{5s} = \frac{2}{s} //$$

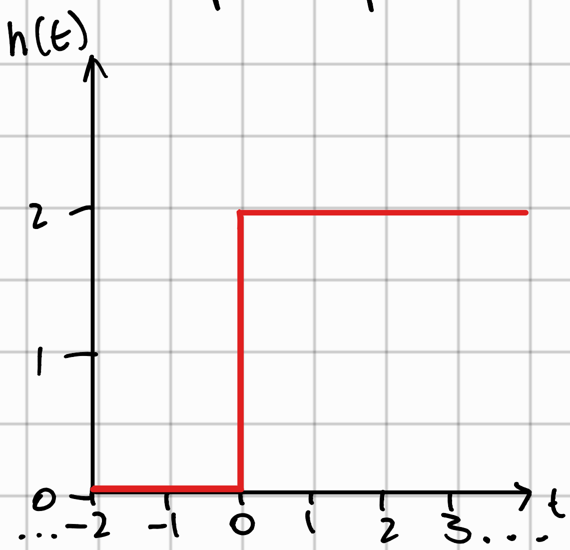
• **Impulse response** $(h(t))$

$\mathcal{L}^{-1}\{H(s)\} \to \mathcal{L}^{-1}\{\frac{2}{s}\}$. from table: $\mathcal{L}^{-1}\{\frac{a}{s}\} = a$

so: $\mathcal{L}^{-1}\{\frac{2}{s}\} = 2$

$h(t) = 2u(t) //$

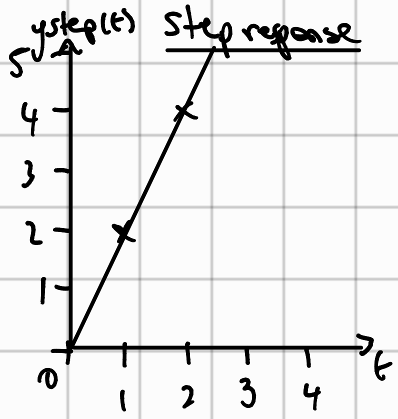
Impulse response



Step response

$\frac{2}{s} \cdot \frac{1}{s} = \frac{2}{s^2}$ $\quad \mathcal{L}^{-1}\{\frac{2}{s^2}\} = 2t\,u(t)$ = step response $y_{step}(t)$

Step response



Frequency transfer function

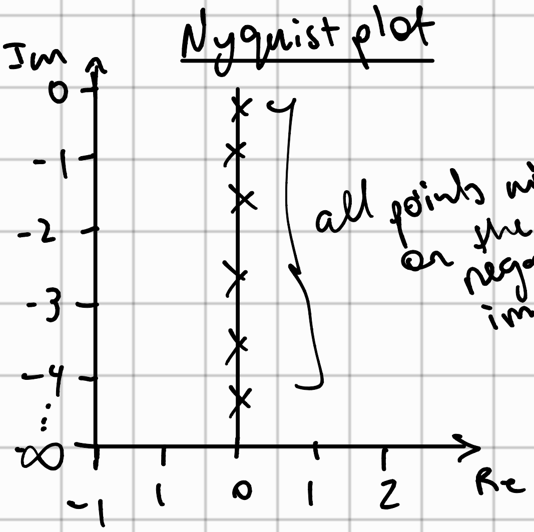
$H(s) = \frac{2}{s}$ $\quad s = j\omega$

$\frac{2}{j\omega} = H(j\omega)$

$\frac{2}{j\omega} \cdot \frac{-j\omega}{-j\omega} = \frac{-j2\omega}{-j^2\omega^2} = \frac{-j2\omega}{\omega^2} = \frac{-j \cdot 2}{\omega} \Rightarrow Re: 0 \quad Im: \frac{-2}{\omega}$

Im < 0

Nyquist plot



all points will lie on the negative imaginary axis.

eg:

	Re	Im
$\omega = 0$	0	0
$\omega = 0.5$	0	-1
$\omega = 1$	0	-2
$\omega = 100$	0	-50
$\omega = 1000$	0	-500
		⋮
		$-\infty$

Amplitude of harmonic output signal.

$\omega = 0.2$

at $\omega = 0.2$, $Re = 0$, $Im = -10$ $\quad$ Amplitude $= \sqrt{(-10)^2} = \sqrt{100} = \underline{10}$

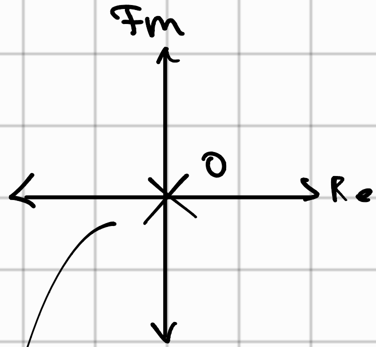
Phase of harmonic output signal

$\arctan\left(\frac{-10}{0}\right) = \arctan(-\infty) = -90^\circ = \text{phase}$

Poles and zeros

$H(s) = \frac{2}{s}$ → numerator $= 2 \neq 0 \Rightarrow$ no finite zeros

→ denominator $= s = 0 \rightarrow$ pole at origin



quasi stable system