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Naming game on small-world networks with geographical effects

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ABSTRACT

The naming game model characterizes the main evolutionary features of languages or more generally of communication systems. Very recently, the combination of complex networks and the naming game has received much attention and the influences of various topological properties on the corresponding dynamical behavior have been widely studied. In this paper, we investigate the naming game on small-world geographical networks. The small-world geographical networks are constructed by randomly adding links to two-dimensional regular lattices, and it is found that the convergence time is a nonmonotonic function of the geographical distance of randomly added shortcuts. This phenomenon indicates that, although a long geographical distance of the added shortcuts favors consensus achievement, too long a geographical distance of the added shortcuts inhibits the convergence process, making it even slower than the moderates.

1. Introduction

The naming game model describes the self-organized emergence of linguistic conventions and the establishment of multi-agent communication systems via pairwise local interactions [1,2]. It is the fundamental model of semiotic dynamics to investigate how linguistic conventions originate, spread, evolve and compete over time in a population [3,4]. Meanwhile, it has been found to have application in the field of artificial intelligence to reproduce the self-organized collective learning processes in populations of artificial agents [5,6]. Recently, models of semiotic dynamics have also been found meaningful for the newly developed web tools, such as the del.icio.us or www.flickr.com, which enables web users to share the classification of information in the web by tags [7,8]. A study of the naming game has both scientific and technological interest [9].

Recently, a minimal version of the naming game was proposed by Baronchelli et al. [10]. This model simplifies the original naming game model but can reproduce the main features of semiotic dynamics and the fundamental experimental phenomena as well. In addition, this minimal model can not only make us gain a deeper understanding of the self-organization process, but also help us extract quantitative scaling properties for systems with a large number of agents [9]. Later on, some modified versions of this model have been studied, which considered some universal features of human sociality or some mechanisms for improving the convergence efficiency, such as finite memory [11], asymmetric negotiation [12], local broadcast [9], reputation [13], connectivity-induced weighted words [14] and “play smart” strategy [15]. Early studies generalized the original minimal naming game to complete graphs [10] and lower-dimensional lattices [16]. A number of empirical studies and statistical analyses have revealed that real complex networks share some common features, the most important of which are called small-world (SW) [17] and scale-free structural properties [18]. Hence, for better reproducing and understanding the evolution of languages or self-organized communication systems,

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several papers have implemented this minimal naming game model or its modified versions on networks with various topological properties and studied their corresponding dynamical behavior [19–23]. Since some real networks are embedded in two- or three- dimensional Euclidean geographical space, their node positions and edge lengths have some important physical meanings, such as power grid networks, traffic lines, the Internet and neuronal networks. From this perspective, pure topological representations of complex networks cannot depict the intra relationships among agents of some real complex systems well. Consequently, some well-known dynamical processes on geographical complex networks have been studied, such as cascading breakdown [24], random walks [25], synchronization [26–28], magnetic models [29], competitive clusters [30], epidemic spreading [31–33], and immunization [34]. Therefore, it is also necessary to study the naming game in geographical complex networks.

In this paper, we investigate the naming game on SW geographical networks. We construct the SW geographical networks by randomly adding links with a fixed geographical distance. Our main result is that the convergence time is a nonmonotonic function of the geographical distance of the added shortcuts, with the fastest convergence in the middle of the range. This nontrivial result indicates that a long geographical distance of the shortcuts contributes significantly to the convergence efficiency of the system. On the other hand, too long a geographical distance of the shortcuts does harm to the consensus achievement. Besides, we study the number of different words, the number of total words and the success rate of negotiations as well for better understanding the evolution of the system.

2. The model

In order to introduce the notion of geographical distance into SW networks, we adopt a modified Newman–Watts (NW) network model [35] described in Ref. [28]. The SW network is generated as follows: At first, a two-dimensional regular lattice with $n \times n$ nodes and periodic boundary conditions is created. Specifically, each node i has a particular pair of integers (x_i, y_i) to represent its coordinates on the lattice. And then, m shortcuts with a fixed geographical distance L are randomly added to this regular lattice. Duplicate connections are avoided. The geographical distance L_{ij} between two nodes i and j is defined as

$$L_{ij} = |x_i - x_j| + |y_i - y_j|. \tag{1}$$

Note that, the periodic boundary conditions are used here and L_{ij} is no larger than n. A more detailed description of the geographical distance can be seen in Ref. [28].

In the minimal version of the naming game, N identical agents are located on a network (herein, we adopt a network generated by the modified NW network model described above, N equals the network size $n \times n$); they observe a single object and try to communicate its name with others. Each agent is assigned an internal inventory or memory to store an unlimited number of different names or opinions. Initially, each agent has an empty memory. The evolutionary rules are as follows:

(i) At each time step, a hearer j is chosen at random and then the hearer randomly chooses one agent i from its neighbors as the speaker, which has been called the reverse naming game. (In the direct strategy, a randomly chosen speaker also randomly selects a hearer among its neighbors. In this strategy, hub nodes have a lower probability of being selected as a speaker than in the reverse strategy [20].)

(ii) If the speaker i's memory inventory is empty, it invents a new name. Otherwise, if i already knows one or more names of the object, it randomly chooses one of them. The invented or selected word is then transmitted to the hearer.

(iii) If the hearer j already has this transmitted name in its inventory, the negotiation is a success, both agents preserve this name and cancel all other terms in their memory. Otherwise, the negotiation fails, and the new name is included in the memory of the hearer without cancelation. By repeating the above process, the system evolves.

3. Simulation results

Relevant quantities in the study of the naming game are the total number of words $N_w(t)$ in the system, which corresponds to the total memory used by the agents, the number of different words $N_d(t)$, the average success rate $S(t)$ of the negotiation and the convergence time t_c. Fig. 1 reports t_c as a function of the geographical distance L of the shortcuts for different numbers of shortcuts m. An interesting result is observed, that is, there exists some values of L, resulting in convergence time staying at a plateau. When L is larger or smaller than these values, t_c will become larger than the value of this plateau. In other words, too short or too long geographical distances of the added shortcuts do not favor a consensus of achievement, only the moderates can lead to fast convergence. (We have checked that when the direct naming game strategy is adopted, a similar nonmonotonic phenomenon is observed. Here in this paper, we only show the numerical results of reverse naming game strategy as a typical example.)

In order to interpret this nonmonotonic phenomenon, we study the average topological distance $\langle l \rangle$ as a function of the geographical distance L of the shortcuts in two-dimensional SW networks for different numbers of shortcuts m. As shown in Fig. 2, the curves of $\langle l \rangle$ against L resemble those in Fig. 1. The convergence time t_c synchronously varies with the average topological distance $\langle l \rangle$ as the geographical distance L increases. This result implies that the average topological distance $\langle l \rangle$ has a strong influence on the convergence time t_c. In the following sections, we will study how the geographical distance L affects the evolutionary progress by the average topological distance $\langle l \rangle$ in detail.

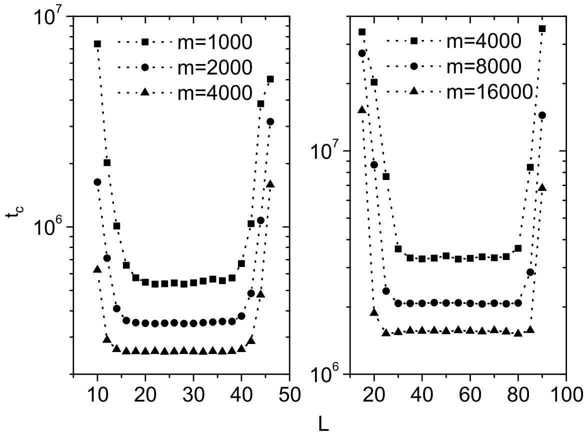

Fig. 1. Convergence time t_c as a function of geographical distance L of the shortcuts. Left panel: for different values of m with network size 50×50. Right panel: for different values of m with network size 100×100. Each data point is obtained by averaging over 1000 runs on each of ten different network realizations.

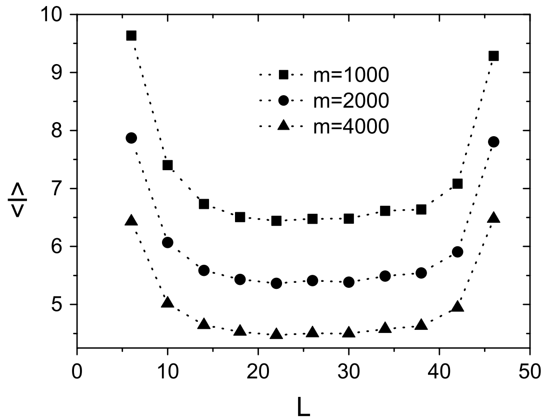

Fig. 2. Average topological distance $\langle l\rangle$ versus the geographical distance L in two dimensional SW networks with periodic boundary conditions for different numbers of shortcuts m. The size of all networks is set to 50×50. $\langle l\rangle$ is defined as the average shortest hop between each pair of nodes [17].

Fig. 3 reports the evolution of the number of different words $N_d(t)$. For different values of L with fixed m, the evolutionary progress shares a very similar behavior at early stage. Initially agents' inventories are empty, so they have to invent some new words and directly transmit the new words to their neighbors. Meanwhile, the transmission of a word from one agent's inventory to another's by many intermediary agents is infrequent, so the number of links plays an important role and the average topological distance of a network has almost no effect in this period. During the latter stage of evolution, the average topological distance $\langle l\rangle$ begins to play an important role for word spreading. A larger $\langle l\rangle$ decreases the velocity of word spreading and consequently favors the establishment of a local consensus, i.e., a larger $\langle l\rangle$ favors word cluster formation. A word cluster is a set of neighboring agents sharing the same word. For $L = 10$ and $L = 44$, because of the competition among the word clusters, it is very hard for one big cluster to invade the others and finally dominate the system with a global consensus. However for the case of $L = 30$, the smaller $\langle l\rangle$ makes the evolutionary behavior maintain a longer spreading progress and finally exhibit a more rapid convergence to the consensus state. Therefore, a larger average topological distance $\langle l\rangle$ does harm to the global convergence. The bottom panel of Fig. 3 reports the results for different numbers of shortcuts m with fixed $L = 30$, and one can find that the convergence time becomes shorter and shorter as m increases. All these results indicate that the mechanisms of convergence towards agreement strongly depend not only on the number of the links, but also on the average topological distance of a network.

The evolution of success rate $S(t)$ is displayed in Fig. 4. For different values of L with fixed m, similar to $N_d(t)$, L does not affect the success rate $S(t)$ remarkably at first. In the middle stage of evolution, the average topological distance $\langle l\rangle$ begins to play an important role in the evolution progress. A larger $\langle l\rangle$ does harm to word spreading and favors word cluster formation. Agents share the same word within each cluster, which enhances the success rate. Therefore, for $L = 30$, the success rate $S(t)$ is lower than those for $L = 10$ and $L = 44$ during this period. At a later stage, for $L = 10$ and $L = 44$, several big word clusters only are left and the inside conversations of big word clusters are of high frequency, so the success rate is very high but hard to achieve 1. For different numbers of shortcuts m with fixed L, after an increment, $S(t)$ reaches a plateau with slowly increasing velocity. All of a sudden, $S(t)$ increases towards 1 over a very short period. The larger the number of

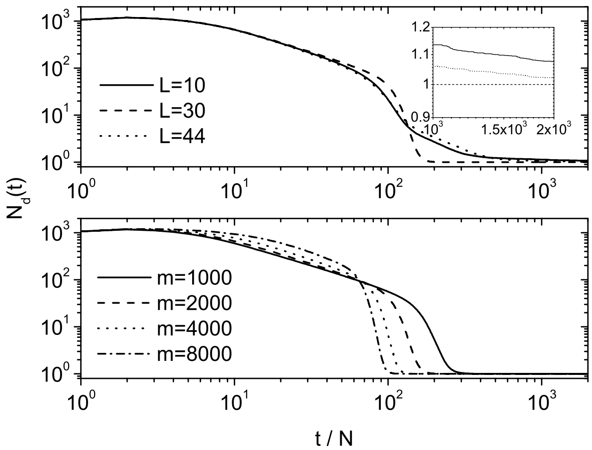

Fig. 3. Evolution of the number of different names $N_d(t)$ versus rescaled time t/N. Upper panel: for different values of L with fixed $m = 2000$. The inset shows the slow convergence for $L = 10$ and $L = 44$. Bottom panel: for different numbers of shortcuts m with fixed $L = 30$. The present results are obtained by averaging over 1000 runs on each of ten different network realizations. The size of all networks is set to 50×50 uniformly.

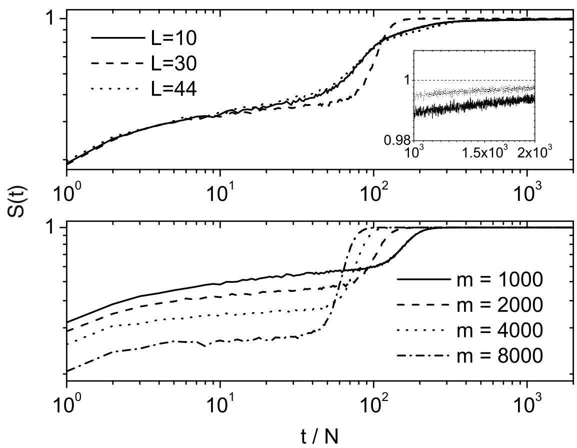

Fig. 4. Time evolution of the success rate $S(t)$. Upper panel: for different values of L with fixed $m = 2000$. The inset shows the slow convergence for $L = 10$ and $L = 44$. Bottom panel: for different numbers of shortcuts m with fixed $L = 30$. The present results are obtained by averaging over 1000 runs on each of ten different network realizations. The size of all networks is set to 50×50 uniformly.

shortcuts m, the lower the values of the plateau, and the earlier it is, the more sudden is the increase. The present results are consistent with those of Refs. [11,20].

Next, we turn to the evolutionary behavior of the total number of names $N_w(t)$ in the system. As shown in the upper panel of Fig. 5, the evolutionary behavior of $N_w(t)$ is almost independent of the geographical distance L of the shortcut at first. A lower success rate is helpful for word spreading, while a higher success rate leads to faster word deletion in the memories of both speakers and hearers. For $L = 10$ and $L = 44$, the total numbers of names $N_w(t)$ in the system are smaller than those of the case of $L = 30$ in the middle stage of evolution, but larger than those at a later stage. These results are in accordance with the above results and support the above analysis. In addition, one can find that the geographical distance L of the shortcuts has no remarkable influence on the maximum total memory of agents $N_w^{\max}$. That is because, the peak of $N_w(t)$ occurs before the time when the average topological distance $\langle l \rangle$ begins to influence the evolutionary progress remarkably. From the bottom panel of Fig. 5, one can find that the maximum total memory of agents $N_w^{\max}$ becomes larger and larger as m increases, which is also in accordance with the analysis that $N_w(t)$ is determined by the average degree of a network in Ref. [11].

4. Conclusion

In conclusion, we have studied the minimal version of the language game in two-dimensional geographical SW networks. We construct the SW geographical networks by randomly adding links with a fixed geographical distance L to two-dimensional regular lattices. Interestingly, by tuning the parameter L, we found the convergence time is a nonmonotonic function of the geographical distance L of the added shortcuts. The result is explained by the combination of two items.

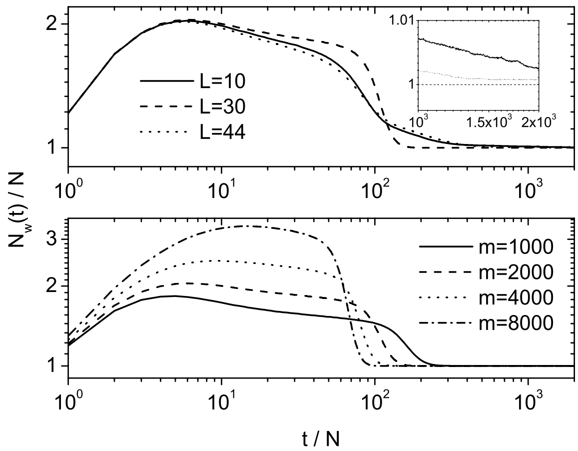

Fig. 5. Time evolution of the average memory per agent $N_w(t)/N$. Upper panel: for different values of L with fixed $m = 2000$. The inset shows the slow convergence for $L = 10$ and $L = 44$. Bottom panel: for different numbers of shortcuts m with fixed $L = 30$. The present results are obtained by averaging over 1000 runs on each of ten different network realizations. The size of all networks is set to 50×50 uniformly.

One is that a moderate geographical length of shortcuts leads to the smallest average topological distance, which has been given by Kleinberg in 2000 [36], and the other is that a smaller average distance will lead to a faster convergence. The latter one is very similar to the synchronization [28,37–40] and consensus problems [41,42]. A smaller average distance leads to better synchronizability, and the consensus process is favored by a shorter average distance. In addition, we have reported the dynamical behavior of statistical quantities, including the total memory (inventory) of agents, success rate of negotiations, and the number of total different invented words. All these results demonstrate that the geographical effect plays an important role in the evolutionary progress of the language game in SW networks. These results also indicate that the mechanisms of convergence towards agreement strongly depend not only on the number of added links, but also on the geographical distance of the added links of an SW network. Since the geographical distance is an important parameter of many real networks, our work may be meaningful in a study of the language game.

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