Homework 7 – Intro to Probability and Statistics

Your name here

Instructions:

Due: 05/28 at 11:59PM.

What am I expecting? An R Markdown with the answers.

Have fun!

Question 1

An urn has two white balls (W) and three red balls (R). You draw a ball from the urn. If it is white, you flip a coin. If it is red, you throw it back in the urn and draw another ball. What is the sample space of this experiment?

Question 2

Suppose you toss two dices. Consider two events:

- A: the sum of the numbers in both dices is equal to 9
- B: the number in the first die is greater than or equal 4.

Define:

1. The elements of A
2. The elements of B
3. The elements of $A \cap B$
4. The elements of $A \cup B$
5. The elements of A^C

Question 3

The probability that student A solves a given problem is $\frac{2}{3}$. The probability that student B solves the same problem is $\frac{3}{4}$. If both try to solve the problem independently, what is the chance that the problem will be solved?

Question 4

Consider the following probability table:

	B	B^C	Total
A	0.04	0.06	0.10
A^C	0.08	0.82	0.90
Total	0.12	0.88	1.00

Note that $P(A) = 0.10$, $P(A \cap B) = 0.04$, and so on. Are the events A and B independent?

Question 5

A company produces phones in three factories. In factory I, the company produces 40% of the phones, while in factories II and III produce 30% of the phones in each. The chance of a phone is assembled broken is 0.01 (factory I), 0.04 (factory II), and 0.03 (factory III). The phones are then taken to a warehouse.

1. If you select a phone randomly in the warehouse, what is the chance that it is broken?
2. Suppose the phone you draw is broken. What is the probability that it was manufactured by factory I?

Question 6

Prove that if A and B are independent, then:

1. A^C and B^C are independent.
2. A and B^C are independent.
3. A^C and B are independent.

Question 7

Let $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$. Prove that:

1. $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$
2. $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

Question 8

Let $P(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$ the probability of k successes in a binomial distribution with n trials p probability of success. Prove that:

$$P(k+1; n, p) = \frac{(n-k)p}{(k+1)(1-p)} P(k; n, p)$$