

Exercise 1 (Kalma Filter)

1.

$$f(X_t \mid X_{t-1}) \sim N(\phi X_{t-1}, \sigma_v^2)$$

$$g(Y_t \mid X_t) \sim N(\phi X_t, \sigma_w^2)$$

2. Note that $X_{t+1} = \phi X_t + V_t$. Then:

$$X_{t+1} \mid Y_{1:t} = \phi X_t + V_t \mid Y_{1:t}$$

Therefore, since $X_t \mid Y_{1:t} \sim N(m_{t|t}, \sigma_{t|t}^2)$ and $V_t \sim N(0, \sigma_v^2)$, we have the sum of normals, hence:

$$X_{t+1} \mid Y_{1:t} \sim N(m_{t|t}, \sigma_v^2 + \phi^2 \sigma_{t|t}^2)$$