Exercise 1 (Monte Carlo for Gaussians)

1. Let's prove that $E[\phi(X)] = E[\phi(X+\theta)exp(\frac{-1}{2}\theta^T\theta - \theta^T X)]$.

$$E[\phi(X+\theta)exp(\frac{-1}{2}\theta^T\theta - \theta^T X)] = \int_{\mathbb{R}^d} \phi(x+\theta)exp(\frac{-1}{2}\theta^T\theta - \theta^T X)\pi(x)dx_1...dx_d =$$

$$\propto \int_{\mathbb{R}^d} \phi(x+\theta)exp\left(\frac{-1}{2}\theta^T\theta - \theta^T X\right)exp(-x^T x/2)dx_1...dx_d =$$

$$\int_{\mathbb{R}^d} \phi(x+\theta)exp\left(\frac{-1}{2}(x-\theta)^T(x-\theta)\right)dx_1...dx_d$$

Finally, making $x - \theta = y$,

$$\int_{\mathbb{R}^d} \phi(y)exp\left(\frac{-1}{2}(y)^T(y)\right)dx_1...dx_d = E[\phi(Y)]$$