Notation: here is a brief summary of the notation used in this worksheet.

- $p(X = x)$ is equal to the probability density function;
- Capital letters such as X stand for the random variable.

Exercise 1 (Inversion and Rejection)

1. Let $F_X(x) = \mathbb{P}(X \leq x)$ and $U \sim Unif[0, 1]$:

$$F_X(x) = 1 - e^{-\lambda(X-a)}\mathbb{I}_{\{X \geq a\}} = U$$

$$-\ln(1 - U) = \lambda(x - a)$$

$$F_X^{-1}(U) = a - \frac{-\ln(1 - U)}{\lambda}$$

To simulate X from U, just simulate value from U and substitute in the formula above.

2.

$$p(X = x \mid a \leq X \leq b) = p(X = x, a \leq X \leq b)$$