Mathematical Short Notes

Davi S. Barreira

Mathematical Short Notes

by

Davi Sales Barreira

TBD

First edition, 2022

ISBN XYZ

Published by TBD

Preface

This "book" is a collection of notes taken during my Masters and PhD on Mathematical Modelling at FGV/EMAp. The main purpose is to better organize the several subjects I've been studying, and avoid forgetting all together, **specially** interesting proofs and results that I've found along the way. I'm sharing this so my classmates can read the lecture notes, and also so it might be useful to some random math student.

I started writing these notes as separate documents, but, Mathematics is very intertwined, and it quickly dawned on me that I was having to regularly repeat definitions, theorems, proofs... Not to mention the issues related to trying to keep a consistent notation across different documents.

Thus, I decided to put everything together in a book format, while keeping the subjects somewhat compartmentalized.

The notes supposed to be an introduction to the many fields of Mathematics, thus, there are almost no motivation nor many examples. The goal is instead to present definitions and results of different areas in an organized manner. Yet, this style is not really consistent across the book. Some chapters just state results without proofs, while others not only have proofs, but very "wordy" proofs.

Finally, the topics covered are really varied, and are based on the courses I've taken. In some notes, there are also some code written in Julia.

Contents

List of Definitions

List of Theorems

Notation

The symbol "⊛" means that such definition or theorem was created by the author, so it should be taken with care.

Unless state otherwise, we are always dealing with the probability space $(\Omega, \mathcal{F}, P)$, where $\mathcal{F}$ is the Borel σ-algebra.

- $\mathbb{N}$ represent the natural numbers (the zero not included of course);
- $\mathbb{Z}$ are the integers, $\mathbb{Z}_+$ the positive integers which include 0, and $\mathbb{Z}_-$ are the negative integers that inlcude 0. $\mathbb{Z}_+^*$ and $\mathbb{Z}_-^*$ exclude the 0;
- For a function $f : \mathbb{R} \to \mathbb{R}$, $f_+(x) = \max 0, f(x)$, which explains the choice of notation above. Note that $f_-(x) = \min 0, f(x)$;
- $\mathbb{Q}, \mathbb{I}, \mathbb{R}, \mathbb{C}$ are, respectively, the rationals, irrationals, reals and complex numbers;
- (x_n) is a sequence of numbers indexed by $n \in \mathbb{N}$;
- $\mathbb{R}^{\mathbb{N}} = \mathbb{R} \times \mathbb{R} \times ....$

- A probability measure P_θ implies that it comes from a parametric family $\mathcal{P} = \{P_\theta : \theta \in \Theta\}$;
- Whenever a probability measure P_θ has a density with respect to another measure that dominates it (e.g. Lebesgue), the density function is usually represented by $p_\theta(x)$;
- $E_\theta[X] = \int_\Omega X(\omega) dP_\theta$;
- C is the space of continuous functions;
- C^1 is the space of continuous functions with at least a continuous first derivative;
- C^∞ is the space of continuous functions with infinite continuous derivates;
- C_b is the space of continuous and bounded functions,
- C_0 is the space of continuous functions that goes to 0 as $|x| \to +\infty$;

- We use $|\cdot|$ to mean the Euclidean norm, and $||\cdot||$ to mean a more general norm, e.g. $||\cdot||_{L^p}$ is the L^p norm.

- If two sets A and B are disjoint, then $A \cup B$ is the same as $A + B$;
- If sets are disjoint, then $\cup A_n$ is equivalent to $\sum A_n$.
- $\cup_n$ might be used instead of $\cup_{n\in\mathbb{N}}$, when it's clear by the context;
- $A_n \uparrow A$ means $A_1 \subset A_2 \subset ... \subset A_n \subset ...$ and $\cup_n A_n = A$;
- $A_n \downarrow A$ means $A_1 \; A_2 \supset ... \supset A_n \supset ...$ and $\cap_n A_n = A$;
- (X, d) usually stands for a metric space of metric, unless stated otherwise.