Partial Differential Equations

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Notes based on Craig [1] and on the Partial Differential Equations course ministered by Emanuel Carneiro on 2011 at IMPA.

1 The Wave Equation

1.1 d'Alembert formula

Consider the following problem:

$$\begin{aligned}
&\partial_t^2 u - \partial_x^2 u = 0,\\
&u(0,x) = f(x), \quad \partial_t u(0,x) = g(x).
\end{aligned}$$

To solve this, we'll use a change of coordinates

$$r := x + t, \quad s := x - t.$$

Now, let's apply the Chain rule for $g(r,s) = (\frac{r-s}{2}, \frac{r+s}{2})$,

$$\begin{aligned}
(\partial_r v, \partial_s v) = D(u \circ g)(r,s) = Du(t,x)Dg(r,s) &= (\partial_t u, \partial_x u)\begin{pmatrix} \partial_r g_1 & \partial_s g_1 \\ \partial_r g_2 & \partial_s g_2 \end{pmatrix}\\
&= (\partial_t u, \partial_x u)\begin{pmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix}\\
&= \left(\frac{\partial_t u + \partial_x y}{2}, \frac{\partial_t u - \partial_x y}{2}\right).
\end{aligned}$$

References

[1] Walter Craig. *A Course on Partial Differential Equations*, volume 197. American Mathematical Soc., 2018.