2.8 Solución

i) $\mathcal{L}\{x(t-t_0)\} = e^{-st_0} X(s)$

$$\mathcal{L}\{x(t-t_0)\} = \int_0^\infty x(t-t_0) e^{-st} dt$$

$u = t - t_0 \quad t \to \infty = u \to \infty$

$t = u + t_0 \quad t \to \infty = u \to -\infty$

$$\mathcal{L}\{x(t-t_0)\} = e^{-st_0} \int_0^\infty x(u) e^{-su} du = e^{-st_0} X(s)$$

ii) $\mathcal{L}\{x(at)\} = \frac{1}{|a|} X(s/a)$

si $a > 0$

$t \to \infty \quad u \to +\infty$

$t \to -\infty \quad u \to -\infty$

$$\mathcal{L}\{x(at)\} = \int_{-\infty}^\infty x(at) e^{-st} dt$$

$$\frac{1}{a} \int_{-\infty}^\infty x(u) e^{-s(u/a)} = \frac{1}{a} X(s/a)$$

si $a < 0$

$$(-1) \frac{1}{a} \int_\infty^{-\infty} x(u) e^{-s(u/a)} = -\frac{1}{a} X(s/a)$$

$$\mathcal{L}\{x(at)\} = \frac{1}{|a|} X(s/a)$$

iii) $\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = s X(s)$

$$\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = \int_{-\infty}^\infty x'(t) e^{-st} dt$$

$u = e^{-st}$

$du = -se^{-st}$

$dv = x'(t)$

$v = x(t)$

$$= e^{-st} x(t) \Big|_0^\infty \int_{-\infty}^\infty s e^{-st} x(t) dt$$

$$= 0 - x(0) + s \mathcal{L}\{x(t)\}$$

$$\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = s\,X(s) - x(0)$$

$$\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = s\,X(s)$$

iv) $\mathcal{L}\{x(t) * y(t)\} = X(s)\,Y(s)$

$$\mathcal{L}\{x(t) * y(t)\} = \int_{-\infty}^{\infty} (x(t) * y(t))\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} x\,T\,y(T-t)\,dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(T)\,y(T-t)\,dT\,e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty} x(T) \int_{-\infty}^{\infty} y(T-t)\,e^{-st}\,dt\,dT$$

$u = T - t$

$du = -dt$

$t = T + u$

$$= \int_{-\infty}^{\infty} x(T)\,(1) \int_{-\infty}^{\infty} y(u)\,e^{-s(T+u)}\,dt\,dT$$

$$= \int_{-\infty}^{\infty} x(T)\,e^{-sT}\,dT \int_{-\infty}^{\infty} y(u)\,e^{-su}\,du$$

$$= X(s) \cdot Y(s)$$