2.9 Solución

i) $e^{-2t} u(t) + e^{-3t} u(t)$

$$= \int_0^\infty e^{-2t} e^{-st} dt + \int_0^\infty e^{-3t} e^{-st} dt$$

$$= \int_0^\infty e^{-(s+2)t} dt + \int_0^\infty e^{-(s+3)t} dt$$

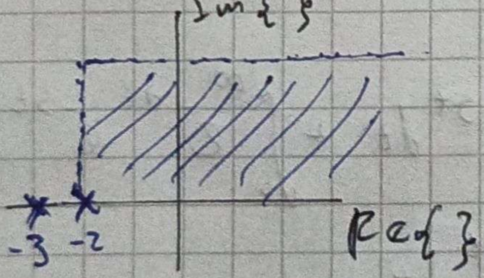
$$= -\frac{e^{-(s+2)t}}{s+2}\Big|_0^\infty - \frac{e^{-(s+3)t}}{s+3}\Big|_0^\infty$$

ROC
$s > -2$
$s > -3$

$$= \frac{1}{s+2} + \frac{1}{s+3} = \frac{s+3+s+2}{(s+2)(s+3)} = \frac{2s+5}{(s+2)(s+3)}$$

Donde los polos son $P_1 = -2$ y $P_2 = -3$

No hay ceros en esta función porque no hay un factor al numerador que se anule de forma natural. por ejm: $(s + 5/2)$



Para señales causales (multiplicadas) por $u(t)$ es

Re(s) > polo más a la derecha

$\mathrm{Re}(s) > -2$

ii) $e^{2t} u(t) + e^{-3t} u(-t)$

$$\mathcal{L}\{e^{2t} u(t)\} + \mathcal{L}\{e^{-3t} u(-t)\}$$

$$= \int_0^\infty e^{2t} e^{-st} + \int_\infty^0 e^{-3t} e^{-st} dt$$

$$= \int_0^\infty e^{-(s-2)t} dt + \int_{-\infty}^0 e^{-(3+s)t} dt$$

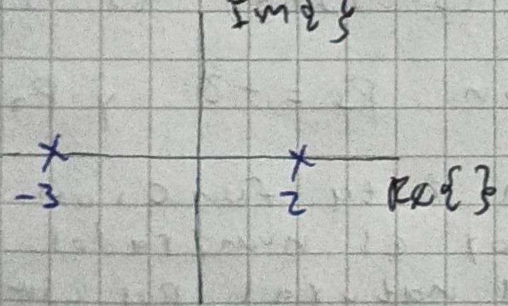
$$= -\frac{e^{-(s-2)t}}{(s-2)}\Bigg|_0^{\infty} - \frac{e^{(-3-s)}}{(3+s)}\Bigg|_{-\infty}^{0}$$

$$= \frac{1}{s-2} - \frac{1}{s+3} = \frac{s+3-s+2}{(s-2)(s+3)}$$

$$\mathcal{L}\{e^{2t}u(t)\} + \mathcal{L}\{e^{-3t}u(-t)\} = \frac{5}{(s-2)(s+3)}$$

$P_1 = 2$ $\quad$ $P_2 = -3$

ROC: $Re(s) > 2$ $\quad$ ROC: $Re(s) < -3$



No hay ROC (vacía)

No hay polos

iii) $e^{-a|t|}$

$$\mathcal{L}\{e^{-a|t|}\} = \int_{-\infty}^{0} e^{-a(-t)} e^{-st}\,dt + \int_0^{\infty} e^{-at} e^{-st}\,dt$$

$$= \int_{-\infty}^{0} e^{(a-s)t}\,dt + \int_0^{\infty} e^{-(a+s)t}\,dt$$

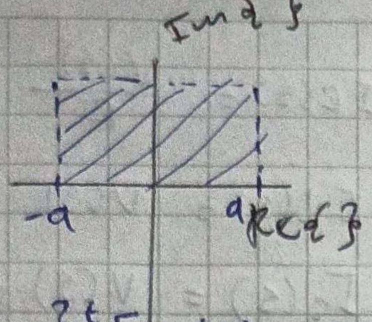
$$= \frac{1}{a-s} + \frac{1}{a+s} = \frac{a+s+a-s}{a^2-s^2} = \frac{2a}{a^2-s^2}$$

ROC: $s < a$ y $s > -a$

La región de convergencia es $(-a, a)$

$P_1 = s = a$ $\quad$ $P_2 = s = -a$

No hay ceros



Roc: $(-a, a)$

iv) $e^{-2t}[u(t)-u(t-5)]$

$$\mathcal{L}\{e^{-2t}[u(t)-u(t-5)]\} = \int_0^5 e^{-2\tau} e^{-st}\, dt$$

$$= \int_0^5 e^{-(2+s)t}\, dt = \left.\frac{e^{-(2+s)}}{2+s}\right|_0^5 = \frac{1-e^{-(5s+10)}}{s+2}$$

Roc $s > -2$

$$1 - e^{-(5s+10)} = 0$$

$$\ln(1) = -5s+10$$

$$0 = -5s+10$$

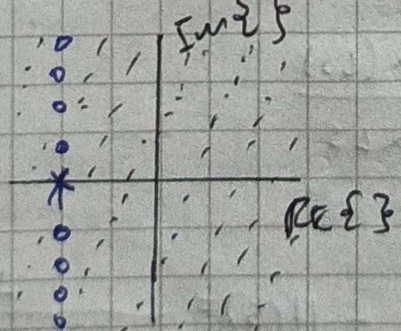
$$s = 2$$

Un polo en $s=-2$

Caso en valores que hacen $e^{-(5s+10)} = 1$

$$s+2 = \frac{2\pi jn}{5}$$

$$s = -2 + \frac{2\pi jn}{5} \quad n \in \mathbb{Z}$$



Roc: Todo el plano complejo $\mathbb{C}$