The Ornstein-Uhlenbeck Process and Its Applications to Pairs Trading

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Objective

Mean reversion processes are stationary stochastic processes, i.e. they tend to move to its average value over time. Its properties are well understood by the studies of the Orenstein-Uhlenbeck (OU) process. The ratio between two suitable stocks is mean reversive. It can hence be described as an OU process and its guiding parameters can be inferred. The properties of such a stock pair can be exploited by Pairs trading. This is a widely applied trading strategy among hedge funds. An exemplary simulation provides insights how the mathematical model of an OU process is linked to the stock market. As a proof of concept, we demonstrate how one could profit with Pairs trading, using historical data from two S&P500 traded stocks.

The OU Process

The OU process is governed by the stochastic differential equation:

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t \tag{1}$$

The formal solution reveals the significance of the parameters:

- μ: The value to which the expectation value of the OU process converges.
- θ: Determines the time scale at which the expectation value of the process converges.
- σ: Defines the amplitude of the stochastic variation.

For the ratio between two stocks, which show the key property of mean reversion and hence behaves like an OU process, these parameters can be inferred with maximum likelihood (ML) estimation:

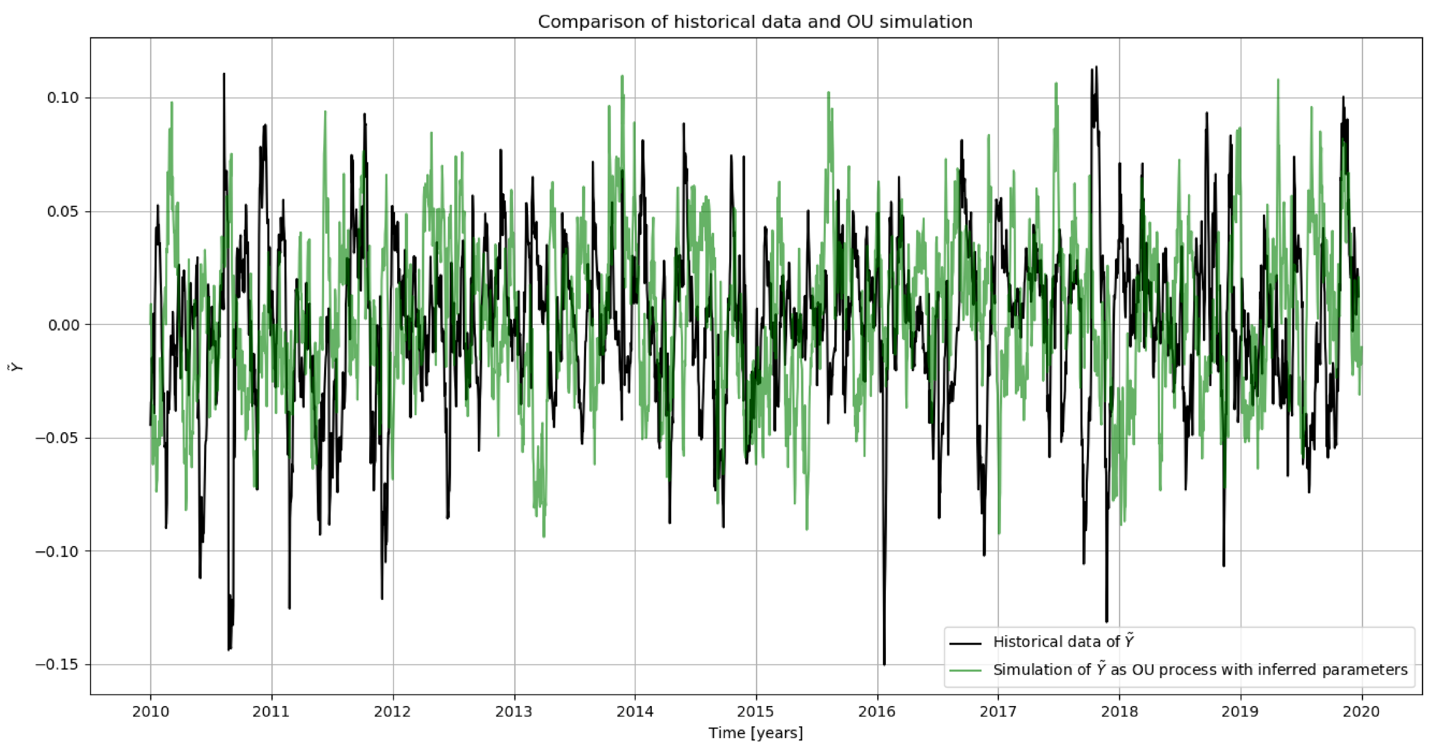

Figure 1:The historical ratio of stocks and OU process with ML inferred parameters agree.

One process model

In order to model two stocks we consider the following model:

$$X_t = x \exp\{F_t\} \tag{2}$$

$$Y_t = k_o X_t \exp\{\tilde{Y}_t\} \tag{3}$$

This model assumes a fairly stable stock X_t around x, with predictable behaviour with respect to external factors captured in F_t. Taking $y = k_o x$ to be representative of the historical price ratio, we obtain:

$$\frac{Y_t}{k_o X_t} = \exp\{\tilde{Y}_t\} \tag{4}$$

where $\tilde{Y}$ is a OU-process with zero drift. We notice, that the ratio of two stock prices seems to imply some degree of mean reversion. It is clear that the hypothetical ratio k_o we expect to revert to is not completely constant in time. Due to changes in fundamentals of the companies considered, the fair market ratio may change over time. We hypothesize a 252-day trailing average as a good initial assumption of fair ratio k_o.

Trading strategy

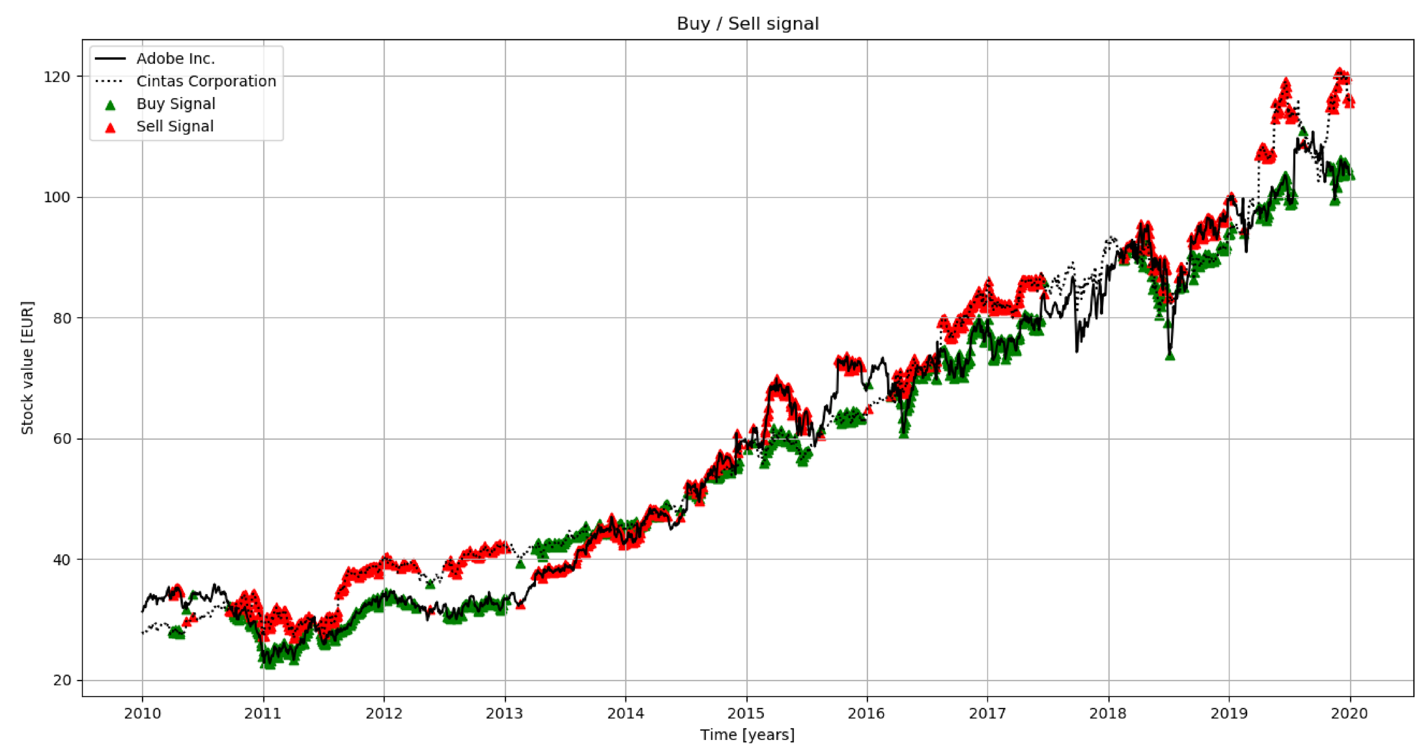

Figure 2:The strategy started with 0 EUR, ended with 3000 EUR.

From the deviation of the market ratio k_t to the historical average k_o, we can calculated the implied underlying OU-process, $\tilde{Y}$. As $\tilde{Y}$ increases, the probability distribution of $\Delta\tilde{Y}$ is shifted down due to the negative correlation of $\Delta\tilde{Y}$ with $\tilde{Y}$. This can be seen from the definition of the OU-process. Based on this knowledge a trading strategy can be devised: When $\tilde{Y}_t$ is positive, Y_t is overpriced with respect to X_t. Thus, at a certain threshold we short Y_t and long X_t. When the ratio shortens we close our position. A buy and sell signal for such a strategy is seen in the figure above. Back-testing the strategy on historical data shows promising results, which we take to be a proof of concept.

Discussion

We conclude that the ratio of two well chosen stock can be described as a mean reverting OU process. This can be exploited by the hedging strategy Pairs trading. Applying this strategy, we could define buying and selling signals, depending on the ratio between the two S&P500 traded stocks in question. An exemplary simulation shows that the strategy is indeed profitable. Further investigations could try to answer the questions:

- How would tuning the free simulation parameters (buying/selling threshold; time window to revert to the mean k) change our profits?
- Would a two process model be a more accurate description for the behavior of two mean reverting stocks? This is: Two stocks share a common dependency on external factors F_t and are further guided by a two dimensional OU Process $\tilde{X}_t$, $\tilde{Y}_t$:

$$X_t = x \exp\{F_t + \tilde{X}_t\} \tag{5}$$

$$Y_t = y \exp\{F_t + \tilde{Y}_t\} \tag{6}$$

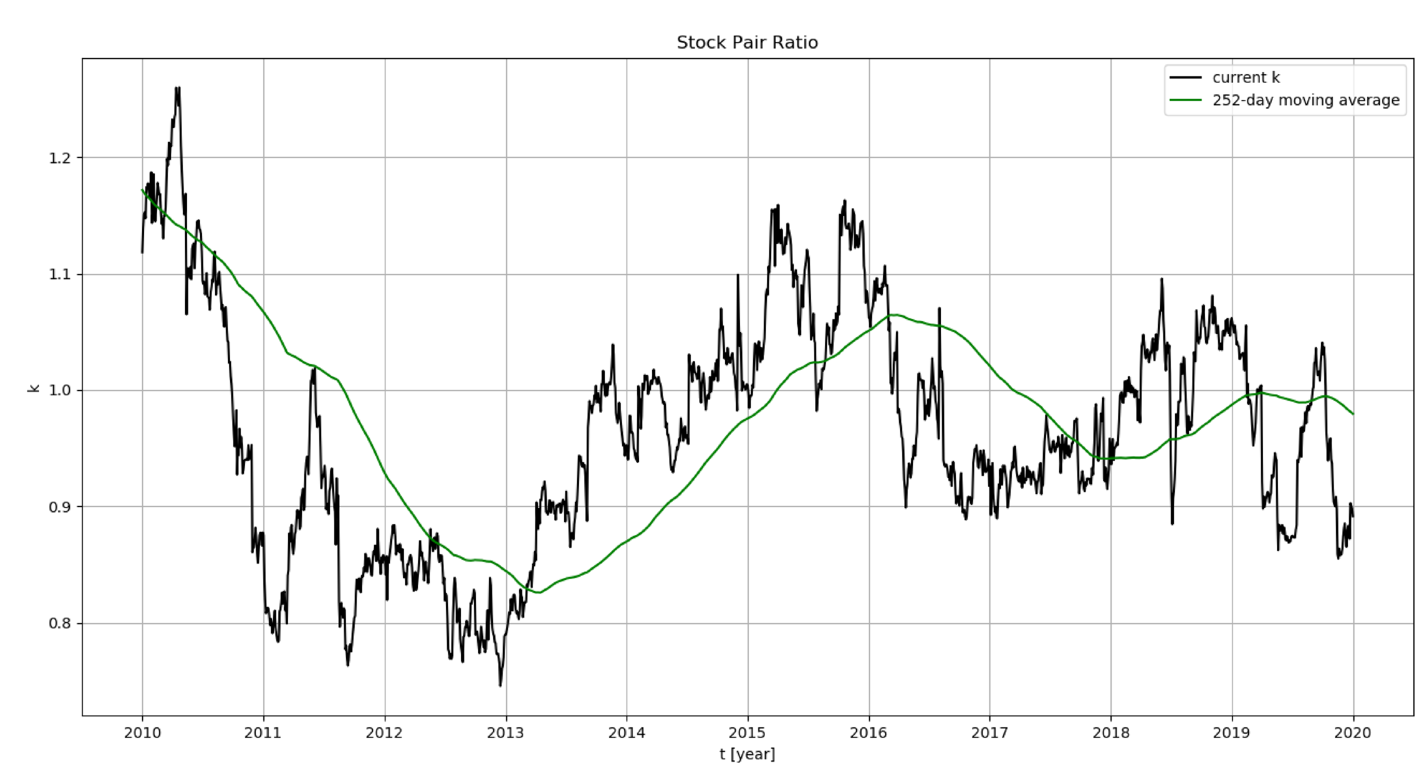

Figure 3:Current and 252-day trailing average.

References

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