Lab 3 Assignment Supplementary Material

In[41]:=
```
<< Screws.m;
<< RobotLinks.m;
```

For Lab 3, I used the configuration of the ur4 presented in Assignment 4 as the zero position. Hence the following formulas will be essentially identical to those of the assignment.

For implementations in MATLAB, see the files "…/lab3/helpers/ur5Parameters.m" "…/lab3/ur5FwdKin.m" and "…/lab3/ur5BodyJacobian.m"

In[37]:=
```
e1 = {1, 0, 0};
e2 = {0, 1, 0};
e3 = {0, 0, 1};
I3 = IdentityMatrix[3];
```

Forward Kinematics Calculations

This code is implemented/used to control the ur5 in matlab. ur5Parameters.m sets up the joint lengths and twist axes. ur5FwdKin.m performs the actual forward kinematics.

In[43]:=
```
ω1 = e3; q1 = {0, 0, 0};
ω2 = e1; q2 = {0, 0, 0};
ω3 = e1; q3 = {0, 0, L1};
ω4 = e1; q4 = {0, 0, L1 + L2};
ω5 = e3; q5 = {L3, 0, 0};
ω6 = e1; q6 = {0, 0, L1 + L2 + L4};

ξ1 = RevoluteTwist[q1, ω1]
ξ2 = RevoluteTwist[q2, ω2]
ξ3 = RevoluteTwist[q3, ω3]
ξ4 = RevoluteTwist[q4, ω4]
ξ5 = RevoluteTwist[q5, ω5]
ξ6 = RevoluteTwist[q6, ω6]
```

Out[49]= {0, 0, 0, 0, 0, 1}

Out[50]= {0, 0, 0, 1, 0, 0}

Out[51]= {0, L1, 0, 1, 0, 0}

Out[52]= {0, L1 + L2, 0, 1, 0, 0}

Out[53]= {0, − L3, 0, 0, 0, 1}

Out[54]= {0, L1 + L2 + L4, 0, 1, 0, 0}

In[55]:=
```
gst0 = RPToHomogeneous[I3, {L3 + L5, 0, L1 + L2 + L4}];
gst0 // MatrixForm
gst[θ1_, θ2_, θ3_, θ4_, θ5_, θ6_] := Simplify[
    ForwardKinematics[
     {ξ1, θ1}, {ξ2, θ2}, {ξ3, θ3}, {ξ4, θ4}, {ξ5, θ5}, {ξ6, θ6}, gst0
    ]
  ];
gst[θ1, θ2, θ3, θ4, θ5, θ6]
```

Out[56]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & L3 + L5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L1 + L2 + L4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[58]=

$$\Big\{\Big\{\text{Cos}[\theta1]\,\text{Cos}[\theta5] - \text{Cos}[\theta2+\theta3+\theta4]\,\text{Sin}[\theta1]\,\text{Sin}[\theta5],\\ -\text{Cos}[\theta6]\,\big(\text{Cos}[\theta2+\theta3+\theta4]\,\text{Cos}[\theta5]\,\text{Sin}[\theta1] + \text{Cos}[\theta1]\,\text{Sin}[\theta5]\big) +\\ \text{Sin}[\theta1]\,\text{Sin}[\theta2+\theta3+\theta4]\,\text{Sin}[\theta6],\\ \text{Cos}[\theta4]\,\text{Cos}[\theta6]\,\text{Sin}[\theta1]\,\text{Sin}[\theta2+\theta3] + \text{Cos}[\theta2+\theta3]\,\text{Cos}[\theta6]\,\text{Sin}[\theta1]\,\text{Sin}[\theta4] +\\ \big(\text{Cos}[\theta2+\theta3+\theta4]\,\text{Cos}[\theta5]\,\text{Sin}[\theta1] + \text{Cos}[\theta1]\,\text{Sin}[\theta5]\big)\,\text{Sin}[\theta6],\\ \text{Cos}[\theta1]\,\big(L3 + L5\,\text{Cos}[\theta5]\big) + \frac{1}{2}\,\text{Sin}[\theta1]\,\big(2\,L1\,\text{Sin}[\theta2] + 2\,L2\,\text{Sin}[\theta2+\theta3] +\\ 2\,L4\,\text{Sin}[\theta2+\theta3+\theta4] + L5\,\text{Sin}[\theta2+\theta3+\theta4-\theta5] - L5\,\text{Sin}[\theta2+\theta3+\theta4+\theta5]\big)\Big\},\\ \Big\{\text{Cos}[\theta5]\,\text{Sin}[\theta1] + \text{Cos}[\theta1]\,\text{Cos}[\theta2+\theta3+\theta4]\,\text{Sin}[\theta5],\ -\text{Cos}[\theta6]\,\text{Sin}[\theta1]\,\text{Sin}[\theta5] +\\ \text{Cos}[\theta1]\,\big(\text{Cos}[\theta2+\theta3+\theta4]\,\text{Cos}[\theta5]\,\text{Cos}[\theta6] - \text{Sin}[\theta2+\theta3+\theta4]\,\text{Sin}[\theta6]\big),\\ \text{Sin}[\theta1]\,\text{Sin}[\theta5]\,\text{Sin}[\theta6] - \text{Cos}[\theta1]\,\big(\text{Cos}[\theta4]\,\text{Cos}[\theta6]\,\text{Sin}[\theta2+\theta3] +\\ \text{Cos}[\theta2+\theta3]\,\text{Cos}[\theta6]\,\text{Sin}[\theta4] + \text{Cos}[\theta2+\theta3+\theta4]\,\text{Cos}[\theta5]\,\text{Sin}[\theta6]\big),\\ \big(L3 + L5\,\text{Cos}[\theta5]\big)\,\text{Sin}[\theta1] - \frac{1}{2}\,\text{Cos}[\theta1]\,\big(2\,L1\,\text{Sin}[\theta2] + 2\,L2\,\text{Sin}[\theta2+\theta3] +\\ 2\,L4\,\text{Sin}[\theta2+\theta3+\theta4] + L5\,\text{Sin}[\theta2+\theta3+\theta4-\theta5] - L5\,\text{Sin}[\theta2+\theta3+\theta4+\theta5]\big)\Big\},\\ \Big\{\text{Sin}[\theta2+\theta3+\theta4]\,\text{Sin}[\theta5],\ \frac{1}{4}\,\big(-2\,\text{Sin}[\theta2+\theta3+\theta4-\theta6] + \text{Sin}[\theta2+\theta3+\theta4-\theta5-\theta6] +\\ \text{Sin}[\theta2+\theta3+\theta4+\theta5-\theta6] + 2\,\text{Sin}[\theta2+\theta3+\theta4+\theta6] +\\ \text{Sin}[\theta2+\theta3+\theta4-\theta5+\theta6] + \text{Sin}[\theta2+\theta3+\theta4+\theta5+\theta6]\big),\\ \frac{1}{4}\,\big(2\,\text{Cos}[\theta2+\theta3+\theta4-\theta6] - \text{Cos}[\theta2+\theta3+\theta4-\theta5-\theta6] - \text{Cos}[\theta2+\theta3+\theta4+\theta5-\theta6] +\\ 2\,\text{Cos}[\theta2+\theta3+\theta4+\theta6] + \text{Cos}[\theta2+\theta3+\theta4-\theta5+\theta6] + \text{Cos}[\theta2+\theta3+\theta4+\theta5+\theta6]\big),\\ L1\,\text{Cos}[\theta2] + L2\,\text{Cos}[\theta2+\theta3] + L4\,\text{Cos}[\theta2+\theta3+\theta4] + \frac{1}{2}\,L5\,\text{Cos}[\theta2+\theta3+\theta4-\theta5] -\\ \frac{1}{2}\,L5\,\text{Cos}[\theta2+\theta3+\theta4+\theta5]\Big\},\ \{0,\,0,\,0,\,1\}\Big\}$$

Body Jacobian Calculations

This code is also implemented/used in to control the ur5 in matlab. ur5Parameters.m is run inside the ur5BodyJacobian.m, for the same reasons as above. The body Jacobian is then computed by the same algorithm used in the Mathematica implementation RobotLinks.m

In[59]:=
```
Jbst = BodyJacobian[{ξ1, θ1}, {ξ2, θ2},
   {ξ3, θ3}, {ξ4, θ4}, {ξ5, θ5}, {ξ6, θ6}, gst0] // Simplify
```

Out[59]=
```
{{1/2 (L1 Sin[θ2 - θ5] + L2 Sin[θ2 + θ3 - θ5] -
      L3 Sin[θ2 + θ3 + θ4 - θ5] + L4 Sin[θ2 + θ3 + θ4 - θ5] + L1 Sin[θ2 + θ5] +
      L2 Sin[θ2 + θ3 + θ5] + L3 Sin[θ2 + θ3 + θ4 + θ5] + L4 Sin[θ2 + θ3 + θ4 + θ5]),
  -(L4 + L2 Cos[θ4] + L1 Cos[θ3 + θ4]) Sin[θ5], -(L4 + L2 Cos[θ4]) Sin[θ5],
  -L4 Sin[θ5], 0, 0},
 {-Sin[θ2] (L1 Cos[θ6] Sin[θ5] - L4 Cos[θ6] Sin[θ3] Sin[θ4] Sin[θ5] +
      Cos[θ3] Cos[θ6] ((L5 + L3 Cos[θ5]) Sin[θ4] + L2 Sin[θ5]) -
      L3 Sin[θ3] Sin[θ4] Sin[θ6] - L5 Cos[θ5] Sin[θ3] Sin[θ4] Sin[θ6] +
      Cos[θ4] (Cos[θ6] ((L5 + L3 Cos[θ5]) Sin[θ3] + L4 Cos[θ3] Sin[θ5]) +
        Cos[θ3] (L3 + L5 Cos[θ5]) Sin[θ6])) +
   Cos[θ2] (-Sin[θ3] (Cos[θ6] ((L5 + L3 Cos[θ5]) Sin[θ4] + (L2 + L4 Cos[θ4]) Sin[θ5]) +
        Cos[θ4] (L3 + L5 Cos[θ5]) Sin[θ6]) + Cos[θ3] (Cos[θ4] (L5 + L3 Cos[θ5]) Cos[θ6] -
        Sin[θ4] (L4 Cos[θ6] Sin[θ5] + (L3 + L5 Cos[θ5]) Sin[θ6]))),
  -(L4 + L2 Cos[θ4] + L1 Cos[θ3 + θ4]) Cos[θ5] Cos[θ6] +
   (L2 Sin[θ4] + L1 Sin[θ3 + θ4] + L5 Sin[θ5]) Sin[θ6],
  -(L4 + L2 Cos[θ4]) Cos[θ5] Cos[θ6] +
   (L2 Sin[θ4] + L5 Sin[θ5]) Sin[θ6],
  -L4 Cos[θ5] Cos[θ6] + L5 Sin[θ5] Sin[θ6],
  L5 Cos[θ6],
  0},
 {Sin[θ2] ((L3 + L5 Cos[θ5]) Cos[θ6] Sin[θ3] Sin[θ4] +
      (Cos[θ4] (L5 + L3 Cos[θ5]) Sin[θ3] + (L1 - L4 Sin[θ3] Sin[θ4]) Sin[θ5]) Sin[θ6]) +
   Cos[θ2] Sin[θ3] (((L5 + L3 Cos[θ5]) Sin[θ4] + L2 Sin[θ5]) Sin[θ6] +
      Cos[θ4] (-(L3 + L5 Cos[θ5]) Cos[θ6] + L4 Sin[θ5] Sin[θ6])) -
   Cos[θ3] (-Sin[θ2] ((L5 + L3 Cos[θ5]) Sin[θ4] + L2 Sin[θ5]) Sin[θ6] +
      Cos[θ2] Sin[θ4] ((L3 + L5 Cos[θ5]) Cos[θ6] - L4 Sin[θ5] Sin[θ6]) +
      Cos[θ4] ((L3 + L5 Cos[θ5]) Cos[θ6] Sin[θ2] +
        (Cos[θ2] (L5 + L3 Cos[θ5]) - L4 Sin[θ2] Sin[θ5]) Sin[θ6])),
  Cos[θ6] ((L2 + L1 Cos[θ3]) Sin[θ4] + L5 Sin[θ5]) +
   Cos[θ5] (L4 - L1 Sin[θ3] Sin[θ4]) Sin[θ6] +
   Cos[θ4] (L1 Cos[θ6] Sin[θ3] + (L2 + L1 Cos[θ3]) Cos[θ5] Sin[θ6]),
  Cos[θ6] (L2 Sin[θ4] + L5 Sin[θ5]) + (L4 + L2 Cos[θ4]) Cos[θ5] Sin[θ6],
  L5 Cos[θ6] Sin[θ5] + L4 Cos[θ5] Sin[θ6],
  -L5 Sin[θ6], 0},
 {Sin[θ2 + θ3 + θ4] Sin[θ5], Cos[θ5],
  Cos[θ5], Cos[θ5], 0, 1},
 {1/4 (-2 Sin[θ2 + θ3 + θ4 - θ6] + Sin[θ2 + θ3 + θ4 - θ5 - θ6] + Sin[θ2 + θ3 + θ4 + θ5 - θ6] +
      2 Sin[θ2 + θ3 + θ4 + θ6] + Sin[θ2 + θ3 + θ4 - θ5 + θ6] + Sin[θ2 + θ3 + θ4 + θ5 + θ6]),
  -Cos[θ6] Sin[θ5], -Cos[θ6] Sin[θ5], -Cos[θ6] Sin[θ5],
```

$$\text{Sin}[\theta 6], 0\},$$
$$\left\{\frac{1}{4}\left(2\,\text{Cos}[\theta 2+\theta 3+\theta 4-\theta 6]-\text{Cos}[\theta 2+\theta 3+\theta 4-\theta 5-\theta 6]-\text{Cos}[\theta 2+\theta 3+\theta 4+\theta 5-\theta 6]+\right.\right.$$
$$\left.\left.2\,\text{Cos}[\theta 2+\theta 3+\theta 4+\theta 6]+\text{Cos}[\theta 2+\theta 3+\theta 4-\theta 5+\theta 6]+\text{Cos}[\theta 2+\theta 3+\theta 4+\theta 5+\theta 6]\right),$$
$$\text{Sin}[\theta 5]\,\text{Sin}[\theta 6], \text{Sin}[\theta 5]\,\text{Sin}[\theta 6], \text{Sin}[\theta 5]\,\text{Sin}[\theta 6], \text{Cos}[\theta 6], 0\}\}$$