Contents


```
% main driver script for lab3

%%%%%BEFORE LAUNCHING THIS SCRIPT%%%%%%%%%
% hide 'base_link' and 'tool0' and show 'base' and 'ee_link' in rviz
% 'base' is the spactial frame, and 'ee_link' is the tool frame
```

Setup

```
clear
clc
close all


rosshutdown
rosinit
ur5 = ur5_interface();

%redefine base frame position based off of construction
%the ur5 configuration from assignment 4 number 2 is defined as gst0
tf_frame('base_link', 'base', [ROTZ(pi/2) [0 0 0.0892]'; 0 0 0 0]);
pause(1)
```

```
Shutting down global node /matlab_global_node_88464 with NodeURI http://david-MSI-Desktop:36653/
The value of the ROS_MASTER_URI environment variable, http://localhost:11311, will be used to connect to the ROS master.
Initializing global node /matlab_global_node_63998 with NodeURI http://david-MSI-Desktop:42983/
Shutting down global node /matlab_global_node_63998 with NodeURI http://david-MSI-Desktop:42983/
The value of the ROS_MASTER_URI environment variable, http://localhost:11311, will be used to connect to the ROS master.
Initializing global node /matlab_global_node_30715 with NodeURI http://david-MSI-Desktop:33987/
```

Part 3 a) Forward Kinematic Map Verification

```
fprintf('\n\nBeginning testing of ur5FwdKin() function:\n')

for i = 1:4

    %generate a rigid transform in the space
    while true
        q = [rand(1,6)*2*pi - pi]'; %generate joint values within limits
        q(2) = -rand * pi;           %force q2 to be positive so that it doesnt intersect the floor

        %tf_frame('base', 'Forward_Kinematics', ur5FwdKin(joints - ur5.home));

        g = ur5FwdKin(q - ur5.home);

        if g(3,4) > 0.1 %check to make sure g is above the floor
            break
        end
    end



    fwdKinToolFrame = tf_frame('base','fwdKinToolFrame',eye(4));
    fwdKinToolFrame.move_frame('base',g);


    %for generating screenshots
    %pause

    %make sure to hide tool0 and show ee_link
    ur5.move_joints(q, 7);
    pause(7.1)
    err = norm(ur5.get_current_transformation('base','ee_link') - g);
    fprintf('\terror between current position and forward map is %d\n', err);
```

```
end

fprintf('Finished testing of ur5FwdKin() function.\n\n')
```

```
Beginning testing of ur5FwdKin() function:
 error between current position and forward map is 4.753129e-04
 error between current position and forward map is 4.318107e-04
 error between current position and forward map is 2.887373e-04
 error between current position and forward map is 4.099708e-04
Finished testing of ur5FwdKin() function.
```

Part 3 b) Body Jacobian Verification

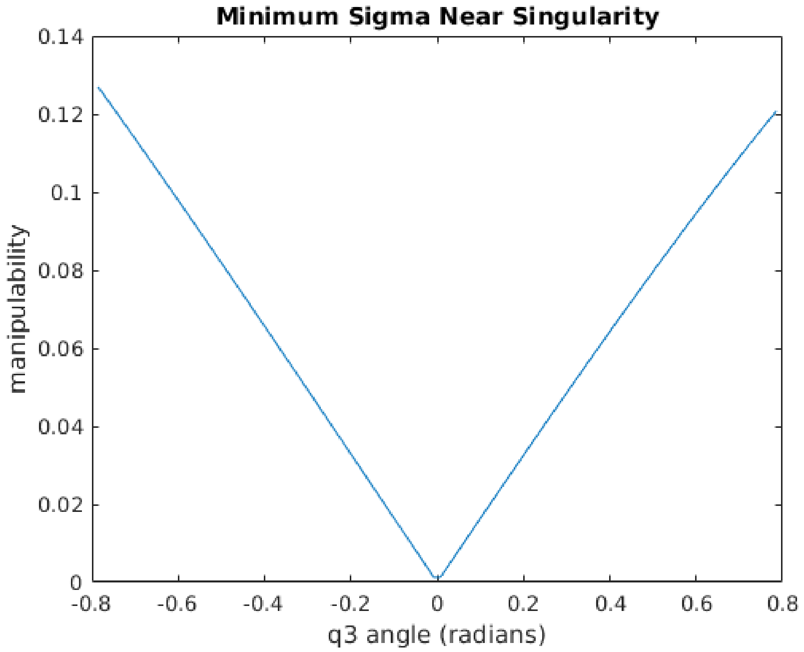
```
fprintf('Beginning testing of ur5BodyJacobian() function:\n')

for i = 1:10
    %generate random valid joints
    q = [rand(1,6)*2*pi - pi]';

    g = ur5FwdKin(q);           %tranform at q
    J = ur5BodyJacobian(q);     %jacobain at q
    Japprox = zeros(6,6);       %matrix for jacobian approximation


    e = eye(6);    %easy access to standard basis vectors in R^6

    for i = 1:6
        ei = e(:,i);    %get the current basis vector
        dgdq_i = 1/2/epsilon * ( ur5FwdKin(q + epsilon*ei) - ur5FwdKin(q - epsilon*ei) );
        xi_hat = rigid_inverse(g)*dgdq_i;

        %twistify xi_hat, and insert into jacobian approximation
        Japprox(:,i) = vee(xi_hat);

    end

    err = norm(J - Japprox);
    fprintf('\terror between Jacobian and central difference approximation is %d\n', err);

end


fprintf('Finished testing of ur5BodyJacobian() function.\n\n')
```

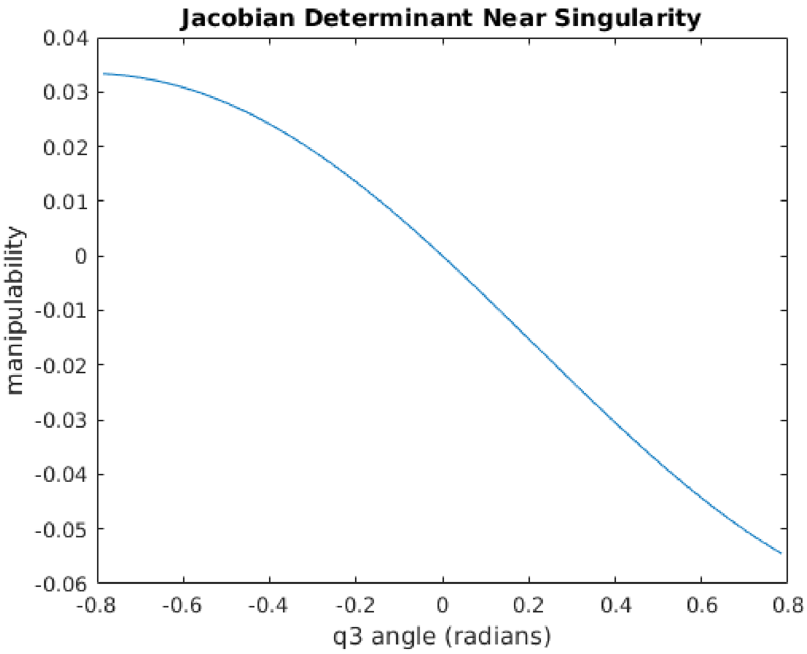
```
Beginning testing of ur5BodyJacobian() function:
 error between Jacobian and central difference approximation is 2.559420e-06
 error between Jacobian and central difference approximation is 5.275562e-06
 error between Jacobian and central difference approximation is 2.000121e-06
 error between Jacobian and central difference approximation is 3.264981e-06
 error between Jacobian and central difference approximation is 1.955514e-06
 error between Jacobian and central difference approximation is 2.977764e-06
 error between Jacobian and central difference approximation is 3.819921e-06
 error between Jacobian and central difference approximation is 4.913408e-06
 error between Jacobian and central difference approximation is 3.514969e-06
 error between Jacobian and central difference approximation is 2.115468e-06
Finished testing of ur5BodyJacobian() function.
```

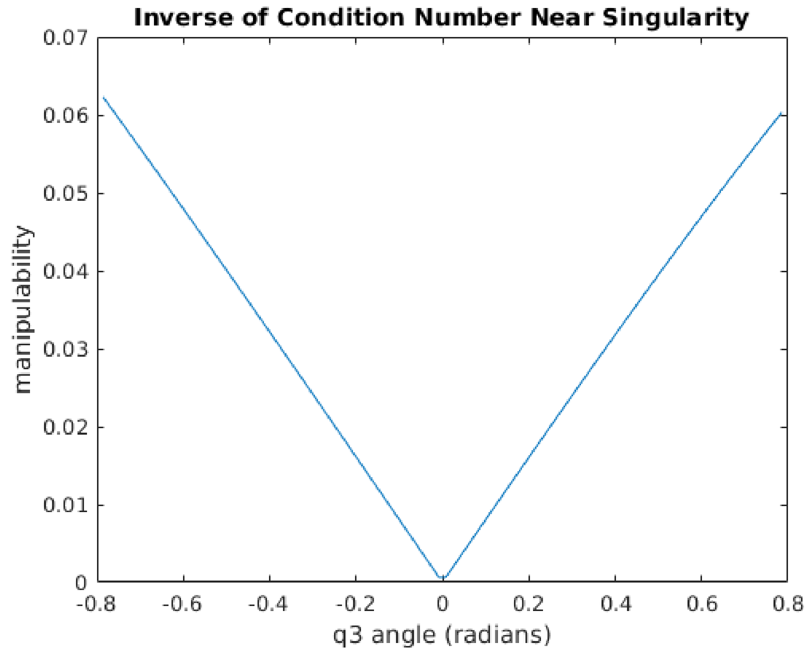
Part 3 c) Manipulability Measure Verification

```
%generate a q value that is not near a singularity
while true
    q = [rand(1,6)*2*pi - pi]';
    if manipulability(ur5BodyJacobian(q), 'invcond') > 0.01
        break
    end
end

%set the joints to a singular configuration (q3 = 0)
q(3) = 0;

pts = 100;   %how many points to plot

sigmamin = zeros(pts,1);
```

```
detjac = zeros(pts,1);
invcond = zeros(pts,1);

i = 1;                          %keep track of index
theta = -pi/4:pi/2/(pts-1):pi/4;    %range to vary q3 over
for q3 = theta
    q(3) = q3;
    sigmamin(i) = manipulability(ur5BodyJacobian(q), 'sigmamin');
    detjac(i) = manipulability(ur5BodyJacobian(q), 'detjac');
    invcond(i) = manipulability(ur5BodyJacobian(q), 'invcond');

    i = i + 1;  %update to next index
end

figure
plot(theta, sigmamin)
title('Minimum Sigma Near Singularity')
xlabel('q3 angle (radians)')
ylabel('manipulability')

figure
plot(theta, detjac)
title('Jacobian Determinant Near Singularity')
xlabel('q3 angle (radians)')
ylabel('manipulability')

figure
plot(theta, invcond)
title('Inverse of Condition Number Near Singularity')
xlabel('q3 angle (radians)')
ylabel('manipulability')
```







Part 3 d) Twist from g Transform Verification

```
fprintf('Beginning testing of getXi() function:\n')

for i = 1:24


    %generate a random twist
    xi = [( rand(3,1)-0.5 ) * 2; ( rand(3,1)-0.5 ) * 2*pi];

    %occasionally force pure translation or pure rotation twists
    if mod(i,3) == 0 xi(1:3) = 0; end
    if mod(i,3) == 1 xi(4:6) = 0; end


    g = expm(wedge(xi));
    xi_comp = getXi(g);

    colinear = norm(proj(xi, xi_comp) - xi_comp);    %are xi and xi_comp colinear
    same_dir = dot(proj(xi, xi_comp), xi) > 0;      %are xi and xi_comp pointing in the same direction

    %compute twist angle, and correct for if xi and xi_comp are pointing opposite
    if same_dir
        angle_diff = norm(xi) - norm(xi_comp);
    else
        angle_diff = 2*pi - norm(xi) - norm(xi_comp);
```

```
    end



    %display warnings if the returned values are different
    if colinear > epsilon
        warning('Returned non-colinear twist')
    elseif angle_diff > epsilon
        warning('different twist angle returned.')

    end


    fprintf('\terror between input and computed twist is %d\n', max(colinear,angle_diff));


end

fprintf('\nthe instances where err is large are caused by the rotations occuring around axes rotated by 180 degrees.\n')
fprintf('I account for this with planer and pure rotation, but haven''t figured out how to do so for general twists\n\n')

fprintf('Finished testing of getXi() function\n\n')
```

```
Beginning testing of getXi() function:
 error between input and computed twist is 2.775558e-17
Warning: Returned non-colinear twist
 error between input and computed twist is 1.290590e+00
 error between input and computed twist is 1.332268e-15
 error between input and computed twist is 0
Warning: Returned non-colinear twist
 error between input and computed twist is 4.648989e-01
 error between input and computed twist is 2.220446e-15
 error between input and computed twist is 0
 error between input and computed twist is 4.422836e-15
 error between input and computed twist is 4.440892e-16
 error between input and computed twist is 0
 error between input and computed twist is 1.387779e-16
 error between input and computed twist is 1.190159e-13
 error between input and computed twist is 0
 error between input and computed twist is 7.791361e-16
 error between input and computed twist is 4.965068e-16
 error between input and computed twist is 0
Warning: Returned non-colinear twist
 error between input and computed twist is 3.994852e-01
 error between input and computed twist is 1.110223e-15
 error between input and computed twist is 0
 error between input and computed twist is 1.776357e-15
 error between input and computed twist is 2.719480e-16
 error between input and computed twist is 1.110223e-16
 error between input and computed twist is 3.390841e-16
 error between input and computed twist is 2.155663e-15

the instances where err is large are caused by the rotations occuring around axes rotated by 180 degrees.
I account for this with planer and pure rotation, but haven't figured out how to do so for general twists

Finished testing of getXi() function
```

Part 3 e) Resolved Rate Controller Test Validation

```
fprintf('Beginning testing of ur5RRcontrol() function.\n')

K = 0.1;    % gain for controller

while true

    fprintf('\tAttempting RR control\n')

    %move the ur5 to a start configuration with good manipulability
    if manipulability(ur5BodyJacobian(force_get_current_joints(ur5) - ur5.home), 'invcond') < 0.01
        %move the ur5 from the singular starting position
        ur5.move_joints(ur5.home + rand(6,1), 5)
        while true
            jstart = rand(6,1)*2*pi - pi;
            gs = ur5FwdKin(jstart);

            %ensure selected transform is above the ground, not over the center,
            %and not (nearly) singular
```

```
            if gs(3,4) > 0.1 & sqrt(gs(2,4)^2 + gs(1,4)^2) > 0.1 & ...
                    manipulability(ur5BodyJacobian(jstart), 'invcond') > 0.01
                break
            end
        end
    pause(5)
    end



    %generate a goal transform to move to
    while true
        jfinal = rand(6,1)*2*pi - pi;
        jfinal(2) = -rand*pi;   %force the transform to be above the ground
        gf = ur5FwdKin(jfinal);

        %ensure selected transform is above the ground, not over the center,
        %and not (nearly) singular
        if gf(3,4) > 0.1 & sqrt(gf(2,4)^2 + gf(1,4)^2) > 0.3 & ...
                manipulability(ur5BodyJacobian(jfinal), 'invcond') > 0.01
            break
        end
    end

    %display the goal frame in rvis
    Frame_goal = tf_frame('base', 'Goal', gf);
    pause(0.3)


    %drive the arm to the goal transform
    finalerr = ur5RRcontrol(gf, K, ur5);

    if finalerr ~= -1
        fprintf('final distance to goal: %0.2f cm\n', finalerr);
        break   % exit loop on successful completion
    else
        fprintf('encountered singularity on trajectory. Retrying\n')
        %stay in loop if unsuccessful
        pause(5)
    end


end




%demonstrate end at a singularity
fprintf('\n\tAttempting controller while starting at a singularity\n')

jstart = ur5.home;   % start at a singularity
ur5.move_joints(jstart, 5)
pause(5.1)

jfinal = rand(6,1)*2*pi - pi;   % end position


gf = ur5FwdKin(jfinal);
ur5RRcontrol(gf, K, ur5);   %this should necessarily fail


fprintf('Finished testing of ur5RRcontrol() function\n\n')
```

```
Beginning testing of ur5RRcontrol() function.
 Attempting RR control
final distance to goal: 0.96 cm

 Attempting controller while starting at a singularity
Warning: UR5 is near a singularity. Resetting ur5 position and exiting
RRcontroller.
Finished testing of ur5RRcontrol() function
```

Lab 3 Assignment Supplementary Material

In[41]:=
```
<< Screws.m;
<< RobotLinks.m;
```

For Lab 3, I used the configuration of the ur4 presented in Assignment 4 as the zero position. Hence the following formulas will be essentially identical to those of the assignment.

For implementations in MATLAB, see the files “.../lab3/helpers/ur5Parameters.m” “.../lab3/ur5FwdKin.m” and “.../lab3/ur5BodyJacobian.m”

In[37]:=
```
e1 = {1, 0, 0};
e2 = {0, 1, 0};
e3 = {0, 0, 1};
I3 = IdentityMatrix[3];
```

Forward Kinematics Calculations

This code is implemented/used to control the ur5 in matlab. ur5Parameters.m sets up the joint lengths and twist axes. ur5FwdKin.m performs the actual forward kinematics.

In[43]:=
```
ω1 = e3; q1 = {0, 0, 0};
ω2 = e1; q2 = {0, 0, 0};
ω3 = e1; q3 = {0, 0, L1};
ω4 = e1; q4 = {0, 0, L1 + L2};
ω5 = e3; q5 = {L3, 0, 0};
ω6 = e1; q6 = {0, 0, L1 + L2 + L4};

ξ1 = RevoluteTwist[q1, ω1]
ξ2 = RevoluteTwist[q2, ω2]
ξ3 = RevoluteTwist[q3, ω3]
ξ4 = RevoluteTwist[q4, ω4]
ξ5 = RevoluteTwist[q5, ω5]
ξ6 = RevoluteTwist[q6, ω6]
```

Out[49]= {0, 0, 0, 0, 0, 1}

Out[50]= {0, 0, 0, 1, 0, 0}

Out[51]= {0, L1, 0, 1, 0, 0}

Out[52]= {0, L1 + L2, 0, 1, 0, 0}

Out[53]= {0, - L3, 0, 0, 0, 1}

Out[54]= {0, L1 + L2 + L4, 0, 1, 0, 0}

In[55]:=
```
gst0 = RPToHomogeneous[I3, {L3 + L5, 0, L1 + L2 + L4}];
gst0 // MatrixForm
gst[θ1_, θ2_, θ3_, θ4_, θ5_, θ6_] := Simplify[
   ForwardKinematics[
    {ξ1, θ1}, {ξ2, θ2}, {ξ3, θ3}, {ξ4, θ4}, {ξ5, θ5}, {ξ6, θ6}, gst0
   ]
  ];
gst[θ1, θ2, θ3, θ4, θ5, θ6]
```

Out[56]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & L3 + L5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L1 + L2 + L4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[58]=
```
{{Cos[θ1] Cos[θ5] - Cos[θ2 + θ3 + θ4] Sin[θ1] Sin[θ5],
  -Cos[θ6] (Cos[θ2 + θ3 + θ4] Cos[θ5] Sin[θ1] + Cos[θ1] Sin[θ5]) +
   Sin[θ1] Sin[θ2 + θ3 + θ4] Sin[θ6],
  Cos[θ4] Cos[θ6] Sin[θ1] Sin[θ2 + θ3] + Cos[θ2 + θ3] Cos[θ6] Sin[θ1] Sin[θ4] +
   (Cos[θ2 + θ3 + θ4] Cos[θ5] Sin[θ1] + Cos[θ1] Sin[θ5]) Sin[θ6],
  Cos[θ1] (L3 + L5 Cos[θ5]) + 1/2 Sin[θ1] (2 L1 Sin[θ2] + 2 L2 Sin[θ2 + θ3] +
      2 L4 Sin[θ2 + θ3 + θ4] + L5 Sin[θ2 + θ3 + θ4 - θ5] - L5 Sin[θ2 + θ3 + θ4 + θ5])},
 {Cos[θ5] Sin[θ1] + Cos[θ1] Cos[θ2 + θ3 + θ4] Sin[θ5], -Cos[θ6] Sin[θ1] Sin[θ5] +
   Cos[θ1] (Cos[θ2 + θ3 + θ4] Cos[θ5] Cos[θ6] - Sin[θ2 + θ3 + θ4] Sin[θ6]),
  Sin[θ1] Sin[θ5] Sin[θ6] - Cos[θ1] (Cos[θ4] Cos[θ6] Sin[θ2 + θ3] +
      Cos[θ2 + θ3] Cos[θ6] Sin[θ4] + Cos[θ2 + θ3 + θ4] Cos[θ5] Sin[θ6]),
  (L3 + L5 Cos[θ5]) Sin[θ1] - 1/2 Cos[θ1] (2 L1 Sin[θ2] + 2 L2 Sin[θ2 + θ3] +
      2 L4 Sin[θ2 + θ3 + θ4] + L5 Sin[θ2 + θ3 + θ4 - θ5] - L5 Sin[θ2 + θ3 + θ4 + θ5])},
 {Sin[θ2 + θ3 + θ4] Sin[θ5], 1/4 (-2 Sin[θ2 + θ3 + θ4 - θ6] + Sin[θ2 + θ3 + θ4 - θ5 - θ6] +
     Sin[θ2 + θ3 + θ4 + θ5 - θ6] + 2 Sin[θ2 + θ3 + θ4 + θ6] +
     Sin[θ2 + θ3 + θ4 - θ5 + θ6] + Sin[θ2 + θ3 + θ4 + θ5 + θ6]),
  1/4 (2 Cos[θ2 + θ3 + θ4 - θ6] - Cos[θ2 + θ3 + θ4 - θ5 - θ6] - Cos[θ2 + θ3 + θ4 + θ5 - θ6] +
     2 Cos[θ2 + θ3 + θ4 + θ6] + Cos[θ2 + θ3 + θ4 - θ5 + θ6] + Cos[θ2 + θ3 + θ4 + θ5 + θ6]),
  L1 Cos[θ2] + L2 Cos[θ2 + θ3] + L4 Cos[θ2 + θ3 + θ4] + 1/2 L5 Cos[θ2 + θ3 + θ4 - θ5] -
   1/2 L5 Cos[θ2 + θ3 + θ4 + θ5]}, {0, 0, 0, 1}}
```

Body Jacobian Calculations

This code is also implemented/used in to control the ur5 in matlab. ur5Parameters.m is run inside the ur5BodyJacobian.m, for the same reasons as above. The body Jacobian is then computed by the same algorithm used in the Mathematica implementation RobotLinks.m

In[59]:=
```
Jbst = BodyJacobian[{ξ1, θ1}, {ξ2, θ2},
   {ξ3, θ3}, {ξ4, θ4}, {ξ5, θ5}, {ξ6, θ6}, gst0] // Simplify
```

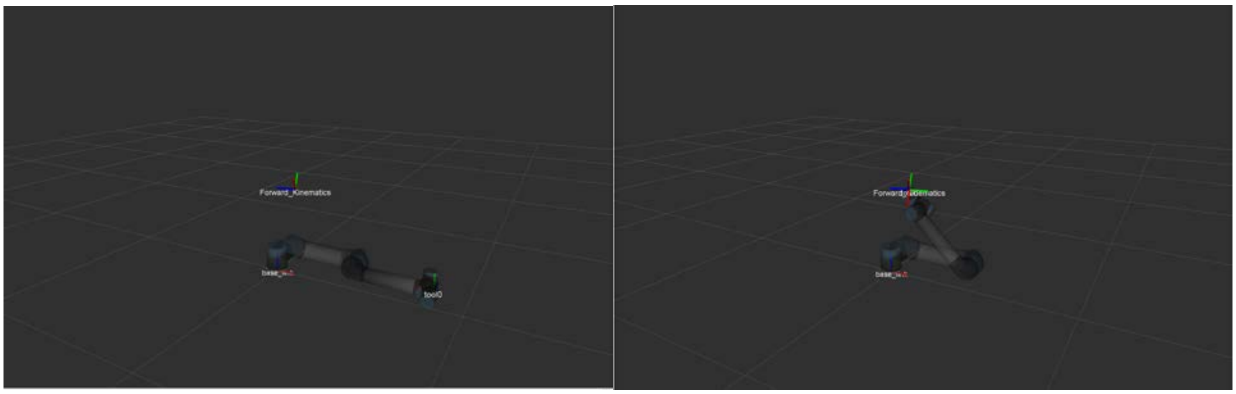
Out[59]=
```
{{1/2 (L1 Sin[θ2 - θ5] + L2 Sin[θ2 + θ3 - θ5] -
     L3 Sin[θ2 + θ3 + θ4 - θ5] + L4 Sin[θ2 + θ3 + θ4 - θ5] + L1 Sin[θ2 + θ5] +
     L2 Sin[θ2 + θ3 + θ5] + L3 Sin[θ2 + θ3 + θ4 + θ5] + L4 Sin[θ2 + θ3 + θ4 + θ5]),
  -(L4 + L2 Cos[θ4] + L1 Cos[θ3 + θ4]) Sin[θ5], -(L4 + L2 Cos[θ4]) Sin[θ5],
  -L4 Sin[θ5], 0, 0},
 {-Sin[θ2] (L1 Cos[θ6] Sin[θ5] - L4 Cos[θ6] Sin[θ3] Sin[θ4] Sin[θ5] +
      Cos[θ3] Cos[θ6] ((L5 + L3 Cos[θ5]) Sin[θ4] + L2 Sin[θ5]) -
      L3 Sin[θ3] Sin[θ4] Sin[θ6] - L5 Cos[θ5] Sin[θ3] Sin[θ4] Sin[θ6] +
      Cos[θ4] (Cos[θ6] ((L5 + L3 Cos[θ5]) Sin[θ3] + L4 Cos[θ3] Sin[θ5]) +
         Cos[θ3] (L3 + L5 Cos[θ5]) Sin[θ6])) +
   Cos[θ2] (-Sin[θ3] (Cos[θ6] ((L5 + L3 Cos[θ5]) Sin[θ4] + (L2 + L4 Cos[θ4]) Sin[θ5]) +
         Cos[θ4] (L3 + L5 Cos[θ5]) Sin[θ6]) + Cos[θ3] (Cos[θ4] (L5 + L3 Cos[θ5]) Cos[θ6] -
         Sin[θ4] (L4 Cos[θ6] Sin[θ5] + (L3 + L5 Cos[θ5]) Sin[θ6]))),
  -(L4 + L2 Cos[θ4] + L1 Cos[θ3 + θ4]) Cos[θ5] Cos[θ6] +
   (L2 Sin[θ4] + L1 Sin[θ3 + θ4] + L5 Sin[θ5]) Sin[θ6],
  -(L4 + L2 Cos[θ4]) Cos[θ5] Cos[θ6] +
   (L2 Sin[θ4] + L5 Sin[θ5]) Sin[θ6],
  -L4 Cos[θ5] Cos[θ6] + L5 Sin[θ5] Sin[θ6],
  L5 Cos[θ6],
  0},
 {Sin[θ2] ((L3 + L5 Cos[θ5]) Cos[θ6] Sin[θ3] Sin[θ4] +
      (Cos[θ4] (L5 + L3 Cos[θ5]) Sin[θ3] + (L1 - L4 Sin[θ3] Sin[θ4]) Sin[θ5]) Sin[θ6]) +
   Cos[θ2] Sin[θ3] (((L5 + L3 Cos[θ5]) Sin[θ4] + L2 Sin[θ5]) Sin[θ6] +
      Cos[θ4] (-(L3 + L5 Cos[θ5]) Cos[θ6] + L4 Sin[θ5] Sin[θ6])) -
   Cos[θ3] (-Sin[θ2] ((L5 + L3 Cos[θ5]) Sin[θ4] + L2 Sin[θ5]) Sin[θ6] +
      Cos[θ2] Sin[θ4] ((L3 + L5 Cos[θ5]) Cos[θ6] - L4 Sin[θ5] Sin[θ6]) +
      Cos[θ4] ((L3 + L5 Cos[θ5]) Cos[θ6] Sin[θ2] +
         (Cos[θ2] (L5 + L3 Cos[θ5]) - L4 Sin[θ2] Sin[θ5]) Sin[θ6])),
  Cos[θ6] ((L2 + L1 Cos[θ3]) Sin[θ4] + L5 Sin[θ5]) +
   Cos[θ5] (L4 - L1 Sin[θ3] Sin[θ4]) Sin[θ6] +
   Cos[θ4] (L1 Cos[θ6] Sin[θ3] + (L2 + L1 Cos[θ3]) Cos[θ5] Sin[θ6]),
  Cos[θ6] (L2 Sin[θ4] + L5 Sin[θ5]) + (L4 + L2 Cos[θ4]) Cos[θ5] Sin[θ6],
  L5 Cos[θ6] Sin[θ5] + L4 Cos[θ5] Sin[θ6],
  -L5 Sin[θ6], 0},
 {Sin[θ2 + θ3 + θ4] Sin[θ5], Cos[θ5],
  Cos[θ5], Cos[θ5], 0, 1},
 {1/4 (-2 Sin[θ2 + θ3 + θ4 - θ6] + Sin[θ2 + θ3 + θ4 - θ5 - θ6] + Sin[θ2 + θ3 + θ4 + θ5 - θ6] +
     2 Sin[θ2 + θ3 + θ4 + θ6] + Sin[θ2 + θ3 + θ4 - θ5 + θ6] + Sin[θ2 + θ3 + θ4 + θ5 + θ6]),
  -Cos[θ6] Sin[θ5], -Cos[θ6] Sin[θ5], -Cos[θ6] Sin[θ5],
```

```
Sin[θ6], 0},
{1/4 (2 Cos[θ2 + θ3 + θ4 - θ6] - Cos[θ2 + θ3 + θ4 - θ5 - θ6] - Cos[θ2 + θ3 + θ4 + θ5 - θ6] +
    2 Cos[θ2 + θ3 + θ4 + θ6] + Cos[θ2 + θ3 + θ4 - θ5 + θ6] + Cos[θ2 + θ3 + θ4 + θ5 + θ6]),
 Sin[θ5] Sin[θ6], Sin[θ5] Sin[θ6], Sin[θ5] Sin[θ6], Cos[θ6], 0}}
```

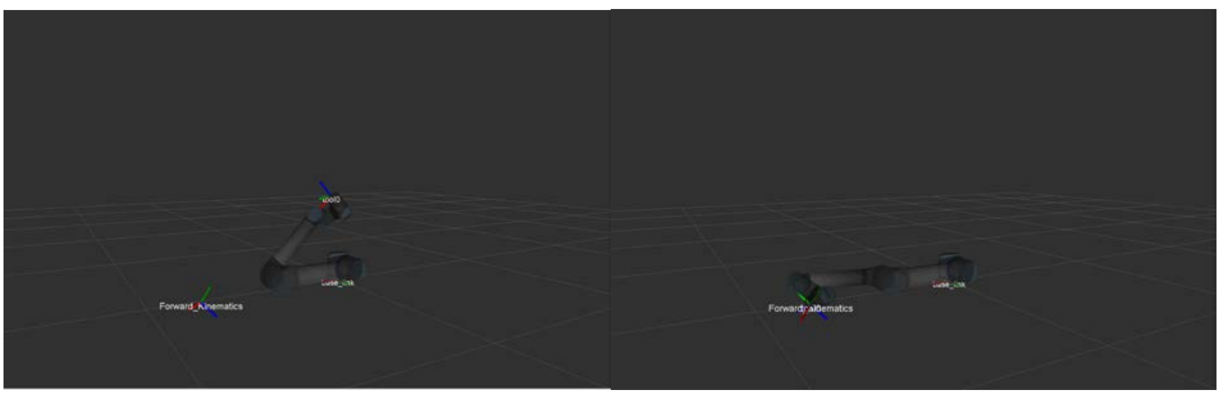
Forward Kinematics Screen Shots

A random joint configuration is generated. The rigid transform g is computed according to the forward map. These images show a frame placed g relative to the base frame, and then ur5 set to the original joints (and how it lines up with the placed frame).

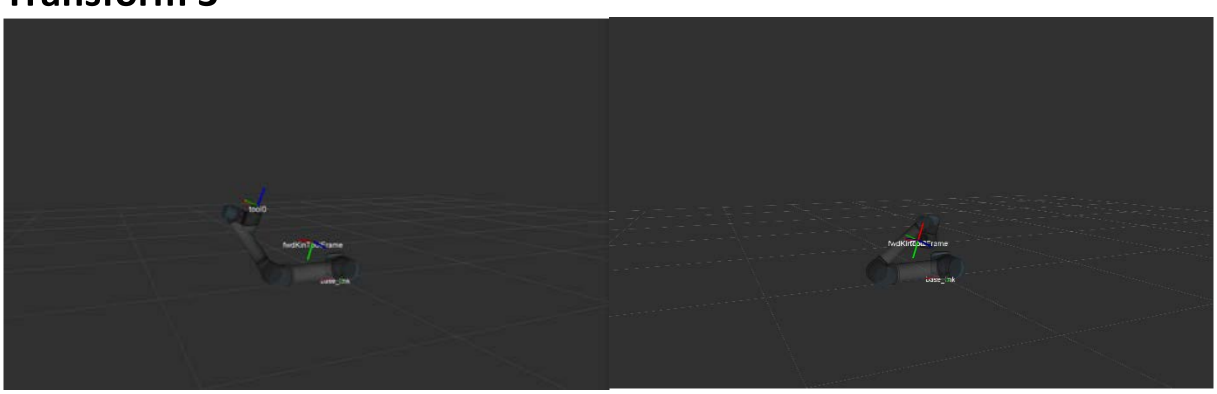
Transform 1



Transform 2



Transform 3



Transform 4