Happy-GLL: modular, reusable and complete top-down parsers for parameterized nonterminals

L. Thomas van Binsbergen Itvanbinsbergen@acm.org Informatics Institute, University of Amsterdam Amsterdam, The Netherlands

Abstract

Parser generators and parser combinator libraries are the most popular tools for producing parsers. Parser combinators use the host language to provide reusable components in the form of higher-order functions with parsers as parameters. Very few parser generators support this kind of reuse through abstraction and even fewer generate parsers that are as modular and reusable as the parts of the grammar for which they are produced. This paper presents a strategy for generating modular, reusable and complete top-down parsers from syntax descriptions with parameterized nonterminals, based on the FUN-GLL variant of the GLL algorithm.

The strategy is discussed and demonstrated as a novel back-end for the Happy parser generator. Happy grammars can contain 'parameterized nonterminals' in which parameters abstract over grammar symbols, granting an abstraction mechanism to define reusable grammar operators. However, the existing Happy back-ends do not deliver on the full potential of parameterized nonterminals as parameterized nonterminals cannot be reused across grammars. Moreover, the parser generation process may fail to terminate or may result in exponentially large parsers generated in an exponential amount of time.

The GLL back-end presented in this paper implements parameterized nonterminals successfully by generating higher-order functions that resemble parser combinators, inheriting all the advantages of top-down parsing. The back-end is capable of generating parsers for the full class of context-free grammars, generates parsers in linear time and generates parsers that find all derivations of the input string. To our knowledge, the presented GLL back-end makes Happy the first parser generator that combines all these features.

This paper describes the translation procedure of the GLL back-end and compares it to the LALR and GLR back-ends of Happy in several experiments.

1 Introduction

Recursive descent parsing is a technique for (manually or mechanically) writing *top-down* parsers based on the description of a context-free grammar. Recursive descent parsers have in common that every (nonterminal and terminal) symbol of the grammar is implemented by a piece of code, that

Damian Frölich dfrolich@acm.org Informatics Institute, University of Amsterdam Amsterdam, The Netherlands

these pieces of code can be placed in a sequence – implementing an alternate of the grammar – and that the choice between a nonterminal's alternates is implemented by branching control-flow. Recursive descent parser generators implement a direct translation from grammar to parser. Every symbol is translated independently (separate compilation) and the code for symbols can be tested independently (semantic modularity). As a result, generated parsers are easy to maintain and debug; updated grammar specifications require minimal recompilation and unexpected behavior can be identified by isolating the parts of the parser and input that cause the unexpected behavior.

The functional characteristics of recursive descent parsers vary depending on the top-down parsing algorithm on which they are based, but recursive descent parsers have historically struggled with left-recursive nonterminals and nonfactorized alternates. Typical implementations only accept LL(k) grammars: the set of context-free grammars for which it holds that, with $k \geqslant 0$ terminal symbols lookahead, no two alternates are simultaneously applicable. Employing a recursive descent parser generator often involves applying grammar transformations to remove left-recursion and/or to apply left-factorization to produce an LL(k) grammar. However, applying grammar transformations may not always be desirable or even possible; there are context-free grammars for which there is no LL(k) equivalent.

Bottom-up parsing is more often possible without applying grammar transformations. Bottom-up parsers are also very fast, benefiting from pre-computed information recorded in a table – the so-called 'parse table'. These properties have made bottom-up parsing more popular than top-down parsing as the basis for parser generators, as evidenced by the large variety of algorithms such as LALR, SLR and GLR [36] and implementations such as by Yacc, Bison, Happy [21], Menhir [24], Rascal [18], Spoofax [17] and SDF [40]. However, the users of bottom-up parser generators do not benefit from separate compilation and semantic modularity.

This paper discusses a new back-end for Happy which generates recursive descent parsers based on the generalized LL (GLL) algorithm. The back-end has the aforementioned advantages of recursive descent parsing, but does not require any grammar transformations. **Generalized parsing algorithms** are *general* in the sense that they accept arbitrary context-free grammars and *complete* in the sense

that they produce all possible derivations of a given input string. The generalized LL (GLL) algorithm [28, 29] relies on intricate bookkeeping within potentially large datastructures to simultaneously ensure that parsers terminate and find all derivations. Despite this added complexity, GLL parsers are still easy to maintain, debug and support separate compilation and semantic modularity (like other recursive descent parsers).

Generalized parser combinators are recent technologies that combine the benefits of generalized parsing and parser combinators [15, 25, 38, 39]. The mentioned approaches have in common that they involve explicit representations of grammar components such as symbols and productions rules – as is required by Earley's generalized parsing algorithm [10], GLR [36] and GLL [28] – whereas conventional parser combinators have no explicit representation of grammar components. The libraries presented by [25] and [38] generate an actual grammar object before providing it as input to a standalone generalized parsing algorithm (voiding separate compilation). This idea of so-called grammar combinators has also been applied outside the context of generalized parsing by [20], [5], and [9].

Grammar combinators blur the line between combinator libraries and parser generators, leaving only a few essential differences: grammar combinator libraries generate parsers at runtime rather than in a separate phase and grammar combinator libraries define *embedded* domain-specific languages (EDSLs) whereas parser generators define external domain-specific languages (DSLs). EDSLs are typically easy to extend, with the power of the host language available to define new operators. This holds true for parser combinators and, to a lesser degree, for grammar combinators [37].

The **FUN-GLL** variant of the GLL algorithm computes the minimal amount of grammar information necessary for generalized parsing [39]. The algorithm can be used by parser combinators because grammar information is computed on an as-needed basis during parsing, rather than in a separate phase. This way, reuse through abstraction with separate compilation is realized in FUN-GLL, and in our Happy backend based on FUN-GLL.

Parser generators, viewed as implementing DSLs, typically provide a *fixed* number of language constructs, often corresponding to some variation of Extended Backus-Naur Form (EBNF). The 'parameterized nonterminals¹' of the Happy parser generator make it possible for users to define their own operators over grammar symbols, akin to macro-grammars [11, 34] and the parameterized nonterminals of the Menhir [24] and PRECC [7] parser generators. However, parameterized nonterminals do not reach their full potential in the existing (LALR and GLR) back-ends of Happy as it is not possible to reuse user-defined operators

across grammars. Moreover, the back-ends rely on an algorithm that effectively performs macro-expansion on all parameterized nonterminals. This algorithm may fail to terminate or may result in exponentially large parsers generated in an exponential amount of time. The GLL back-end for Happy presented in this paper overcomes these problems by generating reusable, higher-order functions, akin to FUN-GLL parser combinators, for the nonterminals of a grammar.

The contributions of this paper are as follows. This paper:

- Presents a strategy for generating modular, reusable and complete top-down parsers from syntax descriptions with parameterized nonterminals
- Adds a back-end to Happy that realizes the full potential of Happy's parameterized nonterminals, making Happy one of the few parser generators to support 'reuse through abstraction' of which it is perhaps the first to generate complete parsers that find all derivations of an input string
- The GLL back-end is to our knowledge the first implementation of GLL in a parser generator for Haskell and the first back-end for Happy with all the benefits of recursive descent parsing
- The GLL back-end is compared to the existing GLR and LALR back-ends in a number of experiments, demonstrating the characteristics of the new back-end

Section 2 motivates the GLL back-end by explaining the advantages of recursive descent parsing and parameterized nonterminals. Section 3 explains how the GLL back-end translates parameterized nonterminals to reusable GLL parsers that compute all interpretations of an input string. Section 4 discusses practical aspects of the implementation of the GLL back-end, including some specific aspects of the Happy grammar formalism such as monadic semantic actions. Section 5 demonstrates certain advantages of the alternative treatment of parameterized nonterminals and compares the running times of the different Happy back-ends in an empirical evaluation. Sections 6 and 7 discuss related work and conclude.

2 Recursive descent and reuse

This section demonstrates the benefits of recursive descent parsing by presenting a translation from Happy grammars to 'recognizer² functions' and motivates parameterized nonterminals by giving examples of reuse. Familiarity with parsing and Happy (or Yacc-like) syntax is assumed. The syntax of Happy is explained in the online documentation by [21] and [14] provide an excellent introduction to parsing.

The Happy grammar in Figure 1 defines the syntax of a tuple-like structure with alphabetical characters as elements. Semantic actions, normally associated with production alternates in Happy grammars, are ignored in this section.

 $^{^1\}mathrm{Referred}$ to as 'parameterized productions' in the user manual of Happy.

²Recognizers merely indicate whether strings are part of a language whereas parsers also provide a proof in the form of a parse tree.

Figure 1. Happy grammar of comma-separated alphabetical characters within parentheses.

The translation generates (possibly higher-order) recognizer functions for the symbols of a grammar (such as *alpha* and *AlphaTuples* in the example). The recognizer functions may apply the recognizer functions generated for other symbols, independent of whether they are defined in the same file. Symbol definitions can thus be spread across source files and only need to be recompiled if their definitions change and crucially not when the symbols used in this definition change. As is described in this section, this improvement over the current (LALR and GLR) back-ends of Happy is achieved by generating top-down parsers and using higher-order functions to implement parameterized nonterminals.

2.1 Recognizer functions

A recognizer function is a function that takes a sequence of tokens inp (referred to as a sentence) and an index into the sequence k and decides whether it recognizes a prefix of the subsequence of inp starting at index k. The type of tokens is left abstract and is denoted in Haskell code by the type variable t. This type is instantiated by the recognizers when there is a %**tokentype** directive in the grammar specification.

```
type Recognizer\ t = [t] \rightarrow Int \rightarrow (Int \rightarrow Bool) \rightarrow Bool
```

The third argument is a continuation function which is to be applied when a prefix is recognized in order to attempt to recognize the rest of the input sentence. The application of continuation-passing style is crucial to the extension to GLL in Section 3. Explanations and applications of continuation-passing for parsing can be found in [16, 20, 31, 32].

The %**token** directives associate an identifier (e.g. *alpha*) with a Haskell pattern written within braces (e.g. ', '). The following recognizer implements token *alpha*:

```
-- t instantiated to Char following the %tokentype declaration p\_alpha:: Recognizer Char p\_alpha = matchPattern\_1 matcher
```

```
where matcher t = \text{case } t \text{ of}
c \mid c \in (['a' ...'z'] + ['A' ...'Z']) \rightarrow \text{Just } t
\longrightarrow \text{Nothing}
```

The %**token** directive determines the recognizer's name and the first case of the **case**-expression. The logic of recognizer functions for tokens is implemented in the function *matchPattern_1*, provided in a separate support library, as follows:

```
\label{eq:matchPattern_1::} (t \to Maybe\ t) \to Recognizer\ t \mbox{matchPattern\_1 matcher inp $k$ c} \mbox{$|k \geqslant 0$, $k < length\ inp, Just $\_\leftarrow$ matcher (inp !!\ k) = c\ (k+1)$} \mbox{$|$ otherwise} = False
```

The continuation c is applied (to k+1, because one token has been recognized) when the token is successfully matched and *False* is returned otherwise.

A recognizer function can be given to the support function run_1 to attempt to recognize a sentence. The function applies the recognizer to the sentence, together with an initial index of 0 and a continuation that checks whether the whole sentence has been recognized.

```
run_1 :: Recognizer \ t \to [t] \to Bool

run_1 rec inp = rec inp \ 0 \ (\lambda k \to k \equiv length inp)
```

The function run_1 is used to implement the %name directive of Happy (indicating the start symbol to be used by the LR back-ends). The following code is generated for the %name directive of the example. The directive determines the name of the function and the used recognizer.

```
run\_tuples :: [t] \rightarrow Bool

run\_tuples = run_1 p\_AlphaTuples
```

The application of run_1 demonstrates one of the advantages of our approach: every recognizer can be used to recognize a sentence, without the need for recompilation.

To generate code for a nonterminal's alternates it is required to place recognizers in sequence so that one is executed 'before' the other. For example, the last alternate of *Alphas* requires an *alpha* to be recognized, then a comma (',') and then more *Alphas*. For every alternate, code is generated that 'chains' several recognizer functions by defining new continuations. For example, if symbol p appears before q in the alternate of a nonterminal, then the recognizer for p is applied with a continuation that applies the recognizer for q. The following code is generated for nonterminal Alphas:

```
p\_Alphas :: Recognizer Char

p\_Alphas inp \ k \ c_0 = p\_alpha inp \ k \ c_0 \lor p\_alpha inp \ k \ c_1

where c_1 \ k = p\_comma \ inp \ k \ c_2

c_2 \ k = p\_Alphas \ inp \ k \ c_0
```

The code for both alternates of the nonterminal defining *Alphas* apply p_alpha . The difference is in the continuation given to p_alpha . The first alternate is done after p_alpha so the given continuation (c_0) is the continuation received by p_Alphas . In the second alternate, alpha is followed by

comma and Alphas and thus³ c_1 k = p_comma inp k c_2 and c_2 k = p_Alphas inp k c_0 . Note that these definitions have shadowing declarations for the variable k in order to keep a consistent naming convention.

The first alternate of *MAlphas* is empty. The code for this alternate therefore directly applies the continuation c_0 :

```
p\_MAlphas :: Recognizer Char

p\_MAlphas inp \ k \ c_0 = c_0 \ k \lor p\_Alphas inp \ k \ c_0
```

The recognizer⁴ for *AlphaTuples* completes the translation of the grammar in Figure 1:

```
p\_AlphaTuples:: Recognizer Char

p\_AlphaTuples inp k c_0 = p\_lparen inp k c_1

where c_1 k = p\_MAlphas inp k c_2

c_2 k = p\_rparen inp k c_0
```

2.2 Discussion

Reuse. The parameterized nonterminals of Happy abstract over symbols. For example, *comma* in the definition of *Alphas* can be replaced by a parameter to abstract over *comma* as a separator. Abstractions and applications are written in the familiar 'functional style', with formal and actual parameters appearing between parentheses and with commas in between. A parameterized nonterminal for the suggested, more general version of *Alphas* is written as follows:

```
SepAlphas (sep) : alpha
| alpha sep SepAlphas (sep)
```

By taking advantage of Haskell's higher-order functions, the abstractions and applications of Happy can be translated more or less directly into Haskell:

```
p\_SepAlphas :: Recognizer\ Char 
ightharpoonup Recognizer\ Char
p\_SepAlphas\ p\_sep\ inp\ k\ c_0 = p\_alpha\ inp\ k\ c_0 \lor p\_alpha\ inp\ k\ c_1
where c_1\ k = p\_sep\ inp\ k\ c_2
c_2\ k = p\_Alphas\ p\_sep\ inp\ k\ c_0
```

Figure 2 shows several parameterized nonterminals for highly reusable patterns such as delimiters, repetition with a separator, and optionality, as well as examples of their usage. Further real-world examples for reuse can be found in [35].

Advantages of recursive descent. LALR and GLR require⁵ the indication of 'entry point nonterminals' via %name directives. This is because LR algorithms require grammars with start symbols. The translation described in this section does not require entry points because run₁ can be applied to arbitrary recognizers. Practically, this means that every nonterminal of a grammar can be debugged individually, making the whole grammar easier to test and maintain. Moreover, the code generated for a nonterminal is independent

```
Tuples \ (elem) : Parens \ (Optional \ (Multiple \ (elem, comma))) \\ Lists \ (elem) : Brackets \ (Optional \ (Multiple \ (elem, comma))) \\ Parens \ (x) : Within \ ('(',')',x) \\ Brackets \ (x) : Within \ ('[',']',x) \\ Within \ (l,r,x) : l \ x \\ Optional \ (x) : \\ | x \\ Multiple \ (elem, sep) : elem \\ | elem \ sep \ Multiple \ (elem, sep)
```

Figure 2. Examples of reusable parameterized nonterminals

of the definitions of the other symbols in the grammar. The translation thus preserves the inherent modularity of non-terminals.

This section has shown that recognizer functions are freely combined to form more complex recognizers in higher-order functions such as *p_SepAlphas*. Recognizers generated from different grammars can also be combined in this way, enabling reuse across files, albeit with the following caveats. Type signatures instantiate the type for tokens to the type mentioned in the **%tokentype** directive. This implies that two grammars with different **%tokentype** directives produces recognizers that cannot be composed. There are at least two ways a Happy user can approach this potential problem, depending on the application.

One approach is to omit the **%tokentype** directive and to rely on Haskell's type-inferencing to automatically assign the most general type possible. If the **%tokentype** directive is removed from the grammar in Figure 1, then the inferred type is *Char* because of the usage of literals in the **%token** directives. Although the definition of *MAlphas* does not refer to token symbols, the token type *Char* is inferred from the type of the recognizer for *Alphas*. If *Alphas* was a parameter – as it is in *Optional* of Figure 2 – then the derived type would have been a type variable. The function and signature generated for *Optional* are as follows⁶:

```
p\_Optional :: Recognizer \ t \rightarrow Recognizer \ t

p\_Optional \ p\_x \ inp \ k \ c_0 = c_0 \ k \lor p\_x \ inp \ k \ c_0
```

Another approach to reusing nonterminal definitions across grammars is to give co-dependent grammars the same type for tokens. This might be practical for projects that involve a single lexer or that have multiple lexers producing tokens of the same type. This approach provides a practical improvement over the LALR and GLR back-ends because large, complex grammars, with perhaps many entry points, can

³The token name ', ' given to the pattern ', ' by the first **%token** directive is replaced by *comma* in the name for the generated recognizer.

⁴The names for '(' and ')' have been replaced by *lparen* and *rparen*.

⁵The %name directive is optional though, because by default the first non-terminal of the file is chosen as the entry point.

⁶The translation by the GLL back-end differs slightly because type inferencing might assign constraints to types. Rather than inferring types, the GLL back-end uses the 'wildcards' of the PartialTypeSignatures extension of Haskell. The wildcards are replaced with (appropriately constrained) types by compilers supporting this extension.

be spread across several files. Moreover, adopting this approach does not rule out defining a reusable library of parameterized nonterminals (such as the ones in Figure 2) in a separate file without %**tokentype** directive. Note that the alternates for *Within, Optional* and *Multiple* in Figure 2 do not refer to any token symbols.

Limitations of basic recursive descent. Recursive descent parsers typically choose an alternate of a nonterminal based on lookahead (the recognizer functions of this section lazily apply all alternates without considering lookahead). Lookahead involves pre-computing for each alternate the set of terminals the alternate is capable of matching initially, and checking, during a parse, whether the next terminal in the input sentence is a member of that set. In general, however, lookahead cannot be used to rule out each, or all but one, alternate. The class of LL(k) grammars is defined to contain those context-free grammars for which it holds that no two alternates are simultaneously applicable using $k \ge 0$ symbols of lookahead. After choosing an alternate, different forms of backtracking can be used to revert the decision and to choose another alternate. For every amount of lookahead k and for every backtracking strategy, however, there are worst-case grammars that require running times exponential in the size of the input. These exponential running times can be avoided with memoization [13, 22].

If a nonterminal is left-recursive, a parse function implementing this non-terminal may end up calling itself without a change of input, resulting in non-termination. Grammar transformations can be used to remove left-recursion and to left-factorize the grammar, turning the grammar into an LL(k) grammar. However, not every context-free grammar has an LL(k) equivalent and transforming a grammar is not desirable if the resulting grammar is further removed from the semantic interpretation of the syntax. Memoization tables can be used to record continuations for parse functions when parse functions are written in continuation-passing style, making it possible to produce complete parsers for leftrecursive grammars [15, 16]. Frost, Hafiz, and Callaghan employ a 'curtailment' strategy to handle left-recursion, making at most as many recursive calls as there are characters left in the sentence [12].

The parsers generated by the GLL back-end of Happy manage continuations in a fashion similar to [15, 16] to avoid all forms of repeated work – thereby preventing non-termination and exponential running times – and to discover all possible derivations. The grammar to GLL parser translation, presented in the next section, also maintains the benefits of recursive descent parsing mentioned in this section.

3 Generating GLL parsers

This section explains how the GLL back-end translates Happy grammars to GLL parsers based on the purely functional (FUN-GLL) variant of GLL presented here [38, 39]. In [39],

it is shown that FUN-GLL can be implemented directly as parser combinators without generating a grammar object as in [38]. The insights that enabled the parser combinators of [39] also enable the translation presented in this section. The FUN-GLL algorithm and its combinator-oriented origins in [38] and [39] are summarized first.

The combinator expressions written with the parser combinators of [38] evaluate in three stages. First, the grammar represented by the combinator expression is extracted. The grammar is then given to a standalone implementation of FUN-GLL which produces an efficient representation of all possible derivations: a set of BSR elements named after the binary subtree representation (BSR) of [31]. An evaluation function is extracted alongside the grammar. The evaluation function is applied to the BSR set to compute a semantic value for each of the derivations by executing the semantic actions of the combinator expression (this step is hereafter referred to as the 'semantic phase'). A similar architecture for developing complete parsers with grammar combinators was first presented by Ridge [25]. In [39] it was shown that parser combinators can use the datastructures of FUN-GLL to compute BSR sets directly, thereby removing the need to compute an intermediate grammar object. The key insight is that the grammar information used by FUN-GLL can be computed locally for each nonterminal. It is this insight that makes it possible to generate complete, modular and reusable top-down parsers from Happy grammars and to translate Happy's parameterized nonterminals directly.

3.1 The FUN-GLL algorithm

The following paragraphs summarize the explanations of the FUN-GLL algorithm in [37–39]. An example run of the algorithm is given in Table 1, with the output shown in Figure 3, demonstrating how the algorithm deals with left-recursion and ambiguity. The example is taken from [39] based on the grammar used by Ridge [25].

Consider a simple recursive descent parser with a parse function for each symbol of a grammar that receives as argument an index *l* into the input sentence and returns an index $r \ge l$. If the parse function for x returns r given l, then x matches the subsentence ranging from l to r-1. The indices l and r are referred to as the left extent and right extent of the match respectively. The FUN-GLL algorithm generalizes such recursive descent parsers by trying all alternates of x in order to find all right extents. The algorithm prevents duplicate work, and thereby non-termination and exponential running times, by labelling parts of the work with descriptors and ensuring that no descriptor is 'processed' twice. A descriptors is a triple $((x, \alpha, \beta), l, k)$ with (x, α, β) a *slot* and $k \ge l$ integers. A slot (x, α, β) identifies the point in the alternate $\alpha\beta$ of x preceded by the symbols α and succeeded by the symbols β . A descriptor $((x, \alpha, \beta), l, k)$ denotes progress towards checking whether the alternate $\alpha\beta$ can be used to match a subsentence with left extent l, indicating that the

#	processed descriptor	action	uset ext .	bsrs ext.	grel extension	prel ext.
1	$\langle E ::= \bullet EEE, 0, 0 \rangle$	descend	1,2,3		$\langle\langle E, 0 \rangle, \langle E ::= E \cdot EE, 0 \rangle\rangle$	
2	$\langle E ::= \bullet a, 0, 0 \rangle$	continue	4	5		
3	$\langle E ::= \bullet, 0, 0 \rangle$	ascend	5	1,2		$\langle\langle E,0\rangle,0\rangle$
4	$\langle E ::= a \cdot, 0, 1 \rangle$	ascend	6	6		$\langle\langle E,0\rangle,1\rangle$
5	$\langle E ::= E \cdot EE, 0, 0 \rangle$	continue	7,8	3,7	$\langle \langle E, 0 \rangle, \langle E ::= EE \cdot E, 0 \rangle \rangle$	
6	$\langle E ::= E \cdot EE, 0, 1 \rangle$	descend	9,10,11		$\langle\langle E, 1 \rangle, \langle E ::= EE \cdot E, 0 \rangle\rangle$	
7	$\langle E ::= EE \cdot E, 0, 0 \rangle$	continue	12,13	4,9	$\langle\langle E, 0 \rangle, \langle E ::= EEE \cdot, 0 \rangle\rangle$	
8	$\langle E ::= EE \cdot E, 0, 1 \rangle$	descend	9,10,11		$\langle \langle E, 1 \rangle, \langle E ::= EEE \cdot, 0 \rangle \rangle$	
9	$\langle E ::= \bullet EEE, 1, 1 \rangle$	descend	9,10,11		$\langle\langle E, 1 \rangle, \langle E ::= E \cdot EE, 1 \rangle\rangle$	
10	$\langle E ::= \bullet a, 1, 1 \rangle$	continuc				
11	$\langle E ::= \bullet, 1, 1 \rangle$	ascend	8,13 ,14	8,10,11,12		$\langle\langle E, 1\rangle, 1\rangle$
12	$\langle E ::= EEE \bullet, 0, 0 \rangle$	ascend	5,7,12	2,3,4		$\langle\langle E,0\rangle,0\rangle$
13	$\langle E ::= EEE \cdot, 0, 1 \rangle$	ascend	6,8,13	6,7,9		$\langle\langle E,0\rangle,1\rangle$
14	$\langle E ::= E \cdot EE, 1, 1 \rangle$	continue	15	13	$\langle\langle E, 1 \rangle, \langle E ::= EE \cdot E, 1 \rangle\rangle$	
15	$\langle E ::= EE \cdot E, 1, 1 \rangle$	continue	16	14	$\langle\langle E, 1 \rangle, \langle E ::= EEE \cdot, 1 \rangle\rangle$	
16	$\langle E ::= EEE \bullet, 1, 1 \rangle$	ascend	8,13,14,15,16	8,10,12,13,14		$\langle\langle E,1\rangle,1\rangle$

Table 1. Example execution of FUN-GLL with $E := EEE \mid a \mid \epsilon$ and string "a". The BSR elements are given in Figure 3.

$\{\langle \mathbf{E} ::= \bullet, 0, 0, 0 \rangle,$	(1)
$\langle \mathbf{E} ::= \mathbf{E} \cdot \mathbf{E} \; \mathbf{E}, 0, 0, 0 \rangle,$	(2)
$\langle \mathbf{E} ::= \mathbf{E} \; \mathbf{E} \cdot \mathbf{E}, 0, 0, 0 \rangle,$	(3)
$\langle \mathbf{E} ::= \mathbf{E} \; \mathbf{E} \; \mathbf{E} \bullet, 0, 0, 0 \rangle,$	(4)
$\langle \mathbf{E} ::= \mathbf{a} \bullet, 0, 0, 1 \rangle,$	(5)
$\langle \mathbf{E} ::= \mathbf{E} \cdot \mathbf{E} \; \mathbf{E}, 0, 0, 1 \rangle,$	(6)
$\langle \mathbf{E} ::= \mathbf{E} \; \mathbf{E} \bullet \mathbf{E}, 0, 0, 1 \rangle,$	(7)
$\langle \mathbf{E} ::= \mathbf{E} \; \mathbf{E} \bullet \mathbf{E}, 0, 1, 1 \rangle,$	(8)
$\langle \mathbf{E} ::= \mathbf{E} \; \mathbf{E} \; \mathbf{E} \bullet, 0, 0, 1 \rangle,$	(9)
$\langle \mathbf{E} ::= \mathbf{E} \; \mathbf{E} \; \mathbf{E} \bullet, 0, 1, 1 \rangle,$	(10)
$\langle E ::= \bullet, 1, 1, 1 \rangle,$	(11)
$\langle \mathbf{E} ::= \mathbf{E} \cdot \mathbf{E} \; \mathbf{E}, 1, 1, 1 \rangle,$	(12)
$\langle \mathbf{E} ::= \mathbf{E} \; \mathbf{E} \bullet \mathbf{E}, 1, 1, 1 \rangle,$	(13)
$\langle \mathbf{E} ::= \mathbf{E} \; \mathbf{E} \; \mathbf{E} \bullet, 1, 1, 1 \rangle \}$	(14)

Figure 3. BSR set for $E := EEE \mid a \mid \epsilon$ and string "a".

symbols in α have matched with left extent l and right extent k. At any time during the execution of the algorithm, the set uset holds the descriptors encountered so far.

The relation *prel* holds the right extents discovered for a pair (x, l) of a nonterminal and a left extent, referred to as a *commencement* (as a dual to 'continuation'). If the element ((x, l), r) is in *prel*, then r is a right extent for the commencement (x, l). Relation *prel* is reminiscent of a memoization table for a parse function of x.

The relation *grel* associates a commencement with zero or more continuation identifiers. A continuation identifier

 $((x, \alpha, \beta), l)$ is essentially a descriptor with a hole for the second index, which is to be filled by every right extent discovered for the sequence of symbols α . If $((y, k), ((x, \alpha, \beta), l))$ is in *grel* and if right extent r is discovered for the commencement (y, k), this indicates that r is a right extent for α and thus that the descriptor $((x, \alpha, \beta), l, r)$ needs to be processed (if not in *uset*). The algorithm is such that if $((y, k), ((x, \alpha, \beta), l))$ is in *grel*, then the last symbol of α is y and there is a descriptor $((x, \alpha', y\beta), l, k)$ in *uset* with $\alpha = \alpha' y$.

The algorithm records that descriptor $((x, \alpha', y\beta), l, k)$ and the right extent r for commencement (y, k) give rise to the descriptor $((x, \alpha'y, \beta), l, r)$ by adding the BSR element $((x, \alpha'y, \beta), l, k, r)$ to the set *bsrs*. The index *k* in some sense connects the two consecutive descriptors $((x, \alpha', y\beta), l, k)$ and $((x, \alpha'y, \beta), l, r)$ and is therefore referred to as a pivot. At any moment, bsrs contains enough information to retrace the entire parse up to that moment because bsrs records the pivots for every pair of consecutive descriptors. The reader is referred to [31] for a full explanation on how a BSR set embeds derivations. As explained later, the semantic phase revisits all traces of complete matches of the sentence (potentially subjected to ambiguity reduction strategies) and executes semantic actions along the way. Based on the introduced sets and relations, the FUN-GLL algorithm can be explained in terms of the actions descend, ascend, process and continue:

- To **descend** a commencement (y, k) with continuation identifier $((x, \alpha, \beta), l)$ means: extending *grel* to contain $((y, k), ((x, \alpha, \beta), l))$ and finding the set R such that for all $r \in R$ it holds that ((y, k), r) is in *prel*.
 - If $R \neq \emptyset$, then **continue** with descriptor $((x, \alpha, \beta), l, r)$ and pivot k, for every $r \in R$ and in any order

- If $R = \emptyset$, **process** the descriptor $((y, \epsilon, \delta), k, k)$, for every alternate δ of y and in any order (ϵ denotes the empty sequence of symbols)
- To **ascend** a commencement (x, k) with right extent r means: extending prel to contain ((x, k), r) and finding all continuation identifiers (s, l) for which it holds that ((x, k), (s, l)) is in grel and **continue** with every descriptors (s, l, r) and pivot k in any order
- To **continue** with a descriptor $((x, \alpha, \beta), l, r)$ and pivot k means: add the BSR element $((x, \alpha, \beta), l, k, r)$ to bsrs and then **process** descriptor $((x, \alpha, \beta), l, r)$
- A descriptor $((x, \alpha, \beta), l, r)$ is processed by adding it to *uset* and only if it was not already in *uset* do the following:
 - If β is the empty sequence of symbols, then **ascend** commencement (x, l) with right extent r.
 - If $\beta = t\beta'$ with t a token, match t it against the token at position r in the sentence and, if successful, **continue** with $((x, \alpha t, \beta'), l, r + 1)$ and pivot r
 - If $\beta = y\beta'$ with y a nonterminal, **descend** (y, r) with continuation identifier $((x, \alpha y, \beta'), l)$

The example run in Table 1 starts with processing the descriptors $\langle E ::= \bullet EEE, 0, 0 \rangle$, $\langle E ::= \bullet a, 0, 0 \rangle$ and $\langle E ::= \bullet, 0, 0 \rangle$. The symbol \bullet marks the position in an alternate identified by a slot. Duplicate set elements are striked (the action for descriptor #10 is striked because the character 'a' is not the second character in the input). The descriptors are processed in the order of first-in first out, meaning that the i-th descriptor to be encountered is also the i-th descriptor to be processed. Choosing this order has enabled the concise tabular form of presenting runs of the algorithm; any other order gives the same output.

3.2 Translating Happy grammars

This subsection explains, by example, how the GLL backend translates the parameterized nonterminals of Happy grammars to higher-order functions that implement the FUN-GLL algorithm and explains how the aforementioned 'semantic phase' is implemented. The example is provided by the definition of *CSV* in Figure 5 and the generated code in Figure 6. The translation of **%token** directives (and other Happy directives) has been omitted. Figure 4 shows the datastructures and operations of the FUN-GLL algorithm as provided by the support library of the implementation.

Every %**token** directive and every nonterminal definition of a Happy grammar generates a function that returns a *Symbol*. If generated for a parameterized nonterminal, the function has a parameter of type *Symbol* for every parameter of the nonterminal. As the example shows, when a symbol is used in the code generated for another symbol, it is

```
data SID
                 = Tn String | App String [SID]
type Slot
                 = (SID, [SID], [SID])
type Comm
                = (SID, Int) -- commencement
type CID
                 = (Slot, Int) -- continuation identifier
type CMap\ t = Map\ CID\ (Cont\ t)
type USet
                 \equiv Set Descr
addDescr
                 :: Descr \rightarrow USet \rightarrow USet
hasDescr
                 :: Descr \rightarrow USet \rightarrow Bool
type GRel
                 \equiv Set (Comm, CID)
addCont
                 :: Comm \rightarrow CID \rightarrow GRel \rightarrow GRel
                 :: Comm \rightarrow GRel \rightarrow [CID]
conts
type PRel
                 \equiv Set (Comm, Int)
addExtent
                 :: Comm \rightarrow Int \rightarrow PRel \rightarrow PRel
                 :: Comm \rightarrow PRel \rightarrow [Int]
extents
                 \equiv Set BSR
type BSRs
type BSR
                 = (Slot, Int, Int, Int)
                 :: BSR \rightarrow BSRs \rightarrow BSRs
addBSR
pivots
                :: (Slot, Int, Int) \rightarrow BSRs \rightarrow [Int]
```

Figure 4. The FUN-GLL datastructures and operations.

```
CSV(v): CSV(v)', CSV(v) \{\$1 + \$3\}
| v {[\$1]}
```

Figure 5. A left-recursive parameterized nonterminal.

irrelevant whether the symbol is made available as a parameter or is defined in the same namespace.

```
data Symbol t \ a = Symbol
\{id_{sm} :: SID, match_{sm} :: Matcher \ t, eval_{sm} :: Evaluator \ t \ a\}
```

Besides an identifier, every Symbol consists of a 'matcher' and an 'evaluator', implementing FUN-GLL and the semantic phase respectively. A symbol identifier (SID, see Figure 4) is either a token name (Tn) or the name of a nonterminal applied to zero or more symbol identifier arguments (App). The type parameters t and a of Symbol are for the type of tokens and the type of semantic values produced by the evaluator. The generated code in Figure 6 shows how the slots (named $s_0, ..., s_5$ in the code) for the definition of CSV are computed based on the symbols and the parameters that are mentioned in the definition of CSV.

A matcher (*Matcher*) receives an index (pivot or right extent) and a continuation paired with its continuation identifier (of type *CID*). A matcher exhibits the necessary effects on the FUN-GLL datastructures (*Data*) as prescribed by either the **continue**, **ascend** or **descend** action, explained in the previous subsection. An immutable array holds the input sentence for fast access.

```
type Matcher t = Int \rightarrow (CID, Cont \ t) \rightarrow Data \ t \rightarrow Data \ t
type Cont t = Int \rightarrow Int \rightarrow Data \ t \rightarrow Data \ t
```

 $^{^{7}}$ Some type definitions use ≡ rather than = to indicate that the datastructures are equivalent to sets but are implemented differently for efficiently.

```
sCSV \ x\_1 = Symbol \ nt \ matcher \ evaluator
where
   nt = App "CSV" [id_{sm} x_1]
   matcher\ l\ (c,cf) = descend\ (nt,l)\ (c,cf)\ (c_0\ l\ l\circ c\_4\ l\ l)
         c_0 \ k \ r = continue (s_0, l, k, r) \ (match_{sm} \ (sCSV \ x_1) \ r \ ((s_1, l), c_1))
         c_1 k r = continue (s_1, l, k, r) (match_{sm} sCom r ((s_2, l), c_2))
         c_2 \ k \ r = continue (s_2, l, k, r) \ (match_{sm} \ (sCSV \ x_1) \ r \ ((s_3, l), c_3))
         c_3 k r = continue(s_3, l, k, r) (ascend(nt, l) r)
         c_4 k r = continue(s_4, l, k, r) (match_{sm} x_1 r ((s_5, l), c_5))
         c \ 5 \ k \ r = continue (s \ 5, l, k \ r) (ascend (nt, l) \ r)
   s_0 = (nt, [], [id_{sm} (sCSV x_1), id_{sm} sCom, id_{sm} (sCSV x_1)])
   s_1 = (nt, [id_{sm} (sCSV x_1)], [id_{sm} sCom, id_{sm} (sCSV x_1)])
   s_2 = (nt, [id_{sm} (sCSV x_1), id_{sm} sCom], [id_{sm} (sCSV x_1)])
   s_3 = (nt, [id_{sm} (sCSV x_1), id_{sm} sCom, id_{sm} (sCSV x_1)], [])
   s_4 = (nt, [], [id_{sm} x_1])
   s_5 = (nt, [id_{sm} x_1], [])
   evaluator in p bsrs l r nts | nt \in nts = []
                                 | otherwise = res
      where
         nts' p q \mid l \not\equiv p \lor r \not\equiv q = empty
                   otherwise
                                   = insert nt nts
         res = [sv_1 + sv_3] -- semantic action of first alternate
                | p_3 \leftarrow pivots (s_3, l, r) bsrs
                , p_2 \leftarrow pivots (s_2, l, p_3) bsrs
                , p_1 \leftarrow pivots (s_1, l, p_2) bsrs -- p_1 \equiv l
                , sv_3 \leftarrow eval_{sm} (sCSV \ x_1) inp bsrs p_3 r (nts' p_3 r)
                , sv_2 \leftarrow eval_{sm} sCom inp bsrs p_2 p_3 (nts' p_2 p_3)
                , sv_1 \leftarrow eval_{sm} (sCSV x_1) inp bsrs l p_2 (nts' l p_2)
                [ [sv_1] -- semantic action of second alternate
                , p_1 \leftarrow pivots (s_5, l, r) bsrs - p_1 \equiv l
                | sv_1 \leftarrow eval_{sm} x_1 \text{ inp bsrs } l r (nts' l r) ]
```

Figure 6. Code generated by the GLL back-end for the production of Figure 5. *sCom* is the *Symbol* for token ', '.

```
data Data t = Data { uset :: USet, bsrs :: BSRs, grel :: GRel , cmap :: CMap t, prel :: PRel, inp :: Array Int t }
```

A continuation identifier is a pair of a slot and a left extent, as in the previous subsection. The continuation itself (Cont) captures the behavior of the **continue** action of the previous section. The functions c_0, \ldots, c_5 in the generated code are the continuation functions defined for CSV, with one for every slot. These definitions are local to the definition of the matcher for CSV because the matcher is called with the left extent l. The slot for which the continuation is defined (e.g. s_0 for c_0), the left extent l (parameter of matcher) and the parameters of the continuation (k and k) form the descriptor and BSR element required to perform the **continue** action. The support function continue receives both as a single (BSR) argument and implements the behavior of **continue**.

```
continue :: BSR \rightarrow (Data\ t \rightarrow Data\ t) \rightarrow Data\ t \rightarrow Data\ t

continue bsr cf d = maybeProcess\ bsr\ cf\ (addBSR\ bsr\ d)
```

```
maybeProcess(s, l, k, r) cf d
| hasDescr(s, l, r) (uset d) = d
| otherwise = cf(d \{uset = addDescr(s, l, r) (uset d)\})
```

The BSR element is always added to bsrs by continue, whereas cf (capturing the effect of the next processing step) is only applied if the descriptor is not already in uset. The second argument of *continue* is the result (a function *Data* $t \rightarrow$ Data t) of applying the matcher for the next symbol to be matched according to the slot, i.e. the first symbol in the third component of the slot (for example, the parameter x_1 in the case of c_4 and s_4). The matcher is applied to the right extent r and the continuation and continuation identifier pair for the 'next' slot (e.g. s_1 in the case of s_0 , s_2 in the case of s_1 , etc.) with the same left extent. Or, if the third component of the slot is empty (e.g. in the case of s_3 and s_5), the support function ascend is applied, corresponding to the ascend action. In other words, the continuation function generated for slot (x, α, β) supplies the matcher function generated for the first symbol of β to *continue* or supplies the function ascend if there is no such symbol. In this way, the **process** action is performed.

The function *ascend* is given a commencement (nt, l) and a right extent r and performs the **ascend** action. The **ascend** action involves looking up all continuation identifiers c stored in *grel* for the given commencement. The continuation identified by c is recorded in the map cmap (see the definition of descend below).

```
ascend :: Comm \rightarrow Int \rightarrow Data\ t \rightarrow Data\ t

ascend (nt, l)\ r\ d = foldr\ ((\$) \circ (\lambda cf \rightarrow cf\ l\ r))\ id\ cs\ d'

where d' = d\ \{prel = addExtent\ (nt, l)\ r\ (prel\ d)\}

cs = map\ (flip\ lookup\ (cmap\ d))\ (conts\ (nt, l)\ (grel\ d))
```

The effects of the different continuations applied to l and r (functions from $Data \rightarrow Data$, one for every c) accumulate by function composition. The order of composition does not influence the outcome since descriptors can be processed in any order by FUN-GLL.

The matcher for a token (not shown here) is defined in terms of a support function like matchPattern (see Section 2). The matcher for a nonterminal with name nt applies descend (see the definition of matcher for CSV) to commencement (nt, l) and a continuation cf paired with its identifier c. The arguments l and (c, cf) are themselves inputs of the matcher, supplied when continuations are applied (see the definitions of c_0, \ldots, c_5 in the code generated for CSV). Function descend also receives the effects of processing the alternates of nt. In the case of the example, the effects of the alternates of CSV are produced by composing the effects of applying the continuations c_0 and c_4 that match the first symbol of each of the two alternates.

```
descend :: Comm \rightarrow (CID, Cont \ t) \rightarrow (Data \ t \rightarrow Data \ t)
\rightarrow Data \ t \rightarrow Data \ t
descend \ (x, l) \ (c, cf) \ alts \ d
```

```
| null rs = alts d'

| otherwise = foldr ((\$) \circ (\lambda r \rightarrow cf \ l \ r)) id rs d'

where rs = extents (x, l) (prel d)

d' = d \{ grel = addCont (<math>x, l) c \ (grel \ d)

, cmap = insert c \ cf \ (cmap \ d) \}
```

Depending on whether there are right extents (rs) stored in prel for the given commencement (nt, l), descend either applies the continuation cf to each of the right extents or applies the effects of the alternates (alts). In both cases grel is extended.

The definition of CSV is left-recursive, and its matcher will call itself without consuming tokens from the input sentence (when continuation c_0 is applied with r=l). This does not result in non-termination however, because the first application of *continue* adds the descriptor (s_0, l, r) (with l=r) to uset, which is 'noticed' during the second application.

Semantic phase. The function *run*, defined⁸ below, turns a *Symbol* into a parser, a function from a sentence to a list of semantic values.

```
run :: Symbol t a \rightarrow Array Int t \rightarrow [a]

run x str = eval_{sm} str (bsrs d_{-}1) 0 (length str) empty

where -- below, all omitted fields are empty

d_{-}0 = Data { inp = str, uset = empty, ... }

-- below, id is the identity function over Data t d_{-}1 = match_{sm} 0 (c_{0}, id) d_{-}0

-- "__START" is an artificial start symbol c_{0} = ((App \text{ "}_{-}START\text{ "}[], [id_{sm} x], []), 0)
```

The result of a parser contains a semantic value for every derivation of the sentence encoded in the BSR set computed by the matcher of the *Symbol*. The semantic values are based on the semantic actions of the grammar and are computed by the evaluator of the *Symbol*. Although not shown here, the GLL back-end allows the user to implement disambiguation strategies as filters over semantic values computed from a BSR set. These strategies can be based on precedence levels, associativity rules or the semantic values themselves. The remainder of this section assumes that no disambiguation strategies are in place.

As described next, without disambiguation strategies, the semantic phase has worst-case exponential running times as grammar exists with exponentially many derivations of sentences. In fact, a grammar with a nonterminal that is both left- and right-recursive can yield infinitely many derivations. However, to prevent non-termination, the semantic phase does not consider all the derivations of such *cyclic nonterminals*. A similar semantic phase was first described by Ridge [25].

```
type Evaluator t a = Array Int t \rightarrow BSRs \rightarrow Int \rightarrow Int \rightarrow Set Symbol \rightarrow [a]
```

An evaluator function (third component of a Symbol) receives the input sentence inp, a set of BSR elements bsrs, a left extent l, a right extent r and a set of (nonterminal) symbols nts. It returns a semantic value for every derivation of the subsentence of *inp* ranging from l to r-1 encoded in bsrs, unless it is an evaluator for a nonterminal that is in nts. The evaluator function for the nonterminal *nt* is defined in terms of the evaluator functions of the symbols occuring in the definition of nt (see the usage of $eval_{sm}$ in Figure 6). The evaluator for *nt* might call itself recursively, as is the case for CSV. It is possible to detect recursive calls, ensuring termination of the semantic phase, by maintaining a set of encountered nonterminals nts and seeing whether the evaluator for nt is called with $nt \in nts$. In order to detect only that the evaluator is simultaneously left-recursive and rightrecursive, the algorithm empties the set nts whenever l or r changes between two evaluator calls (see the definition of nts' in the example code).

The result of the evaluator for nt (res in Figure 6) is the concatenation of the results for the alternates of nt. If an alternate has k >= 1 symbols, then the evaluator finds all the k-length splits of the subsentence of inp ranging from l to r-1 such that every *i*-th element of the split is matched by the *i*-th symbol of the alternate. In essense, finding one or more such splits is what parsing algorithms do. However, in this case, the information necessary to compute the splits is already encoded as the pivots in *bsrs*. The function *pivots* (see Figure 4) is given a triple (s, l, r) and returns all the pivots k such that $(s, l, k, r) \in bsrs$. The slots s are of the form $(x, \alpha y, \beta)$. By producing pivot k, the BSR set informs the evaluator that α matches with left extent l and right extent k and γ matches with left extent k and right r. The evaluator finds all splits by continuing with the triple $((x, \alpha, y\beta), l, k)$ and so on for all k and until all slots have been seen (except those of the form (x, ϵ, β)). For every split, the evaluator of the i-th symbol of the alternate is called with the left and right extents of the *i*-th element of the split. The semantic values produced by these calls form the inputs of the semantic action that is attached to the alternate.

If k=0, then a singleton list with the result of the semantic action is returned, or the empty list, if there is no BSR element $((nt, \epsilon, \epsilon), l, l, r) \in bsrs$ (which can only be true if l=r).

The evaluator for a token (not shown) checks whether the given left and right extent are one apart, i.e. whether l+1=r, and if so returns the singleton list with the matched token in it (and the empty list otherwise).

4 Implementation

The supplementary material of this paper contains a version of Happy that implements the GLL back-end. Aspects of this

⁸The support library defines variants of run to make partial parsers (and implementing the %partial directive) and parsers that return a value of type $Either\ [String\]\ [a]$ of which the left component is a sequence of errors.

implementation are discussed in this section. The completeness with respect to the LALR back-end is discussed, as well as several avenues for improvements and extensions.

Datastructures and operations. The previous section describes the essential data structures of the FUN-GLL algorithm as sets, omitting their actual implementation and focusing on the method of generating parsers. However, the efficiency and worst-case complexity of FUN-GLL are strongly influenced by the datastructures and their operations. A direct implementation as sets (e.g. from *Data.Set*) is inefficient. The type *USet* is therefore defined as nested integer tries [23], with a nesting-level for the two integers of descriptors:

```
type USet = IntMap (IntMap (Set Slot))
```

This approach is not sufficient however: the evaluation section shows that the GLL back-end is significantly slower than in the LALR and GLR back-ends when applied to LALR grammars. The implementation work supporting this paper has been used to demonstrate the capabilities of the GLL back-end and to build up a collection of examples that can be used for testing faster implementations. Alternative implementations may be explored in future work. Faster parser generation strategies are also possible if modularity and reuse are not required.

Lookahead. The GLL back-end does not currently implement a form of lookahead. However, lookahead sets can be computed dynamically for every slot in a way similar to how the slot itself is computed. A possible approach is to extend the type *Symbol* with lookahead sets and to generate expressions that computes these sets. Lookahead sets can be computed statically if modularity and reuse are not required.

Monadic actions. Figure 7 shows a Happy grammar defining a small expression language for integer arithmetic by using several additional features of Happy implemented by the GLL back-end. The semantic actions define an interpreter for this language. The %monad directive influences the type signatures of the generated functions as well as the treatment of semantic actions. In the example, the semantic values produced by the semantic actions are computations in the SeedM monad, as specified by the %monad directive. The language is defined such that whenever a '#' is encountered, a fresh integer is generated by applying enroll.

The example shows four kinds of semantic actions, distinguished by the zero, one, two or three percentage symbols that precede the action. The first kind { ... } is used to write semantic actions in the same way with or without %monad directive (e.g. the alternate for multiplication). Since there is a %monad directive in the example, the action is transformed before it is inserted in the generated parser, to form an equivalent of:

```
do v_1 \leftarrow x_1
v_2 \leftarrow x_2
```

```
%error { error ∘ show }
%monad { SeedM Int }
                   -- token directives have been elided
%token ...
%left '-' '+'
                  -- order of %left directives
%left '*' '/' -- determines precedence
%%
Expr
   : Expr '+' Expr {%
                              return~(~\$1+~\$3)\}
   | Expr '-' Expr \{\%\% (-) \langle \$ \rangle \$1 \langle * \rangle \$3 \}
   | Expr '*' Expr {
                               $1 * $3}
   | Expr'' | Expr {\%\% [ div \$ $ $1 \* $ $3
                               | 0 \neq giveValue \$3 ] 
   (' Expr')'
   | digit
                               $1}
                       {
   | '#'
                               enroll }
data SeedM s a = SeedM \{ runSeedM :: (s \rightarrow (s, a)) \}
instance Monad (SeedM s) where
   return a = SeedM \ (\lambda i \rightarrow (i, a))
   (SeedM \ p) \gg mq = SeedM \ \$ \ \lambda seed \rightarrow
     let (seed', pv) = p seed
     in runSeedM (mq pv) seed'
giveValue :: SeedM Int a \rightarrow a
giveValue\ sm = snd\ (runSeedM\ sm\ 1)
enroll :: Enum \ s \Rightarrow SeedM \ s \ s
enroll = SeedM (\lambda i \rightarrow (succ i, i))
```

Figure 7. Example grammar demonstrating additional Happy features implemented by the GLL back-end.

```
v_3 \leftarrow x_3
return (v_1 * v_3) -- for the action \{\$1 * \$3\}
```

Informally, the three monadic parse results for the three symbols of the alternate are 'run' to yield their values. The semantic action written by the programmer is the returned expression, after replacing \$1 and \$3 with the identifiers binding the semantic values of the first and third symbol. The second kind {%...} of semantic action differs in that there is no implicit application of return; the user expression must be of the right monadic type. This means that the programmer has access to the monad in the semantic action (e.g. see enroll in the alternate for '#'). In the third kind {%%...}, the action parameters \$1, \$2, \$3, etc. are replaced by the identifiers binding the monadic parse results of the alternate's symbols without 'running' them. The programmer thus has complete control over the monadic computation constructed by the semantic action (e.g. see the alternate for subtraction). On top of this, the fourth⁹ kind { %%%...} gives the programmer access to the list-monad of

 $^{^9\}mathrm{If}$ there is no $\%\mathbf{monad}$ directive, the behavior of the fourth kind is given to the second kind of semantic action.

the evaluation phase. This powerful feature can be used to implement arbitrary disambiguation strategies based on the semantic values of sub-expressions. In the example, this feature is used to rule out expressions in which a division by zero error would occur.

Error handlers. The example also shows that the **%error** directive can be used to specify a handler for parse errors. This is not trivial, because the GLL algorithm explores all possible interpretations of an input sentence. In the case of a parse failure, there are likely several points of failure and it is not clear which should be reported as errors. A wrapper function is generated for each production of a grammar that applies the parser for the production and yields errors as the left component of an *Either*. There are *n* error values ¹⁰, one for each of the *n* parsing attempts reaching the furthest into the input sentence. The number *n* is determined by a configuration option during parser generation or at runtime.

Disambiguation. The disambiguation directives %left, %right and %nonassoc behave slightly different in the GLL back-end than the LALR and GLR back-ends. This is because the GLL back-end implements the directives by filtering BSR sets rather than resolving conflicts in a parse table. The precise difference in behavior is to be investigated further.

Threaded lexers. The %lexer directive of Happy makes it possible to parse based on a 'threaded lexer monad' that propagates the input sentence in its state. This enables error handlers and semantic actions that interact with the lexer state, for example to produce messages based on the line and column number of the next character in the input. This feature has not been implemented. An implementation would require the parsing algorithm to be able to 'reset' lexer state in order to explore the possibly many ways a sentence can be parsed.

Attribute grammars. The semantic actions of Happy allow the programmer to write arbitrary syntax-directed translations in the style of 'The Dragon Book' [3]. The Attribute Grammar formalism [19] can be explained as a particular way of writing syntax-directed translations in which semantic equations define attributes in terms of each other. The equations are solved by an evaluator that traverses a program by 'visiting' its components possibly many times until all attributes are assigned a value. Haskell's lazy-evaluation makes it possible to generate evaluation functions for each production of the attribute grammar in a modular fashion [26, 33]. The evaluator fails to terminate, however, if the dependencies between attributes form a cycle. Happy employs this strategy and implements attribute grammars as an alternative to semantics actions. There should be no problem making attribute grammar evaluation available to the GLL

back-end as Happy's evaluation strategy for attribute grammars is a natural fit with the semantic phase of the generated GLL parsers. However, the current implementation does not yet demonstrate this.

5 Evaluation

This section evaluates characteristics of the GLL back-end implementation in comparison to the LALR and GLR back-ends. The modularity and compositionality aspects that are unique to the GLL back-end have been discussed in Section 2. All experiments have been executed under Linux mint 20.3 on a laptop with an Intel i7-8565U (8) @ 4.600GHz CPU and 16GiB of RAM using version 8.6.5 of the Glasgow Haskell Compiler (GHC). The tools and files necessary to reproduce the results of the experiments are provided as supplementary material.

Beyond context-free grammars. Happy's parameterized nonterminals can be used to write grammars for languages that are not context-free. For example, the following code fragment shows a grammar for the language $a^nb^nc^n$ with $n \ge 1$ (this example is taken from [11]).

```
Start : F ('a', 'b', 'c')
F (x, y, z) : F (Seq (x, 'a'), Seq (y, 'b'), Seq (z, 'c'))
| x y z
Seq (p, q) : p q
```

The LALR and GLR back-ends depend on a fix-point algorithm that replaces parameterized nonterminals with specialized (non-parameterized) variants in a process similar to macro-expansion. However, this algorithm cannot be used to remove parameterized nonterminals that apply themselves recursively with arguments that change with every recursive call. In this example, the algorithm tries to compute the grammar that has infinitely many specializations of F, one for every choice of n. Happy does not detect this and fails to terminate on this example without warning.

As discussed in Sections 2 and 3, the GLL back-end generates parsers that take advantage of Haskell's abstraction mechanism to implement parameterized nonterminals so that applications are executed dynamically. In the case of this example, a GLL parser is generated, but it fails to terminate on any input sentence. This is because each recursive call to the parser implementing F produces a fresh nonterminal name and thus a descriptor that is unique to the call and therefore not already in the descriptor set. This can be seen as a higher-order variant of the problem of left-recursion.

The grammar in the following fragment generates the context-sensitive language $\sum_{i=0}^{\infty} (b^i a c^i)$. That is, the language $\{"a", "a(a)", "a(a)((a))", ...\}$ with '(' instead of 'b' and ')' instead of 'c' for clarity.

```
Start : List ('a')
List (e) : e
```

¹⁰Their type is inferred from the error handler.

```
| e List (Parens (e))
Parens (e) : '(' e ')'
```

The recursion of *List* is on the right and the GLL parser generated for this grammar indeed recognizes the language. This is because every recursive call to the parser for *List* receives an index into the input sentence that is closer towards the end of the sentence. No further recursive calls are made once the whole sentence is 'consumed'. This observation suggests that a variant of the curtailment procedure of [12] can overcome the problem of higher-order left-recursion by making at most as many recursive calls as there are tokens left in the input sentence. The other back-ends cannot handle this example for the reason mentioned before.

Permutation phrases. The next experiment is about parsing 'permutation phrases' [4]. A similar experiment has been performed in [39]. A permutation phrase is a sequence of permutable elements in which each element occurs exactly once and in any order [8]. In the permutation phrases considered here, elements occur at most once. Real-world examples of such permutation phrases are the modifiers associated with fields and methods in Java [35] and the declaration specifiers of C [8]. The syntax of permutation phrases can be captured by a nonterminal with an alternate for each of the possible permutations. Such a grammar is not practical as the number of alternates grows exponentially with the number of permutable elements. The following fragment shows how the syntax of permutation phrases is captured conveniently with parameterized nonterminals. This formulation is based on the PermP3 example of [35]. The permutable elements are the digits 1 to 4. The character '\$' is assumed not to occur in any input sentence, thus ensuring that the parser for Nul always fails. Each alternate of Choose chooses one of the elements and makes a recursive call to continue choosing. In the recursive call, the chosen element is replaced with Nul so that it can no longer be chosen.

```
Permutations: Choose ('1', '2', '3', '4') { $1}

Choose (a, b, c, d): {[]}

| a Choose (Nul, b, c, d) { ( $1: $2)}

| b Choose (a, Nul, c, d) { ( $1: $2)}

| c Choose (a, b, Nul, d) { ( $1: $2)}

| d Choose (a, b, c, Nul) { ( $1: $2)}

Nul : '$' { '$'}
```

Happy's algorithm for removing parameterized nonterminals successfully generates an equivalent grammar for this example. However, the resulting grammar has exponentially many alternates relative to the number of permutable elements. Table 2 demonstrates the exponential growth by showing data about the LALR parsers generated for the syntax of permutation phrases with four, five and six elements. The table shows the time it took to generate the parser, the size of the generated parser in kilobytes, the time it took GHC to compile this parser and the time it took the parser to

Alg.	#	Generate	Size	Compile	Parse
LALR	4	0.009s	45.0KiB	1.226s	0.001s
	5	0.054s	88.0KiB	3.003s	0.001s
	6	0.679s	191.0KiB	11.751s	0.001s
GLL	4	0.004s	34.0KiB	0.964s	0.001s
	5	0.004s	35.0KiB	0.983s	0.002s
	6	0.004s	37.0KiB	1.026s	0.002s

Table 2. Parsing permutation phrases.

Alg.	194		934		
LALR	0.0s	0.0s	0.01s	0.01s	0.01s
GLR	0.01s	0.02s	0.04s	0.06s	0.08s
GLL	0.02s	0.04s	0.11s	0.17s	0.29s

Table 3. Running times for LBNF pipelines.

parse a permutation. The data for the GLL parsers generated from the same grammars, without removing parameterized nonterminals, show a small linear growth instead.

LALR grammars. In the next experiment, the running times of the parsers generated by the three different backends are compared with LALR grammars. The BNF Convertor (BNFC) generates lexers, parsers and abstract syntax from a single grammar description written in the Labelled BNF (LBNF) formalism [1]. In fact, the convertor generates a small pipeline that runs the lexer and parser, creates an abstract syntax tree and pretty-prints it. The BNFC tool is capable of generating two types of Happy grammars from an LBNF grammar, one for the LALR back-end - to which the GLL back-end can also be applied - and another specialized for the GLR back-end, making it the perfect tool for generating the inputs of this experiment. The BNFC project includes a substantial amount of LBNF grammars for realworld languages such as Java, ANSI-C and Prolog. However, the examples do not come with many tests and it is not always clear for which version of the language the syntax has been described. Experiments have been performed with pipelines for ANSI-C and for LBNF itself.

The experiment with the LBNF pipeline has been conducted with the LBNF grammars for Prolog, LBNF itself, OCL (Object Constraint Language), GF (Grammatical Framework) and ANSI-C as test inputs. Table 3 shows the running times of executing the pipeline on these inputs. The column headers contain the number of tokens produced by the lexer of the pipeline. The only difference between the LALR, GLR and GLL rows of the table are the back-end used to generate the parser of the pipeline.

The experiment with the ANSI-C pipeline has been conducted in a similar way, with two small programs and one large program taken from the BNFC repository. The largest program is large indeed, with over 7500 lines of code and

Alg.	46	271	7261	15425(x2.1)	26106(x1.7)
LALR	0.0s	0.0s	0.05s	0.1s(x2.0)	0.18s(x1.8)
GLR	0.01s	0.02s	0.57s	1.23s(x2.2)	2.0s(x1.6)
GLL	0.04s	0.23s	7.13s	15.71s(x2.2)	28.41s(x1.8)

Table 4. Running times for ANSI-C pipelines.

187KB in size. Two smaller, but still large, programs have been constructed by copying to separate files the first 2500 and 5000 lines of code of the largest program. This has been done to demonstrate linear growth in running times. Table 4 shows the running times of executing the pipeline for ANSI-C on the five input programs. The last and second to last columns have multipliers indicating the growth of the value in the cell with respect to the previous column.

Tables 3 and 4 show that the LALR parsers are significantly faster than the GLR parsers which in turn are significantly faster than the GLL parsers on LALR grammars. The implementation of the GLL back-end has not yet been optimized for performance. Various possible approaches to speeding up the GLL parsers have been discussed in the previous section. For example, lookahead sets can be computed dynamically (to preserve modularity). Alternatively, GLL parsers can be generated without a concern for modularity and parameterized nonterminals, thereby allowing more optimizations such as statically computed lookahead sets.

Highly ambiguous grammars. In the next experiment, the GLL and GLR back-ends are tested on highly ambiguous grammars. The chosen grammars have been used by Ridge [25] in a comparison with the GLR back-end of Happy. The grammars form a significant stress-test for complete parsers. The LALR back-end is not used in the experiment because it does not produce complete parsers. Each grammar consists of a single non-terminal and generates the language a^n with $n \ge 0$. The nonterminals are defined as follows:

$$S_1$$
: 'a' S_1 S_1 S_2 : S_2 S_2 'a' $E: E E E$ $|$ 'a'

The grammars are such that the number of derivations grows exponentially relative to the size of the input. Efficiently *enumerating* all derivations is therefore not possible and the GLR and GLL parsers are used for recognition only. The GLL parsers still compute a BSR set that embeds all derivations.

Table 5 shows the running times of the GLR and GLL parsers given inputs that contain between 20 to 200 repetitions of the character 'a'. The GLL parsers for these grammars are significantly faster than the GLR parsers. In fact, the GLR parsers for nonterminals S_1 and E seem to suffer from exponential blow up (the GLR parsers for S_1 and E ran

Alg.	Nt.	20	30	40	50	100	200
GLR	S_1	0.11s	0.81s	4.18s	13.96s	-	1
	S_2	0.004s	0.01s	0.02s	0.03s	0.32s	3.46s
	E	0.17s	1.73s	8.18s	34.96s	-	-
GLL	S_1	0.006s	0.02s	0.05s	0.08s	0.82s	7.43s
	S_2	0.007s	0.01s	0.03s	0.06s	0.45s	3.1s
	E	0.012s	0.03s	0.08s	0.12s	1.06s	7.72s

Table 5. Recognition times for highly ambiguous grammars.

unsuccessfully with a timeout of 500 seconds on 100 tokens) although the GLR algorithm theoretically does not [30, 36].

6 Related work

The GLL algorithm is a relatively new addition to the parsing landscape [27, 28], with several variations [2, 29, 38, 39], that has rekindled the interest in top-down parsing. Implementations of the algorithm are not yet widespread, but can be found in parser generators [6, 29] and combinator libraries [15, 38, 39]. To our knowledge, the GLL back-end for Happy is the first parser generator that combines GLL parsing with a facility for abstraction and reuse, benefitting directly from the top-down nature of the algorithm. The PRECC compiler generator combines $LL(\infty)$ parsing with a facility for abstraction and reuse, but does not generate complete parsers [7]. The OCaml parser generator Menhir generates LR(1) parsers based on grammars with parameterized rules and includes a warning for rules with unbounded growth that cause nontermination [24]. Menhir offers a library of reusable rules for optionality, sequences and lists. As embedded domainspecific languages, parser combinator libraries [32] and grammar combinator libraries [9] support reuse naturally but vary wildly in the class of languages they accept and by the ease with which parsers are written. For a lengthier discussion on this topic, the reader is referred to [37, 39].

Introduced by Fischer in 1968 [11], macro-grammars extend context-free grammars by introducing 'macro-like productions', similar to the parameterized nonterminal definitions of Happy. An argument of a macro is a sequence of symbols, whereas an argument of a parameterized nonterminal is a single symbol. So although macro-grammars are more expressive, it should be possible to write a Happy grammar for every macro-grammar in the way demonstrated by the nonterminal F in Section 5. Thiemann and Neubauer discuss generating LR parsers for restricted macro-grammars [34] and describe an algorithm for checking whether a macrogrammar can be transformed into a context-free grammar [35]. A variant of this algorithm can be implemented in Happy to prevent non-termination of the algorithm that removes parameterized nonterminals for the LALR and GLR back-ends. Perhaps this algorithm can be extended to detect what was called 'higher-order left-recursion' in Section 5 in order to

prevent the GLL back-end from generating parsers for parameterized nonterminals that fail to terminate.

7 Conclusion

This paper has presented a strategy for generating modular, reusable and complete top-down parsers from syntax descriptions with parameterized nonterminals, based on the FUN-GLL variant of the GLL algorithm. The strategy has been implemented in Haskell as a new back-end for Happy. The ideas in this paper should be transferable to other grammar formalisms and host languages with higher-order functions. The generated parsers are easy to test and debug because each grammar symbol is directly implemented as an executable parse function. Moreover, the GLL back-end supports parameterization directly by generating higher-order parse functions reminiscent of parser combinators. As a result, the back-end can generate practical parsers for a large class of grammars, including all context-free grammars and certain grammars that describe context-sensitive languages.

The new Happy back-end has been developed as a practical alternative to the LALR and GLR back-ends, whilst extending the functionality and usability of Happy by inheriting the positive aspects of recursive descent parsing, such as modularity, and realizing the full potential of 'reuse through abstraction'. The runtime efficiency of the generated GLL parsers can be improved, however. The GLL parsers are significantly faster than the GLR parsers for highly ambiguous grammars but significantly slower than the LALR and GLR parsers for LALR grammars. The running times of the generated GLL parsers can perhaps be improved by reimplementing the datastructures provided by the support library and by including lookahead tests. However, some of the envisioned efficiency improvements require precomputing information based on the grammar as a whole, resulting in parsers that are not reusable across files and projects. These possible runtime improvements are to be explored in future work.

References

- [1] Andreas Abel and Grégoire Détrez. 2010. BNFC repository on GitHub. https://github.com/BNFC. [Online; accessed 15 May 2020].
- [2] Ali Afroozeh and Anastasia Izmaylova. 2015. Faster, Practical GLL Parsing. In Compiler Construction. Springer Berlin Heidelberg, 89– 108
- [3] Alfred V. Aho, Monica S. Lam, Ravi Sethi, and Jeffrey D. Ullman. 2006. Compilers: Principles, Techniques, and Tools (2nd Edition). Addison-Wesley Longman Publishing Co., Inc., USA.
- [4] Arthur I. Baars, Andres Löh, and S. Doaitse Swierstra. 2004. Parsing Permutation Phrases. *Journal of Functional Programming* 14, 6 (Nov. 2004), 635–646. https://doi.org/10.1017/S0956796804005143
- [5] Arthur I. Baars and S. Doaitse Swierstra. 2004. Type-safe, Self Inspecting Code. In Proceedings of the 2004 ACM SIGPLAN Workshop on Haskell (Snowbird, Utah, USA) (Haskell '04). ACM, 69–79.
- [6] Bas Basten, Jeroen van den Bos, Mark Hills, Paul Klint, Arnold Lankamp, Bert Lisser, Atze van der Ploeg, Tijs van der Storm, and Jurgen J. Vinju. 2015. Modular Language Implementation in Rascal

- Experience Report. *Sci. Comput. Program.* 114, C (Dec. 2015), 7–19. https://doi.org/10.1016/j.scico.2015.11.003
- [7] Peter T. Breuer and Jonathan P. Bowen. 1995. A prettier compiler-compiler: Generating higher-order parsers in C. Software: Practice and Experience 25, 11 (1995), 1263–1297.
- [8] Robert D. Cameron. 1993. Extending Context-free Grammars with Permutation Phrases. ACM Letters on Programming Languages and Systems 2, 1-4 (March 1993), 85–94. https://doi.org/10.1145/176454.176490
- [9] Dominique Devriese and Frank Piessens. 2011. Explicitly Recursive Grammar Combinators. In Proceedings of the 13th International Symposium on Practical Aspects of Declarative Languages. Springer Berlin Heidelberg, 84–98.
- [10] Jay Earley. 1970. An Efficient Context-free Parsing Algorithm. Commun. ACM 13, 2 (Feb. 1970), 94–102.
- [11] Michael J. Fischer. 1968. Grammars with macro-like productions. In 9th Annual Symposium on Switching and Automata Theory (SWAT 1968). 131–142. https://doi.org/10.1109/SWAT.1968.12
- [12] Richard A. Frost, Rahmatullah Hafiz, and Paul Callaghan. 2008. Parser Combinators for Ambiguous Left-Recursive Grammars. In Practical Aspects of Declarative Languages. Springer Berlin Heidelberg, 167– 181
- [13] Richard A. Frost and Barbara Szydlowski. 1996. Memoizing Purely Functional Top-down Backtracking Language Processors. Science of Computer Programming 27, 3 (1996), 263–288.
- [14] Dick Grune and Ceriel Jacobs. 2010. Parsing Techniques: A Practical Guide (2nd ed.). Springer Publishing Company, Incorporated.
- [15] Anastasia Izmaylova, Ali Afroozeh, and Tijs van der Storm. 2016. Practical, General Parser Combinators. In Proceedings of the 2016 ACM SIG-PLAN Workshop on Partial Evaluation and Program Manipulation (St. Petersburg, FL, USA) (PEPM 2016). ACM, 1–12.
- [16] Mark Johnson. 1995. Memoization in Top-down Parsing. Computational Linguistics 21, 3 (1995), 405–417.
- [17] Lennart C. L. Kats and Eelco Visser. 2010. The Spoofax Language Workbench: Rules for Declarative Specification of Languages and IDEs. In International Conference on Object Oriented Programming Systems Languages and Applications (OOPSLA 2010). ACM, 444–463. https://doi.org/10.1145/1869459.1869497
- [18] P. Klint, T. v. d. Storm, and J. Vinju. 2009. Rascal: A Domain Specific Language for Source Code Analysis and Manipulation. In 2009 Ninth IEEE International Working Conference on Source Code Analysis and Manipulation. 168–177.
- [19] Donald E. Knuth. 1968. Semantics of context-free languages. Mathematical systems theory 2, 2 (1968), 127–145. https://doi.org/10.1007/BF01692511
- [20] Peter Ljunglöf. 2002. Pure Functional Parsing. Ph.D. Dissertation. Chalmers University of Technology and Göteborg University.
- [21] Simon Marlow and Andy Gill. 2001. Happy The Parser Generator for Haskell. https://www.haskell.org/happy/. [Online, accessed 28 April 2020].
- [22] Peter Norvig. 1991. Techniques for Automatic Memoization with Applications to Context-free Parsing. Computational Linguistics 17, 1 (1991), 91–98.
- [23] Chris Okasaki and Andrew Gill. 1998. Fast Mergeable Integer Maps. In In Workshop on ML. 77–86.
- [24] Francois Pottier and Yann Régis-Gianas. 2020. Menhir Reference Manual. http://gallium.inria.fr/~fpottier/menhir/manual.pdf. [Online, accessed 13 May 2020].
- [25] Tom Ridge. 2014. Simple, Efficient, Sound and Complete Combinator Parsing for All Context-Free Grammars, Using an Oracle. In Software Language Engineering. Springer International Publishing, 261–281.
- [26] João Saraiva. 1999. Purely Functional Implementation of Attribute Grammars: Zuiver Functionele Implementatie Van Attributengrammatica's. IPA. http://books.google.nl/books?id=yFqTAAAACAAJ

- [27] Elizabeth Scott and Adrian Johnstone. 2010. GLL Parsing. Electronic Notes in Theoretical Computer Science 253, 7 (2010), 177 – 189. Proceedings of the Ninth Workshop on Language Descriptions Tools and Applications (LDTA 2009).
- [28] Elizabeth Scott and Adrian Johnstone. 2013. GLL parse-tree generation. Science of Computer Programming 78, 10 (2013), 1828 – 1844.
- [29] Elizabeth Scott and Adrian Johnstone. 2016. Structuring the GLL parsing algorithm for performance. *Science of Computer Programming* 125 (2016), 1 22.
- [30] Elizabeth Scott, Adrian Johnstone, and Rob Economopoulos. 2007. BRNGLR: a cubic Tomita-style GLR parsing algorithm. Acta Informatica 44, 6 (2007), 427–461.
- [31] Elizabeth Scott, Adrian Johnstone, and L. Thomas van Binsbergen. 2019. Derivation Representation using Binary Subtree Sets. Science of Computer Programming 175 (2019), 63 84. https://doi.org/10.1016/j.scico.2019.01.008
- [32] S. Doaitse Swierstra. 2009. Combinator Parsing: A Short Tutorial. In Language Engineering and Rigorous Software Development. Springer Berlin Heidelberg, 252–300.
- [33] S. Doaitse Swierstra, Pablo R. Azero Alcocer, and João Saraiva. 1999. Designing and implementing combinator languages. In Advanced Functional Programming. Springer Berlin Heidelberg, 150–206.
- [34] Peter Thiemann and Matthias Neubauer. 2004. Parameterized LR Parsing. Electronic Notes in Theoretical Computer Science 110 (2004), 115

- 132. https://doi.org/10.1016/j.entcs.2004.06.007 Proceedings of the Fourth Workshop on Language Descriptions, Tools, and Applications (LDTA 2004).
- [35] Peter Thiemann and Matthias Neubauer. 2008. Macros for Context-free Grammars. In Proceedings of the 10th International ACM SIGPLAN Conference on Principles and Practice of Declarative Programming (Valencia, Spain) (PPDP 2008). ACM, 120–130. https://doi.org/10.1145/1389449.1389465
- [36] Masaru Tomita. 1985. Efficient Parsing for Natural Language: A Fast Algorithm for Practical Systems. Kluwer Academic Publishers.
- [37] L. Thomas van Binsbergen. 2019. Executable Formal Specification of Programming Languages with Reusable Components. Ph.D. Dissertation. Royal Holloway, University of London.
- [38] L. Thomas van Binsbergen, Elizabeth Scott, and Adrian Johnstone. 2018. GLL Parsing with Flexible Combinators. In Proceedings of the 11th ACM SIGPLAN International Conference on Software Language Engineering (SLE 2018).
- [39] L. Thomas van Binsbergen, Elizabeth Scott, and Adrian Johnstone. 2020. Purely Functional GLL Parsing. Journal of Computer Languages (2020)
- [40] Mark G. J. van den Brand, J. Heering, P. Klint, and P. A. Olivier. 2002. Compiling Language Definitions: The ASF+SDF Compiler. ACM Trans. Program. Lang. Syst. 24, 4 (2002), 334–368.