Infrared Multilayer Laboratory

Absorption and extinction coefficient theory

The velocity of propagation of a electromagnetic wave through a solid is given by the frequency-dependent complex refractive index $N = n - ik$ where the real part, n is related to the velocity, and k, the extinction coefficient is related to the decay, or damping of the oscillation amplitude of the incident electric field. The optical properties of the solid are therefore governed by the interaction between the solid and the electric field of the electromagnetic wave.

If a plane wave of frequency (f) propagates through a solid with velocity (v) in a direction defined by (x), the electric field (E) is described by the following progressive wave equation:

$$E = E_0 \exp\{i 2\pi f[t - (x / v)]\}$$

Where, (E_0)is the incident electric field vector and

$$\{i 2\pi f[t - (x / v)]\}$$

is the displacement at time t after a disturbance, created by the electric field at a point situated at x along the line of propagation.

From Maxwell's equations on electromagnetic theory, the speed of light in a vacuum c is related to the permittivity of free space ε_0, (the degree to which a medium can resist the flow of charge, defined by the ratio of the electric displacement to the intensity of the electric field that produces it), and the permeability of free space μ_0 (the ratio of the magnetic flux density in a solid to the external magnetic field strength inducing it, μ = **B/H**.) by the equation $c=1/(\mu_0\varepsilon_0)^{1/2}$.

The velocity of propagation through the solid of complex refractive index $N = n - ik$ is related to the speed of light in a vacuum, c, by $V = C / N$, then:

$$\frac{1}{v} = \frac{n}{c} - \frac{ik}{c}$$

Therefore, substituting $1/V$ into equation above produces:

$$E = E_0\, exp\,(i 2\pi f t)\, exp\left(\frac{-i 2\pi x n}{c}\right)\, exp\left(\frac{-2\pi f k x}{c}\right)$$

where the last term

$$(-2\pi f k x / c)$$

is a measure of the damping factor, or extinction coefficient (k).

As the power (P) or intensity of an incident wave through a solid is the conductivity (σ) of the solid multiplied by the square of the electric field vector ($P=\sigma E^2$), then using the damping factor term, the fraction of the incident power that has propagated from position (o) to a distance (x) through the material with conductivity (σ) is given by:

$$\frac{P(x)}{P(0)} = \frac{\sigma E^2(x)}{\sigma E^2(0)} = \exp\left(\frac{-4\pi fkx}{c}\right)$$

from which the absorption coefficient (α) can be expressed in terms of the extinction coefficient (*k*) as:

$$\alpha = \frac{4\pi fk}{c}$$

As the velocity of light in a vacuum, $c = f\lambda$, then $\alpha = 4\pi k/\lambda$, and the power or intensity is $P = P_o exp^{-\alpha x}$. This equation is known as Bouguer's law or Lambert's law of absorption, by which radiation is absorbed to an extent that depends on the wavelength of the radiation and the thickness and nature of the medium. The absorption coefficient is therefore described as the reciprocal of the depth of penetration of radiation into a bulk solid, i.e., it is equal to the depth at which the energy of the radiation has decreased by the factor of $e^{-\alpha x}$, or alternatively, the intensity of the incident radiation is attenuated by the solid to $1/e$ of its initial value at a distance from the surface boundary defined by $\lambda/4\pi k$.

When electromagnetic radiation passes from one medium into another the values of the relative permittivity ε_r and relative permeability μ_r must alter according to the characteristics of the materials. In addition to this, boundary conditions are required to be defined to ensure waves in the two media match at the interface. This requires the tangential components of **E** (electric field vector) and **H** (magnetic field vector) to be continuous across the boundary, and the normal components of **D** (electric displacement vector) and **B** (magnetic flux density vector) to also be continuous across the boundary. Hence, $\varepsilon_{0\varepsilon 1}\mathbf{E}_1 = \varepsilon_{0\varepsilon 2}\mathbf{E}_2$ and $\mu_{0\mu 1}\mathbf{H}_1 = \mu_{0\mu 2}\mathbf{H}_2$.

The optical impedance of a material is another useful parameter in considering reflection and transmission of electromagnetic waves across an interface, $Z=\mathbf{E}_x/\mathbf{H}_y=\mathbf{E}_y/\mathbf{H}_x=(\mu_{0\mu r}/\varepsilon_{0\varepsilon r})^{1/2}$. By substituting values for ε_0 ($8.854\text{x}10^{-12}$ Fm^{-1}) and μ_0 ($1.257\text{x}10^{-6}$ Hm^{-1}), the impedance of free space $Z_0 = (\mu_0/\varepsilon_0)^{1/2} = 377\Omega$. The optical admittance of free space, Y, is given by $Y = 1/Z_0 = (\varepsilon_0/\mu_0)^{1/2} = 2.654\text{x}10^{-3\Omega-1}$. In a dielectric with a relative permittivity given by ε_r and a relative permeability given by μ_r which is at unity, the admittance is given by $y = (\varepsilon_{0\varepsilon r}/\mu_0)^{1/2} = Y\,\varepsilon_{r 1/2} = Y\,N = Y\,(n-ik)$

The effects of thin-film interfaces can be calculated in terms of **E** and **H**, parallel to the boundary, however this notation can become cumbersome, particularly where exact values of ε_r and μ_r are not well quantified, therefore a modified optical admittance η is introduced to connect **H** and **E** (η = **H**/**E**). At normal incidence,

$\eta = y = Y\,N$, while at oblique incidence where the incident wave becomes polarised

$\eta_p = y/\cos\theta = YN/\cos\theta$ and $\eta_s = y\cos\theta = NY\cos\theta$.

In the case of an absorbing material, the behaviour of a beam of radiation incident in a medium of refractive index n_1 on an absorbing medium of complex refractive index $n_2 = n_2\text{-}ik_2$, with an angle of incidence θ_1, using Snell's law in complex form is then defined by:

$n_1\sin\theta_1 = n_2\sin\theta_2 = (n_2 - ik_2)\sin\theta_2$(2-6)

Substrate optical theory

[Incoherent multiple internal reflection]
[Reduced substrate-temperature effects]
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