Physical mechanics

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1 Cinematics

Having:

- $\mathcal{R}$ as rotation matrix of S' seen from S
- $\mathbf{R}$ the relative position of S' seen from S

the space coordinates of a point are:

$$r = \mathcal{R}^T r' + \mathbf{R}$$
$$r' = \mathcal{R}(r - \mathbf{R})$$

We can then define the antisymmetric **angular momentum matrices** as:

$$\hat{w} = \dot{\mathcal{R}}^T \mathcal{R} = \begin{pmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{pmatrix}$$
$$\hat{w}' = \mathcal{R}\dot{\mathcal{R}}^T = \begin{pmatrix} 0 & -w'_z & w'_y \\ w'_z & 0 & -w'_x \\ -w'_y & w'_x & 0 \end{pmatrix}$$

And the a can then define the dual vectors too:

$$w = (w_x, w_y, w_z)$$
$$w' = (w'_x, w'_y, w'_z)$$

We get the relationships:

$$\hat{w} q = w \times q$$
$$\hat{w}' q = w' \times q$$
$$\hat{w} = \mathcal{R}^T \hat{w}' \mathcal{R}$$
$$\hat{w}' = \mathcal{R} \hat{w}$$

Defining $\mathbf{V}$ as the speed of S' as seen from S, we have the velocities (deriving the position one):

$$v = \mathcal{R}^T (v' + w' \times r') + \mathbf{V}$$
$$v' = \mathcal{R}[v - \mathbf{V} - w \times (r - \mathbf{R})]$$

Then with α as the angular acceleration, A as the acceleration of S' as seen by S, and deriving this once more we get the acceleration:

$$\begin{aligned} a' &= -\alpha' \times r' ;\ \text{acimutal} \\ &\quad -2w' \times v' ;\ \text{coriolis} \\ &\quad -w' \times (w' \times r') ;\ \text{centrifugal} \\ &\quad +\mathcal{R} a \\ &\quad -\mathcal{R}\mathbf{A} ;\ \text{drag} \end{aligned}$$

$$\begin{aligned} a &= \mathbf{A} + \mathcal{R}^T [a' \\ &\quad +\alpha' \times r' \\ &\quad +1w' \times v' \\ &\quad +w' \times (w' \times r')] \end{aligned}$$