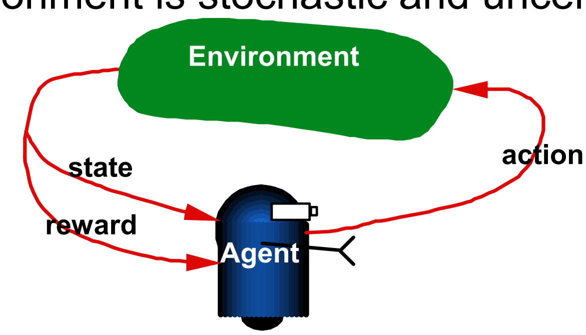
Reinforcement Learning

- Reinforcement Learning Problem
- Temporal Difference Learning
- SARSA
- Function Approximation

Complete Agent

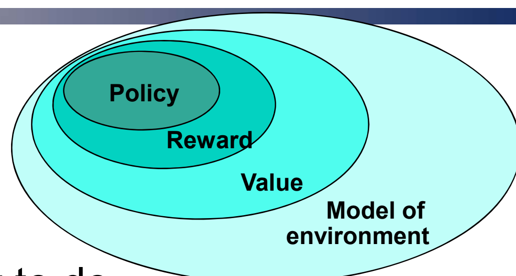
- Temporally situated
- Continual learning and planning
- Object is to ***affect*** the environment
- Environment is stochastic and uncertain



Reinforcement Learning

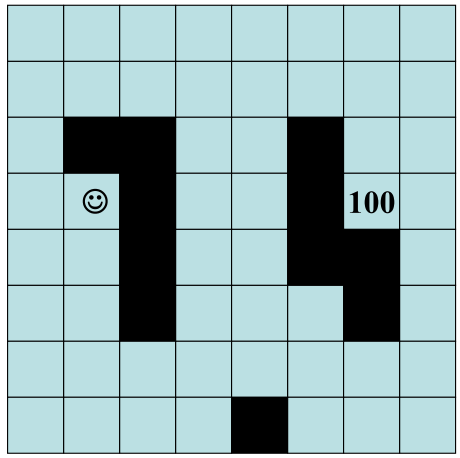
- Goal: learn an optimal strategy for maximizing future reward
  - Agent has little prior knowledge and no immediate feedback
  - Possible need to learn multiple tasks with same sensors/effectors
- *Temporal/Action Credit Assignment*
  - Rewards and punishments may be delayed – sacrifice short term for long term
- Exploration/Exploitation
- Planning and Acting under Uncertainty
  - Can handle deterministic or probabilistic state transitions
  - Possibility that state only partially observable
- Two basic agent designs
  - Agent learns utility function $V(s)$ on states
  - Agent learns action-value $Q(s,a)$ function

Elements of RL

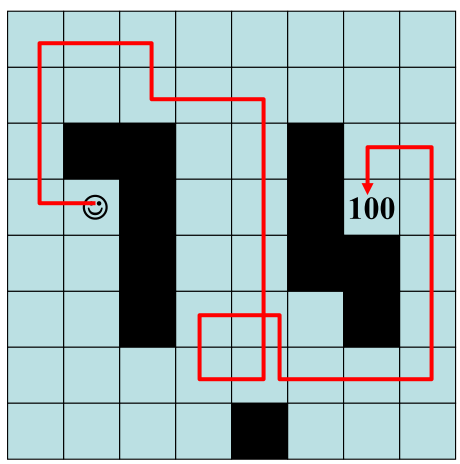


- **Policy**: what to do
- **Reward**: what is good
- **Value**: what is good because it *predicts* reward
- **Model**: what follows what

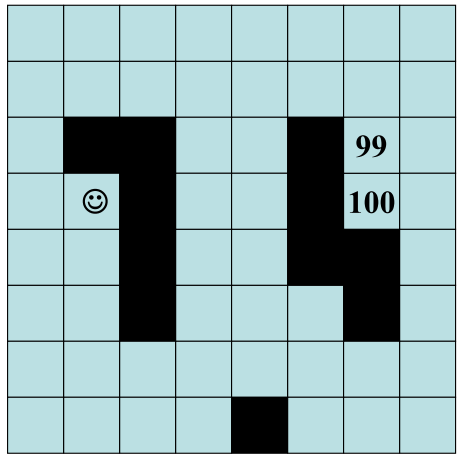
Example



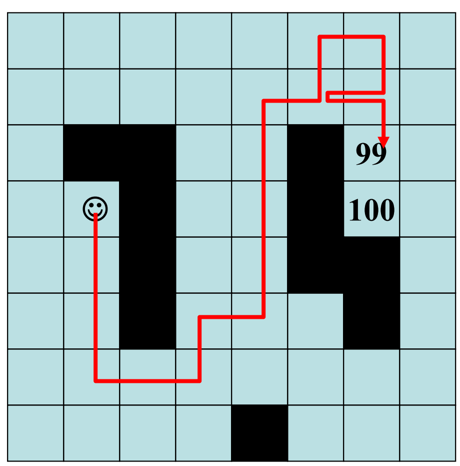
Example



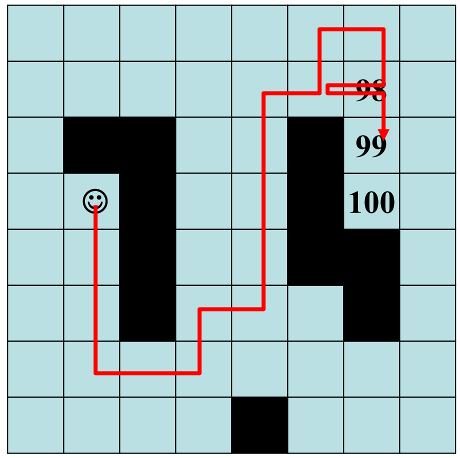
Example



Example



Example



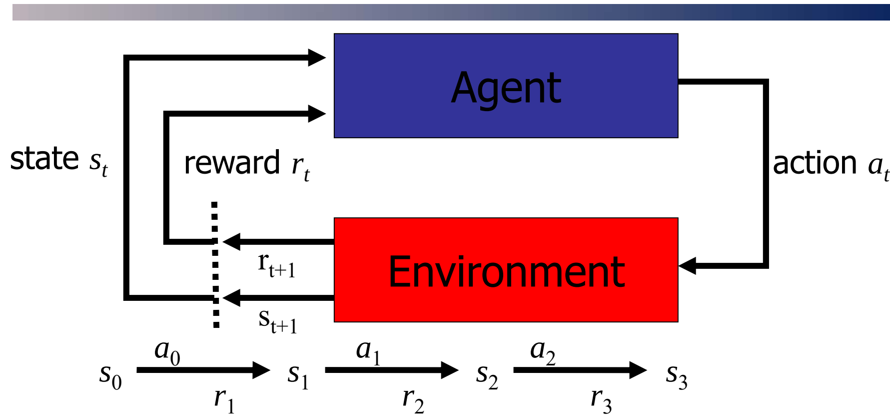
Example

91	92	93	94	95	96	97	96
92	93	94	95	96	97	98	97
91			94	95		99	98
90	☺		93	94		100	99
89	88		92	93			98
88	89		91	92	93		97
89	90	91	92	93	94	95	96
88	89	90	91		93	94	95

Some Notable RL Applications

- TD-Gammon – world's best backgammon program (Tesauro)
- Elevator Control (Crites & Barto)
- Inventory Management – 10-15% improvement over industry standard methods (Van Roy, Bertsekas, Lee, and Tsitsiklis)
- Dynamic Channel Assignment – high performance assignment of radio channels to mobile telephone calls (Singh and Bertsekas)

Reinforcement Learning Problem



- Goal: Learn to choose actions a_t that maximize future rewards $r_1+\gamma r_2+\gamma^2 r_3+\ldots$, where $0<\gamma<1$ is a discount factor

Approaches for RL

- Model-base Learning
  - These use a model's prediction or distributions of next state and reward in order to calculate optimal actions
    - Policy iteration, value iteration, V(s) based TD learning
- Model-free Learning
  - Relies on samples from the environment to gain experience and learn which actions to take for a state to gain rewards
    - SARSA, Q(s,a) based TD learning, Actor-Critic

Markov Decision Process (MDP)

- S = Finite set of states ($|S| = n$)
- A = Finite set of actions ($|A| = m$)
- $\Pr(s_t,a,s_{t+1})$ = Transition function (τ)
  - At each time step the agent observes state $s_t \in S$ and chooses action $a \in A$
  - Represented by set of stochastic matrices (non-deterministic)
    - Also includes events – occurrences not caused by the agent
- $R(s_t,a,s_{t+1})$ = Reward/cost function
- *Markov assumption*
  - Current state s_t encapsulates all previous state and action history $h = (s_0,a_0,\ldots s_{t-1},a_{t-1})$
  - Optimal action only depends on the current state and action a_t (and in some cases resultant state s_{t+1})
  - Functions $\Pr(s_t,a,s_{t+1})$ and $R(s_t,a,s_{t+1})$ not necessarily known to agent (model based reinforcement learning)

Temporal Difference Learning

- Temporal Difference (TD) learning methods
  - Can be used when accurate models of the environment are unavailable – neither state transition function $Pr(s,a,s')$ nor reward function $R(s,a,s')$ are known
  - Can be extended to work with implicit representations of action-value functions
  - Are among the most useful reinforcement learning methods

TD Prediction

- Policy evaluation (the prediction problem): for a given policy π, compute the state-value function V^*
  - The simplest TD method, TD(0)

$$V(s_t) \leftarrow V(s_t) + \alpha\big[\underbrace{r_{t+1} + \gamma V(s_{t+1})}_{\text{target: an estimate of the return}} - V(s_t)\big]$$

- TD method is *bootstrapping* by using the existing estimate of the next state $V(s_{t+1})$ for updating $V(s_t)$
- α is the learning rate $(0 \le \alpha \le 1)$

TD(0): Policy Evaluation

Initialize:

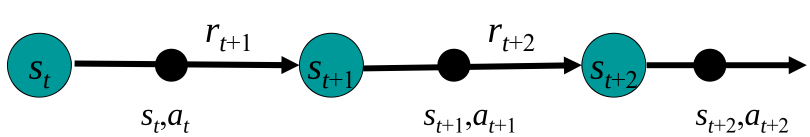
- π = policy to be evaluated
- $V(s)$ = an arbitrary state-value function

Repeat for each episode

- Initialize s
- Repeat for each step of episode
  - $a \leftarrow$ action given by π for s
  - Take action a, observe reward r, and next state s'
  - $V(s) \leftarrow V(s_t) + \alpha[r + \gamma V(s') - V(s)]$
  - $s \leftarrow s'$
- Until s is terminal

TD(0): Policy Iteration

- $Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha\, [r_{t+1} + \gamma Q(s_{t+1},a_{t+1}) - Q(s_t,a_t)]$



- The update rule uses a quintuple of events $(s_t,a_t,r_{t+1},s_{t+1},a_{t+1})$, therefore called SARSA.
- Problem: Unlike in the deterministic case we can not choose a completely greedy policy $\pi(s)=\max_a Q(s,a)$, as due to the unknown transition $\Pr(s_t,a,s_{t+1})$ and reward functions and $R(s_t,a,s_{t+1})$ we cannot be sure if another action might eventually turn out to be better

ε-greedy policy

- Soft policy: $\pi(s,a) > 0$ for all $s \in S$, $a \in A(s)$
  Non-zero probability of choosing every possible action
- ε-greedy policy: Most of the time with probability $(1-\varepsilon)$ follow the optimal policy

  $\pi(s) = \max_a Q(s,a)$

  but with probability ε pick a random action:

  $\pi(s,a) \geq \varepsilon/|A(s)|$
- Let $\varepsilon \to 0$ go to zero as $t \to \infty$ for example $\varepsilon = 1/t$ so that ε-greedy policy converges to the optimal deterministic policy

SARSA Policy Iteration

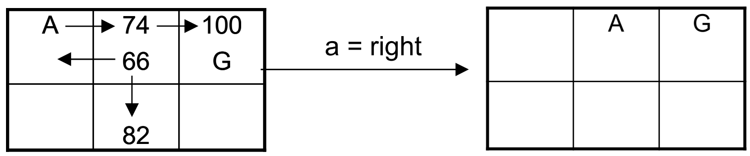
Initialize $Q(s,a)$ arbitrarily:

Repeat for each episode

- Initialize s
- Choose a from using ε-greedy policy derived from Q
- Repeat for each step of episode
  - Take action a, observe reward r, and next state s'
  - $Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma Q(s',a') - Q(s,a)]$
  - $s \leftarrow s'$, $a \leftarrow a'$
- Until s is terminal

Example

- Consider grid world where goal state (G) yields reward of 100 and other states yield reward of 0.



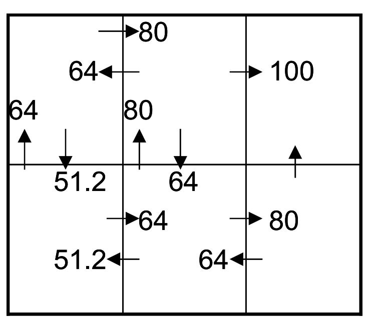
Initial State s1 State s2

- Let learning rate be 0.8.

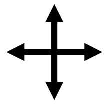
Q(s1, right) = 74 + 0.8(0 + max{(Q(s2,left),Q(s2,right), Q(s2,down),Q(s2,up)} - 74)
= 74 + 0.8(0 + max{66, 82, 100} - 74)
= 74 + 0.8(26)
= 94.8

Example

- Learned Q(s,a) values

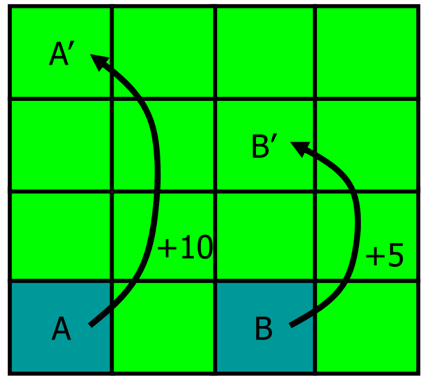


SARSA Example

Actions 

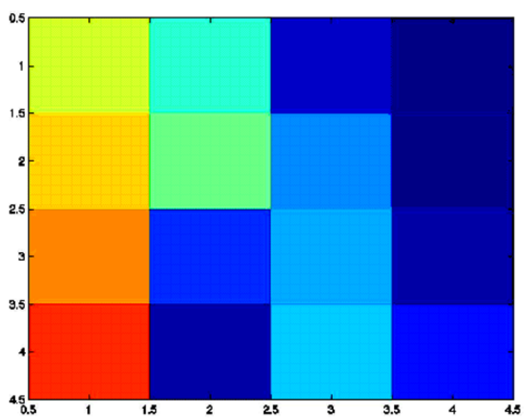
Reward:

- +10 for A→A′
- +5 for B→B′
- -1 for falling off the grid
- 0 otherwise



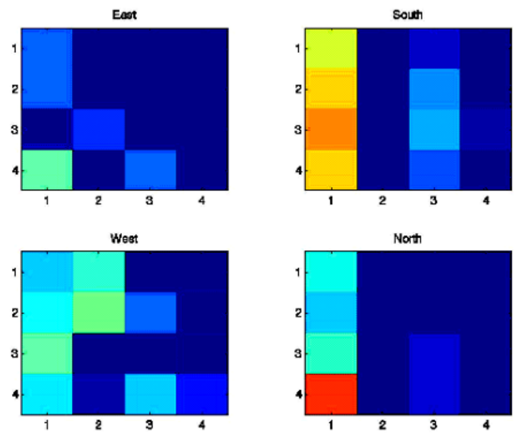
SARSA Example V(s)

V(s) after 100, 200, 1000, 2000, 5000, 10000 SARSA steps



SARSA Example Q(s,a)

Q(s,a) after 100, 200, 1000, 2000, 5000, 10000 SARSA steps



Q-Learning (Off-Policy TD)

- Approximates the optimal value functions *V**(*s*) or *Q**(*s*,*a*) independent of the policy being followed.
- The policy determines which state-action pairs are visited and updated

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha\left[r_{t+1} + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t)\right]$$

Advantages of TD Learning

- TD methods do not require a model of the environment, only experience
- TD methods can be fully incremental
  - You can learn before knowing the final outcome
    - Less memory
    - Less peak computation
  - You can learn without the final outcome
    - From incomplete sequences
- TD converges (under certain assumptions)

Optimality of TD(0)

- Batch Updating: train completely on a finite amount of data, e.g., train repeatedly on 10 episodes until convergence.
  - Compute updates according to TD(0), but only update estimates after each complete pass through the data
- For any finite Markov prediction task, under batch updating, TD(0) converges for sufficiently small α

Extending TD

So far TD uses one-step look-ahead but why not use 2-steps or *n*-steps look-ahead to update $Q(s,a)$.

- 2-step look-ahead

$Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha\,[r_{t+1} + \gamma r_{t+2} + \gamma^2 Q(s_{t+2},a_{t+2}) - Q(s_t,a_t)]$

- *N*-step look-ahead

$Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha[r_{t+1} + \gamma r_{t+2} + \ldots + \gamma^{n-1} r_{t+n} + \gamma^n Q(s_{t+n},a_{t+n}) - Q(s_t,a_t)]$

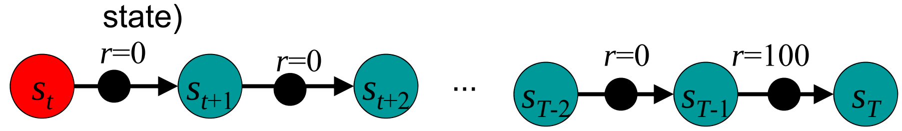
- Monte-Carlo method

$Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha[r_{t+1} + \gamma r_{t+2} + \ldots + \gamma^{N-1} r_{t+N} - Q(s_t,a_t)]$

(compute total reward R^T at the end of the episode)

Temporal Difference Learning

- Drawback of one-step look-ahead:
  - Reward is only propagated back to the successor state (takes long time to finally propagate to the start state)



Initial *V*:

$V(s)=0$ $V(s)=0$ $V(s)=0$ $V(s)=0$ $V(s)=0$

After first epsiode:

$V(s)=0$ $V(s)=0$ $V(s)=0$ $V(s)=0$ $V(s)=a*100$

After second epsiode:

$V(s)=0$ $V(s)=0$ $V(s)=0$ $V(s)=a*\gamma*100$ $V(s)=a*100+..$

Monte-Carlo Method

Drawback of Monte-Carlo method

- Learning only takes place after an episode terminated
- Performs a random walk until goal state is discovered for the first time as all state-action values seem equal
- It might take long time to find the goal state by random walk
- TD-learning actively explores state space if each action receives a small default penalty

N-Step Return

- Idea: blend TD learning with Monte-Carlo method
- Define:
- $R_t^{(1)} = r_{t+1} + \gamma V_t(s_{t+1})$
- $R_t^{(2)} = r_{t+1} + \gamma r_{t+2} + \gamma^2 V_t(s_{t+2})$
- …
- $R_t^{(n)} = r_{t+1} + \gamma r_{t+2} + \ldots + \gamma^{n-1} r_{t+n} + \gamma^n V_t(s_{t+n})$
- The quantity $R_t^{(n)}$ is called the n-step return at time t.

TD(λ)

- TD(λ): use average of *n*-step returns for backup

$$R_t^\lambda = (1-\lambda)\sum_{n=1}^{\infty}\lambda^{n-1}R_t^n$$
$$= (1-\lambda)\sum_{n=1}^{T-t-1}\lambda^{n-1}R_t^n + \lambda^{T-t-1}R_t$$

- *n*-step backup: $V_t(s_t) \leftarrow V_t(s_t) + \alpha[R_t^{(n)} - V_t(s_t)]$
  - (if s_T is a terminal state)
- The weight of the *n*-step return decreases by a factor of λ, $0 < \lambda < 1$
  - TD(0): one-step temporal difference method
  - TD(1): Monte-Carlo method

Online (passive) TD(λ)

Initialize π an MDP, V a table of values, and s,a,r previous state, action, and reward.

Repeat for each episode

- Repeat
  - If s' is new then $V(s') = r'$
  - If s is not new then do
    - $V(s) = r + \gamma((1-\lambda)V(s) + \lambda V(s'))$
  - $s,a,r \leftarrow s'\ \pi(s'),r'$
- Until s is terminal

Another View (Sutton)

- Initialize $V(s)$ arbitrarily and $e(s)=0$, for all $s \in S$
- Repeat (for each episode):
  - Initialize s
  - Repeat (for each step of episode):
    - $a \leftarrow$ action given by π for s
    - Take action *a*, observe reward, r, and next state, s'
    - $\delta \leftarrow r + \gamma V(s') - V(s)$
    - $e(s) \leftarrow e(s)+1$
    - For all s:
      - $V(s) \leftarrow V(s) + \alpha\delta e(s)$
      - $e(s) \leftarrow \gamma\lambda e(s)$
    - $s \leftarrow s'$
  - until s is terminal

Q-Learning

Initialize Q a table of state-action pairs, and s,a,r previous state, action and reward.

Repeat for each episode

- Repeat
  - If s' is new then $Q(s',a) = r'$
  - If s is not null then do
    - $Q(s,a) = r + \gamma\left((1-\lambda)\max_a Q(s,a) + \lambda Q(s',a')\right)$
  - $s,a,r \leftarrow s', \max_{a'} Q(s',a'), r'$
- Until s is terminal

Q-Learning with Exploration

Initialize Q a table of state-action pairs, and s,a,r previous state, action and reward, and N_{sa} a table of frequencies for state-action pairs.

Repeat for each episode

- Repeat
  - If s' is new then $Q(s',a) = r'$
  - If s is not null then do
    - Increment $N_{sa}(s,a)$
    - $Q(s,a) = r + N_{sa}(s,a)\gamma\left((1-\lambda)\max_a Q(s,a) + \lambda Q(s',a')\right)$
  - $s, a, r \leftarrow s', \max_{a'} f\left(Q(s', a'), N_{sa}(s', a')\right), r'$
  - Until s is terminal

Exploratory Function

$$f(u,n) = \begin{cases} R+ & \text{if } n < N_e \\ u & \text{otherwise} \end{cases}$$

- Where R^+ is an optimistic estimate of the best possible reward obtainable in any state
- N_e is a fixed threshold
- Mitchell alternative:

$$P(a_i \mid S) = \frac{k^{Q(s,a_i)}}{\sum_j k^{Q(s,a_j)}}$$

Function Approximation

- So far we assumed that the functions $Q(s,a)$ is represented as a table.
- Limited to problems with a small number of states and actions
- For large state spaces a table based representation requires large memory and large data sets to fill them accurately
- Generalization: Use any supervised learning algorithm to estimate $Q(s,a)$ from a limited set of action value pairs
  - Neural Networks (Neuro-Dynamic Programming)
  - Linear Regression
  - Nearest Neighbors

Function Approximation

- Minimize the mean-squared error between training examples $Q_t(s_t,a_t)$ and the true value function $Q^\pi(s,a)$

$$\sum_{s \in S} Pr(s)[Q^\pi(s,a) - Q_t(s_t,a_t)]^2$$

- Notice that during policy iteration $P(s)$ and $Q^\pi(s,a)$ change over time
- Parameterize $Q^\pi(s,a)$ by a vector $\boldsymbol{\theta}=(\theta_1,\ldots,\theta_n)$ for example weights in a feed-forward neural network

Stochastic Gradient Descent

- Use gradient descent to adjust the parameter vector θ in the direction that reduces the error for the current example

  $\theta_{t+1} = \theta_t + \alpha[Q^{\pi}(s_t, a_t) - Q_t(s_t, a_t)]\Delta_{\theta_t} Q_t(s_t, a_t)$
- The target output q_t of the t-th training example is not the true value of QP but some approximation of it.

  $\theta_{t+1} = \theta_t + \alpha[v_t - Q_t(s_t, a_t)]\Delta_{\theta_t} Q_t(s_t, a_t)$
- Still converges to a local optimum if

  $E(v_t) = Q^{\pi}\ (s_t, a_t)$ if $a \rightarrow 0$ for $t \rightarrow \infty$
- Example: AlphaZero

Subtleties and Ongoing Research

- Scalability
- Generalize utilities from visited states to other states (inductive learning)
- Handle case where state only partially observable
- Design optimal exploration strategies
- Extend to continuous actions, states
- Inverse Reinforcement Learning

Summary Reinforcement Learning

- Different Learning Problem: Learn policy for action selection that maximizes future reward
- Solution: Learn value function
- Two approaches: Recursive backups and supervised learning. Q-Learning is an example of the former.
- TD(λ): mixes both of these approaches