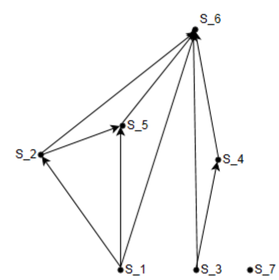
Discrete Mathematics - CSCE 531 Fall 2017
In-Class Work, Day 17 (4 December 2017)

From Section 10.1

1. (Problem 33) Construct a precedence graph for the following program:

S_1: `x := 0`
S_2: `x := x + 1`
S_3: `y := 2`
S_4: `z := y`
S_5: `x := x + 2`
S_6: `y := x + z`
S_7: `z := 4`

Following Example 8, which models read-after-write hazards (but not write-after-read or write-after-write hazards) and includes transitive relationships, we obtain the following:



This is a silly problem. Why would we want to model transitivity and not write-after-read and not write-after-write hazards? Merkle should make up his own problem and ignore Example 8.

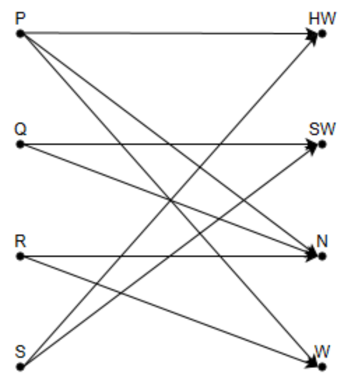
From Section 10.2

2. (Problem 27) Suppose that there are four employees in the computer support group of the School of Engineering of a large university. Each employee will be assigned to support one of four different areas: hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware, networking, and wireless; Quiggley is qualified to support software and networking; Ruiz is qualified to support networking and wireless, and Sitea is qualified to support hardware and software.
    a. Use a bipartite graph to model the four employees and their qualifications.

Let $V_1 = \{P, Q, R, S\}$, $V_2 = \{HW, SW, N, W\}$, $V = V_1 \cup V_2$, and

$$E = \{(P, HW), (P, N), (P, W), (Q, SW), (Q, N), (R, N), (R, W), (S, HW), (S, SW)\}.$$

Then $G = (V, E)$ is a bipartite graph with bipartition (V_1, V_2) modeling the four employees and their qualifications.



b. Use Hall's theorem to determine whether there is an assignment of employees to support areas so that each employee is assigned one area to support.

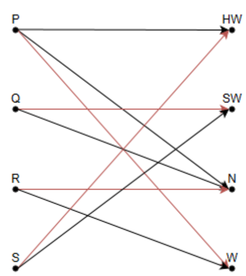
We have the following:

$$\begin{aligned}
|N(\emptyset)| &= |\emptyset| = 0 \geq 0 = |\emptyset| \\
|N(\{S\})| &= |\{HW, SW\}| = 2 \geq 1 = |\{S\}| \\
|N(\{R\})| &= |\{N, W\}| = 2 \geq 1 = |\{R\}| \\
|N(\{R, S\})| &= |\{HW, SW, N, W\}| = 4 \geq 2 = |\{R, S\}| \\
|N(\{Q\})| &= |\{SW, N\}| = 2 \geq 1 = |\{Q\}| \\
|N(\{Q, S\})| &= |\{HW, SW, N\}| = 3 \geq 2 = |\{Q, S\}| \\
|N(\{Q, R\})| &= |\{SW, N, W\}| = 3 \geq 2 = |\{Q, R\}| \\
|N(\{Q, R, S\})| &= |\{HW, SW, N, W\}| = 4 \geq 3 = |\{Q, R, S\}| \\
|N(\{P\})| &= |\{HW, N, W\}| = 3 \geq 1 = |\{P\}| \\
|N(\{P, S\})| &= |\{HW, SW, N, W\}| = 4 \geq 2 = |\{P, S\}| \\
|N(\{P, R\})| &= |\{HW, N, W\}| = 3 \geq 2 = |\{P, R\}| \\
|N(\{P, R, S\})| &= |\{HW, SW, N, W\}| = 4 \geq 3 = |\{P, R, S\}| \\
|N(\{P, Q\})| &= |\{HW, SW, N, W\}| = 4 \geq 2 = |\{P, Q\}| \\
|N(\{P, Q, S\})| &= |\{HW, SW, N, W\}| = 4 \geq 3 = |\{P, Q, S\}| \\
|N(\{P, Q, R\})| &= |\{HW, SW, N, W\}| = 4 \geq 3 = |\{P, Q, R\}| \\
|N(\{P, Q, R, S\})| &= |\{HW, SW, N, W\}| = 4 \geq 4 = |\{P, Q, R, S\}|
\end{aligned}$$

Thus, $|N(A)| \geq |A|$ for every $A \subseteq V_1$. By Hall's Marriage Theorem, there is a complete matching from V_1 to V_2.

c. If an assignment of employees to support areas so that each employee is assigned to one support area exists, find one.

One such assignment is $\{(P, W), (Q, SW), (R, N), (S, HW)\}$.



From Section 10.3

3. (Problem 51) Find a self-complementary simple graph with five vertices.

One self-complementary simple graph with five vertices is C_5. Are there others?

Observe that any self-complementary simple graph $G = (V, E)$ must have the property $|E| = \frac{1}{4}|V|(|V| - 1)$, i.e. it must have exactly one half of the possible edges for a graph with $|V|$ vertices.

From Section 10.4

4. (Problem 47) How many nonisomorphic connected simple graphs are there with n vertices when n is

Enumeration yields the following:

   a. 2? One
   b. 3? Two
   c. 4? Six
   d. 5? Twenty-one

From Section 10.5

5. (Problem 9) Suppose that in addition to the seven bridges of Königsberg (shown in Figure 1) there were two additional bridges, connecting regions B and C and regions B and D, respectively. Could someone cross all nine of these bridges exactly once and return to the starting point?

The additional bridges do not affect the degree of vertex A (for example), which is odd. Therefore, even with the additional bridges, no Euler circuit exists, so a person cannot cross all nine of these bridges exactly once and return to the starting point.

From Section 10.6

6. (Problem 15) Extend Dijkstra's algorithm for finding the length of a shortest path between two vertices in a weighted simple connected graph so that the length of a shortest path between the vertex a and every other vertex of the graph is found.

The only change required is the termination criterion. Instead of terminating when z is added to S, continue until S contains all the vertices of the graph.

From Section 10.7

7. (Problem 13) Suppose that a connected planar graph has six vertices, each of degree four. Into how many regions is the plane divided by a planar representation of this graph?

We are given that $G = (V, E)$ is a connected planar graph with $v = |V| = 6$ and $e = |E| = \frac{1}{2} \cdot 6 \cdot 4 = 12$. By Euler's formula, the edges divide the plane into $r = e - v + 2 = 12 - 6 + 2 = 8$ regions.

From Section 10.8

8. (Problem 13) Which graphs have a chromatic number of 1?

Only graphs with no edges have a chromatic number of 1.