Discrete Mathematics - CSCE 531 Fall 2018
In-Class Work, Day 13 (19 Nov 18)

Section 8.5 (Inclusion-Exclusion)

1. (Problem 1) Find $|A_1 \cup A_2|$ if $|A_1| = 12$, $|A_2| = 18$, and

   We will use $|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2| = 12 + 18 - |A_1 \cap A_2|$ for each part of the problem.

   a. $A_1 \cap A_2 = \emptyset$?

   $$|A_1 \cup A_2| = 12 + 18 - 0 = 30$$

   b. $|A_1 \cap A_2| = 1$?

   $$|A_1 \cup A_2| = 12 + 18 - 1 = 29$$

   c. $|A_1 \cap A_2| = 6$?

   $$|A_1 \cup A_2| = 12 + 18 - 6 = 24$$

   d. $A_1 \subseteq A_2$?

   Since $\mathrm{A}_1 \subseteq \mathrm{A}_2$, we have $\mathrm{A}_1 \cap \mathrm{A}_2 = A_1$, and therefore $|\mathrm{A}_1 \cap \mathrm{A}_2| = |\mathrm{A}_1|$. Thus,

   $$|A_1 \cup A_2| = 12 + 18 - 12 = 18.$$

2. (Problem 7) There are 2504 computer science students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Linux, and 345 have taken a course in C. Further, 876 have taken courses in both Java and Linux, 231 have taken courses in both Linux and C, and 290 have taken course in both Java and C. If 189 of these students have taken courses in Linux, Java, and C, how many of these 2504 students have not taken a course in any of these three programming languages?

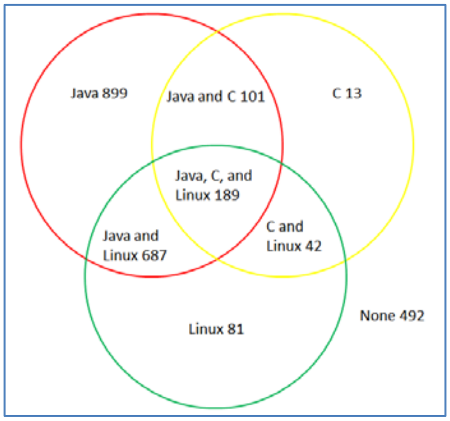
Following the notation of Section 8.6, let P_1 be the property that a student has taken a course in Java, P_2 the property that a student has taken a course in Linux, and P_3 the property that a student has taken a course in C.

$$\begin{aligned} N &= 2504 \\ N(P_1) &= 1876 \\ N(P_2) &= 999 \\ N(P_3) &= 345 \\ N(P_1P_2) &= 876 \\ N(P_1P_3) &= 290 \\ N(P_2P_3) &= 231 \\ N(P_1P_2P_3) &= 189 \end{aligned}$$

Finally,

$$N(P_1', P_2', P_3') = N - \sum_{1 \le i \le n} N(P_i) + \sum_{1 \le i < j \le n} N(P_i P_j) - \sum_{1 \le i < j < k \le n} N(P_i P_j P_k)$$
$$N(P_1', P_2', P_3') = N - (N(P_1) + N(P_2) + N(P_3)) + (N(P_1 P_2) + N(P_1 P_3) + N(P_2 P_3)) - N(P_1 P_2 P_3)$$
$$N(P_1', P_2', P_3') = 2504 - (1876 + 999 + 345) + (876 + 290 + 231) - 189$$
$$N(P_1', P_2', P_3') = 492$$

Alternatively, we can use the relationships between various



3. (Problem 17) How many elements are in the union of four sets if the sets have 50, 60, 70, and 80 elements, respectively, each pair of the sets has 5 elements in common, each triple of the sets has 1 common element, and no element is in all four sets.

$$50 + 60 + 70 + 80 - 5 \cdot \binom{4}{2} + 1 \cdot \binom{4}{3} - 0 \cdot \binom{4}{4} = 234$$

<u>From Section 8.6 (Applications of Inclusion-Exclusion)</u>

4. (Problem 11) In how many ways can seven different jobs be assigned to four different employees so that each employee is assigned to at least once job and the most difficult job is assigned to the best employee? *Hint: Begin by ignoring the requirement that the most difficult job is assigned to the best employee. Then use a symmetry argument.*

Ignoring the requirement that the most difficult job is assigned to the best employee, the number of assignments is the same as the number of onto functions from a set of $m = 7$ elements to a set of $n = 4$ elements. By Theorem 1, this is $4^7 - C(4,1)3^7 + C(4,2)2^7 - C(4,3)1^7$. By symmetry, the number of assignments in which the most difficult job is assigned to the best employee is

$$\frac{1}{n} \cdot [4^7 - C(4,1)3^7 + C(4,2)2^7 - C(4,3)1^7].$$

Here is an alternative solution provided by Capts Crowl and Mochocki along with Lts Echeverry, Matsui, and Vagedes:

We first found the number of possibilities in which the best worker has two or more jobs:

$$4^6 - \binom{4}{1}3^6 + \binom{4}{2}2^6 - \binom{4}{3}1^6 = 1560$$

Then we found the number of possibilities in which the best worker only has one job:

$$3^6 - \binom{3}{1}2^6 + \binom{3}{2}1^6 = 540$$

There is no overlap of the two, so you can simply add them together to get 2100

5. (Problem 25) How many of the derangements of {1,2,3,4,5,6} begin with the integers 1, 2, and 3 in some order?

There are two derangements of 1, 2, 3. There are two derangements of 4, 5, 6. Thus, there are $2 \cdot 2 = 4$ derangements of the set with 1, 2, and 3 at the beginning.