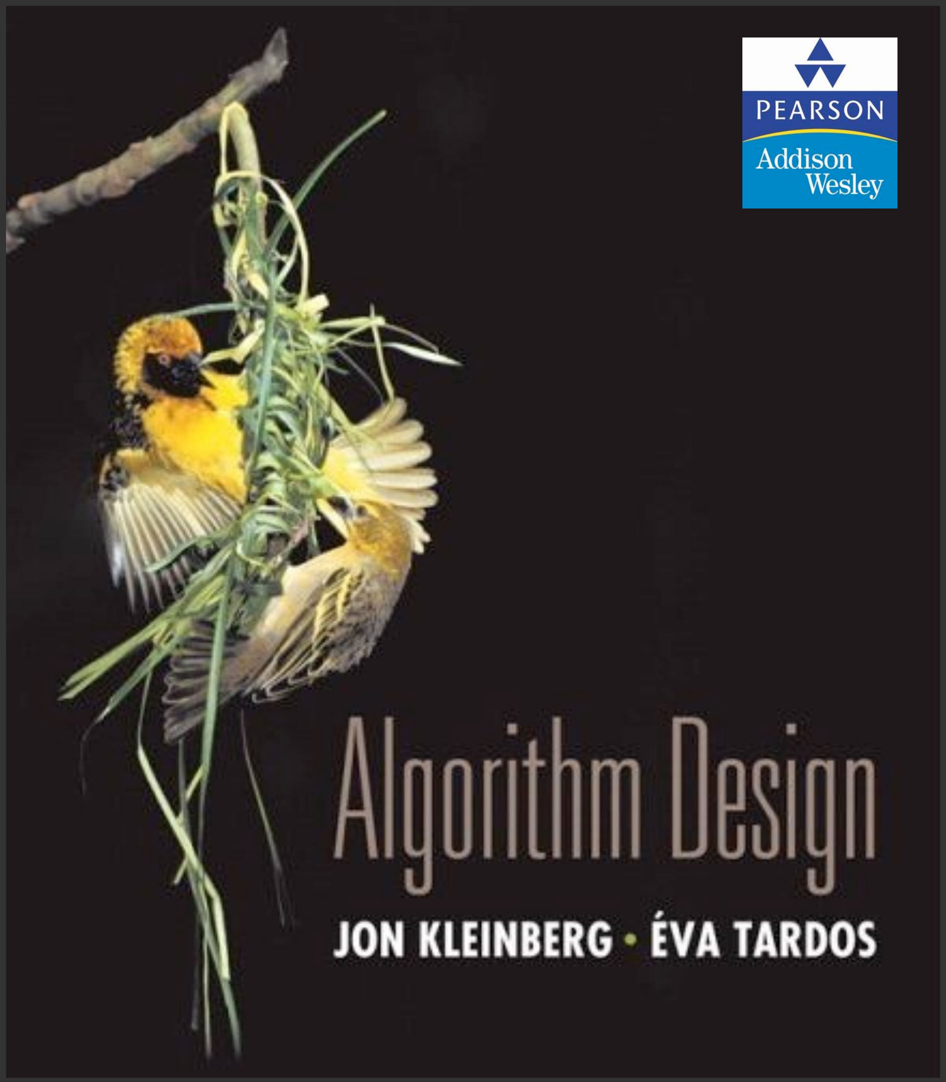


7. Network Flows I

- *Ford–Fulkerson pathological example*

Lecture slides by Kevin Wayne

http://www.cs.princeton.edu/~wayne/kleinberg-tardos

Last updated on 7/25/17 11:04 AM

Ford–Fulkerson pathological example

Intuition. Let r satisfy $r^2 = 1 - r$.

- Initial capacities are $\{ 1, r \}$.
- After some augmentations, residual capacities are $\{ 1, r, r^2 \}$.
- After some more, residual capacities are $\{ 1, r, r^2, r^3 \}$.
- After some more, residual capacities are $\{ 1, r, r^2, r^3, r^4 \}$.

$1 - r$

$r - r^2$

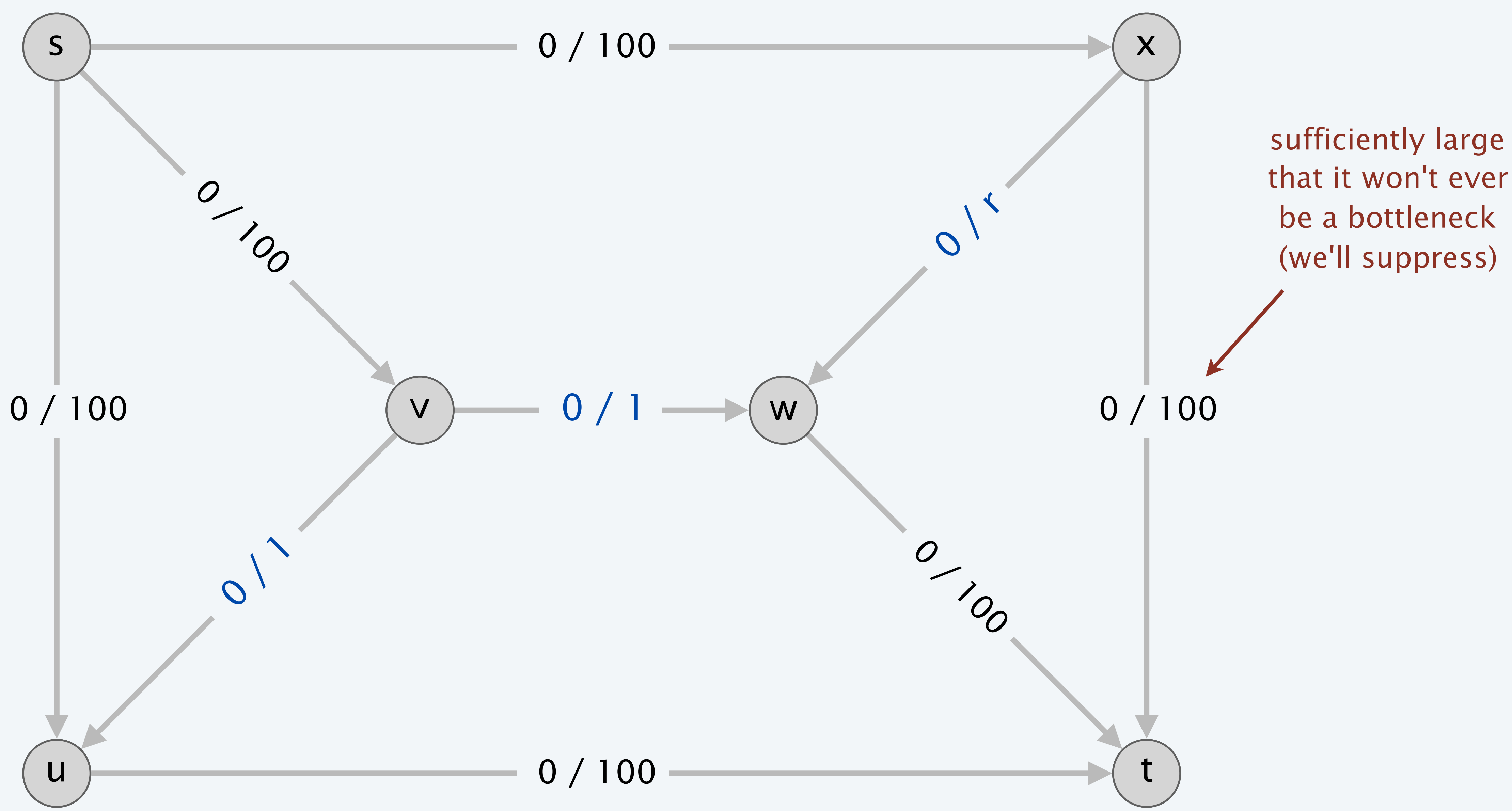
$r^2 - r^3$

$$r = \frac{\sqrt{5} - 1}{2} \implies r^2 = 1 - r$$

$$r \approx 0.618 \implies r^4 < r^3 < r^2 < r < 1$$

Ford–Fulkerson pathological example

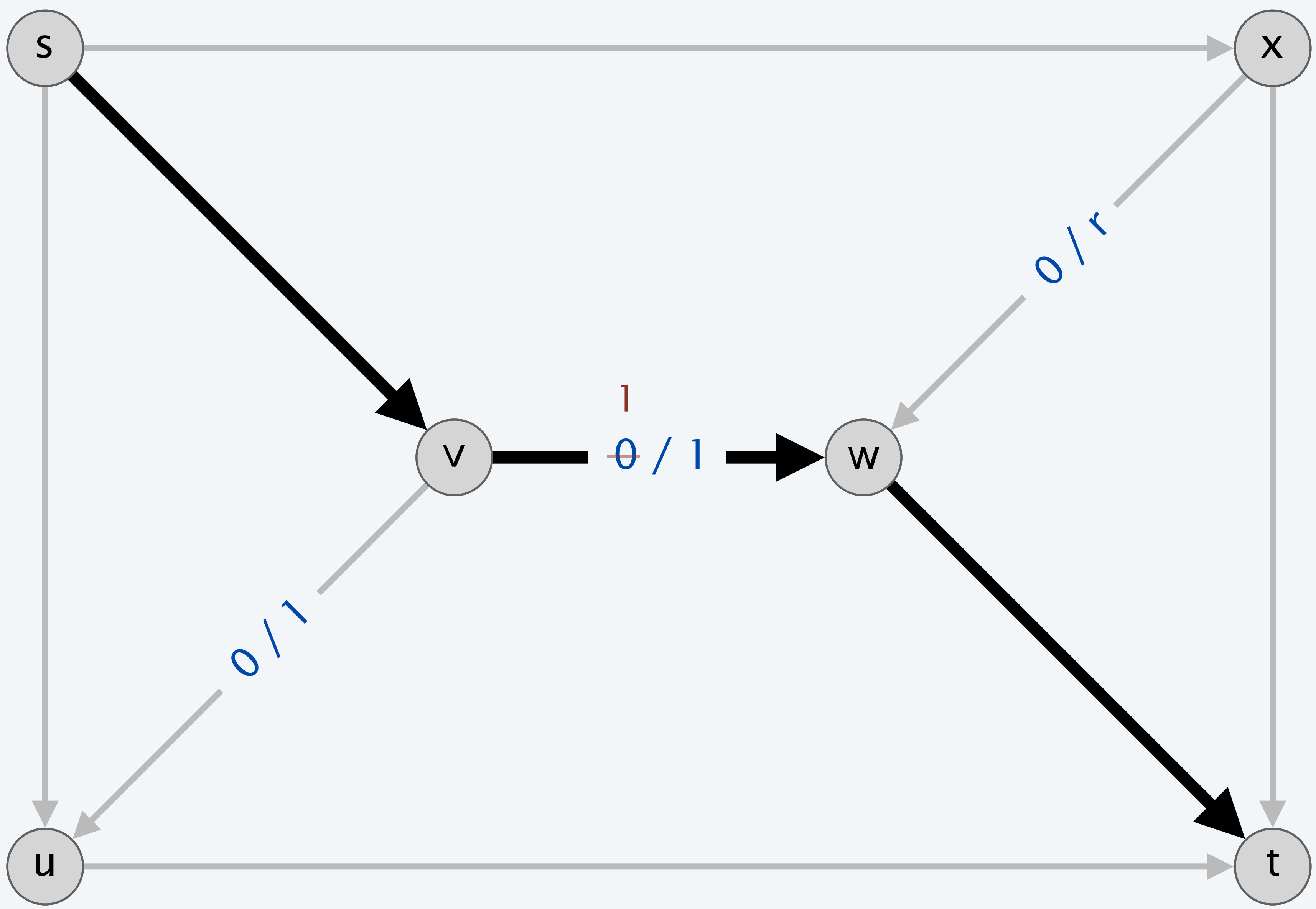
network G



$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

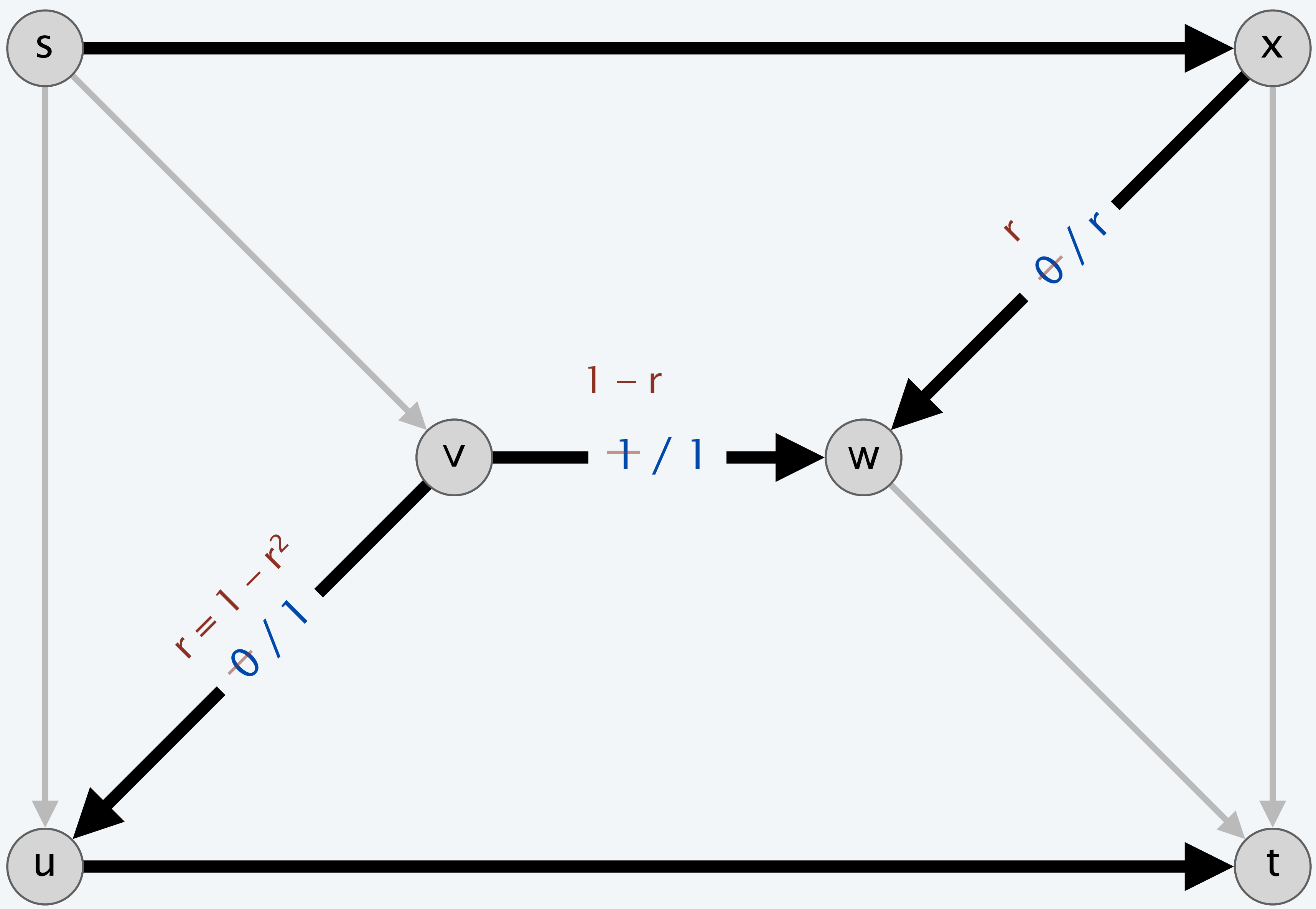
augmenting path 1: s→v→w→t (bottleneck capacity = 1)



$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

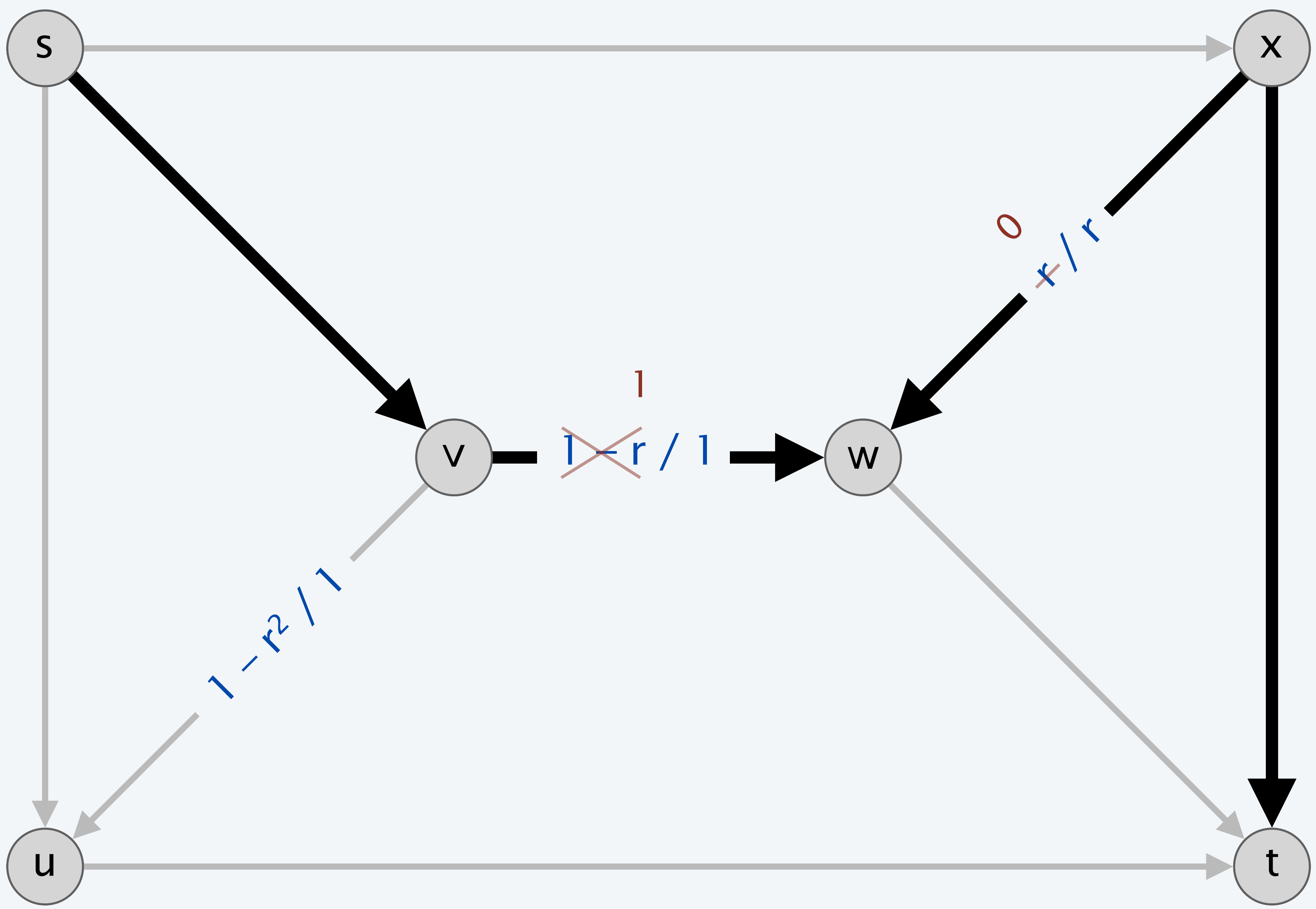
augmenting path 2: s→x→w→v→u→t (bottleneck capacity = r)



$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

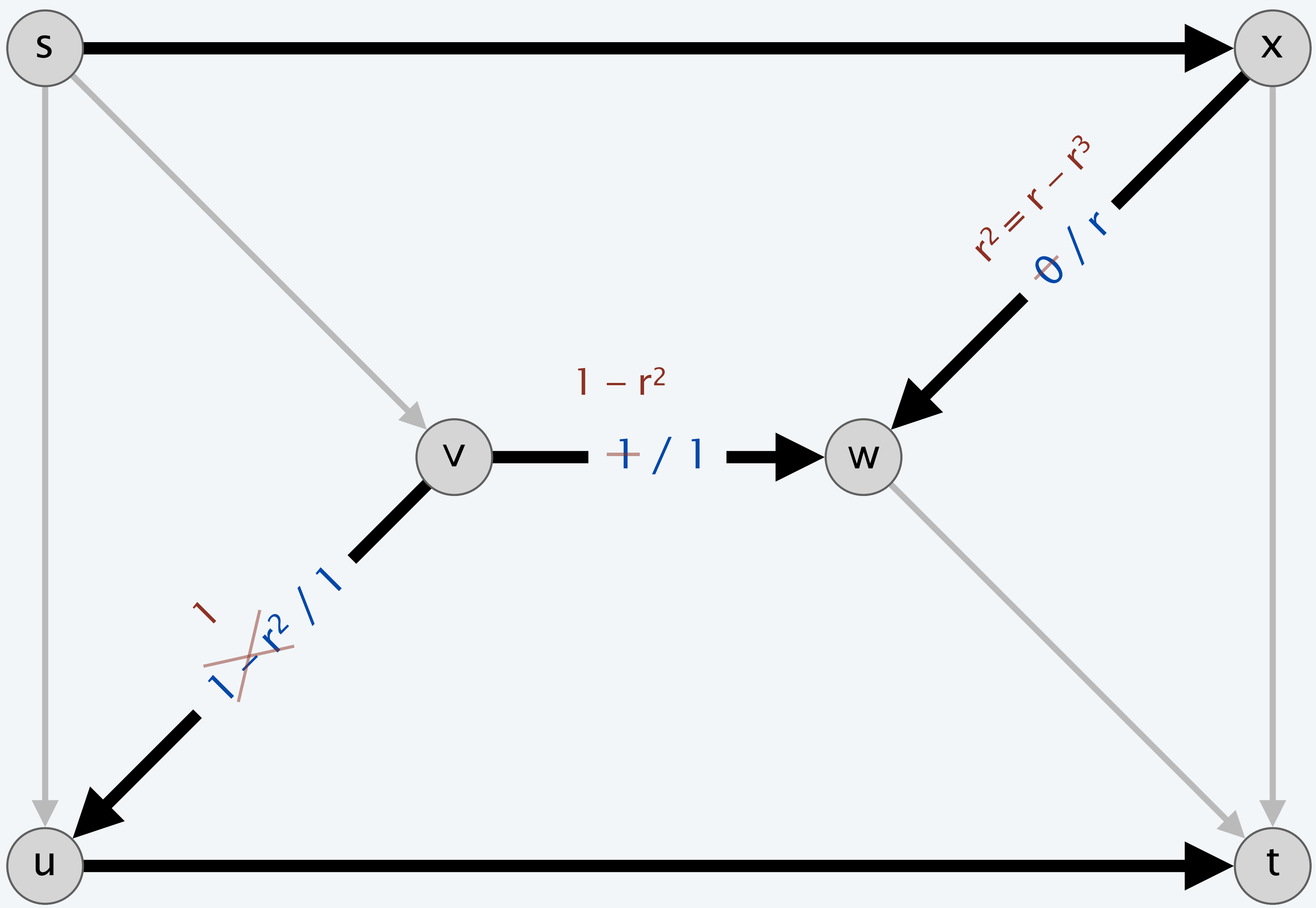
augmenting path 3: s→v→w→x→t (bottleneck capacity = r)



$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

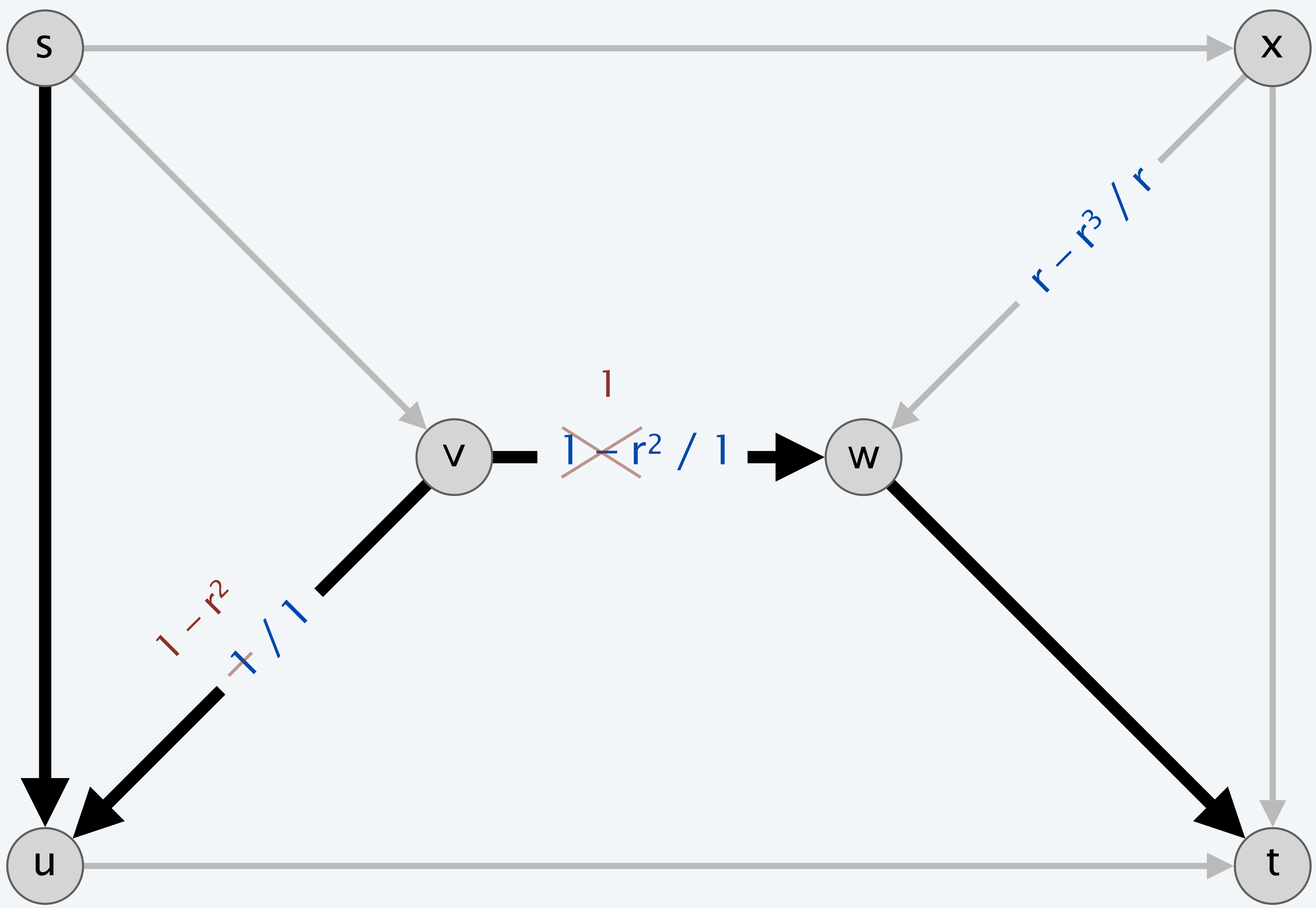
augmenting path 4: s→x→w→v→u→t (bottleneck capacity = r^2)



$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

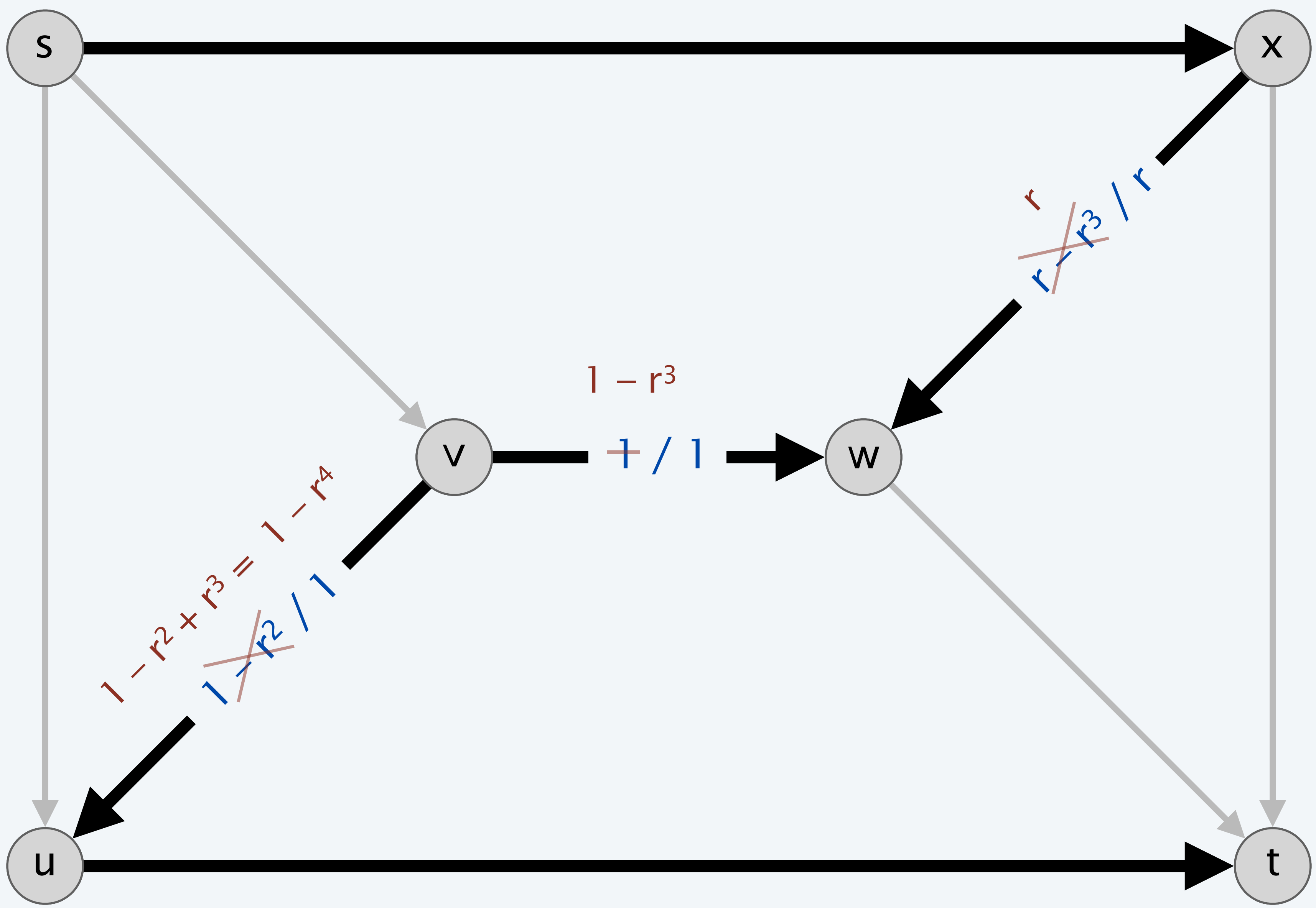
augmenting path 5: s→u→v→w→t (bottleneck capacity = r^2)



$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

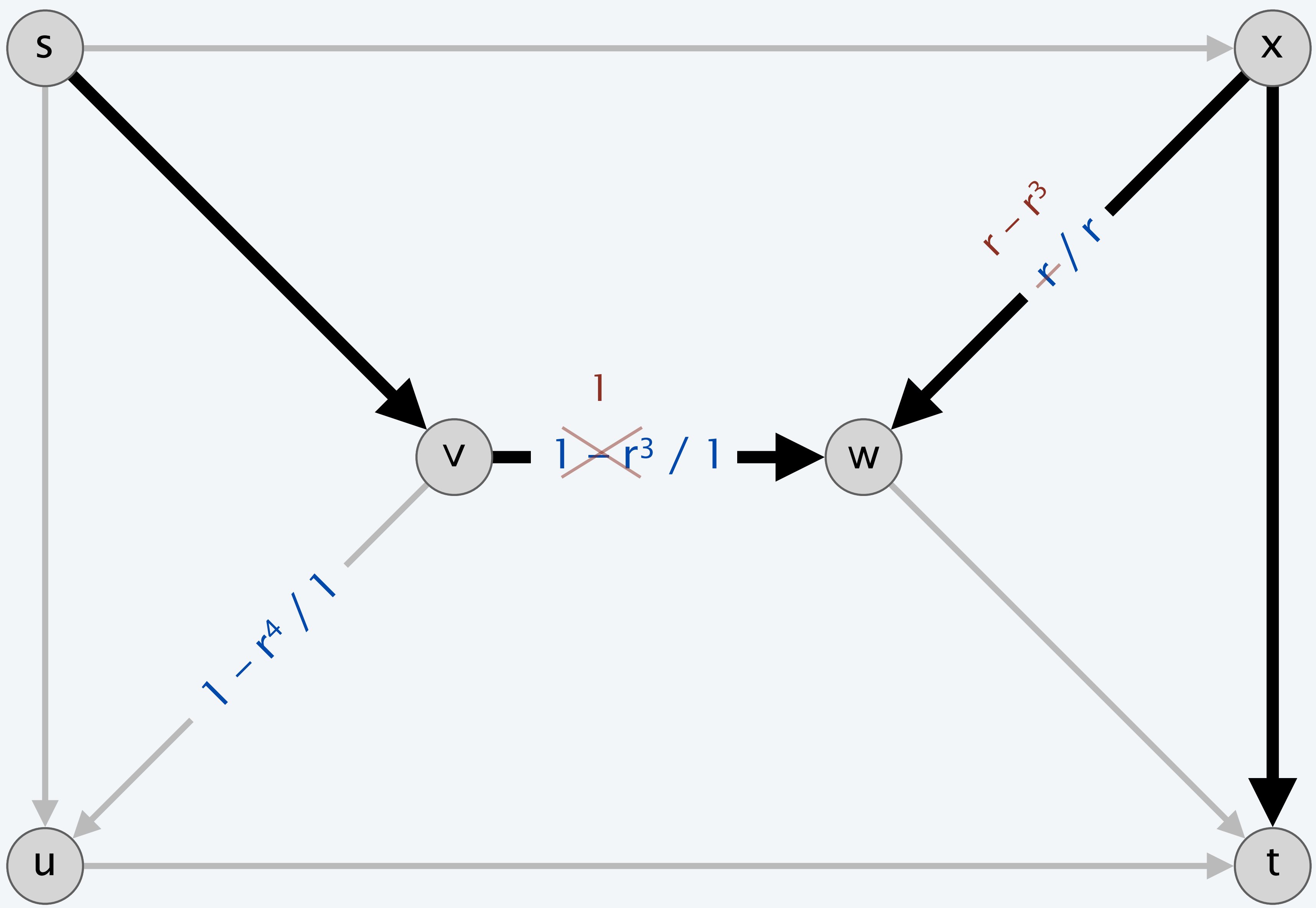
augmenting path 6: s→x→w→v→u→t (bottleneck capacity = r^3)



$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

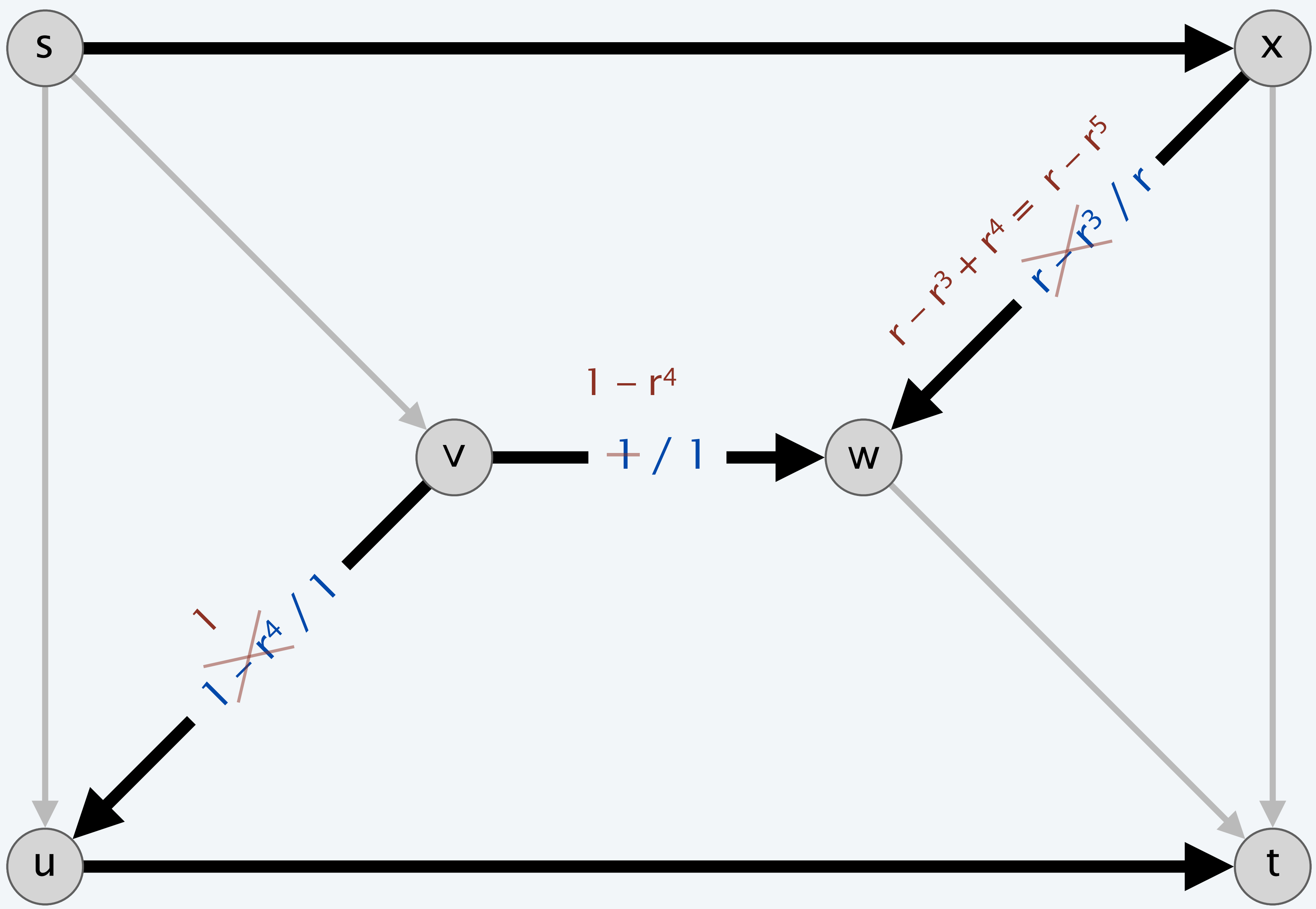
augmenting path 7: s→v→w→x→t (bottleneck capacity = r^3)



$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

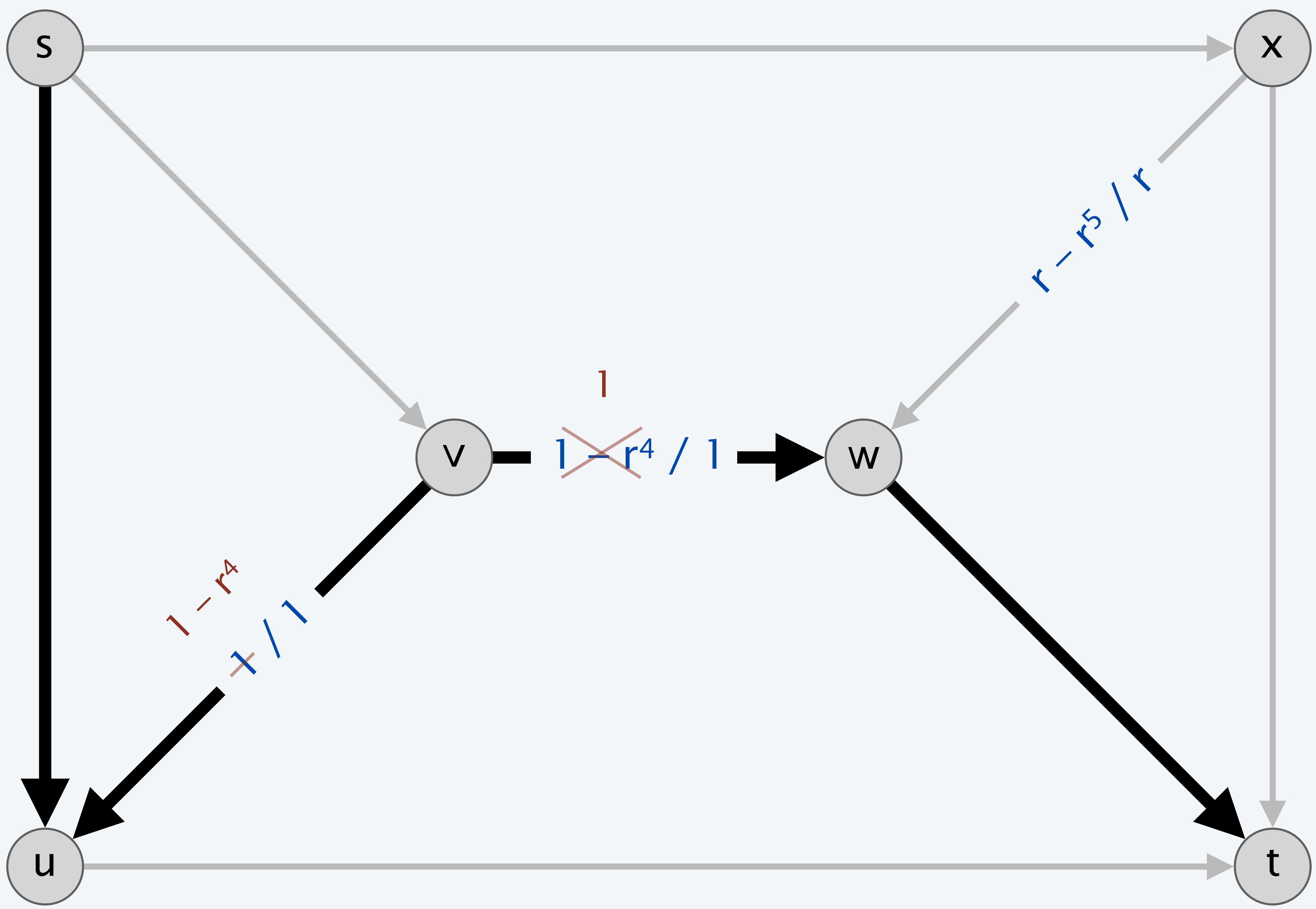
augmenting path 8: s→x→w→v→u→t (bottleneck capacity = r^4)



$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

augmenting path 9: s→u→v→w→t (bottleneck capacity = r⁴)



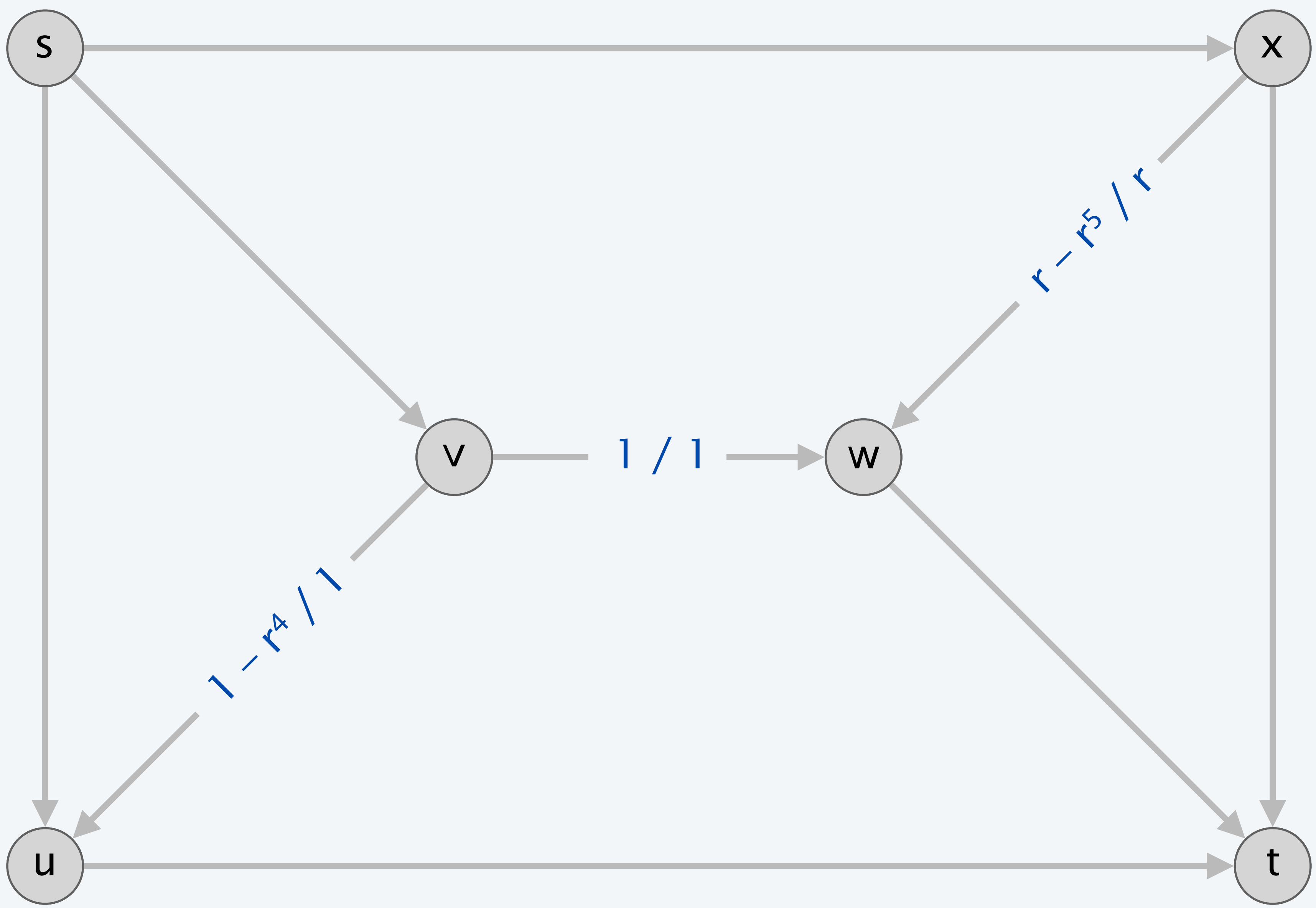
$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

after augmenting path 1: { $1 - r^0$, 1, $r - r^1$ } (flow value = 1)

after augmenting path 5: { $1 - r^2$, 1, $r - r^3$ } (flow value = $1 + 2r + 2r^2$)

after augmenting path 9: { $1 - r^4$, 1, $r - r^5$ } (flow value = $1 + 2r + 2r^2 + 2r^3 + 2r^4$)



$$r^2 = 1 - r$$

Ford–Fulkerson pathological example

Theorem. The Ford–Fulkerson algorithm may not terminate; moreover, it may converge to a value not equal to the value of the maximum flow.

Pf.

- Using the given sequence of augmenting paths, after $(1+4k)^{th}$ such path, the value of the flow

$$\begin{aligned} &= 1 + 2\sum_{i=1}^{2k} r^i \\ &\leq 1 + 2\sum_{i=1}^{\infty} r^i \\ &= 3 + 2r \\ &< 5 \end{aligned}$$

$$r = \frac{\sqrt{5}-1}{2}$$

- Value of maximum flow = $200 + 1$. ▪