Discrete Mathematics - CSCE 531 Fall 2018
In-Class Work, Day 02 (3 October 2018)

<u>From Section 1.4</u>

1. (Inspired by Problem 9) Let

   - $A(x)$ denote the statement "x attends the Artificial Intelligence course", and
   - $B(x)$ the statement "x attends the Biologically-Inspired Algorithms course."

   Using quantifiers and logical connectives as necessary, express each of the following statements in terms of $A(x)$ and $B(x)$ where the domains consists of all students in your school. *Note: some of the sentences are ambiguous. Learn from them so that you can avoid the same mistakes.*

   a. **There is at least one** student in your school who attends the Artificial Intelligence course.

   $$\exists x A(x)$$

   b. **Every** student in your school attends the Biologically-Inspired Algorithms class.

   $$\forall x B(x)$$

   c. **Either** the Artificial Intelligence course **or** the Biologically-Inspired Algorithms course is attended by **all** of the students in your school.

   $$[\forall x A(x)] \vee [\forall x B(x)]$$

   d. **All** students in your school attend the Artificial Intelligence course **or** the Biologically-Inspired Algorithms course.

   $$\forall x [A(x) \vee B(x)]$$

   e. **Some** student in your school attends the Artificial Intelligence course **but** not the Biologically-Inspired Algorithms course.

   $$\exists x [A(x) \wedge \neg B(x)]$$

   f. **No** student in your school attends the Artificial Intelligence course **and** the Biologically-Inspired Algorithms course.

   $$\neg \exists x [A(x) \wedge B(x)]$$

   g. **Each** of the courses is attended by **some** student.

   $$[\exists x A(x)] \wedge [\exists x B(x)]$$

2. (Inspired by Problem 35) Find a counterexample, if possible, to these universally quantified statements, where the domain of x for all statements is $\mathcal{Z}$, the set of all integers. If no counterexample exists, write “This is a true statement.”

   a. $\forall x(x^2 \geq x)$

This is a true statement.

   b. $\forall x[(x > 0) \vee (x < 0)]$

$x = 0$ is a counterexample.

   c. $\forall x(x = 1)$

$x = 2$ is a counterexample.

Section 1.5

3. (Problem 27) Determine the truth value (True or False) of each of these statements if the domain of each variable is $\mathcal{Z}$, the set of all integers. Give an example, counterexample, equation, etc. demonstrating the correctness of each of your answers (but don’t write complete proofs).

   a. $\forall n \exists m(n^2 < m)$

True (e.g. $m = n^2 + 1$)

   b. $\exists n \forall m(n \leq m^2)$

True ($n = 0$)

   c. $\forall n \exists m(n + m = 0)$

True ($m = -n$)

   d. $\exists n \forall m(nm = m)$

True ($n = 1$)

   e. $\exists n \exists m(n^2 + m^2 = 5)$

True (e.g. $n = 1$, $m = 2$)

   f. $\exists n \exists m(n^2 + m^2 = 6)$

False (Because $n^2 \geq 0$, we have $m^2 \leq 6$ and conversely. Thus $-2 \leq n \leq 2$ and $-2 \leq m \leq 2$. Enumeration completes the proof.)

   g. $\exists n \exists m(n + m = 4 \wedge n - m = 1)$

False (Adding the equations yields $2n = 5$, but $\frac{5}{2}$ is not an integer.)

h. $\exists n \exists m(n + m = 4 \wedge n - m = 2)$

True ($n = 3$, $m = 1$)

i. $\forall n \forall m \exists p \left(p = \frac{m+n}{2}\right)$

False (e.g. $n = 0, m = 1$, since $\frac{1}{2}$ is not an integer)

<u>Section 1.6</u>

4. (Inspired by Problem 9) Complete the table below showing conclusions that can be drawn from the following collection of premises. Indicate which rules of inference you used to draw each conclusion.

   "If I take the day off, it either rains or snows."
   "I took Monday off or I took Friday off"
   "It was sunny on Monday."
   "It did not snow on Friday"

Let

- $T(x)$ denote the statement "I take x off,"
- $R(x)$ the statement "it rains on x," and
- $S(x)$ the statement "it snows on x."

Step	Proposition	Reason
1	$\forall x\{T(x) \to [R(x) \vee S(x)]\}$	Premise
2	$T(Mon) \to [R(Mon) \vee S(Mon)]$	Universal instantiation of (1)[1]
3	$\neg T(Mon) \vee [R(Mon) \vee S(Mon)]$	Logical equivalence $p \to q \equiv \neg p \vee q$ applied to (2)
4	$T(Fri) \to [R(Fri) \vee S(Fri)]$	Universal instantiation of (1)
5	$\neg T(Fri) \vee [R(Fri) \vee S(Fri)]$	Logical equivalence $p \to q \equiv \neg p \vee q$ applied to (4)
6	$T(Mon) \vee T(Fri)$	Premise
7	$T(Fri) \vee [R(Mon) \vee S(Mon)]$	Resolution of (3) and (6)
8	$T(Mon) \vee [R(Fri) \vee S(Fri)]$	Resolution of (5) and (6)
9	$\neg R(Mon) \wedge \neg S(Mon)$	Premise
10	$\neg R(Mon)$	Simplification of (9)
11	$T(Fri) \vee S(Mon)$	Disjunctive syllogism applied to (7) and (10)
12	$\neg S(Mon)$	Simplification of (9)
13	$T(Fri)$	Disjunctive syllogism applied to (11) and (12)
14	$R(Fri) \vee S(Fri)$	Disjunctive syllogism applied to (5) and (13)
15	$\neg S(Fri)$	Premise
16	$R(Fri)$	Disjunctive syllogism applied to (14) and (15)

[1] We could also instantiate (1) for Sunday, Tuesday, Wednesday, Thursday, and Saturday, but those instantiations don't lead to further conclusions, so they are not shown.

5. (Inspired by Problem 19) Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?

   a. If $n \in \mathcal{R}$ such that $n > 1$ then $n^2 > 1$.
      Suppose that $n^2 > 1$. Then $n > 1$.

Let p be the proposition $n > 1$ and q the proposition $n^2 > 1$ (in both of which $n \in \mathcal{R}$). Then the given argument takes the form

$$\begin{array}{l} p \to q \\ \underline{q \quad\quad} \\ \therefore p \end{array}$$

This is the fallacy of affirming the conclusion.

   b. If n is a real number such that $n > 3$ then $n^2 > 9$.
      Suppose that $n^2 \leq 9$. Then $n \leq 3$.

Let p be the proposition $n > 3$ and q the proposition $n^2 > 9$ (in both of which $n \in \mathcal{R}$). Then the given argument takes the form

$$\begin{array}{l} p \to q \\ \underline{\neg q \quad\quad} \\ \therefore \neg p \end{array}$$

This is the law of modus tollens.

   c. If n is a real number such that $n > 2$ then $n^2 > 4$.
      Suppose that $n \leq 2$. Then $n^2 \leq 4$.

Let p be the proposition $n > 2$ and q the proposition $n^2 > 4$ (in both of which $n \in \mathcal{R}$). Then the given argument takes the form

$$\begin{array}{l} p \to q \\ \underline{\neg p \quad\quad} \\ \therefore \neg q \end{array}$$

This is the fallacy of denying the hypothesis.

6. Identify the error or errors in this argument that supposedly shows that if $\exists x P(x) \wedge \exists x Q(x)$ then $\exists x[P(x) \wedge Q(x)]$is true:

1.	$\exists x P(X) \wedge \exists x Q(x)$	Premise
2.	$\exists x P(x)$	Simplification from (1)
3.	$P(c)$	Existential instantiation from (2)
4.	$\exists x Q(x)$	Simplification from (1)
5.	$Q(c)$	Existential instantiation from (4)
6.	$P(c) \wedge Q(c)$	Conjunction from (3) and (5)
7.	$\exists x[P(x) \wedge Q(x)]$	Existential generalization of (6)

If we know that an element exists that satisfies a property, e.g. statements (2) and (4), then existential instantiation allows us to conclude that there is a specific element that satisfies the property. However, that element must either be one for which we already know the property is satisfied (which is rare) or one about which we know nothing. Statement (5) violates this constraint by “reusing” c.