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THE COORDINATE FRAMES OF THE US SPACE OBJECT CATALOGS

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Abstract

Using analytical theories known as General Perturbations (GP), the US Space Command maintains databases of artificial satellite orbital elements known as the Space Object Catalog. The GP Space Catalog is familiar to the general public as the NORAD or NASA two-line orbital elements. These elements are referenced to an unconventional reference frame based on approximations to the classical temporal equinox, specifically the "uniform equinox". This frame is rarely discussed in detail because the uncertainty of the GP orbital elements is usually larger than the subtle distinctions between non-standard Catalog frames and the conventional International Astronomical Union (IAU) frames. Advances in computing technology now allow for Space Catalog maintenance using higher accuracy numerical techniques, sometimes called Special Perturbations (SP). The SP Catalog is also maintained with respect to a temporal equinox. However, pending IAU resolutions will formally redefine conventional Earth orientation theory with new models that are independent of the equinox. The authors review the fundamentals of the old and new IAU transformations, including the transformation of satellite velocity, and relate the current Space Catalog frames to these conventions. Because the uniform equinox is already uncommon outside the US Space Commands, and because it will be rendered obsolete by upcoming IAU conventions, operational reliance on the uniform equinox should be phased out for high accuracy applications such as the SP Space Catalog.

The US Space Catalog

The United States Department of Defense (DoD) has maintained a database of satellite states since the launch of the first Sputnik in 1957, known as the Space Object Catalog[1], or simply the Space Catalog. These satellite states are regularly updated with observations from the Space Surveillance Network[2], a globally distributed network of interferometer, radar and optical tracking systems. Two separate catalog databases are maintained under the US Space Command: a primary catalog by the Air Force Space Command (AFSPC), and an alternate catalog by the Naval Space Command (NSC). The number of cataloged objects is approaching 10,000.

Different astrodynamic theories are used to maintain these catalogs. The so-called General Perturbations (GP) theory[3] provides a general analytical solution of the satellite equations of motion. The orbital elements and their associated partial derivatives are expressed as series expansions in terms of the initial conditions of these differential equations. The GP theories operated efficiently on the earliest electronic computing machines, and were therefore adopted as the primary theory for Space Catalog orbit determination. Assumptions must be made to simplify these analytical theories, such as truncation of the Earth's gravitational potential to a few zonal harmonic terms[4]. The atmosphere is usually modeled as a static, spherical density field that exponentially decays[5]. Third body influences and resonance effects are partially modeled[6]. Increased accuracy of GP theory usually requires significant development efforts [7].

NASA maintains civilian databases of GP orbital elements, also known as NASA or NORAD two-line elements"[8 9]. The GP element sets are "mean" element sets that have specific periodic features removed to enhance long-term prediction performance, and require special software to reconstruct the compressed trajectory[10 11].

The GP reference frame has been a recurring source of confusion for analysts and satellite owner-operators because the GP reference frame has not been precisely defined in a form accessible to a larger audience. Additionally, the reference frame is not widely acknowledged as an astrodynamic standard and the adopted descriptors are uncommon outside the US Space Commands. Because the GP elements are not terribly accurate[12], misinterpretation of this reference frame is probably within the uncertainty of the orbital elements themselves and has not warranted distinction.

General Perturbations theory is not sufficiently accurate to support every mission. Higher accuracy methods, sometimes known as Special Perturbations (SP)[13], follow from the direct integration of the satellite equations of motions using numerical methods. Although SP orbit determination is more computationally intensive than GP theory, it is simpler to develop and more readily modified to afford improvements in accuracy.

The capacity of computing machines has increased at a must faster rate than the size of the Space Catalog[14]. In 1997, the Naval Research Laboratory and the NSC demonstrated the Special Perturbations orbit determination concept by maintaining the entire Space Catalog for one month using a distributed machine architecture[15]. This processor, known as SPeCIAL-K, passed Initial Operational Capability in July, 1999 and is expected to be certified as having Final Operational Capability in mid-2000. In September 1999, the AFSPC began using a Special Perturbations orbit determination processor known as the Astrodynamics Support Workstation (ASW) to maintain its supplemental "high accuracy catalogue"[16] [17]. Both systems are expected to replace the older Special Perturbations orbit determination systems at their respective command centers. Their supplemental SP catalogs will eventually supersede the operational GP catalogs.

The SP Space Catalogs are a relatively new product. The final standards regarding general distribution, format, and possible compression are evolving as military and civilian applications confer their diverse requirements. SP catalog states already support certain civilian missions such as collision avoidance for the Space Shuttle and International Space Station[18]. Ever-increasing accuracy of Special Perturbations is likely to expand the military and civilian customer base for the SP Space Catalog.

At the time of this writing, SP satellite states are often supplied in different frames. Recent studies of SP catalog accuracy[19] showed that the prediction errors in select satellite states were well below the errors that may be introduced due to simple confusion of the frame. GP and SP frames are based on older IAU conventions that are expected to change soon. The authors therefore advocate adoption of a more accurately defined reference standard for distributing the SP catalog, one that will be widely recognized by both civilian and military agencies and more consistent with the upcoming IAU system.

Coordinate Systems And Frames

The ordinary differential equations that describe satellite motion are most simply defined in a Newtonian-inertial space. In this case, the time derivatives of the coordinate axes are negligible, eliminating Coriolis effects. The initial conditions of these differential equations (the satellite state) are therefore estimated in an inertial frame of reference. However, satellite motion is usually observed from stations fixed to the surface of the rotating Earth. For geocentric orbit determination then, the Earth's orientation with respect to inertial space must be well approximated. The motion of the Earth's spin axis with respect to celestial objects has been refined through astrometric observation over many decades, and the temporal equinox has long been the basis of these observations and their corresponding celestial reference systems. It is therefore convenient to adopt a celestial reference system as the Newtonian-inertial space for orbit determination. By so doing, the satellite state is ultimately tied to a specific celestial convention and a specific realization of its temporal equinox and pole.

A conventional coordinate *system* is a set of prescriptions that defines a triad of orthogonal axes at any time, whereas a conventional *frame* is the practical realization of a system at a specific epoch based on the system prescriptions[20]. These conventions are not necessarily the most rigorous or correct definitions; rather, they are practical representations that are commonly accepted. Conventional reference systems are approved by the International Astronomical Union (IAU) and partly maintained by the International Earth Rotation Service (IERS). Before 1998, the Fundamental Katalog 5 (FK5) system was the basis of the IAU celestial reference system[21]. The FK5 theory is defined by the IAU 1976 Precession model and 1980 Theory of Nutation, and was primarily realized from stellar observations taken at optical wavelengths.

Beginning January 1, 1998, the International Astronomical Union (IAU) adopted the International Celestial Reference System (ICRS)[22]. The ICRS is a system that defines a triad of orthogonal axes whose origin resides at the barycenter of the solar system. Its axes are realized by observations of extragalactic radio sources from the Very Long Baseline Interferometry (VLBI) network[23], and the origin in right ascension is based on the FK5 J2000 value adopted for radio source 3C 273[24]. At optical wavelengths, the ICRS has been astrometrically tied to the HIPPARCOS star catalog[25]. As estimates of the relative positions between the defining sources improves and more defining sources are added, the frame of the ICRS (known as the ICRF) will be maintained such that there is no net rotation introduced with respect to previous realizations. A significant difference between the ICRF and the FK5 theory is that the axes of the FK5 system move as a function of time with respect to inertial space, while the axes of the ICRF remain fixed for all practical purposes.

The geocentric counterpart to the ICRF will be known as the Geocentric Celestial Reference Frame (GCRF)[26], which has been the celestial reference for the IERS since January 1, 1997[27]. The axes of the GCRF are close to the frame of J2000, as prescribed by the FK5 theory, to provide a high level of continuity between the former and current IAU systems. Because there was no official IAU nutation theory compatible with the ICRS when it was adopted, the IERS has continued to maintain tabulated corrections for the IAU FK5 theory to relate it to the GCRF[28]. The relationship between these frames at J2000 is approximated using the quasi-orthogonal rotation:

$$\mathbf{r}_{\mathrm{GCRF}} = \begin{bmatrix} 1 & 0.000273\times10^{-8} & -9.740996\times10^{-8} \\ -0.000273\times10^{-8} & 1 & -1.324146\times10^{-8} \\ 9.740996\times10^{-8} & 1.324146\times10^{-8} & 1 \end{bmatrix} \mathbf{r}_{\mathrm{J2000/FK5}} \tag{1}$$

where

$\mathbf{r}_{\mathrm{GCRF}}$ is a three dimensional vector with respect to the GCRF basis, and
$\mathbf{r}_{\mathrm{J2000/FK5}}$ is a three dimensional vector with respect to the FK5 basis at J2000.

The change of basis implied by Eq.(1) is within the formal uncertainty of the original FK5 prescriptions[29] (50 mas or 1.5 m/ER[*] in pole and 80 mas or 1.5 m/ER in equinox). Neither frame is exactly coincident with the current best estimates of the mean pole and equinox at J2000, as these directions are determined by observation rather than convention[30].

Orbit determination requires both celestial references frames (which define the Newtonian inertial space in which the differential equations of motion are valid), and terrestrial references frames (from which the satellite observations are taken). The conventional terrestrial frame, known as the International Terrestrial Reference Frame (ITRF), has its origin at the center of mass of the Earth[†]. The directions of the axes are realized by the adopted coordinates of defining fiducial stations on the surface of the Earth. The relative station coordinates are affected by plate tectonic motion on the order of centimeters per year, such that they must be annually re-estimated through VLBI, Satellite Laser Ranging (SLR), GPS, and DORIS observations[31]. The ITRF is a weighted, global combination of several analysis center solutions, adjusted such that there is no net rotation or frame shift with respect to previous realizations of the ITRF.

The WGS-84 terrestrial frame is primarily used by the US DoD. It is realized through GPS observations, although the fundamental WGS-84 stations are usually constrained by their adopted ITRF coordinates during solution. Thus, the WGS-84 and ITRF terrestrial frames agree at the few cm level[32], and are often considered to be identical.

The terrestrial reference frame is related to the celestial reference frame through a series of rotations, known here as an Earth orientation model. The total model is sometimes prescribed by partial sequences of rotations, and intermediate frames are often defined as the bases between these partial sequences. There are two conventional methods for modeling Earth orientation: the classical transformation and the non-rotating origin transformation. The latter is expected to be the basis of the upcoming IAU theory, but the Space Catalog reference is currently defined within the context of the classical transformation.

Classical Transformation

The classical transformation between a celestial basis and a terrestrial or "Earth fixed" basis[33] is well established. This transformation, also known as Option 1, takes the following vector-matrix form[34]:

$$\mathbf{r}_{\mathrm{TRF}} = \mathbf{[W]\ [R]\ [N]\ [P]}\ \mathbf{r}_{\mathrm{CRF}} \tag{2}$$

where

$\mathbf{r}_{\mathrm{TRF}}$ is a three dimensional position vector with respect to the terrestrial reference frame,
$\mathbf{r}_{\mathrm{CRF}}$ is a three dimensional position vector with respect to the celestial reference frame,

$\mathbf{[P]} = \mathbf{ROT3}(-\zeta_A)\ \mathbf{ROT2}(\theta_A)\ \mathbf{ROT3}(-z_A)$ is the precession matrix,

$\mathbf{[N]} = \mathbf{ROT1}(-\varepsilon_A)\ \mathbf{ROT3}(?\psi)\ \mathbf{ROT1}(\varepsilon_A+?\varepsilon)$ is the nutation matrix,

$\mathbf{[R]} = \mathbf{ROT3}(\theta_{\mathrm{GST}})$ is the sidereal rotation matrix, and

$\mathbf{[W]} = \mathbf{ROT2}(-x_{\mathrm{p}})\ \mathbf{ROT1}(-y_{\mathrm{p}})$ is the polar motion matrix.

θ_{GST} is a function of Universal Time UT1 (the primary measure of Earth rotation), and **ROT1**, **ROT2**, and **ROT3** are rotations about the X, Y, and Z axes respectively[35][36]. The time dependent angles ζ_A, θ_A, z_A, ε_A, $?\psi$, $?\varepsilon$, and θ_{GST} are defined according to the IERS Conventions 1996. Tabulated values of x_{p}, y_{p}, and UT1-UTC, compatible with the IERS system and known as Earth Orientation Parameters (EOP), are available through the IERS.

[*] Small angles are usually supplemented here in units of distance per Earth radii, such as cm/ER. "mas" implies milliarcseconds.

[†] In future definitions of the ITRF datum, it will be necessary to modify the basic transformation to account for cm level geocenter motion with respect to the fiducial stations (see IERS Gazette No. 50).

Table 1
FRAME DESCRIPTORS OF THE CLASSICAL TRANSFORMATION OF DATE

Abbrv.	Common Names	Other Designators*	Rotation
TEF	(True) Earth fixed, body fixed	Earth Centered Rotating (ECR)	⇦ **[W]**
PEF	Pseudo Earth fixed, Pseudo body fixed	Earth Fixed Greenwich (EFG) Earth Centered Earth Fixed (ECEF)	⬀ ⇦ **[ℜ]**
UOD	Uniform (Equinox) of Date	Earth Centered Inertial (ECI) True Equator and Mean Equinox	⬀ ⇦ **[Q]**
TOD	True (Equinox) of Date, True Equator and True Equinox		⬀ ⇦ **[N]**
MOD	Mean (Equinox) of Date, Mean Equator and Mean Equinox		⬀ ⇦ **[P]**
J2000	Mean of 2000		⬀ ⇦ **[**Eq.1**]**^{T}
GCRF	Geocentric Celestial Reference Frame		⬀

* AFSPC Operating Instruction 60-102 11-Mar-1996, TP SCC 008

The classical transformation separates the motion of precession and nutation to predict the nominal direction of the Earth's rotational pole with respect to the "fixed stars" on the celestial sphere. This rotational pole is currently known as the Celestial Ephemeris Pole (CEP)[37]. Precession **[P]** describes the large-scale secular motion of the pole while nutation **[N]** describes the quasi-periodic variability of the CEP with respect to its precessional drift. A simple sidereal rotation **[R]** from the temporal equinox about the CEP largely fulfills the change of basis from a celestial to a terrestrial framework. An additional set of small-angle rotations known as polar motion **[W]** compensates for the fact that the axis of Earth rotation is slightly offset from the adopted terrestrial origin for 90° latitude, and continually migrates in a slow but somewhat unpredictable way[38].

In concept, the temporal equinox is defined by the line of intersection between the plane of the equator and the plane of the ecliptic. The equator is the plane perpendicular to the CEP passing through the center of mass of the central body — Earth. The equator can be either the "mean equator" (the plane defined by the pole before **[N]** is applied) or the "true equator" (the plane defined by the pole after **[N]** is applied). The equinox can therefore be described as a "mean" or "true" equinox of date, depending upon the equator to which is it referenced.

The "uniform equinox" is an infrequently acknowledged fiducial direction in astrodynamics, defined by the true equinox of date minus the so-called "Equation of the Equinoxes"[39] (Eq_{equinox}). The uniform equinox is not widely recognized as a conventional basis for satellite states because it is not a *bone fide* equinox in the sense defined above. To define the uniform equinox within the classical transformation, one must sub-divide the Greenwich Sidereal angle θ_{GST} about the CEP into Greenwich Mean Sidereal time θ_{GMST}[40] and Eq_{equinox}, such that Eq. (2) becomes:

$$\mathbf{r}_{\mathrm{TRF}} = \mathbf{[W]}\ \mathbf{[\Re]}\ \mathbf{[Q]}\ \mathbf{[N]}\ \mathbf{[P]}\ \mathbf{r}_{\mathrm{CRF}} \qquad (3)$$

where

$$\mathbf{[\Re]} = \mathbf{ROT3}(\theta_{\mathrm{GMST}}) \text{ and } \mathbf{[Q]} = \mathbf{ROT3}(EQ_{\mathrm{equinox}})\ .$$

The intermediate bases resulting from the classical transformation in Eq. 3 are sometimes described by the naming conventions in Table 1.

The uniform equinox has been historically interpreted as the projection of the mean equinox of date onto the true equator of date. For this reason, it is sometimes cited in early astrodynamics literature as the "true equator and mean equinox of date"[41] (TEME), if it is specified at all. Consequently, it is often described as a rotation with respect to the mean-of-date equinox (Figure 1). For example, AOES[42] defines this TEME rotation as:

$$\mathbf{r}_{\mathrm{TEME}} = \begin{bmatrix} 1 & 0 & -\Delta\psi\sin(\varepsilon) \\ 0 & 1 & -\Delta\varepsilon \\ \Delta\psi\sin(\varepsilon) & \Delta\varepsilon & 1 \end{bmatrix} \mathbf{r}_{\mathrm{MOD}}$$

which comes from a small angle approximation[43] to the series of rotations:

$$\mathbf{r}_{\mathrm{TEME}} = \mathbf{ROT1}(-90°)\ \mathbf{ROT3}(-?\psi\sin(\varepsilon_A+?\varepsilon))\ \mathbf{ROT1}(90°-?\varepsilon)\ \mathbf{r}_{\mathrm{MOD}}\ . \qquad (4)$$

This set of rotations is nearly the same as the series implied by Eq. (3) for the uniform equinox, namely:

$$\mathbf{r}_{\mathrm{UOD}} = \mathbf{ROT3}(EQ_{\mathrm{equinox}})\ \mathbf{ROT1}(\varepsilon_A+?\varepsilon)\ \mathbf{ROT3}(-?\psi)\ \mathbf{ROT1}(\varepsilon_A)\ \mathbf{r}_{\mathrm{MOD}}\ , \quad (5)$$

The maximium differences between Eq. (4) and Eq. (5) are below the milliarcsecond level (~1 cm/ER) prior to 1997.

However, a distinction now exists between the geometric and kinematic interpretations of the Equation of the Equinoxes[44]. Effective January 1, 1997, the Equation of the Equinoxes is:

$$EQ_{\mathrm{equinox}} = \theta_{\mathrm{GST}} - \theta_{\mathrm{GMST}} = ?\psi\cos(\varepsilon_A+?\varepsilon) + ?eq_{\mathrm{equinox}}$$

$$?eq_{\mathrm{equinox}} = 2.64\mathrm{mas}\cdot\sin(?) - 0.009\mathrm{mas}\cdot\sin(2?)$$

where ? is the mean longitude of the ascending node of the Moon. Under this kinematic interpretation, the rotation from the mean-of-date equinox basis to the uniform-equinox basis would have an approximate small angle form:

$$\mathbf{r}_{\mathrm{UOD}} = \begin{bmatrix} 1 & \Delta eq_{\mathrm{equinox}} - \Delta\psi\sin(\varepsilon) \\ -\Delta eq_{\mathrm{equinox}} & 1 & -\Delta\varepsilon \\ \Delta\psi\sin(\varepsilon) & \Delta\varepsilon & 1 \end{bmatrix} \mathbf{r}_{\mathrm{MOD}}$$

The long term difference between kinematic and geometric interpretations due to the correction to the Equation of the Equinoxes $?eq_{\mathrm{equinox}}$ is about 20 times larger than the higher order terms neglected in the small angle forms of these matrices, but never get larger than 3 mas (10 cm/ER).

For the sake of clarity, the term "uniform equinox" is reserved here for the intermediate basis between the application of **[ℜ]** and **[Q]** in Eq. (3), consistent with the kinematic interpretation. The term "true equator and mean equinox of date" (TEME) is considered to described the geometric intepretation.

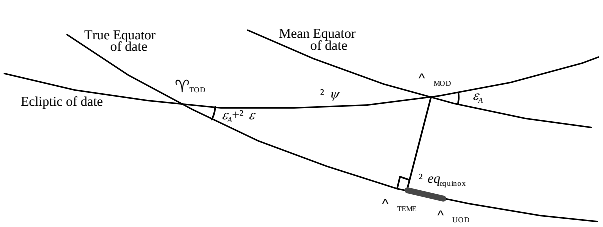

Figure 1. The Mean, True, and Uniform Equinoxes of Date, as viewed from outside the Celestial Sphere.

The conventional transformation of velocity between the Earth fixed frame and the celestial frame is not explicitly prescribed by the IERS Conventions 1996, but it is partly implied by the time derivative of Eq. (2) or Eq. (3). Because the UOD, MOD, and TOD intermediate frames are inertial, $\frac{d}{dt}$**[Q]** $\frac{d}{dt}$**[N]** and $\frac{d}{dt}$**[P]** are zero by definition.

The time derivative of Eq. (2) or Eq. (3) also implies the use of $\dot{\theta}_{\mathrm{GMST}}$ in $\frac{d}{dt}$**[ℜ]** or $\dot{\theta}_{\mathrm{GST}}$ in $\frac{d}{dt}$**[R]**. As a result, $\dot{\theta}_{\mathrm{GMST}}$ is often used to define the Earth's angular velocity instead of ω ($\dot{\theta}_{\mathrm{GMST}}$ being equal to $7.292115855307\times10^{-5}$ rad/sec on January 1, 2000 0.0 UT1). However, $\dot{\theta}_{\mathrm{GMST}}$ defines the Earth's rotation rate with respect to the precessing mean equinox, not inertial space. Sidereal rotation and rate are directly measured with respect to a quasi-inertial space through the observations of stars, quasi-stellar radio sources, and satellite motion in an inertial frame. For this reason, and because all realizations of the equinox are considered inertial by the practiced convention, the angular velocity of the Earth with respect an inertial frame should be used in $\frac{d}{dt}$**[ℜ]** or $\frac{d}{dt}$**[R]**. The "stellar angle" θ represents the angle of a reference meridian on the Earth with respect to a celestial non-rotating origin in the plane of the equator. It can be derived from the conventional expression of θ_{GMST} as a function of UT1[45]:

$$\theta = 2\pi(0.779057273264 + 1.00273781191135448\cdot T_u\cdot 36525)\ \mathrm{rad} \quad (6)$$

where T_u is the Julian Centuries of UT1, The time derivative with respect to a uniform time scale is:

$$\omega = 7.292115146706979\times10^{-5}\cdot(1 - XLOD/86400)\ \mathrm{rad/sec} \quad (7)$$

XLOD is the excess length of day (in seconds) and is estimated or predicted by the IERS. *XLOD* is the instantaneous rate of change of UT1 with respect to a uniform time scale (such as UTC or TAI) about the CEP. Like UT1, it is a measured quantity subject to sub-millisecond zonal tide variations. Eq (7) is consistent with the angular velocity defined in the Explanatory Supplement to IERS Bulletins A and B[46].

Although $\dot{\theta}_{\mathrm{GMST}}$ is less compatible with an inertial equinox, the difference is small relative to the requirements of most Earth fixed satellite applications ($\sim10^{-11}$ rad/sec). However, the differences between $\dot{\theta}_{\mathrm{GMST}}$ and ω are of the same order of *XLOD*. Applications that use length of day corrections will likely require ω in lieu of $\dot{\theta}_{\mathrm{GMST}}$.

Velocity due to polar motion $\frac{d}{dt}\mathbf{[W]}$ calls for the numerical derivatives of the tabulated angles provided by the IERS. Polar motion rates are nominally smaller than UT1 rate by two orders of magnitude and are not provided by the IERS. For these reasons, $\frac{d}{dt}\mathbf{[W]}$ is usually neglected. Based on these conventions, the classical transformation of satellite velocity is:

$$\mathbf{v}_{\mathrm{TRF}} = \mathbf{[W]\,[\Re]\,[Q]\,[N]\,[P]}\,\mathbf{v}_{\mathrm{CRF}} + \mathbf{[W]}\,\frac{d}{dt}\mathbf{[\Re]\,[Q]\,[N]\,[P]}\,\mathbf{r}_{\mathrm{CRF}}$$

where

$\mathbf{v}_{\mathrm{TRF}}$ is a three dimensional velocity vector with respect to the terrestrial reference frame,

$\mathbf{v}_{\mathrm{CRF}}$ is a three dimensional velocity vector with respect to the celestial reference frame, and

$$\frac{d}{dt}\mathbf{[\Re]} = \begin{bmatrix} -\omega\sin(\theta_{\mathrm{GMST}}) & \omega\cos(\theta_{\mathrm{GMST}}) & 0 \\ -\omega\cos(\theta_{\mathrm{GMST}}) & -\omega\sin(\theta_{\mathrm{GMST}}) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (8)$$

An expression for $\frac{d}{dt}\mathbf{[R]}$, compatible with Eq. (2), would be identical to Eq. (8) except θ_{GMST} is replaced with θ_{GST}.

Non-Rotating Origin Transformation

The classical transformation has an intermediate dependence on the ecliptic of date. However, VLBI and SLR techniques employed for modern Earth orientation are insensitive to the ecliptic plane[47]. This deficiency is alleviated by using an alternative transformation making explicit use of the non-rotating origin[48]. In matrix-vector form, the transformation of satellite position appears similar to the classical transformation:

$$\mathbf{r}_{\mathrm{ITRF}} = \mathbf{[W']\,[R']\,[N'P]}\,\mathbf{r}_{\mathrm{CRF}} \qquad (9)$$

where

$\mathbf{r}_{\mathrm{TRF}}$ is a three dimensional position vector with respect to a terrestrial reference frame,

$\mathbf{r}_{\mathrm{CRF}}$ is a three dimensional position vector with respect to a celestial reference frame,

$$\mathbf{[N'P]} = \mathbf{ROT3}(-s)\begin{bmatrix} 1-aX^2 & -aXY & -X \\ -aXY & 1-aY^2 & -Y \\ X & Y & 1-a(X^2+Y^2) \end{bmatrix}\mathbf{r}_{\mathrm{MOD}}$$

is the precession-nutation matrix,

$\mathbf{[R']} = \mathbf{ROT3}(\theta)$ is the sidereal rotation matrix using the stellar angle, and

$\mathbf{[W']} = \mathbf{ROT2}(-x_p)\,\mathbf{ROT1}(-y_p)\,\mathbf{ROT3}(s')$ is the polar motion matrix.

The time varying quantites a, s, θ, s', X and Y will be defined according to the IERS Conventions 2000 (stellar angle θ was defined in Eq.(6)). The Earth Orientation Parameters x_p, y_p, and UT1–UTC are currently the same as those for the classical transformation, and X and Y are the direction cosines of the celestial pole with respect to the GCRF.

The non-rotating origin transformation, known as Option 2, is kinematically correct as it segregates the terrestrial and celestial motion of the pole from the rotation of the Earth. The non-rotating origin transformation method is computationally efficient and conceptually simple relative to the classical transformation. The explicit use of the temporal equinox is notably absent in the non-rotating origin transformation, as the ecliptic plane (which defines the equinox) is no longer employed.

The non-rotating origin transformation of velocity takes the same form as the classical transformation, namely

$$\mathbf{v}_{\mathrm{TRF}} = \mathbf{[W']\,[R']\,[N'P]}\,\mathbf{v}_{\mathrm{CRF}} + \mathbf{[W']}\,\frac{d}{dt}\mathbf{[R']\,[N'P]}\,\mathbf{r}_{\mathrm{CRF}}\ ,$$

if the velocity due to polar motion is ignored. The form of $\frac{d}{dt}\mathbf{[R']}$ will be identical to that of $\frac{d}{dt}\mathbf{[\Re]}$ of Eq.(8), if θ_{GMST} replaces θ.

Frame of the General Perturbations Catalog

The orbital elements of the earliest programmed GP theories were typically referenced to the mean equinox of B1950.0[49], this fiducial direction being tied to the Fundamental Katalog 4 (FK4) star catalog and its associated system of constants. However, the early designers of the operational Space Catalog chose a fiducial direction closer to the "true equator and mean equinox" of the date of the orbital element set epoch for computational expediency.

The repetitive evaluation of trigonometric functions presented a formidable challenge to early digital computers. For GP orbit determination, the relationship between the terrestrial frame and the TEME integration frame is approximated by holding both ?ψ and ?ε constant over the time interval of GP solution to reduce the number of trigonometric evaluations. Specifically, observations from the terrestrial frame are related to the inertial integration frame using an invariant matrix $\mathbf{[N}(t_{\mathrm{epoch}})\mathbf{]}$ initially evaluated at the epoch of solution:

$$\mathbf{r}_{\mathrm{TEF}}(t_i) = \mathbf{[W}(t_i)\mathbf{]\,[R}(t_i)\mathbf{]\,[N}(t_{\mathrm{epoch}})\mathbf{]\,[P}(t_i,t_{\mathrm{epoch}})\mathbf{]}\,\mathbf{[N}(t_{\mathrm{epoch}})\mathbf{]^T}\,\mathbf{r}_{\mathrm{TEME}}(t_{\mathrm{epoch}})\ ,$$

TABLE 2
UNCERTAINTY IN VARIOUS REALIZATIONS OF THE UNIFORM EQUINOX
Assessed from Once-per-day Lageos States from 1988 to 2000

Frame	**Probable Error** mas (cm/ER)	**Standard Error** mas (cm/ER)	**Peak-to-Peak** mas (cm/ER)
IERS observed	-	~ 0.02 (0.05)	-
Small angle matrix form	0.05 (0.2)	0.2 (0.6)	0.6 (2)
TEME Small angle matrix form	0.98 (3)	1.3 (4)	5.0 (15)
106 term FK5* (SP)	2.2 (6.8)	3.3 (10)	13 (40)
4 term FK5* (GP at t_{epoch})	29 (91)	45 (140)	218 (695)
4 term FK5* $t_i = t_{epoch}$ + 1 day	38 (119)	57 (177)	291 (900)
4 term FK5* $t_i = t_{epoch}$ + 4 days	99 (307)	147 (456)	605 (1870)
4 term FK5* $t_i = t_{epoch}$ + 10 days	154 (475)	216 (669)	918 (2840)
4 term FK5* $t_i = t_{epoch}$ + 30 days	286 (884)	511 (1580)	1370 (5360)

* These estimates omit the secular trends and biases of the 1976 Precession model and 1980 Theory of Nutation (-3.0 mas/yr in ?ψ, -0.23 mas/yr in ?ε), which are statistically significant.

where (t_i) is the date of observation and (t_{epoch}) is the epoch the of integration frame. Because the nutation angles are fixed to the epoch of GP solution, the epoch of the TEME reference frame is always coincident with the epoch of the latest updated initial conditions.

Additional GP approximations include truncation of the nutation series to the four largest terms, thus ignoring coefficients whose individual contributions are smaller than 200 mas[50]. (6 m/ER). Usually, the polar motion matrix **[W]** is ignored or roughly estimated in GP solutions, and the mean obliquity of the ecliptic is considered constant or truncated to a be linear function of time. Table 2 contains numerical estimates of the uncertainty due to GP frame approximations for one test satellite.

Frame Of The Special Perturbations Catalog

Through the adoption of modern constants and an unabbreviated FK5 theory, the SP Catalog is more precise in its handling of coordinate frames than its GP ancestory[51]. The NSC software system (SPeCIAL-K) currently conforms to the IAU 1976 Precession and 1980 Theory of Nutation and performs numerical integration in the true-of-date frame, with the capability to transform satellite states to all the intermediate bases listed in Table 1 (except the GCRF). The AFSPC software system (ASW) also conforms to the FK5 theory, and is capable of integrating the satellite equations of motion in the J2000 frame. ASW also has options to output in many frames, and maintains backward compatibility with several approximate coordinate systems including the GP TEME frame.

At the time of this writing, evidence suggests that the two SP systems maintain different definitions of the Equation of the Equinoxes. SPeCIAL-K's definition of $EQ_{equinox}$ is consistent with the kinematic interpretation of the uniform equinox, while ASW's definition appears consistent with the geometric interpretation of the TEME. The difference, never larger than 3 mas (10 cm/ER), is within the uncertainty of the FK5 theory used by both systems.

The Space Catalog is maintained near real time and therefore requires *predicted* Earth orientation parameters x_p, y_p, and UT1–UTC. The United States Naval Observatory is responsible for time keeping, navigation and Earth orientation for the US government and DoD, operating the IERS Sub-bureau for Rapid Service and Predictions. Total Earth orientation is now predicted to an accuracy of about 6.1 mas (19 cm/ER) in standard deviation after four days (1998 IERS Annual Report), with UT1 prediction error dominating the forecast error budget. Although the (uncorrected) FK5 theory has been adopted as the standard for the SP Catalog, it is less suitable for high accuracy work near real time since its errors are now larger than the predicted Earth Orientation Parameters. High quality EOP forecasts depend upon improved nutation theories, such as the IERS 1996 Theory of Nutation[52], which predicts the motion of the CEP with an uncertainty of about 0.3 mas (0.9 cm/ER). In contrast, the uncertainty of the current IAU FK5 theory is about 3 mas, excluding a -3 mas/year (-9 cm/ER/year) error in precession ?ψ and an error rate of -0.3 mas/year (-0.9 cm/ER/year) in obliquity ?ε.[53]

IAU 2000 Earth Orientation Model

IAU Colloquium 180 endorsed the conclusions of the IAU-IUGG Working Group on Non-rigid Earth Nutation Theory[54], which points out that the FK5 theory is not accurate enough for present day needs. It has been recommended (in the form of an resolution to be ratified by the IAU XXIV General Assembly in August 2000) that the scientific community should replace the IAU 1976 Precession Model and the 1980 Theory of Nutation with the IAU 2000 precession-nutation model as of January 1, 2003. This model will be published in the upcoming IERS Conventions 2000, and will be accurate to 0.2 mas. This model will reference the GCRF.

An alternative IAU 2000B model is planned that will be accurate to the 1 mas level. This model is anticipated to be less numerically intensive than the present FK5 theory while providing higher accuracy, and is expected to rapidly replace the existing FK5 theory for many space based applications that require the higher accuracy.

Most importantly, the equinox will no longer be part of the definition of the celestial reference system[55]. Although the final forms of the IAU 2000 models have not been published at the time of this writing, it is likely that the recommended transformation between the terrestrial and celestial frames will be based on the non-rotating origin and the direction cosines of the pole with respect to the GCRF. For this reason, there is legitimate concern that the equinox has now become an obsolete and inconvenient reference frame for high accuracy orbit determination, including Space Catalog operations. Under the new system, the equinox can likely be maintained only through a change of variables or an alternative theory. These alternative methods will be more complex and potentially less accurate than the non-rotating origin formulation.

Implications for the Space Catalog

Conformity to the international conventions has not been a pressing issue for the Space Catalog in the past because Catalog standards have not required a high level of compliance with these conventions. The less precise nature of the GP Space Catalog has not stipulated frame requirements other than for satellite identification purposes. Given the age of the FK5 theory, conformity of the Space Catalog to FK5 constants is actually quite recent. Many enhancements have been motivated by the desire to evaluate system performance against alternative sources of high accuracy ephemerides which do conform to IAU and IERS conventions.

As more users outside the US Space Commands become interested in the higher accuracy SP Space Catalog, full compliance with internationally recognized standards and constants has greater relevance. However, an adoption of the internationally recognized conventions should simultaneously embrace less ambiguous terminology by the military. For example, the commonly used designator “Earth Centered Inertial” usually refers to a generic geocentric-equatorial basis[56] and not the uniform equinox or TEME of date. In the past, the “ECI” designator has referenced approximate forms of the nutation theory under both the FK4 and FK5 systems of constants. Arguably, the exact meaning of the term “true equator and mean equinox” is unclear below the few mas (sub-meter) level due to the kinematic correction to the geometric Equation of the Equinoxes.

There is already concern that this overall lack of specificity confuses some users of NORAD element sets into mistaking the TEME for one of the *bone fide* equinoxes (true-of-date or mean-of-date). Such confusion can easily result in a frame bias of several hundred meters per Earth radii, which far exceeds the accuracy levels being maintained by an SP catalog (Table 3).

TABLE 3
ERROR DUE TO MISINTERPRETATION OF THE UNIFORM EQUINOX
Assessed w/ Once-per-day Lageos States (1988-2000)

Frame Differences	Probable Error arcsec (m/ER)	Standard Error arcsec (m/ER)
MOD — UOD	2.0 (63)	7.0 (220)
TOD — UOD	5.8 (180)	8.0 (250)

The potential for errors resulting from the frame basis may not be readily apparent. For differential correction operations, the necessity to perform intermediate matrix operations (Eq xx needs to be numbered in “frame of the GP catalog”) introduces a bias for observational data that is farthest from the chosen epoch for the solution. Because typical estimation fit spans are between 3 and 5 days for most low Earth satellites, this introduces little additional error. The effect is more pronounced with geosynchronous satellites using fit spans approaching a month.

Propagation operations suffer a similar difficulty. Suppose a conjunction analysis is calculated for two satellites whose element set epoch age is one week old. The GP propagation techniques do not take into account the motion of the two coordinate systems

(TEME) over that period of time. Consequently, using Table 1, just the secular rates of the FK5 theory produce an error of about 235 m/ER. This error is large enough to initiate unnecessary maneuver planning. This effect is relevant for all satellite operations including launch avoidance, conjunction assessment, laser clearinghouse, maneuver planning, and others.

A tremendous benefit of the transition to a SP catalog is the capability for more accurate long range mission planning. Accurate predictions of several weeks are beginning to become common. Coordinate system errors nullify this enhancement, and could potentially result in erroneous planning.

The capabilities of modern computers have mitigated the need to maintain the Space Catalog with respect to the uniform equinox. As the mean, true, and uniform equinoxes are further moved into obsolescence by the IAU, these intermediate frames present an additional layer of operational overhead to high accuracy users that maintain consistency with modern IAU conventions. It is therefore recommended that at least one of the Space Commands maintains a high accuracy SP Catalog with respect to GCRF to support those applications requiring an indisputable frame designation.

To the level of accuracy of the FK5 theory currently used by the SP cataloging software systems, the FK5 J2000 frame approximates the GCRF[57]. Therefore, switching the default output frame in both software systems from the uniform equinox to the J2000 basis will go far to operationally accomodate the GCRF without substantial software modifications[*].

Although the FK5 J2000 frame might temporarily serve as a proxy for the GCRF, the IERS currently recommends that the IERS 1996 Theory of Nutation be used for high accuracy prediction of nutation[58]. This theory is based on empirical estimates that absorbed the precession rate errors of the IAU 1976 theory. The IERS 1996 theory existed prior to the adoption of the GCRF and references its own fundamental basis very close to (within a few mas of) the FK5 J2000 basis. Compatibility with the GCRF requires static corrections to nutations in longitude and obliquity[59]:

$$?\psi_{GCRF} = ?\psi_{IERS1996} - 43.1\text{mas},$$
$$?\varepsilon_{GCRF} = ?\varepsilon_{IERS1996} - 5.1\text{mas}$$

[*] While the J2000 frame is accurate to the 50 mas level, the GCRF is accurate to the sub-mas level. Use of the term GCRF is best reserved for realizations accurate to that level.

Software for the 1996 Theory is provided by the IERS, but not all versions are with respect to the GCRF.[60]

Because much of the existing software for the 1996 theory is dependent upon the classical transformation, it may be more computationally intensive than the anticipated IAU 2000 theory. However, it is fairly easy to apply computation-saving techniques traditionally employed for nutation. The most successful accuracy-preserving technique involves rearranging the evaluation of Fourier series terms in an optimal fashion[61], which is still used today for the FK5 theory[62]. Other techniques truncate the numbers of terms in the nutation series[63], or pre-compute precession and nutation over the time interval of interest and interpolate tables [64]. Results are tabulated in Table 4.

The IAU 2000A model will certainly require many more floating point operations than the current FK5 theory. Because the Space Catalog relies on predicted EOP values that are only accurate to a few mas anyway, the IAU 2000B theory would be a strong candidate to replace the existing FK5 theory used by the SP Space Catalog. It would not be difficult to employ the computation-saving techniques in Table 4 for the IAU 2000A theory. The computational expense of interpolation, for example, is the same regardless of theory, once the interpolation tables have been built.

TABLE 4
RELATIVE TIMING OF PRECESSION / NUTATION MODEL EVALUATION

Frame	Relative Timing	Error arcsec (m/ER)
4 term FK5 Theory (GP)	0.44	
FK5 Theory (SP)	3.25?	
FK5 Theory $?\psi$, $?\varepsilon$ Int	0.11	
IERS 1996 Theory	13.0	
IERS 1996 $?\psi$, $?\varepsilon$ Int	0.11	
IERS 1996 **[NP]** – **[I]** Int		

Int = interpolated

Summary And Conclusions

Precession and nutation orient the spin axis of the Earth with respect to a quasi-inertial reference frame. The classical theories of Earth orientation are defined according to a temporal pole and equinox on the celestial sphere, requiring an epoch to be specified. The GP Space Catalog uses an approximation to the uniform equinox based on the FK5 theory within the classical coordinate transformation. The SP catalog is also based on FK5 theory. SP satellite states may be referenced to any of the temporal equinoxes of the FK5 system, including the uniform equinox.

A new high accuracy precession and nutation theory is slated for adoption by the IAU in mid-2000. The IAU 2000A theory will be independent of a temporal equinox and will reference the GCRF. A supplemental IAU 2000B theory will be accurate to the milliarcsecond level and it is expected to evaluate as fast or faster than the existing FK5 theory. The availablility of a fast, highly accurate model should accelerate the adoption of the new IAU system worldwide.

As SP catalog availability increases, use of the uniform equinox should be withdrawn. This basis is not widely recognized outside the US Space Commands, the terminology is enigmatic, and maintaining intermediate coordinate systems adds operating costs. There will be no requirement for a temporal equinox in the new IAU system.

Satellite owner-operators and orbital analysts are anticipating the new IAU theory and the astrodynamics community should expect to see increasing standarization toward the GCRF. The recent operational SP upgrades to the US Space Catalog provide an excellent opportunity to standardize the SP Catalog to the conventional GCRF, and while operational reliance on the SP Catalog still remains in its infancy.

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