Recall Matrix-Multiply

procedure matrix-multiply (A, B matrices)

for $i := 1$ to m

 for $j := 1$ to n

  $c_{ij} := 0$

  for $q := 1$ to k

   $c_{ij} = c_{ij} + a_{iq} b_{qj}$

return C $\{C = [c_{ij}]$ is the product of A and $B\}$

> Complexity:
> for every one of the $m \times n$ entries in C we do k multiplicat and $k-1$ additions:
> $O(m \times n \times (2k-1))$
> $= O(m \times n \times k)$

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Let A be 3×9

   B : 9×4

   C : 4×2

Q-n What is the best order for $ABC = (A \cdot B) \cdot C$ or $A \cdot (B \cdot C)$?

Answer

- $A \cdot B$ takes $3 \times 9 \times 4 = 108$ multiplications
  and produces 3×4 matrix.
  Then $(A \cdot B) \times C$ takes $3 \times 4 \times 2 = 24$ multiplications
  For a total of $108 + 24 = 132$ operations

- $B \cdot C$ takes $9 \times 4 \times 2 = 72$ multiplications
  and produces 9×2 matrix
  Then $A \cdot (B \cdot C)$ takes $3 \times 9 \times 2 = 54$ multiplication
  For a total of $72 + 54 = 126$ multiplications

- Since $126 < 132$, prefer the order $A \cdot (B \cdot C)$