3.1 (16) Algorithm to find the smallest integer in a finite sequence of numbers.

```
procedure smallest (a_1, ..., a_n : integers)
min := a_1
for i:=2 to n
    if a_i < min then min:= a_i
return min
```

3.1 (42) Selection Sort in pseudocode

```
procedure selection (a_1, a_2, ..., a_n)
for i := 1 to n-1
    min_index := i        //place where remaining minimum element
                            is found
    for j:= i+1 to n
        if a_j < a_min_index then min_index:= j
    exchange a_min_index and a_i
{ the list is now sorted }
```

Growth Functions

3.2 (10) Show that x^3 is $O(x^4)$ but that x^4 is NOT $O(x^3)$.

- x^3 is $O(x^4)$ because:

  $x^3 \leq x^4$ whenever $x \geq 1$

  take as witnesses $C = 1$, $k = 1$

recall definition

$f(x) \leq C \; g(x)$

$x^3 \leq 1 \cdot x^4$

- x^4 is not $O(x^3)$ because:

  Suppose it is true that

  $$x^4 \leq C \cdot x^3 \quad \text{for some witnesses } C, k$$

  But then it follows that (since x is assumed positive)

  $$x \leq C \quad \text{// divide both sides by } x^3$$

  But this can not hold since C is a constant and x grows without bound ($\because x > k$).

  This contradiction completes proof. ∎

3.2 12 Show that $x \log x$ is $O(x^2)$ but that x^2 is not $O(x \log x)$

- **$x \log x$ is $O(x^2)$ because:**

$x \cdot \log x \leq x \cdot x$ for all positive x

take $C=1$, $k=0$ as witnesses

(Aside: The fact that $\log x < x$ can be seen by looking at the graphs of the two functions, for example. Or it can be proved formally, by induction: It follows from the fact that $x < 2^x$, and that $\log x$ is a monotonically increasing function in x.)

- **x^2 is not $O(x \log x)$ because:**

Suppose it is true that

$x^2 \leq C \cdot x \cdot \log x$ for some C, k whenever $x > k$

Or, equivalently, that

$$\frac{x}{\log x} \leq C$$

But this can not hold because $\frac{x}{\log x}$ is unbounded:

This can be seen by noting, for example, that

$\log x < \sqrt{x}$ for $x > 16$ // look at graphs or // by calculus

And therefore

$$\frac{x}{\log x} > \frac{x}{\sqrt{x}} = \sqrt{x}$$, and $\sqrt{x}$ is clearly unbounded: