STAT 8003 Mid-Term Exam

Name:
Group Number:

The exam lasts for 80 minutes, 5:30p-6:50p. Each sup-problem is 10 points (Total points = 80).

Problem 1. One market monitoring organization would like to compare the life time of two brands of bulbs, Brand A and Brand B. They design the experiment in this way. Let X_i and Y_i be the life time of ith bulb in Brand A and Brand B respectively, which can be approximated by independent random variables with exponential distributions with expectations λ and μ respectively. They pair X_i and Y_i. In the ith experiment, instead of letting both two bulbs burn until they die out, they stop when one of the bulbs burn out, and record the burning time Z_i and indicator W_i of which one burns out. They repeat the experiment n times. Mathematically, Z_i and W_i can be defined as

$$Z_i = \min(X_i, Y_i) \text{ and } W_i = \begin{cases} 1 & \text{if } Z_i = X_i, \\ 0 & \text{if } Z_i = Y_i; \end{cases} \qquad i = 1, \ldots, n$$

a). Find the joint distribution of Z_1 and W_1.

b). Suppose the organization is intested to know the ratio of two rates $\theta = \mu/\lambda$. What is the MLE of θ?

Problem 2. Consider an *i.i.d.* sample of random varialbes with density function

$$f(x \mid \sigma) = \frac{1}{2\sigma} \exp\left(-\frac{|x|}{\sigma}\right).$$

a). Find the method of moments estimate of σ.

b). Find the maximum likelihood estimate of $\exp(\sqrt{\sigma} + 1)$.

c). Use the pivot method to construct a $(1 - \alpha)\%$ confidence interval of σ.

Problem 3. The Poisson distribution has been used by traffic engineers as a model for light traffic, based on the rationale that if the rate is approximately constant and the traffic is light (so the individual cars move independently of each other), the distribution of counts of cars in a given time interval or space area should be nearly Poisson (Gerlough and Schuhl 1955). The following table shows the number of right turns during 300 3-min intervals at a specific intersection.

n	Frequency
0	14
1	30
2	36
3	68
4	43
5	43
6	30
7	14
8	10
9	6
10	4
11	1
12	1
≥ 13	0

a). Use the pivot method to construct a 95% confidence interval of the rate. Plug in the data after your get the expression.

b). Use the variance stanbilization method to construct a 95% confidence interval of the rate. Plug in the data after your get the expression.

Problem 4. Consider the same setting as Problem 2. Now assume that the traffic detectors are not sensitive to light traffic. After colleting data for non-rash hours, we got the following table.

n	Frequency
≤ 1	16
2	5
3	4
≥ 4	0

What is the MLE of the rate during the non-rash hours? Plug in the data after your get the expression.