Stat 8003, HW5

Due: Thursday, Oct 9th, 2014

1. A coin is thrown independently 10 times to test the hypothesis that the probability of heads is 1/2 versus the alternative that the probability is not 1/2. The test rejects if either 0 or 10 heads are observed.

(a) What is the significance level of the test?

(b) If in fact the probability of heads is .1, what is the power of the test?

2. Let $X_1, \cdots, X_n$ be a random sample from an exponential distribution with the density function

$$f(x|\theta) = \theta \exp(-\theta x).$$

We want to test the hypothesis

$$H_0 : \theta = 1, vs \quad H_a : \theta \neq 1.$$

Set the desired level of significance as $\alpha = 5\%$.

(a) Derive a generalized likelihood ratio test and show that the rejection region is of the form $\mathcal{R} = \{\bar{X} \exp(-\bar{X}) \leq c\}$;

(b) Suppose $n = 10$. Show that the rejection region in (a) is of the form $\mathcal{R} = \{\bar{X} \leq x_0\} \cup \{\bar{X} \geq x_1\}$, where x_0 and x_1 are determined by c;

(c) When $\theta = 1$, it is known that $\sum_i X_i$ follows $Gamma(n, \frac{1}{\theta})$. How could this knowledge be used to choose c?

Here, $X \sim Gamma(\alpha, \beta)$ where the density is given as

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} \exp(-x/\beta).$$

3. Suppose, to be specific, that in Problem 2, the observed data are the following:

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1.07   0.88   0.66   0.55   1.15   0.65   3.45   3.55   3.51   0.48
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(a) Based on the result in Problem 2, will you reject H_0? What's your p-value?

(b) If we start from generalized likelihood ratio test, and use the asymptotic distribution of $2 \log \Lambda$, will you reject H_0? What's your p-value?