Stat 8003, HW4

Due: Thursday, Sep 25th, 2014

1. Suppose X is a discrete random variables with $P(X = 1) = \theta$ and $P(X = 2) = 1 - \theta$. Three independent observations of X are made: $x_1 = 1, x_2 = 2, x_3 = 2$.

(a) Find the method of moment estimate of θ;

(b) What is the likelihood function?

(c) What is the MLE of θ?

2. Consider an i.i.d. sample of random variables with density function

$$f(x|\sigma) = \frac{1}{2\sigma}\exp(-\frac{|x|}{\sigma}).$$

(a) Find the MOM of σ;

(b) Find the MLE of σ.

3. Consider the space shuttle example. Let X_i denote the number of damaged o-rings and t_i be the temperature, where $i = 1, 2, \cdots, n$. Assume the model as

$$\begin{cases} X_i|p_i \sim Bin(2, p_i), \\ p_i = \frac{\exp(\beta_0+\beta_1 t_i)}{1+\exp(\beta_0+\beta_1 t_i)}. \end{cases}$$

(a) Derive the log-likelihood function;

(b) Set the equations for the maximum likelihood estimator of β_0, β_1;

(c) Derive the steps for the Newton-Raphson algorithm;

(d) Using the Newton-Raphson algorithm to calculate the maximum likelihood estimator of β_0 and β_1.

(e) On January 28, 1986, the outside temperature is 31 degree. Based on your estimated β_0 and β_1, what is the probability that an o-ring will be damaged?

(f) Based on your estimator, plot the probability p against the temperature by letting temperature go from 30 degrees to 90 degrees.

You can load the data with the following command:

```
shuttle <- read.csv("http://astro.temple.edu/~zhaozhg/Stat8003/data/shuttle.txt")
```