Stat 8113, HW1

Due: Thursday, Sep 4th, 2014

1. Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be two matrices defined as

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \\ -2 & 1 \end{pmatrix}.$$

Calculate

- $\boldsymbol{AB}$;
- $\boldsymbol{B}^T\boldsymbol{A}$;

Use R to check your calculation.

2. If $\boldsymbol{A}$ is invertible, prove that $det(\boldsymbol{A}^{-1}) = (det(\boldsymbol{A}))^{-1}$.

3.

(a) If the matrix $\boldsymbol{P}$ is idempotent, then $\boldsymbol{Q} = \mathbf{I} - \boldsymbol{P}$ is also idempotent;

(b) If X is a $n \times m$ matrix with rank m, show that the following matrix $\boldsymbol{P}$ is idempotent,

$$\boldsymbol{P} = \boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T.$$

4. Given a matrix $\boldsymbol{A} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$. Is $\boldsymbol{A}$ positive-definite? Prove it or disprove it.

5. The Gamma function $\Gamma(\alpha)$ is defined as

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} exp(-x) dx.$$

1. Prove that $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$;
2. Calculate $\Gamma(n)$ where n is a positive integer;
3. Calculate $\int_0^\infty x^{-\alpha-1}\exp(-\frac{\beta}{x})dx$, express your result using Gamma function.