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Bernoulli pmf and its Likelihood

Most stat textbooks define Bernoulli r.v. as:

$$X = \begin{cases} 1 & \text{with probability } \theta \\ 0 & \text{with probability } 1-\theta \end{cases}$$

Then its `pmf` is simply given by

$$p_X(x) = \theta^x (1-\theta)^{1-x}$$

And the likelihood function given the observed values of $X = x_1, x_2, \dots x_n$ is simply

$$L(\theta; x) = \theta^{\,\sum x_i}\,(1-\theta)^{n-\sum x_i}$$

In general, for a *shifted Bernoulli*, i.e. when X takes on the values not 0 or 1, but

$$X = \begin{cases} a & \text{with probability } \theta \\ b & \text{with probability } 1-\theta \end{cases}$$

Then its easy to show that

$$p_X(x) = \theta^{(x-b)/(a-b)}\,(1-\theta)^{(a-x)/(a-b)}$$

And thus

$$L(\theta; x) = \theta^{(\sum x_i - nb)/(a-b)}\,(1-\theta)^{(na-\sum x_i)/(a-b)}$$

Example: X is a discrete random variables with $P(X=1)=\theta$ and $P(X=2)=1-\theta$. Three independent observations of X are made: $x_1 = 1; x_2 = 2; x_3 = 2$.

Write down the likelihood:

$$\begin{aligned} L(\theta\ ; x) &= \theta^{(1+2+2-3\times 2)/(1-2)}\,(1-\theta)^{(3\times 1-(1+2+2))/(1-2)} \\ &= \theta\,(1-\theta)^2 \end{aligned}$$

(It's easy to check that this function is maximized at $\frac{1}{3} = \hat{\theta}$, the `MLE` estimate of θ.)