Stat 8003, HW3

Due: Thursday, Sep 18th, 2014

1. The covariance of X and Y is $\text{Cov}(X, Y) = E[X - E(X)(Y - E(Y)]$ and the correlation coefficient of X and Y is

$$\rho_{XY} = \frac{\text{Cov(X, Y)}}{\sqrt{\text{Var}(X)\text{Var}(Y)}}.$$

Consider a bivariate distribution with $P(X = 1, Y = 2) = 0.4$, $P(X = 2, Y = 3) = 0.6$. Find the correlation coefficient between X and Y.

2. Find two random variables X and Y, such that $Cov(X, Y) = 0$ but X and Y are not independent.

3. In the Example of GDP. Assume that the data follows a gamma distribution $\Gamma(\alpha, \beta)$.

a. Derive the estimator of α, β using the methods of moments;

b. Compare the density of the data vs the fitted curve.

4. For any random variable X, let $M_X(t) = E\exp(Xt)$ and $S_X(t) = \log(M_X(t))$. $M_X(t)$ is called the moment generating function and $S_X(t)$ is the cumulant generating function. It is known that (Can you prove it? Not required.)

$$\frac{d}{dt}S_X(t)|_{t=0} = EX \quad \text{and} \quad \frac{d^2}{dt^2}S_X(t)|_{t=0} = Var(X).$$

Use this fact to answer the following questions.

a. Assume that X follows a Gamma distribution with parameter α and β. Calculate the cumulant generating function;

b. Calculate $E\ln X$ and $Var(\ln X)$. Write your final result by using the digamma function $\psi(x)$ and trigamma function $\psi_1(x)$ where $\psi(x) = (\ln\Gamma(x))'$ and $\psi_1(x) = (\ln\Gamma(x))''$.

c. Match the first and second moment of $\log(X)$, and derive the MOM estimator of α and β. (Hint: in R, you can use digamma(x), trigamma(x), and limma::trigammaInverse(x).)

d. Apply your estimator to the GDP dataset and estimate the parameter of α and β.