

Index

- Adjusted R^2 , 162
- Alias matrix, 170
- Analysis of covariance, 443–478
 - assumptions, 443–444
 - covariates, 444
 - estimation, 446–448
 - model, 444–445
 - one-way model with one covariate, 449–451
 - estimation of parameters, 449–450
 - model, 449
 - testing hypotheses, 448, 450–451
 - equality of treatment effects, 450–452
 - homogeneity of slopes, 452–456
 - interpretation, 456
 - slope, 452
 - one-way model with multiple covariates, 464–472
 - estimation of parameters, 465–468
 - model, 464–465
 - testing hypotheses, 468–469
 - equality of treatment effects, 468–469
 - homogeneity of slope vectors, 470–472
 - slope vector, 470
 - power, 444
 - testing hypotheses, 448
 - two-way model with one covariate, 457–464
 - model, 457
 - testing hypotheses, 458–464
 - homogeneity of slopes, 463–464
 - main effects and interactions, 458–462
 - slope, 462
 - unbalanced models, 473–474
 - cell means model, 473
 - constrained model, 473–474
- Analysis of variance, 295–338
 - estimability of β in the empty cells model, 432, 434–435
 - estimability of β in the non-full-rank model, 302–304
 - estimable functions $\lambda'\beta$, 305–308
 - conditions for estimability of $\lambda'\beta$, 305–307
 - estimators of $\lambda'\beta$, 309–313
 - BLUE properties of, 313
 - covariance of, 312
 - variance of, 311
 - estimation of σ^2 in the non-full-rank model, 313–314
 - model, 3–4, 295–301
 - one-way. *See* One-way model
 - two-way. *See* Two-way model
 - normal equations, 302–303
 - solution using generalized inverse, 302–303
 - normal model, 314–316
 - estimators of β and σ^2 , 314–315
 - properties of, 316
 - and regression, 4
 - reparameterization to full-rank model, 318–320
 - side conditions, 320–322, 433
 - SSE in the non-full-rank model, 313–314
 - testable hypotheses, 323–324
 - testable hypotheses in the empty cells model, 433
 - testing hypotheses, 323–329
 - full and reduced model, 324–326
 - general linear hypothesis, 326–329
 - treatments or natural groupings of units, 4
 - unbalanced data. *See* Unbalanced data in ANOVA
- Angle between two vectors, 41–42, 136, 163, 238

- Asymptotic inference for large samples, 260–262, 491, 515
- Augmented matrix, 29
- Bayes' theorem, 278–279
- Bayesian linear model, 279–284, 480
- Bayesian linear mixed model, 497
- Best linear predictor, 499
- Best linear unbiased estimators (BLUE), 147, 165, 313
- Best quadratic unbiased estimators, 151, 486
- Beta weights, 251
- BIC. *See* Information criterion
- BLUE. *See* Best linear unbiased estimators
- Causality, 3, 130–131, 443
- Chi-square distribution, 112–114
 - central chi-square, 112
 - moment-generating function, 112–113
 - noncentral chi-square, 112–114
 - noncentrality parameter, 112, 124
- Cluster correlation, 479–480, 481–485
- Coefficient of determination
 - in multiple regression, 161–164
 - in simple linear regression, 133–134
- Coefficient(s), regression, 2, 127
- Conditional density, 73, 95–99, 278–284, 498–499
- Confidence interval(s)
 - for β_1 in simple linear regression, 133
 - in Bayesian regression, 278, 285
 - in linear mixed models, 491, 495
 - in multiple regression. *See* Regression, multiple linear with fixed x 's, confidence interval(s)
 - in random- x regression, 261–262
- Contrasts, 308, 341, 357–371
- Control of output, 3
- Correlation
 - bivariate, 134
- Correlation matrix (matrices)
 - population, 77–78
 - relationship to covariance matrix, 77–78
 - sample, 247
 - relationship to covariance matrix, 247–248
- Covariance matrix (matrices)
 - for $\hat{\beta}$, 145
 - for partitioned random vector, 78
 - population, 75–76
 - sample, 156, 246–247
 - for two random vectors, 82
- Data space, 153, 163, 316–317
- Dependent variable, 1, 137, 295
- Derivative, matrix and vector, 56–59, 91, 109, 142, 158, 495
- Determinant, 37–41
- Determination, coefficient of. *See* Coefficient of determination
- Diagnostics, regression, 227–238 *also* Hat matrix; Influential observations; Outliers; Residual(s)
- Diagonal matrix, 8
- DIC. *See* Information criterion
- Distance
 - Mahalanobis, 77
 - standardized, 77
- Distribution(s)
 - chi-square, 112–114
 - F , 114–116
 - gamma, 280
 - inverse gamma, 284
 - multivariate t , 282–283, 285
 - normal. *See* Normal distribution
 - t , 216, 283
- Effect of each variable on R^2 , 262–265
- Eigenvalues. *See* Matrix, eigenvalues
- Eigenvectors. *See* Matrix, eigenvectors
- Empty cells, 432–439
- Error sum of squares. *See* SSE
- Error term, 1, 137
- Estimated best linear unbiased predictor, 499
- Estimated generalized least squares estimation, 490
- Exchangeability, 277
- Expected mean squares, 173–174, 179, 182, 312–317, 362–367, 433
- Expected value
 - of bilinear form $[E(\mathbf{x}'\mathbf{A}\mathbf{y})]$, 111
 - of least squares estimators, 131–132
 - of quadratic form $[E(\mathbf{y}'\mathbf{A}\mathbf{y})]$, 107
 - of R^2 , 162
 - of random matrix, 75–76
 - of random variable $[E(y)]$, 70
 - of random vector $[E(y)]$, 75–76
 - of sample covariance $[E(s_{xy})]$, 112
 - of sample variance $[E(s^2)]$, 108, 131, 150
 - of sum of random variables, 70
 - of sum of random vectors, 75–76
- Exponential family, 514

- F*-Distribution, 114–116
 - central *F*, 114
 - mean of central *F*, 115
 - noncentral *F*, 115
 - noncentrality parameter, 115
 - variance of central *F*, 115
- F*-Tests. *See also* Regression, multiple
 - linear with fixed *x*'s, tests of hypotheses; Tests of hypotheses
 - general linear hypothesis test, 198–203
 - for overall regression, 185
 - power, 115
 - subset of the β 's, 189
- False discovery rate, 206
- First order multivariate Taylor series, 495
- Fixed effects models, 480

- Gauss-Markov theorem, 146–147, 276. *See also* Best linear unbiased estimators
- Generalized least squares, 164–169, 285–286, 479, 503
- Generalized linear models, 513–516
 - exponential family, 514
 - likelihood function, 512
 - linear predictor, 513–514
 - link function, 514
 - model, 514
- Generalized inverse, 32–37, 302–303, 343, 384
 - of symmetric matrix, 33
- Generalized variance, 77, 88–89
- Geometry of least squares, 151–154, 163, 316–317
 - angle between two vectors, 163
 - prediction space, 153–154, 163, 316–317
 - data space, 153, 163, 316–317
 - parameter space, 152, 154, 316–317
- Gibbs sampling, 289, 291

- Hadamard product, 16, 425
- Hat matrix, 230–231
- Hessian matrix, 495
- Highest density interval, 279, 285
- Hyperprior distribution, 280, 287
- Hypothesis tests. *See* Tests of hypotheses

- Idempotent matrix
 - for chi-square distribution, 117–118
 - definition and properties, 54–55
 - in linear mixed models, 487
- Identity matrix, 8

- Independence
 - of contrasts, 358–362
 - independence and zero covariance, 93–94
 - of linear functions and quadratic forms, 119–120
 - of quadratic forms, 120–121
 - of random variables, 71, 94
 - of random vectors, 93, 94
 - of SSR and SSE, 187
- Influential observations, 235–238
 - Cook's distance, 236–237
 - leverage, 236
- Information criterion, 286
- Iterative methods for finding estimates, 490
- Invariance
 - of *F*, 149, 200
 - of maximum likelihood estimators, 247–248
 - of R^2 , 149
 - of s^2 , 149
 - of *t*, 149
 - of \hat{y} , 148–149
- Inverse matrix. *See* Matrix, inverse

- j** vector, 8
- J** matrix, 8

- Kenward–Roger adjustment, 496–497

- Lagrange multiplier, 60, 68, 179, 201, 220, 223, 429
- Least squares, 128, 131, 141, 143, 145–151, 302, 507
 - properties of estimators, 129–133, 143, 145–147
- Likelihood function, 158, 513–514
- Likelihood ratio tests, 258–262
- Linear estimator, 143. *See also* Best linear unbiased estimators
- Linear mixed model, 480
 - randomized blocks, 481–482
 - subsampling, 482
 - split plot studies, 483–484, 492–494
 - one-way random effects, 484, 489
 - random coefficients, 484–485
 - heterogeneous variances, 485–486
- Linear model, 2, 137
- Linear models, generalized. *See* Generalized linear models
- Logistic regression, 508–511
 - binary *y*, 508
 - estimation, 510

Logistic regression (*Continued*)

logit transformation, 509

model, 509–510

polytomous model, 511

categorical, 511

ordinal, 511

several x 's, 510

Logit transformation, 509

Loglinear models, 511–512

contingency table, 511

likelihood ratio test, 512

maximum likelihood estimators, 512

LSD test, 209

Mahalanobis distance, 77

Markov Chain Monte Carlo, 288–289,
291–292

Matrix (matrices), 5–68

addition of, 9–10

algebra of, 5–60

augmented matrix, 29

bilinear form, 16

Cholesky decomposition, 27

conditional inverse, 33

conformable matrices, 9

definition, 5

derivatives, 56–58

determinant, 37–41

of partitioned matrix, 38–40

diagonal of a matrix, 7

diagonal matrix, 8

diagonalizing a matrix, 52

differentiation, 56–57

eigenvalues, 46–53, 496

characteristic equation, 47

and determinant, 51–52

of functions of a matrix, 49–50

of positive definite matrix, 53

square root matrix, 53

of product, 50–53

of symmetric matrix, 51

and trace, 51

eigenvectors, 46–47, 496

equality, 6

generalized inverse, 32–37, 302, 343,
384, 391–395

of symmetric matrix, 36

Hadamard product, 16, 425

idempotent matrix, 54

and eigenvalues, 54

identity matrix, 8

inverse, 21–23

conditional inverse, 33

generalized inverse, 32–37

of partitioned matrix, 23–24

of product, 22

 \mathbf{j} vector, 8 \mathbf{J} matrix, 8

multiplication of, 10

conformal matrices, 10

nonsingular matrix, 21

notation, 5

 \mathbf{O} (zero matrix), 8

orthogonal matrix, 41–43

partitioned matrix, 16–18

multiplication of, 17

positive definite matrix, 24–28

positive semidefinite matrix, 25–28

product, 10

commutativity, 10

as linear combination of columns, 17

matrix and diagonal matrix, 16

matrix and \mathbf{j} , 12

matrix and scalar, 10

product equal to zero, 20

rank of product, 21

quadratic form, 16. *See also* Quadratic
form(s)

random matrix, 69

rank, 19–21. *See also* Rank of a matrixspectral decomposition, 51, 360, 362,
495–496

square root matrix, 53

sum of, 9

symmetric matrix, 7

spectral decomposition, 51

trace, 44–46

transpose, 7

of product, 13

triangular matrix, 8

vector(s). *See* Vector(s)zero matrix (\mathbf{O}) and zero vector ($\mathbf{0}$), 8Matrix product. *See* Matrix, product

Maximum likelihood estimators

for β and σ^2 in ANOVA, 315for β and σ^2 in fixed- x regression,
158–159

properties, 159–161

for β_0 , β_1 , and σ^2 in random- x
regression, 245–248

properties, 248–249

invariance of, 249

in loglinear models, 511

for partial correlation, 266–268

MCMC. *See* Markov Chain Monte CarloMean. *See also* Expected valuesample mean. *See* Sample mean

population mean, 70

- Missing at random, 432
- Misspecification of $\text{cov}(\mathbf{y})$, 167–169. *See also* Generalized least squares
- Misspecification of model, 169–174
 alias matrix, 170
 overfitting, 170–172
 underfitting, 170–172
- Model diagnostics, 227–238. *See also* Hat matrix; Influential observations; Outliers; Residual(s)
- Model, linear, 2, 137
- Model validation, 227–238. *See also* Hat matrix; Influential observations; Outliers; Residual(s);
- Moment-generating function, 90–92, 96, 99–100, 103–104, 108
- Multiple linear regression, 90–92, 108, 112–114, 117–119, 122. *See* Regression, multiple linear with fixed x 's
- Multivariate delta method, 495
- Multivariate normal distribution, 87–103
 conditional distribution, 95–97
 density function, 88–89
 independence and zero covariance, 93–94
 linear functions of, 89
 marginal distribution, 93
 moment generating-function of, 90–92
 partial correlation, 100–101
 properties of, 92–100
- Noncentrality parameter
 for chi-square, 112
 for F , 114, 187, 192, 325
 for t , 116, 132
- Nonlinear regression, 507
 confidence intervals, 507
 least squares estimators, 507
 tests of hypotheses, 507
- Nonsingular matrix, 21
- Normal distribution
 multivariate. *See* Multivariate normal distribution
 univariate, 87–88
 standard normal, 87
- Normalizing constant, 278, 281, 284
- O** (zero matrix), 8
- One-way model (balanced), 3, 295–298, 339–376
 contrasts, 357–371
 and eigenvectors, 360–362
 hypothesis test for, 344–351
 orthogonal contrasts, 358–371
 independence of, 363–364
 orthogonal polynomial contrasts, 363–371
 partitioning of sum of squares, 360–361
 estimable functions, 340–341
 contrasts, 341
 estimation of σ^2 , 343–344
 expected mean squares, 351–357
 full-reduced–model method, 352–354
 general linear hypothesis method, 354–356
 normal equations, 341–344
 solution using generalized inverse, 343
 solution using side conditions, 342–343
 overparameterized model, 297
 assumptions, 297–298
 parameters not unique, 297
 reparameterization, 298
 side conditions, 298
 SSE, 314
 testing the hypothesis $H_0: \mu_1 = \mu_2 = \dots = \mu_k$, 344–351
 full and reduced model, 344–348
 general linear hypothesis, 348–351
- Orthogonal matrix, 41–43
- Orthogonal polynomials, 363–371
- Orthogonal vectors, 40
- Orthogonal x 's in regression models, 149, 174–178
- Orthogonality of columns of \mathbf{X} in balanced ANOVA models, 333–335
- Orthogonality of rows of \mathbf{A} in unbalanced ANOVA models, 293–296
- Orthogonalizing the x 's in regression models, 174–178
 and partial regression coefficients, 175–176
- Outliers, 232–235
 mean shift outlier model, 235
 PRESS (prediction sum of squares), 235
- Overfitting, 170–172
- p -Value
 for F -test, 188–189
 for t -test, 132
- Parameter space, 152, 154, 316–317
- Partial correlation(s), 100–101, 266–273
 matrix of (population) partial correlations, 100–101
 sample partial correlations, 177–178, 266–173

- Partial interaction constraints, 434
- Poisson distribution, 512
- Poisson regression, 512–513
likelihood function, 513
model, 513
- Polynomials, orthogonal. *See* Orthogonal polynomials
- Positive definite matrix, 24–28
- Positive semidefinite matrix, 25–28
- Posterior distribution, 278–284
conditional, 289
marginal, 282
- Posterior predictive distribution, 279, 290–292
- Prediction, 2–3, 137, 142, 148, 156, 161
- Precision, 280
- Prediction of a random effect, 497–499
- Prediction interval, 213–215
- Prediction space, 153–154, 163, 316–317
- Prediction sum of squares (PRESS), 235
- PRESS (prediction sum of squares), 235
- Prior distribution, 278–284
diffuse, 281, 287
informative, 281
conjugate, 281, 289
specification, 280
- Projection matrix, 228
- Quadratic form(s), 16, 489
distribution of, 117–118
expected value of, 107
idempotent matrix, 106
independence of, 119–121
moment-generating function of, 108
variance of, 108
- r^2 in simple linear regression, 133–134
- R^2 (squared multiple correlation), 161–164, 254–257
effect of each variable on R^2 , 262–265
fixed x 's, 161–164
adjusted R^2 , 162
angle between two vectors, 163
properties of R^2 and R , 162
random x 's, 254–257
population multiple correlation, 254
properties, 255
sample multiple correlation, 256
properties, 256–257
- Random matrix, 69
- Random model, 480
- Random variable(s), 69
correlation, 74
covariance, 71
and independence, 71–74
expected value (mean), 70
independent, 71, 94
mean (expected value), 70
standard deviation, 71
variance, 70
- Random vector(s), 69–74
correlation matrix, 77–78
covariance matrix, 75–76, 83
linear functions of, 79–83
mean of, 80
variances and covariances of, 81–83
mean vector, 75–76
partitioned, 78–79
- Random x 's in regression. *See* Regression, random x 's
- Rank of a matrix, 19–21
full rank, 19
rank of product, 20–21
- Regression coefficients (β 's), 2, 138, 251
partial regression coefficients, 138
standardized coefficients (beta weights), 251
- Regression, logistic. *See* Logistic regression
- Regression, multiple linear with fixed x 's, 2–3, 137–184
assumptions, 138–139
centered x 's, 154–157
coefficients. *See* Regression coefficients
confidence interval(s)
for β , 209
for $E(y)$, 211–212
for one $a'\beta$, 211
for one β_j , 210–211
for σ^2 , 215
for several $a_i'\beta$'s, 216–217
for several β_j 's, 216
- design matrix, 138
- diagnostics, 227–238. *See also* Diagnostics, regression
- estimation of $\beta_0, \beta_1, \dots, \beta_k$, 141–145
with centered x 's, 154–157
least squares, 2, 143–144
maximum likelihood, 158–159
properties of estimators, 145–149
with sample covariances, 157

- estimation of σ^2
 - maximum likelihood estimator, 158–159
 - minimum variance unbiased estimator, 158–159
 - unbiased estimator, 149–151
 - best quadratic unbiased estimator, 151
- generalized least squares, 164–169
- minimum variance estimators, 158–159
- misspecification of error structure, 151–153
- misspecification of model, 169–174. *See also* Misspecification of model
- model, 137–140
- multiple correlation (R), 161–162
- normal equations, 141–142
- orthogonal x 's, 149, 174–178
- orthogonalizing the x 's, 174–178
- outliers, 232–235. *See also* Outliers
- partial regression, 141
- prediction. *See* Prediction
- prediction equation, 142
- prediction interval, 213–215
- properties of estimators, 145–149
- purposes of, 2–3
- random x 's. *See* Regression, random x 's
- residuals, 227–230. *See also* Residuals
- sufficient statistics, 159–160
- tests of hypotheses
 - all possible $\mathbf{a}'\boldsymbol{\beta}$, 193–194
 - expected mean squares, 173–174
 - general linear hypothesis test
 - $H_0: \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$, 198–203
 - estimation under reduced model, 324–326
 - full and reduced model, 324–326
 - $H_0: \mathbf{C}\boldsymbol{\beta} = \mathbf{t}$, 203–204
 - likelihood ratio tests, 217–221
 - distribution of likelihood ratio, 218–219
 - likelihood ratio, 218
 - for $H_0: \boldsymbol{\beta} = \mathbf{0}$, 219–220
 - for $H_0: \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$, 220–221
 - linear combination $\mathbf{a}'\boldsymbol{\beta}$, 204–205
 - one β_j , 204–205
 - F -test, 204–205
 - t -test, 205
 - overall regression test, 185–189
 - in terms of R^2 , 196–198
 - several $\mathbf{a}'\boldsymbol{\beta}$'s, 205
 - several β_j 's
 - Bonferonni method, 206–207
 - experimentwise error rate, 206
 - overall α -level, 206
 - Scheffé method, 207–209
 - subset of the β 's, 189–196
 - expected mean squares, 193, 196
 - full and reduced model, 190
 - noncentrality parameter, 192–193
 - quadratic forms, 190–193, 195
 - in terms of R^2 , 196
 - weighted least squares, 168
 - \mathbf{X} matrix, 138–139
- Regression, nonlinear. *See* Nonlinear regression
- Regression, Poisson. *See* Poisson regression
- Regression, random x 's, 243–273
 - multivariate normal model, 244
 - confidence intervals, 258–262
 - estimation of β_0 , β_1 , and σ^2 , 245–249
 - properties of estimators, 249
 - standardized coefficients (beta weights), 251
 - in terms of correlations, 249–154
 - R^2 , 254–257. *See also* R^2 , random x 's
 - effect of each variable on R^2 , 262–265
 - tests of hypotheses, 258–262
 - comparison with tests for fixed x 's, 258
 - correlations, tests for, 260–261
 - Fisher's z -transformation, 261
 - likelihood ratio tests, 258–260
 - nonnormal data, 265–266
 - estimation of β_0 and β_1 , 266
 - sample partial correlations, 266–273
 - maximum likelihood estimators, 268
 - other estimators, 269–271
- Regression, simple linear (one x), 1, 127–136
 - assumptions, 127
 - coefficient of determination r^2 , 133–134
 - confidence interval for β_0 , 134
 - confidence interval for β_1 , 132–133
 - correlation r , 133–134
 - in terms of angle between vectors, 135
 - estimation of β_0 and β_1 , 128–129
 - estimation of σ^2 , 131–132

- Regression, simple linear (*Continued*)
 model, 127
 properties of estimators, 131
 test of hypothesis for β_0 , 119
 test of hypothesis for β_1 , 132–133
 test of hypothesis for ρ , 134
 Regression sum of squares. *See* SSR
 Regression to the mean, 498
 Residual(s), 131, 227–230
 deleted residuals, 234
 externally studentized residual, 234
 hat matrix, 228, 230–232
 in linear mixed models, 501–502
 plots of, 230
 properties of, 237–230
 residual sum of squares (SSE), 131, 150–151. *See* SSE
 studentized residual, 233
 Response variable, 1, 137, 150
 Robust estimation methods, 232
- Sample mean
 definition, 105–106
 independent of sample variance, 119–120
 Sample space (data space), 152–153
 Sample variance (s^2), 107–108
 best quadratic unbiased estimator, 151
 distribution, 118
 expected value, 108, 127
 independent of sample mean, 120
 Satterthwaite, 494
 Scalar, 6
 Scientific method, 1
 Selection of variables, 2, 172
 Serial correlation, 479
 Shrinkage estimator, 287, 500
 Significance level (α), 132
 Simple linear regression. *See* Regression, simple linear
 Singular matrix, 22
 Small sample inference for mixed linear models, 491–491, 494–497
 Span, 153
 Spectral decomposition, 51, 495–496
 Square root matrix, 53
 SSE (error sum of squares)
 balanced ANOVA
 one-way model, 343–344
 two-way model, 385, 390–391
 independence of SSR and SSE, 187
 multiple regression, 150–156, 179
 non-full-rank model, 313–314
 simple linear regression, 131–132
 unbalanced ANOVA
 one-way model, 417
 two-way model
 constrained, 428
 unconstrained, 432
 SSH (for general linear hypothesis test)
 in ANOVA, 326–329, 348–351, 401–403
 in regression, 199, 203
 SSR (regression sum of squares), 133–134, 161, 164, 186–189
 Standardized distance, 77
 Subspace, 153, 317
 Sufficient statistics, 159–160
 Sum(s) of squares
 Analysis of covariance, 449–463, 468–473
 ANOVA, balanced
 one-way, 345–346, 348–351
 contrasts, 358–363, 367–331
 two-way, 388–395, 395–403
 ANOVA, unbalanced
 one-way, 417
 contrasts, 417–421
 two-way, 426, 431–432
 full-and-reduced-model test in ANOVA, 324–326
 SSE. *See* SSE
 SSH (for general linear hypothesis test). *See* SSH
 SSR (for overall regression test). *See* SSR
 as quadratic form, 105–107
 test of a subset of β 's, 190–192
 Symmetric matrix, 7
 Systems of equations, 28–32
 consistent and inconsistent, 29
 and generalized inverse, 37–39
- t -Distribution, 116–117, 123
 central t , 117
 noncentral t , 116–117, 132
 noncentrality parameter, 116–117, 132
 p -value. *See* p -Value
 t -Tests, 123, 131–132, 134, 205
 p -value. *See* p -Value
 Tests of hypotheses. *See also* Analysis of variance, testing hypotheses;
 One-way model (balanced), testing the hypothesis $H_0: \mu_1 = \mu_2 = \dots = \mu_k$;
 Two-way model (balanced), tests of hypotheses
 for β_1 in simple linear regression, 131–132

- in Bayesian regression, 286
- F*-tests. *See F*-Tests
- general linear hypothesis test, 198–204
- for individual β 's or linear combinations.
 - See* Regression, multiple linear with fixed x 's, tests of hypotheses
- likelihood ratio tests, 217–221
- in linear mixed models, 491, 495
- overall regression test, 185–189, 196
- for ρ in bivariate normal distribution, 134
- regression tests in terms of R^2 ,
 - 196–198
- significance level (α), 132
- subset of the β 's, 189–196
- t*-tests. *See t*-Tests
- Trace of a matrix, 44–46
- Transpose, 7
- Treatments, 4, 295, 339, 377
- Triangular matrix, 8
- Two-way model (balanced), 3,
 - 299–301, 377–408
 - estimable functions, 378–382
 - estimates of, 382–384
 - interaction terms, 380
 - main effect terms, 380–381
 - estimation of σ^2 , 384–385
 - expected mean squares, 403–408
 - quadratic form approach, 405
 - sums of squares approach, 403–405
 - interaction, 301, 377
 - model, 377–378
 - assumptions, 378
 - no-interaction model, 329–335
 - estimable functions, 330–331
 - testing a hypothesis, 331–333
 - normal equations, 382–384
 - orthogonality of columns of \mathbf{X} , 333–335
 - reparameterization, 299–300
 - side conditions, 300–301, 381
 - SSE, 384, 390
 - tests of hypotheses
 - interaction
 - full-and-reduced-model test, 388–391
 - generalized inverse approach, 391–395
 - hypothesis, 385–388
 - main effects
 - full-and-reduced-model approach, 395–401
 - general linear hypothesis approach, 401–403
 - hypothesis, 396
- Unbalanced data in ANOVA
 - cell means model, 414
 - one-way model, 415–421
 - contrasts, 417–421
 - conditions for independence, 418
 - orthogonal contrasts, 418
 - weighted orthogonal contrasts, 419
 - estimation, 415–416
 - SSE, 416
 - testing $H_0: \mu_1 = \mu_2 = \dots = \mu_k$, 416
 - overparameterized model, 414
 - serial correlation, 479
 - two-way model, 421–432
 - cell means model, 421, 422
 - constrained model, 428–432
 - estimation, 430
 - model, 429
 - SSE, 431
 - testing hypotheses, 431–432
 - type I, II and III sums of squares, 414
 - unconstrained model, 421–428
 - contrasts, 424–425
 - estimator of σ^2 , 423
 - Hadamard product, 425
 - SSE, 423
 - testing hypotheses, 425–428
 - two-way model with empty cells, 432–439
 - estimability of empty cell means, 435
 - estimation for the partially constrained model, 434
 - isolated cells, 432
 - missing at random, 432
 - testing the interaction, 433–434
 - SSE, 433
 - weighted squares of means, 414
- Underfitting, 170–172
- Validation of model, 227–238. *See also* Hat matrix; Influential observations; Outliers; Residual(s)
- Variable(s)
 - dependent, 1, 137
 - independent, 1, 137
 - predictor, 1, 137
 - response, 1, 137
 - selection of variables, 2, 172
- Variance
 - of estimators of $\mathbf{X}'\boldsymbol{\beta}$, 311
 - generalized, 77
 - of least squares estimators, 130–131
 - population, 70–71
 - of quadratic form, 107
 - sample, 95. *See also* Sample variance

- Variance components, 480
 - estimating equations, 488
 - estimation, 486–489
- Vector(s)
 - angle between two vectors, 41–42, 136, 163, 238
 - column vector, 6
 - j** vector, 8–9
 - length of, 12
 - linear independence and dependence, 19
 - normalized vector, 42
 - notation, 6
 - orthogonal vectors, 37
 - orthonormal vectors, set of, 38
 - product of, 10–11
 - random vector. *See* Random Vectors
 - row vector, 6
 - zero vector (**0**), 8
- Weighted least squares, 168
- Zero matrix (**O**), 8
- Zero vector (**0**), 8