STAT 8004 – Statistical Methods II

Spring 2015

Homework Assignment 8 (Due on 4/2/2015 before the end of the day)

- Reading assignment
  - All in-class coverage about linear mixed models.
  - Please read material in Chapter 9 of Faraway (2006) on R for linear mixed model with repeated measurements data.
- Comprehensive coverage for the ANOVA approach for linear mixed model is in Chapter 25 onwards of
  - Kutner, M., Nachtsheim, C., Neter, J. and Li, W. (2004). Applied Linear Statistical Models, 5th Edition. McGraw-Hill/Irwin.
- The following exercises are to be collected. Please upload your homework to the Blackboard. Following the requirement of STAT 8003, please typeset your homework with Latex and upload both the pdf and latex files.

1. This is Problem 3 of Faraway (2006), Chapter 8.

   The ratdrink data consist of five weekly measurements of body weight for 27 rats. The first 10 rats are on a control treatment while seven rats have thyroxine added to their drinking water. Ten rats have thiouracil added to their water. Build a model for the rat weights that shows the effect of the treatment.

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   > library(faraway)
   > data(ratdrink)
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   (a) Model the weights of the rate, incorporating the treatment effects and random effect. Use R to fit the model.

   (b) What is the implication of the random effect on the correlations between weights of the same rat? Is that implication reasonable? It would be nice to support your argument with data evidence.

2. The article "Variability of Sliver Weights at Different Carding Stages and a Suggested Sampling Plan for Jute Processing" by A. Lahiri (Journal of the Textile Institute, 1990) concerns the partitioning of variability in "sliver weight." (A sliver is a continuous strand of loose, untwisted wool, cotton, etc., produced along the way to making yarn.) For a particular mill, 3 (of many) machines were studied, using 5 (10 mm) pieces of sliver cut from each of 5 rolls produced on the machines. The weights of the (75) pieces of sliver were determined and a standard hierarchical (balanced data) ANOVA table was produced as below. (The units of weight were not given in the original article.)

Source	SS	df
Machines	1966	2
Rolls	644	12
Pieces	2890	60
Total	2890	74

The model is

$$y_{ijk} = \mu + \alpha_i + u_{ij} + \varepsilon_{ijk}$$

for the kth piece of the jth roll on the ith machine, where $\alpha_i \overset{iid}{\sim} N(0, \sigma_a^2)$, $u_{ij} \overset{iid}{\sim} N(0, \sigma_u^2)$, and $\varepsilon_{ijk} \overset{iid}{\sim} N(0, \sigma_e^2)$.

(a) Find estimates for σ_a^2, σ_u^2, and σ_e^2.

(b) Make 95% confidence intervals for each of the 3 standard deviations σ_a, σ_u, and σ_u. Based on these, where do you judge the largest part of variation in measured weight to come from? You need to use the Cochran- Satterthwaite approximation for σ_a and σ_u.

(c) Suppose for sake of illustration that the grand average of all 75 weight measurements was in fact $\bar{y}_{...} = 1/75 \sum_{ijk} y_{ijk} = 35.0$. Use this and information from the ANOVA table to make a 95% confidence interval for the model parameter μ.