Variational Bayes: Unified M1/M3 Analysis

Code ▾

Show

1 Scenario 1: Conditional on Model Type

Show

```
## === Running M3 Analysis ===
```

Show

Show

```
## === SCENARIO 1: 30 levels (10 obs each) ===
```

Show

```
## Converged at iteration 9
## Max relative change: 6.10e-06
```

Show

```
##
## Exact posterior parameters:
```

Show

```
## E[beta]: 5.253169 0.4526869 0.4673524 0.4262505
```

Show

```
## E[tau_e]: 0.5052656
```

Show

```
##
## Running Laplace approximation...
```

Show
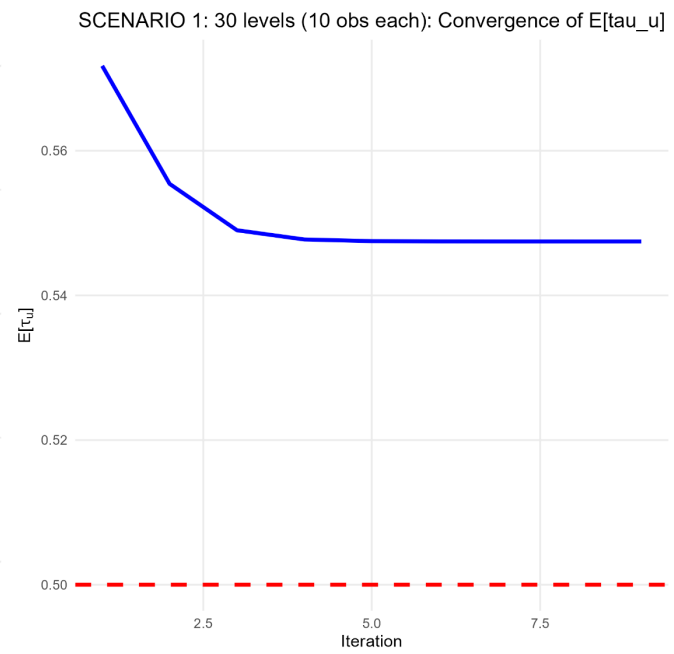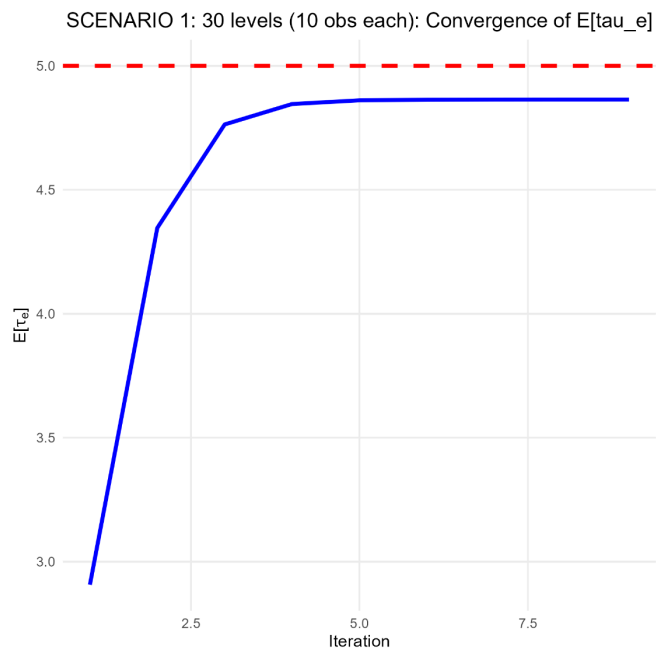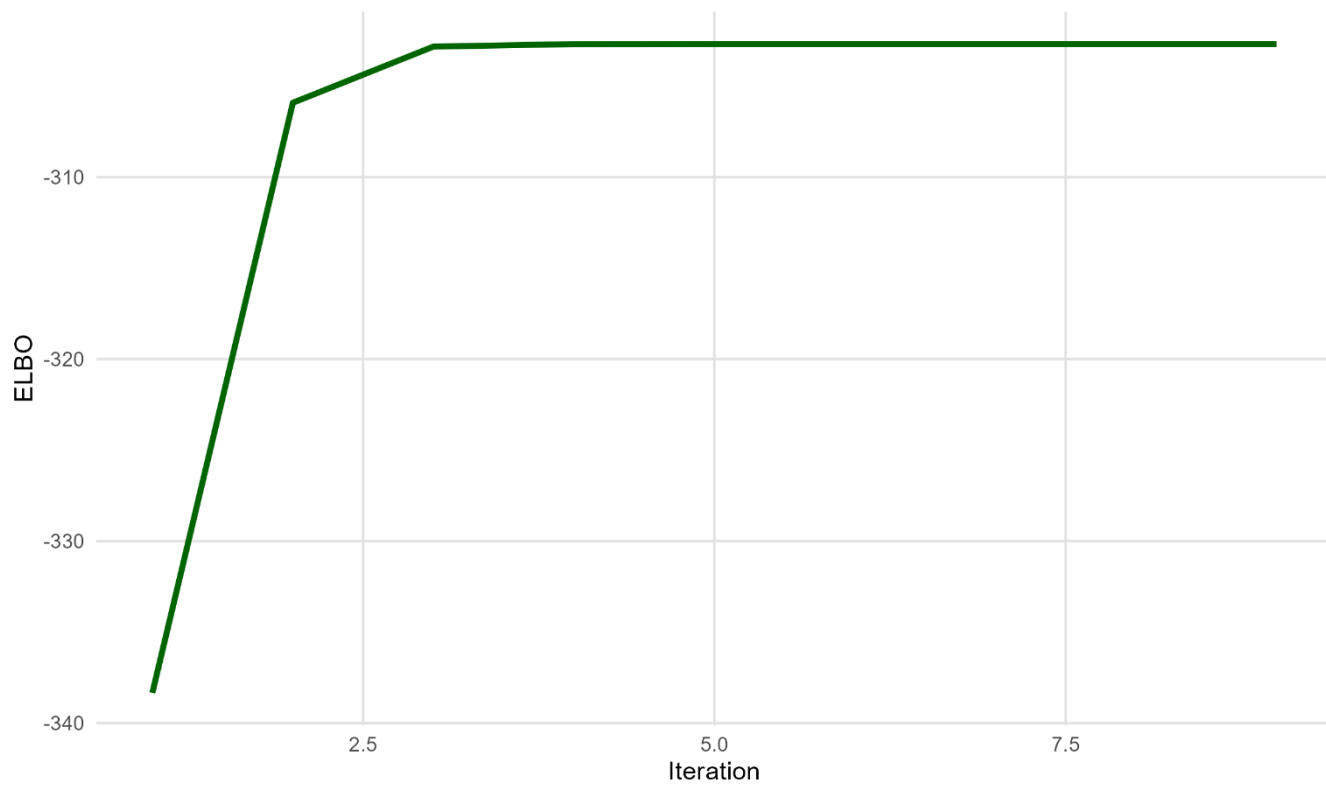
```
## Laplace MAP estimates:
```

Show

```
## beta: 5.254224 0.4904948 0.5289162 0.4710682
## tau_e: 5.853474
## tau_u: 0.5431257
```

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SCENARIO 1: 30 levels (10 obs each): Convergence of E[tau_e]

SCENARIO 1: 30 levels (10 obs each): Convergence of E[tau_u]

SCENARIO 1: 30 levels (10 obs each): ELBO Convergence


```
## M3: 8-panel ggplot saved
```

3-Method Comparison for β_0

3-Method Comparison for β_1

3-Method Comparison for β_2

3-Method Comparison for β_3

VB for u_1

VB for u_2

VB for τ_c

VB for τ_u
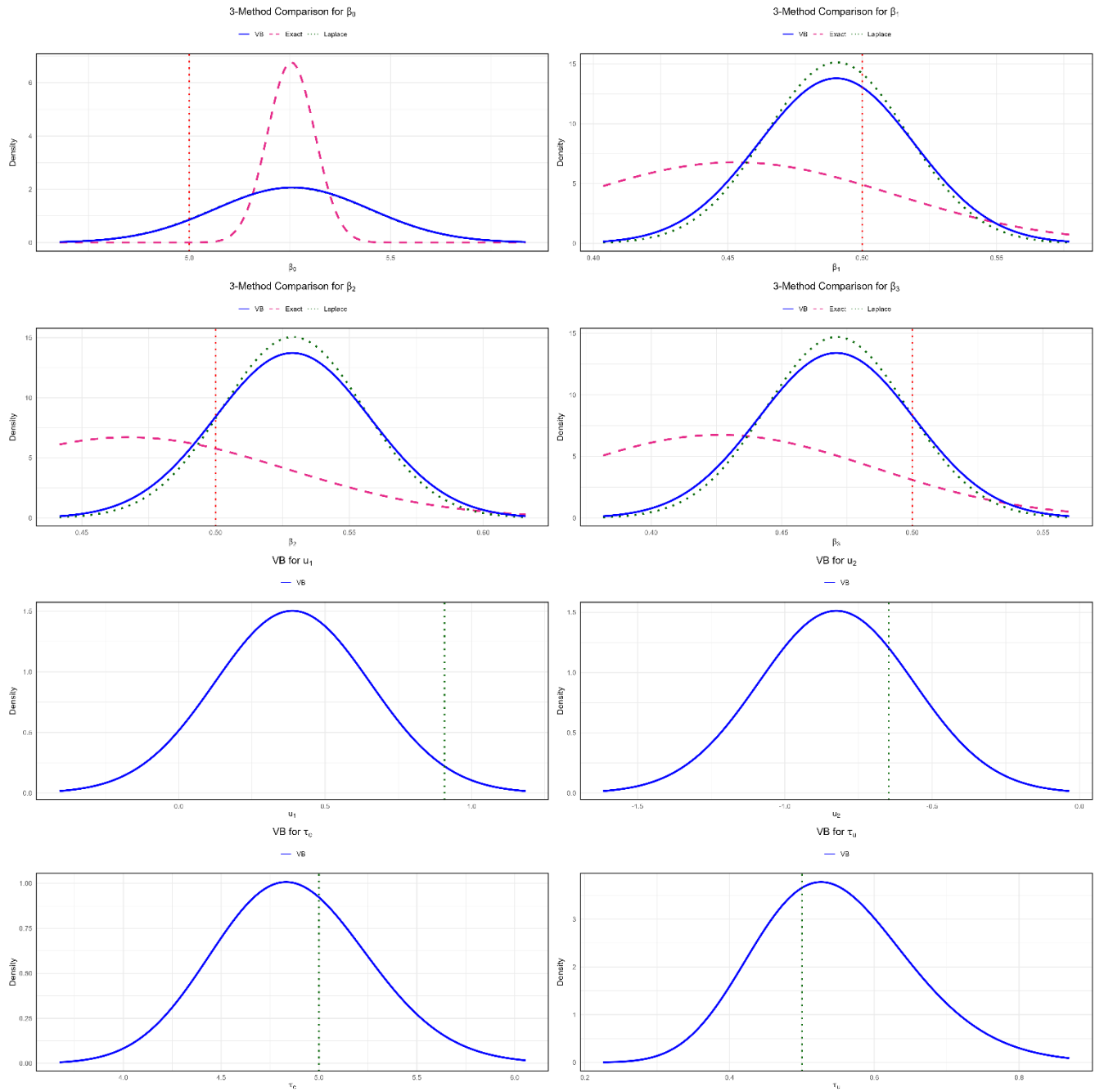
2 Scenario 2: Conditional on Model Type (M3 only)

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<div align="right">Show</div>

```
##
## === SCENARIO 2: 6 levels (50 obs each) ===
```

<div align="right">Show</div>

```
## Converged at iteration 7
## Max relative change: 6.30e-06
```

<div align="right">Show</div>

```
##
## Exact posterior parameters:
```

<div align="right">Show</div>

```
## E[beta]: 3.974663 0.4981715 0.4861898 0.4321623
```

<div align="right">Show</div>

```
## E[tau_e]: 0.42704
```

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```
##
## Running Laplace approximation...
```

<div align="right">Show</div>

```
## Laplace MAP estimates:
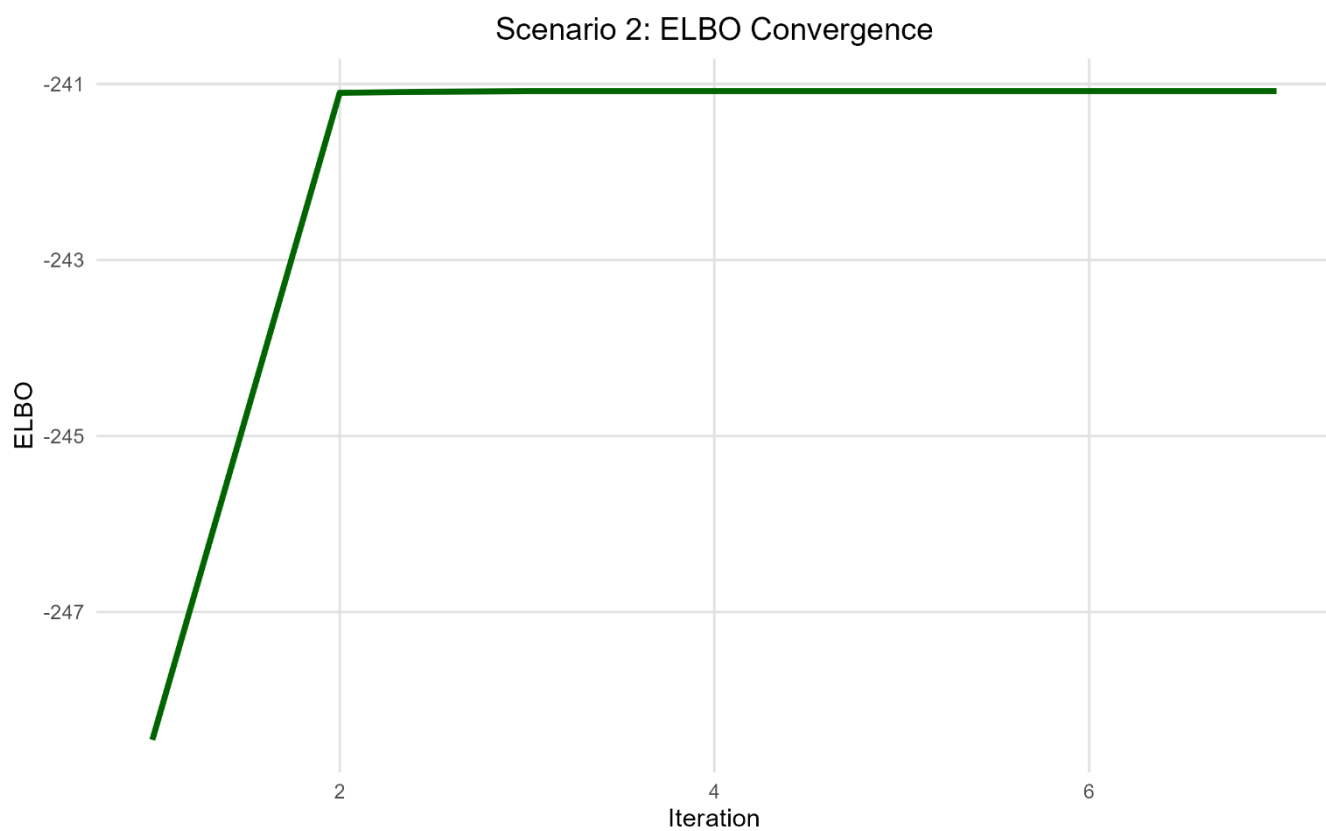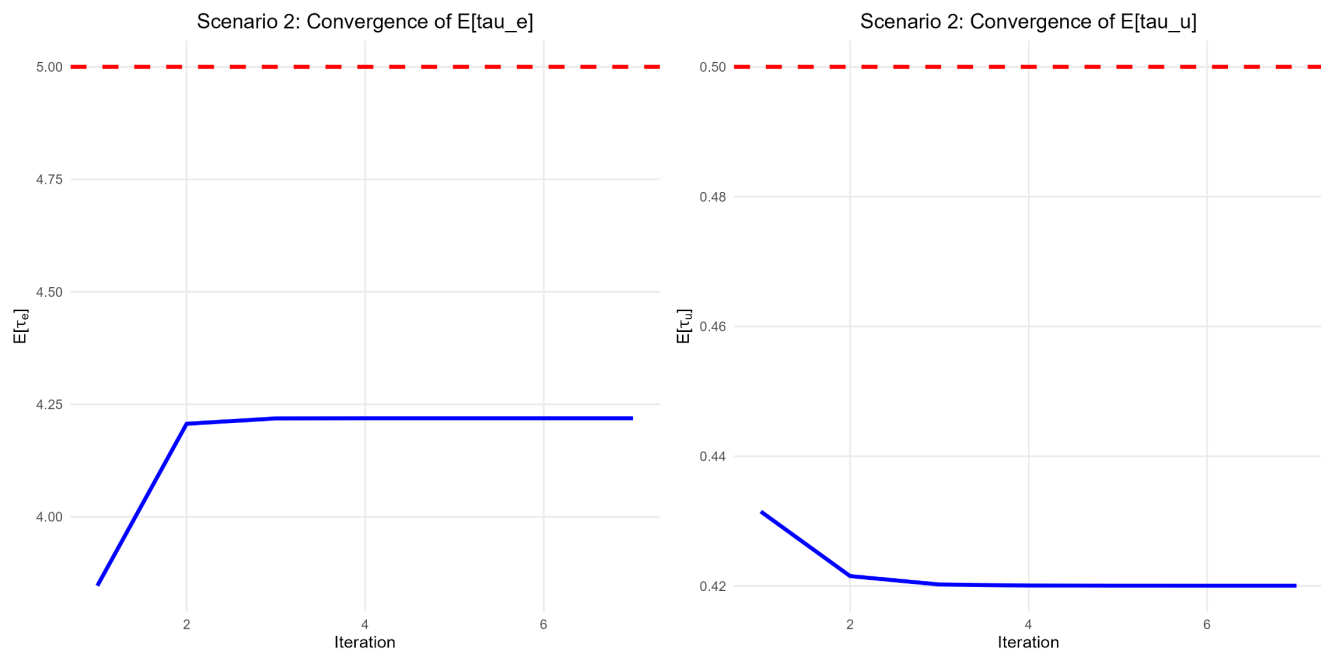```

<div align="right">Show</div>

```
## beta: 3.957505 0.5074171 0.4734573 0.4890945
```

<div align="right">Show</div>

```
## tau_e: 4.320447
```

<div align="right">Show</div>

```
## tau_u: 0.3603174
```

Show

Scenario 2: Convergence of E[tau_e]

Scenario 2: Convergence of E[tau_u]

Scenario 2: ELBO Convergence

```
## 6 levels: 8-panel ggplot saved
```

3-Method Comparison for β_0

3-Method Comparison for β_1

3-Method Comparison for β_2

3-Method Comparison for β_3

VB for u_1

VB for u_2

VB for τ_c

VB for τ_u
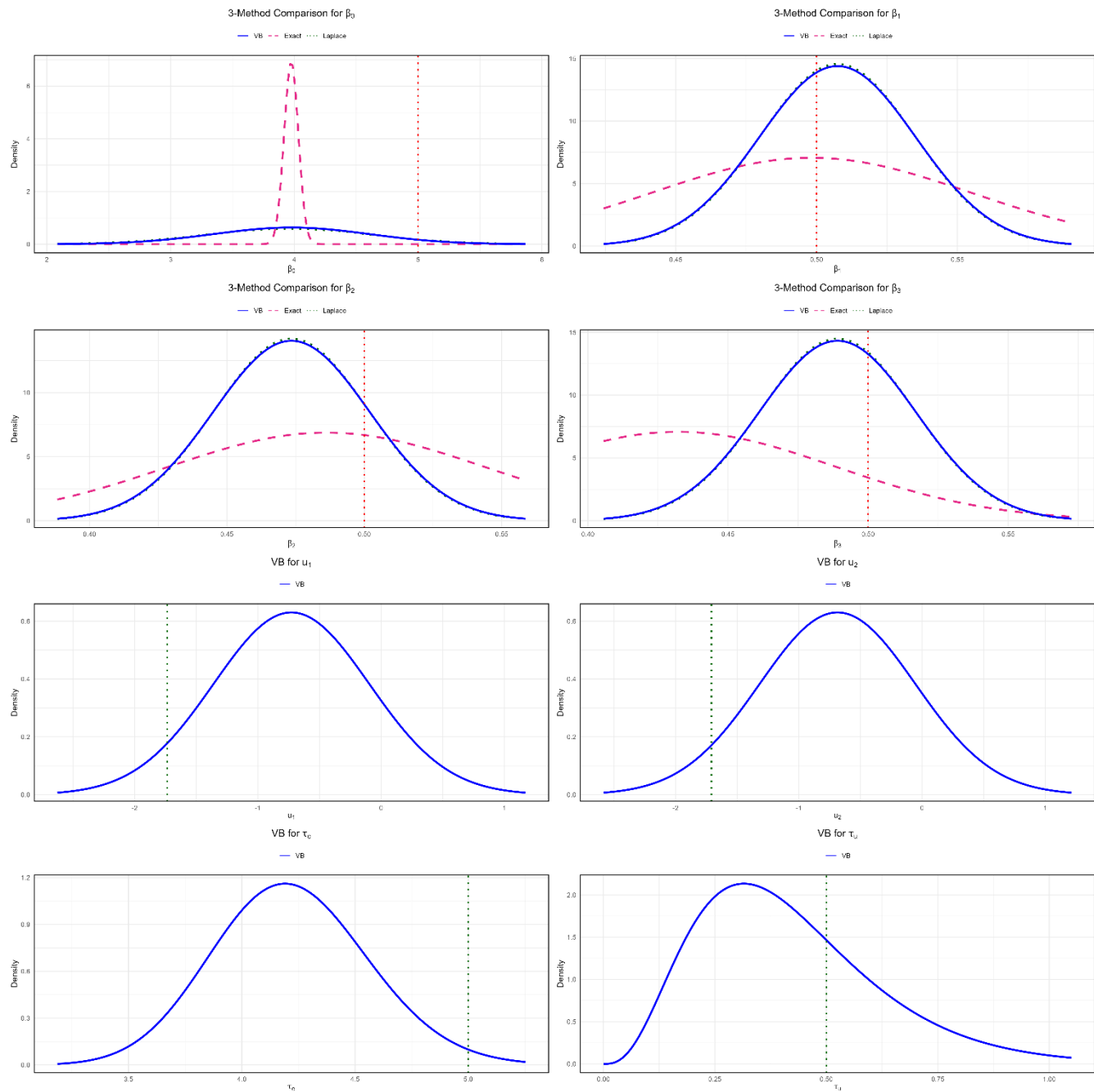
3 Comparison Between Scenarios (M3 only)

```
##    Parameter True_Value Scenario_30 Scenario_6
## 1  E[tau_e]        5.0   4.8640457  4.2190771
## 2  E[tau_u]        0.5   0.5474557  0.4200384
## 3 sigma^2_e        0.2   0.2055902  0.2370187
## 4 sigma^2_u        2.0   1.8266317  2.3807349
```

```
##
## Under-dispersion in tau_u estimates:
```
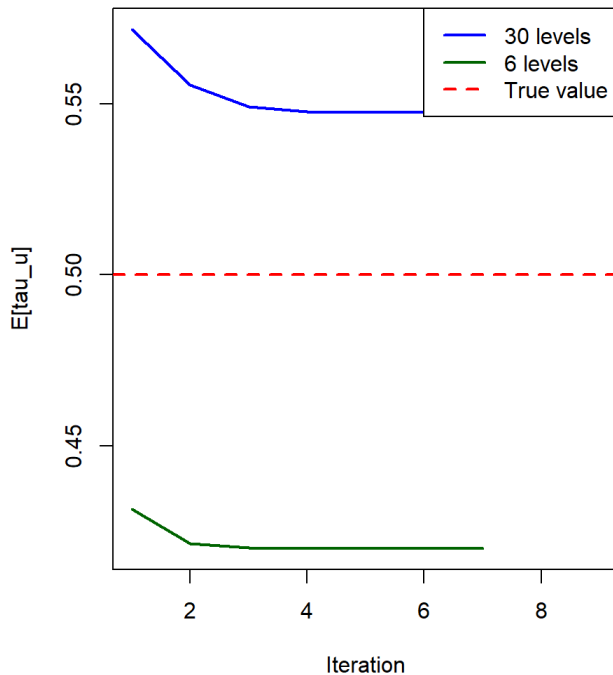
```
## 30 levels: VB tau_u = 0.5475 vs True = 0.5 (ratio: 1.0949 )
```

```
## 6 levels:  VB tau_u = 0.42 vs True = 0.5 (ratio: 0.8401 )
```
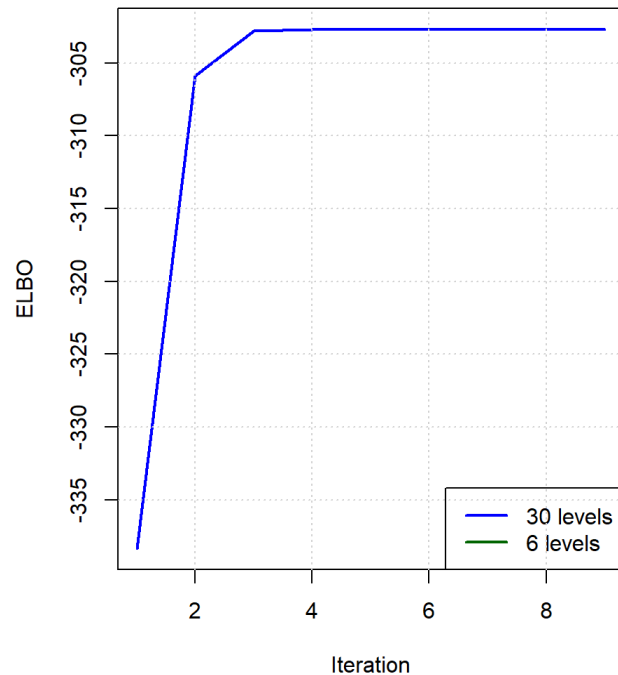
Comparison: E[tau_u] Convergence   **Comparison: ELBO Convergence**

Sample Size Effects on τ_u (Multi-Configuration Analysis)

```
##
## =======================================
```

Show

```
## Multi-Configuration τ_u Analysis
```

Show

```
## =======================================
```

Show

```
##
## --- Running: Q=5 (n=60 per group) ---
## Converged at iteration 8
## Max relative change: 1.44e-05
## VB E[tau_u]: 1.4992
##
## --- Running: Q=10 (n=30 per group) ---
## Converged at iteration 7
## Max relative change: 1.71e-05
## VB E[tau_u]: 2.6993
##
## --- Running: Q=20 (n=15 per group) ---
## Converged at iteration 7
## Max relative change: 1.59e-05
## VB E[tau_u]: 5.0595
##
## --- Running: Q=50 (n=6 per group) ---
## Converged at iteration 9
## Max relative change: 2.64e-05
## VB E[tau_u]: 11.2881
```

Show
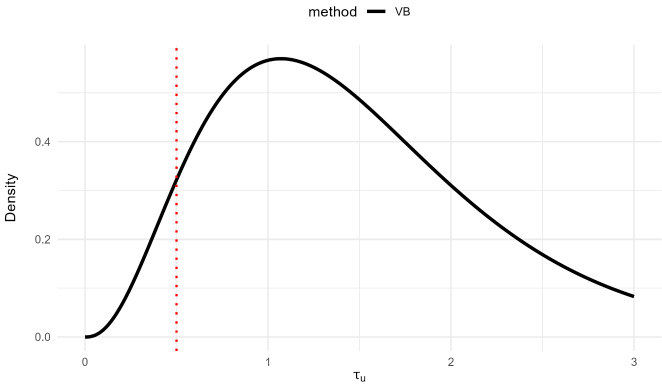
```
##
## =======================================
```

Show

```
## τ_u comparison plot saved to figs/tau_u_sample_size_comparison.png
```
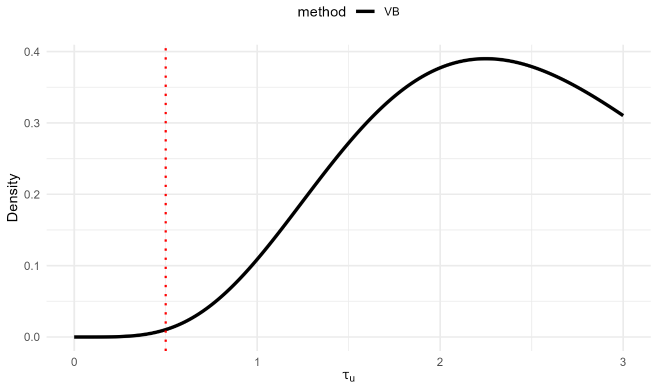
Show

Effect of Sample Size Per Group on τ_u Posterior

VB approximation improves as observations per random effect level increase

Q=5 (n=60 per group)

VB E[τ_u] = 1.499

method ▬ VB

Q=10 (n=30 per group)

VB E[τ_u] = 2.699

method ▬ VB

Q=20 (n=15 per group)

VB E[τ_u] = 5.059

method ▬ VB
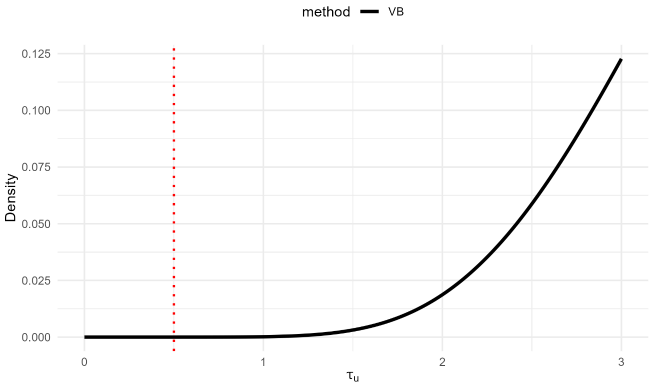
Q=50 (n=6 per group)

VB E[τ_u] = 11.288

method ▬ VB
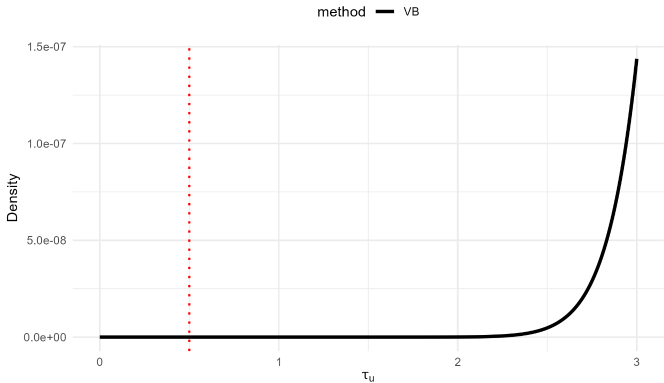
[Show]

```
## 
## =======================================
```

[Show]

```
## Diagnostic Summary: τ_u Across Configurations
```

[Show]

```
## =======================================
```

[Show]

```
##    Q n_per_group  VB_tau_u VB_SD_tau_u
## 1  5          60  1.499189   0.8013504
## 2 10          30  2.699301   1.1019850
## 3 20          15  5.059461   1.5254848
## 4 50           6 11.288077   2.2137740
```

[Show]

```
## <div id="iqqeytkkmt" style="padding-left:0px;padding-right:0px;padding-top:10px;p
adding-bottom:10px;overflow-x:auto;overflow-y:auto;width:auto;height:auto;">
##   <style>#iqqeytkkmt table {
##   font-family: system-ui, 'Segoe UI', Roboto, Helvetica, Arial, sans-serif, 'Appl
e Color Emoji', 'Segoe UI Emoji', 'Segoe UI Symbol', 'Noto Color Emoji';
##   -webkit-font-smoothing: antialiased;
##   -moz-osx-font-smoothing: grayscale;
## }
##
## #iqqeytkkmt thead, #iqqeytkkmt tbody, #iqqeytkkmt tfoot, #iqqeytkkmt tr, #iqqeytk
kmt td, #iqqeytkkmt th {
##   border-style: none;
## }
##
## #iqqeytkkmt p {
##   margin: 0;
##   padding: 0;
## }
##
## #iqqeytkkmt .gt_table {
##   display: table;
##   border-collapse: collapse;
##   line-height: normal;
##   margin-left: auto;
##   margin-right: auto;
##   color: #333333;
##   font-size: 16px;
##   font-weight: normal;
##   font-style: normal;
##   background-color: #FFFFFF;
##   width: auto;
##   border-top-style: solid;
##   border-top-width: 2px;
##   border-top-color: #A8A8A8;
##   border-right-style: none;
##   border-right-width: 2px;
##   border-right-color: #D3D3D3;
##   border-bottom-style: solid;
##   border-bottom-width: 2px;
##   border-bottom-color: #A8A8A8;
##   border-left-style: none;
##   border-left-width: 2px;
##   border-left-color: #D3D3D3;
## }
##
## #iqqeytkkmt .gt_caption {
##   padding-top: 4px;
##   padding-bottom: 4px;
```

```
## }
##
## #iqqeytkkmt .gt_title {
##     color: #333333;
##     font-size: 125%;
##     font-weight: initial;
##     padding-top: 4px;
##     padding-bottom: 4px;
##     padding-left: 5px;
##     padding-right: 5px;
##     border-bottom-color: #FFFFFF;
##     border-bottom-width: 0;
## }
##
## #iqqeytkkmt .gt_subtitle {
##     color: #333333;
##     font-size: 85%;
##     font-weight: initial;
##     padding-top: 3px;
##     padding-bottom: 5px;
##     padding-left: 5px;
##     padding-right: 5px;
##     border-top-color: #FFFFFF;
##     border-top-width: 0;
## }
##
## #iqqeytkkmt .gt_heading {
##     background-color: #FFFFFF;
##     text-align: center;
##     border-bottom-color: #FFFFFF;
##     border-left-style: none;
##     border-left-width: 1px;
##     border-left-color: #D3D3D3;
##     border-right-style: none;
##     border-right-width: 1px;
##     border-right-color: #D3D3D3;
## }
##
## #iqqeytkkmt .gt_bottom_border {
##     border-bottom-style: solid;
##     border-bottom-width: 2px;
##     border-bottom-color: #D3D3D3;
## }
##
## #iqqeytkkmt .gt_col_headings {
##     border-top-style: solid;
##     border-top-width: 2px;
##     border-top-color: #D3D3D3;
##     border-bottom-style: solid;
```

```
##     border-bottom-width: 2px;
##     border-bottom-color: #D3D3D3;
##     border-left-style: none;
##     border-left-width: 1px;
##     border-left-color: #D3D3D3;
##     border-right-style: none;
##     border-right-width: 1px;
##     border-right-color: #D3D3D3;
## }
##
## #iqqeytkkmt .gt_col_heading {
##     color: #333333;
##     background-color: #FFFFFF;
##     font-size: 100%;
##     font-weight: normal;
##     text-transform: inherit;
##     border-left-style: none;
##     border-left-width: 1px;
##     border-left-color: #D3D3D3;
##     border-right-style: none;
##     border-right-width: 1px;
##     border-right-color: #D3D3D3;
##     vertical-align: bottom;
##     padding-top: 5px;
##     padding-bottom: 6px;
##     padding-left: 5px;
##     padding-right: 5px;
##     overflow-x: hidden;
## }
##
## #iqqeytkkmt .gt_column_spanner_outer {
##     color: #333333;
##     background-color: #FFFFFF;
##     font-size: 100%;
##     font-weight: normal;
##     text-transform: inherit;
##     padding-top: 0;
##     padding-bottom: 0;
##     padding-left: 4px;
##     padding-right: 4px;
## }
##
## #iqqeytkkmt .gt_column_spanner_outer:first-child {
##     padding-left: 0;
## }
##
## #iqqeytkkmt .gt_column_spanner_outer:last-child {
##     padding-right: 0;
## }
```

```
##
## #iqqeytkkmt .gt_column_spanner {
##   border-bottom-style: solid;
##   border-bottom-width: 2px;
##   border-bottom-color: #D3D3D3;
##   vertical-align: bottom;
##   padding-top: 5px;
##   padding-bottom: 5px;
##   overflow-x: hidden;
##   display: inline-block;
##   width: 100%;
## }
##
## #iqqeytkkmt .gt_spanner_row {
##   border-bottom-style: hidden;
## }
##
## #iqqeytkkmt .gt_group_heading {
##   padding-top: 8px;
##   padding-bottom: 8px;
##   padding-left: 5px;
##   padding-right: 5px;
##   color: #333333;
##   background-color: #FFFFFF;
##   font-size: 100%;
##   font-weight: initial;
##   text-transform: inherit;
##   border-top-style: solid;
##   border-top-width: 2px;
##   border-top-color: #D3D3D3;
##   border-bottom-style: solid;
##   border-bottom-width: 2px;
##   border-bottom-color: #D3D3D3;
##   border-left-style: none;
##   border-left-width: 1px;
##   border-left-color: #D3D3D3;
##   border-right-style: none;
##   border-right-width: 1px;
##   border-right-color: #D3D3D3;
##   vertical-align: middle;
##   text-align: left;
## }
##
## #iqqeytkkmt .gt_empty_group_heading {
##   padding: 0.5px;
##   color: #333333;
##   background-color: #FFFFFF;
##   font-size: 100%;
##   font-weight: initial;
```

```
##    border-top-style: solid;
##    border-top-width: 2px;
##    border-top-color: #D3D3D3;
##    border-bottom-style: solid;
##    border-bottom-width: 2px;
##    border-bottom-color: #D3D3D3;
##    vertical-align: middle;
## }
##
## #iqqeytkkmt .gt_from_md > :first-child {
##    margin-top: 0;
## }
##
## #iqqeytkkmt .gt_from_md > :last-child {
##    margin-bottom: 0;
## }
##
## #iqqeytkkmt .gt_row {
##    padding-top: 8px;
##    padding-bottom: 8px;
##    padding-left: 5px;
##    padding-right: 5px;
##    margin: 10px;
##    border-top-style: solid;
##    border-top-width: 1px;
##    border-top-color: #D3D3D3;
##    border-left-style: none;
##    border-left-width: 1px;
##    border-left-color: #D3D3D3;
##    border-right-style: none;
##    border-right-width: 1px;
##    border-right-color: #D3D3D3;
##    vertical-align: middle;
##    overflow-x: hidden;
## }
##
## #iqqeytkkmt .gt_stub {
##    color: #333333;
##    background-color: #FFFFFF;
##    font-size: 100%;
##    font-weight: initial;
##    text-transform: inherit;
##    border-right-style: solid;
##    border-right-width: 2px;
##    border-right-color: #D3D3D3;
##    padding-left: 5px;
##    padding-right: 5px;
## }
##
```

```
## #iqqeytkkmt .gt_stub_row_group {
##   color: #333333;
##   background-color: #FFFFFF;
##   font-size: 100%;
##   font-weight: initial;
##   text-transform: inherit;
##   border-right-style: solid;
##   border-right-width: 2px;
##   border-right-color: #D3D3D3;
##   padding-left: 5px;
##   padding-right: 5px;
##   vertical-align: top;
## }
##
## #iqqeytkkmt .gt_row_group_first td {
##   border-top-width: 2px;
## }
##
## #iqqeytkkmt .gt_row_group_first th {
##   border-top-width: 2px;
## }
##
## #iqqeytkkmt .gt_summary_row {
##   color: #333333;
##   background-color: #FFFFFF;
##   text-transform: inherit;
##   padding-top: 8px;
##   padding-bottom: 8px;
##   padding-left: 5px;
##   padding-right: 5px;
## }
##
## #iqqeytkkmt .gt_first_summary_row {
##   border-top-style: solid;
##   border-top-color: #D3D3D3;
## }
##
## #iqqeytkkmt .gt_first_summary_row.thick {
##   border-top-width: 2px;
## }
##
## #iqqeytkkmt .gt_last_summary_row {
##   padding-top: 8px;
##   padding-bottom: 8px;
##   padding-left: 5px;
##   padding-right: 5px;
##   border-bottom-style: solid;
##   border-bottom-width: 2px;
##   border-bottom-color: #D3D3D3;
```

```
## }
##
## #iqqeytkkmt .gt_grand_summary_row {
##   color: #333333;
##   background-color: #FFFFFF;
##   text-transform: inherit;
##   padding-top: 8px;
##   padding-bottom: 8px;
##   padding-left: 5px;
##   padding-right: 5px;
## }
##
## #iqqeytkkmt .gt_first_grand_summary_row {
##   padding-top: 8px;
##   padding-bottom: 8px;
##   padding-left: 5px;
##   padding-right: 5px;
##   border-top-style: double;
##   border-top-width: 6px;
##   border-top-color: #D3D3D3;
## }
##
## #iqqeytkkmt .gt_last_grand_summary_row_top {
##   padding-top: 8px;
##   padding-bottom: 8px;
##   padding-left: 5px;
##   padding-right: 5px;
##   border-bottom-style: double;
##   border-bottom-width: 6px;
##   border-bottom-color: #D3D3D3;
## }
##
## #iqqeytkkmt .gt_striped {
##   background-color: rgba(128, 128, 128, 0.05);
## }
##
## #iqqeytkkmt .gt_table_body {
##   border-top-style: solid;
##   border-top-width: 2px;
##   border-top-color: #D3D3D3;
##   border-bottom-style: solid;
##   border-bottom-width: 2px;
##   border-bottom-color: #D3D3D3;
## }
##
## #iqqeytkkmt .gt_footnotes {
##   color: #333333;
##   background-color: #FFFFFF;
##   border-bottom-style: none;
```

```
##     border-bottom-width: 2px;
##     border-bottom-color: #D3D3D3;
##     border-left-style: none;
##     border-left-width: 2px;
##     border-left-color: #D3D3D3;
##     border-right-style: none;
##     border-right-width: 2px;
##     border-right-color: #D3D3D3;
## }
##
## #iqqeytkkmt .gt_footnote {
##     margin: 0px;
##     font-size: 90%;
##     padding-top: 4px;
##     padding-bottom: 4px;
##     padding-left: 5px;
##     padding-right: 5px;
## }
##
## #iqqeytkkmt .gt_sourcenotes {
##     color: #333333;
##     background-color: #FFFFFF;
##     border-bottom-style: none;
##     border-bottom-width: 2px;
##     border-bottom-color: #D3D3D3;
##     border-left-style: none;
##     border-left-width: 2px;
##     border-left-color: #D3D3D3;
##     border-right-style: none;
##     border-right-width: 2px;
##     border-right-color: #D3D3D3;
## }
##
## #iqqeytkkmt .gt_sourcenote {
##     font-size: 90%;
##     padding-top: 4px;
##     padding-bottom: 4px;
##     padding-left: 5px;
##     padding-right: 5px;
## }
##
## #iqqeytkkmt .gt_left {
##     text-align: left;
## }
##
## #iqqeytkkmt .gt_center {
##     text-align: center;
## }
##
```

```
## #iqqeytkkmt .gt_right {
##   text-align: right;
##   font-variant-numeric: tabular-nums;
## }
##
## #iqqeytkkmt .gt_font_normal {
##   font-weight: normal;
## }
##
## #iqqeytkkmt .gt_font_bold {
##   font-weight: bold;
## }
##
## #iqqeytkkmt .gt_font_italic {
##   font-style: italic;
## }
##
## #iqqeytkkmt .gt_super {
##   font-size: 65%;
## }
##
## #iqqeytkkmt .gt_footnote_marks {
##   font-size: 75%;
##   vertical-align: 0.4em;
##   position: initial;
## }
##
## #iqqeytkkmt .gt_asterisk {
##   font-size: 100%;
##   vertical-align: 0;
## }
##
## #iqqeytkkmt .gt_indent_1 {
##   text-indent: 5px;
## }
##
## #iqqeytkkmt .gt_indent_2 {
##   text-indent: 10px;
## }
##
## #iqqeytkkmt .gt_indent_3 {
##   text-indent: 15px;
## }
##
## #iqqeytkkmt .gt_indent_4 {
##   text-indent: 20px;
## }
##
## #iqqeytkkmt .gt_indent_5 {
```

```
##     text-indent: 25px;
## }
##
## #iqqeytkkmt .katex-display {
##     display: inline-flex !important;
##     margin-bottom: 0.75em !important;
## }
##
## #iqqeytkkmt div.Reactable > div.rt-table > div.rt-thead > div.rt-tr.rt-tr-group-h
eader > div.rt-th-group:after {
##     height: 0px !important;
## }
## </style>
##     <table class="gt_table" data-quarto-disable-processing="false" data-quarto-boot
strap="false">
##     <thead>
##       <tr class="gt_heading">
##         <td colspan="4" class="gt_heading gt_title gt_font_normal" style>τ_u Estima
tes Across Group Configurations</td>
##       </tr>
##       <tr class="gt_heading">
##         <td colspan="4" class="gt_heading gt_subtitle gt_font_normal gt_bottom_bord
er" style>Demonstrating sample size effects on variance component estimation</td>
##       </tr>
##       <tr class="gt_col_headings">
##         <th class="gt_col_heading gt_columns_bottom_border gt_right" rowspan="1" co
lspan="1" scope="col" id="Q">Q</th>
##         <th class="gt_col_heading gt_columns_bottom_border gt_right" rowspan="1" co
lspan="1" scope="col" id="n_per_group">n_per_group</th>
##         <th class="gt_col_heading gt_columns_bottom_border gt_right" rowspan="1" co
lspan="1" scope="col" id="VB_tau_u">VB_tau_u</th>
##         <th class="gt_col_heading gt_columns_bottom_border gt_right" rowspan="1" co
lspan="1" scope="col" id="VB_SD_tau_u">VB_SD_tau_u</th>
##       </tr>
##     </thead>
##     <tbody class="gt_table_body">
##       <tr><td headers="Q" class="gt_row gt_right">5.0000</td>
## <td headers="n_per_group" class="gt_row gt_right">60.0000</td>
## <td headers="VB_tau_u" class="gt_row gt_right">1.4992</td>
## <td headers="VB_SD_tau_u" class="gt_row gt_right">0.8014</td></tr>
##       <tr><td headers="Q" class="gt_row gt_right">10.0000</td>
## <td headers="n_per_group" class="gt_row gt_right">30.0000</td>
## <td headers="VB_tau_u" class="gt_row gt_right">2.6993</td>
## <td headers="VB_SD_tau_u" class="gt_row gt_right">1.1020</td></tr>
##       <tr><td headers="Q" class="gt_row gt_right">20.0000</td>
## <td headers="n_per_group" class="gt_row gt_right">15.0000</td>
## <td headers="VB_tau_u" class="gt_row gt_right">5.0595</td>
## <td headers="VB_SD_tau_u" class="gt_row gt_right">1.5255</td></tr>
##       <tr><td headers="Q" class="gt_row gt_right">50.0000</td>
```

```
## <td headers="n_per_group" class="gt_row gt_right">6.0000</td>
## <td headers="VB_tau_u" class="gt_row gt_right">11.2881</td>
## <td headers="VB_SD_tau_u" class="gt_row gt_right">2.2138</td></tr>
##   </tbody>
##
## </table>
## </div>
```

Show

```
##
## =======================================
```

Show

```
## Key Finding:
```

Show

```
## As observations per group increase (Q decreases),
```

Show

```
## VB approximation improves (SD_ratio → 1.0)
```

Show

```
## =======================================
```