A QCNN for Quantum State Preparation

Carnegie Vacation Scholarship

David Amorim

Weeks 7-8
(12/08/2024 - 23/08/2024)

Aims for the Week

The following aims were set at the last meeting (14/08/2024):

New Phase Encoding Approach

Investigate a new approach to phase encoding using linear piecewise phase functions without explicit function evaluation.

Handover

Hand over the slides, documentation, code and the poster for the Carnegie Trust.

Table of Contents

Preliminaries

- Consider an n-qubit register with computational basis states $|j\rangle = |j_0 j_1 ... j_{n-1}\rangle$ representing n-bit strings

- Let p of the register qubits be precision qubits so that

$$j = \sum_{k=0}^{n-1} j_k 2^{k-p} \tag{1}$$

- Consider a phase function Ψ over the domain $\Omega = \{j\}$ and construct an M-fold partition ($M = 2^m$, $m \leq n \in \mathbb{N}$) into equal sub-domains Ω_u:

$$\Omega = \bigcup_{u=1}^{M} \Omega_u, \quad \Omega_u \cap \Omega_v = \emptyset, \quad |\Omega_u| = |\Omega_v| \tag{2}$$

- On each sub-domain, approximate Ψ using a linear function:

$$\Psi(j) = \alpha_u j + \beta_u, \quad j \in \Omega_u \tag{3}$$

Phase Encoding within a Sub-domain

> **Aim 1**
>
> For $j \in \Omega_u$ construct an operator $\hat{O}_u$ such that $|j\rangle \mapsto e^{i(\alpha_u j + \beta_u)} |j\rangle$.

- Consider the single-qubit operators

$$\hat{P}^{(k)}(\varphi) = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & e^{i\varphi} \end{pmatrix}, \quad \hat{R}^{(k)}(\varphi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \tag{4}$$

each acting on the kth qubit

- Then

$$\hat{O}_u \equiv \bigotimes_{k=0}^{n-1} \hat{P}^{(k)}(\beta_u/n) \hat{R}^{(k)}\left(\alpha_u 2^{k-p}\right) \tag{5}$$

transforms

$$|j\rangle \mapsto \exp\left[i\left(\sum_{k=0}^{n-1} \alpha_u j_k 2^{k-p} + \beta_u\right)\right] |j\rangle = e^{i(\alpha_u j + \beta_u)} |j\rangle \tag{6}$$

Selecting the Subdomain

- It is straight-forward to construct $\hat{O}_u$ for each of the sub-domains Ω_u
- More challenging is applying the correct $\hat{O}_u$ based on the sub-domain corresponding to each $|j\rangle$

Aim 2

Construct a system of controls such that $\hat{O}_u$ is applied to $|j\rangle$ if and only if $j \in \Omega_u$

Sample Case: $M = 2$

- Start with the simplest possible case, a 2-fold partition ($M = 2$):

$$j \in \begin{cases} \Omega_1 & j_0 = 0 \\ \Omega_2 & j_0 = 1 \end{cases} \quad (7)$$

- Using an ancilla qubit, $\hat{O}_1$ is applied for $j \in \Omega_1$ and $\hat{O}_2$ for $j \in \Omega_2$

- The ancilla is required since the operation applied to $|j_0\rangle$ is conditional on $|j_0\rangle$ itself
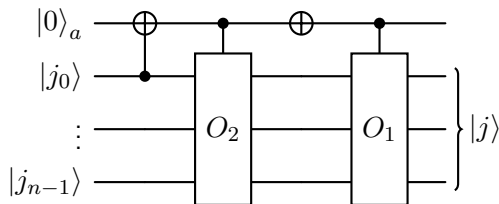

Figure 1: Circuit diagram for $M = 2$

Generalising the Approach

- The approach shown on the previous slide requires $1 \leq \log_2 M \leq n$ ancilla qubits

- The number of controls required is $\mathcal{O}(M \log M)$ as there are M operators, each controlled by all ancillas
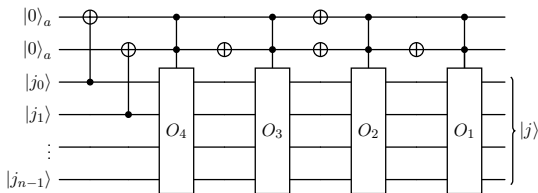
- For $M \sim 2^n$ the gate cost is exponential

Figure 2: Circuit diagram for $M = 4$. Note that $j \in \Omega_u$ if $j_0 j_1 = u - 1$ (e.g. $j \in \Omega_3$ if $j_0 j_1 = 10$).

A Recursive Approach

- Since $M = 2^m$ for some $m \leq n \in \mathbb{N}$ we can view the partition of Ω as a recursive process, splitting the domain into halves m times

- Associate with each control qubit, g, two operators, $\hat{O}_0^{(g)}$ and $\hat{O}_1^{(g)}$ EACH ACTING ON THE REMAINING QUBITS! (CASCADE)

- Is it possible to construct

$$\hat{O}_u = \prod_{g=1}^{m} \hat{O}^{(g)}??$$

(8)

- DEFINE THE APPROPRIATE O AND U OPERATORS !!!
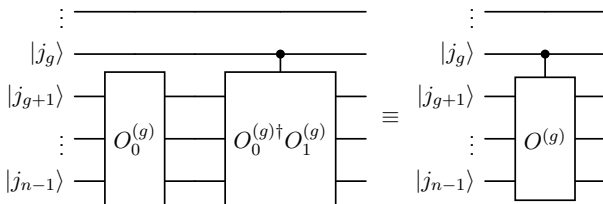
A Recursive Approach

Figure 3: The controlled operator $\hat{O}^{(g)}$ $(0 \leq g \leq m-1)$
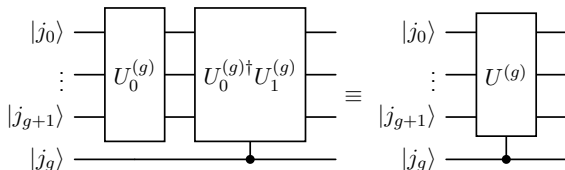

Figure 4: The controlled operator $\hat{U}^{(g)}$ $(1 \leq g \leq m-1)$
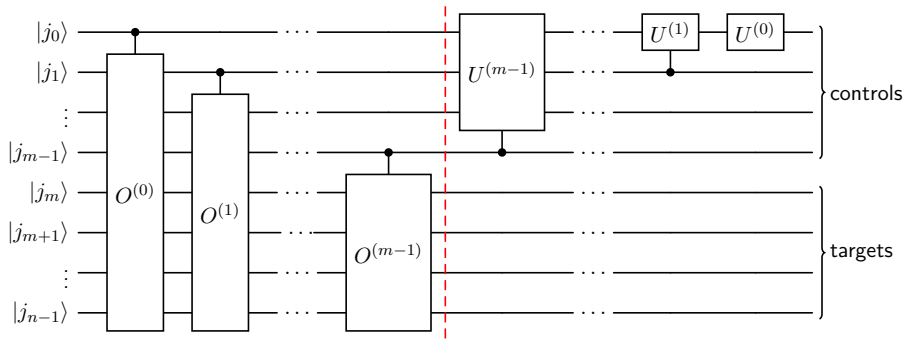
A Recursive Approach

Figure 5: Cascaded controls: $2m - 1$ controlled operations

Comments Sarah

For the other problem, yes this is exactly what I had in mind, this is the right idea. A couple of suggestions:

- I think that you can get rid of the ancillae entirely, and just condition directly on the first few (log M) qubits.

- I agree that you should be able to simplify a bit more by thinking about applying the operators recursively.

- Sadly it will always scale exponentially if we take $M \sim O(2^n)$. It would be helpful to consider breaking the circuit (conceptually, not literally...) into m = log M qubits (which will be the controls) and n-m remaining qubits (the targets). If m $\ll$ n, how does the complexity scale with m and n?

- Finally, how does the complexity of applying the phase in this way compare to the previous method of calculating the phase in an ancilla register, applying the phase, and then uncomputing the result in the ancilla.

Table of Contents

Handover

The code, documentation, slides, and poster are all available on GitHub:

`https://github.com/david-f-amorim/PQC_function_evaluation`

- The source code is found in the directory `pqcprep`
- The slides and poster are found in the directory `slides`
- The documentation is hosted externally here, which is linked on GitHub