PQC Function Evaluation

Carnegie Vacation Scholarship

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Week 4
(22/07/2024 - 26/07/2024)

Aims for the Week

The following aims were set at the last meeting (22/07/2024):

1. Change Input Layer Structure

Improve the connectivity of input layers. Each input qubit should ideally control each target qubit at some point in the network.

2. Fix Parameters

Add the option to keep parameters fixed for each type of network layer.

3. Improve Loss Function

Develop a distance measure taking into account digital encoding. Either incorporate this into weights for an existing loss function or define a new loss function on this basis.

Glossary

Acronym	Meaning
CL	convolutional layer
AA-CL	all-to-all convolutional layer
NN-CL	neighbour-to-neighbour convolutional layer
IL	input layer

Table 1: Acronyms and short-hands used in the following.

Variable	Meaning
n	input register size
m	target register size
L	number of network layers

Table 2: Variables used in the following.

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Changing Input Layer Structure

- Previously, the jth input qubit controlled an operation on the jth target qubit (with wrap-around for $n > m$)
- An optional shift parameter, s, has now been added so that the jth input qubit controls an operation on the $j + s$th target qubit
- This shift parameter is incremented for each successive IL
- The QCNN is padded with additional ILs to ensure that the number of ILs is $\geq m$
- Thus, each input qubit now controls an operation on each target qubit at some point in the QCNN
- Note that ILs still alternate between control states 0 and 1
- FIND A WAY TO COMPARE THIS TO PREVIOUS OUTPUT! GRAPH! WHAT IS THE EFFECT OF THIS??

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Fixing Parameters

- Implemented the option to fix parameters for each layer type
- This means that each instance of a layer type (IL, AA-CL, NN-CL) uses the same set of parameters
- This significantly reduces the number of trainable parameters at large L
- Initial testing shows this to slightly decrease network performance (DO MORE TESTING!; GET A GRAPH?!)
- Surprisingly, reducing the parameter space produces no noticeable speed-up (so-called qiskit primitives, i.e. the sampler, take up roughly 95% of the computational time)

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Preliminary Definitions

- Consider a computational basis state, $|j\rangle$, in a p-qubit register:

$$|j\rangle = \bigotimes_{\alpha=0}^{p-1} |j_\alpha\rangle, \quad |j_\alpha\rangle \in \{|0\rangle, |1\rangle\} \tag{1}$$

- Define

$$j_\alpha \equiv \begin{cases} 0 & \text{if } |j_\alpha\rangle = |0\rangle \\ 1 & \text{if } |j_\alpha\rangle = |1\rangle \end{cases} \tag{2}$$

- Two digitally encoded binary numbers can be associated with $|j\rangle$:

$$j \equiv \sum_{\alpha=0}^{p-1} j_\alpha 2^\alpha \qquad (0 \leq j \leq 2^p - 1), \tag{3}$$

$$j' \equiv \sum_{\alpha=0}^{p-1} j_\alpha 2^{\alpha-p} \qquad (0 \leq j' \leq 1) \tag{4}$$

Preliminary Definitions

- Consider an n-qubit input register and an m-qubit target register, denoted with subscripts i and t, respectively
- A computational basis state of the combined system, $|k\rangle_{i+t}$, can be decomposed into

$$|k\rangle_{i+t} = |j\rangle_i \otimes |l\rangle_t \,, \tag{5}$$

  for computational basis states $|j\rangle_i$, $|l\rangle_t$ of the two registers
- Define

$$\text{input}(|k\rangle_{i+t}) \equiv |j\rangle_i \tag{6}$$

$$\text{target}(|k\rangle_{i+t}) \equiv |l\rangle_t \tag{7}$$

$$\tag{8}$$

- A general state of the two-register system is then

$$|z\rangle = \sum_{k=0}^{2^{n+m}-1} z_k \, |k\rangle_{i+t} \tag{9}$$

Preliminary Definitions

- For training in superposition, the two-register target state is

$$|y\rangle = \sum_{j=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |j\rangle_i \, |\Psi'(j)\rangle_t \equiv \sum_{k=0}^{2^{n+m}-1} y_k \, |k\rangle_{i+t} \, , \tag{10}$$

with $y_k = 0$ if target$(|k\rangle_{i+t})$

Preliminary Definitions

- FIND GOOD NOTATION FOR ALL THIS !!!
- THINK ABOUT THIS ALL MORE DEEPLY! MAYBE HAVE TO SWITCH TO WEIGHTED L1/2 LOSS INSTEAD? (WILL)

Improving WIM

- Recall the definition of WIM (WeIghted Mismatch):

$$\mathsf{WIM}(x, y) = \left| 1 - \sum_{k=0}^{2^{n+m}-1} \tilde{w}_k x_k y_k \right|, \tag{11}$$

  where
  - $|x\rangle = \sum_{k=0}^{2^{n+m}-1} x_k |k\rangle$ is the output state produced by the QCNN,
  - $|y\rangle = \sum_{k=0}^{2^{n+m}-1} y_k |k\rangle$ is the target state,
  - $\tilde{w}_k \in \mathbb{R}_+$ are weighting factors
- .... calculate w_k
- $\tilde{w}$

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Results

Show phase encoding with improved methods show the full waveform [new frame]

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Next Steps

- ...