

PQC Function Evaluation

Weeks 1-3

David Amorim

01/07/2024 - 19/07/2024

Table of Contents

① Background

② Approach: a QCNN

- Convolutional Layers

- Input Layers

- Summary: QCNN Structure

③ Training the QCNN

④ Initial Tests

⑤ Results

- Encoding the Amplitude

- Encoding the Phase

Background

- Hayes 2023¹ presents a scheme to encode a complex vector $\mathbf{h} = \{\tilde{A}_j e^{i\Psi(j)} | 0 \leq j < N\}$ as the state

$$|h\rangle = \frac{1}{|\tilde{A}|} \sum_{j=0}^{2^n-1} \tilde{A}(j) e^{i\Psi(j)} |j\rangle, \quad (1)$$

using $n = \lceil \log_2 N \rceil$ qubits

- This requires operators \hat{U}_A and \hat{U}_Ψ such that

$$\hat{U}_A |0\rangle^{\otimes n} = \frac{1}{|\tilde{A}|} \sum_{j=0}^{2^n-1} \tilde{A}(j) |j\rangle, \quad (2)$$

$$\hat{U}_\Psi |j\rangle = e^{i\Psi(j)} |j\rangle \quad (3)$$

¹<https://arxiv.org/pdf/2306.11073>

Background

- \hat{U}_Ψ is constructed via an operator \hat{Q}_Ψ that performs **function evaluation** in an ancilla register:

$$\hat{Q}_\Psi |j\rangle |0\rangle_a^{\otimes m} = |j\rangle |\Psi'(j)\rangle_a, \quad (4)$$

with $\Psi'(j) \equiv \Psi(j)/2\pi$

- Currently, \hat{Q}_Ψ is implemented using gate-intensive *linear piecewise functions (LPFs)*

Aim

Implement \hat{Q}_Ψ in a gate-efficient way using a parametrised quantum circuit (PQC)

Remark

The n -qubit register containing the $|j\rangle$ and the m -qubit register containing the $|\Psi'(j)\rangle$ will be referred to as the **input register** and **target register**, respectively.

Approach: a QCNN

- A *quantum convolutional neural network* (QCNN) is used to tackle the problem
- A QCNN is a parametrised quantum circuit involving multiple **layers**
- Two types of network layers are implemented:
 - **Convolutional layers (CL)** involve multi-qubit entanglement gates
 - **Input layers (IL)**² involve controlled single-qubit operations on target qubits
- Input qubits only appear as controls throughout the QCNN

²Replacing the conventional QCNN *pooling layers*

Convolutional Layers (CLs)

- Each CL involves the cascaded application of a **two-qubit operator** on the target register
- A general two-qubit operator involves 15 parameters
- To reduce the parameter space, the canonical **three-parameter operator**

$$\mathcal{N}(\alpha, \beta, \gamma) = \exp(i[\alpha X \otimes X + \beta Y \otimes Y + \gamma Z \otimes Z]) \quad (5)$$

is applied, at the cost of restricting the search space

- This can be decomposed³ into 3 CX , 3 R_z , and 2 R_y gates
- A two-parameter real version, $\mathcal{N}_{\mathbb{R}}(\lambda, \mu)$, can be obtained by removing the R_z

³<https://arxiv.org/pdf/quant-ph/0308006>

Convolutional Layers (CLs)

- Two types of convolutional layers are implemented:
 - **Neighbour-to-neighbour / linear CLs**: the \mathcal{N} (or $\mathcal{N}_{\mathbb{R}}$) gate is applied to neighbouring target qubits
 - **All-to-all /quadratic CLs**: the \mathcal{N} (or $\mathcal{N}_{\mathbb{R}}$) gate is applied to all combinations of target qubits
- The \mathcal{N} -gate cost of neighbour-to-neighbour (NN) layers is $\mathcal{O}(m)$ while that of all-to-all (AA) layers is $\mathcal{O}(m^2)$
- Currently, the QCNN uses alternating linear and quadratic CLs

Input Layers (ILs)

- ILs, replacing pooling layers, feed information about the input register into the target register
- An IL involves a sequence of controlled generic single-qubit rotations (*CU3 gates*) on the target qubits, with input qubits as controls
- For an IL producing states with *real* amplitudes, the *CU3* gates are replaced with *CR_y gates*
- Each input qubit controls precisely one *CU3* (or *CR_y* operation), resulting in an $\mathcal{O}(n)$ gate cost (no CX gates!)
- ILs are inserted after every second convolutional layer, alternating between control states 0 and 1

Summary: QCNN Structure

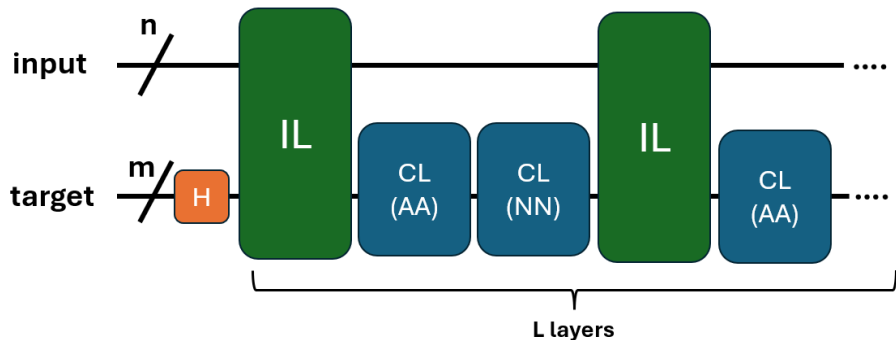


Figure 1: Schematic of QCNN structure

Training the QCNN

- For training, the QCNN is wrapped as a *SamplerQNN* object and connected to PyTorch's **Adam optimiser** via *TorchConnector*
- The optimiser determines improved parameter values for each training run (**epoch**) based on the **loss** between output and target state
- Beyond loss, **mismatch** is an important metric:

$$M = 1 - | \langle \psi_{\text{target}} | \psi_{\text{out}} \rangle | \quad (6)$$

- There are two ways to train the QCNN on input data:⁴
 - 1 Training on **individual states**
 - 2 Training in **superposition**

⁴One can also train the QCNN to produce a target distribution independent of the input register, which is equivalent to constructing \hat{U}_A

Training the QCNN

1. Training on Individual States

- One of the 2^n input states, $|j\rangle$, is randomly chosen each epoch
- The network is taught to transform $|j\rangle |0\rangle \mapsto |j\rangle |\Psi'(j)\rangle$ for each of the states individually

2. Training in Superposition

- The same input state is chosen each epoch
- The network is taught to transform

$$\left(\frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} |j\rangle \right) |0\rangle \mapsto \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} |j\rangle |\Psi'(j)\rangle \quad (7)$$

- By linearity, this teaches the network to transform $|j\rangle |0\rangle \mapsto |j\rangle |\Psi'(j)\rangle$ for each $|j\rangle$

Initial Tests

- Initial tests need to be carried out to inform QCNN design choices regarding:
 - a Number of layers
 - b Number of epochs
 - c Training mode (individually versus in superposition)
 - d Use of \mathcal{N} and $CU3$ versus $\mathcal{N}_{\mathbb{R}}$ and R_y
 - e Choice of loss function
 - f Network structure
- The case $n = m = 2$, $\Psi(x) = x$ (the simplest non-trivial configuration) is an ideal **benchmark problem**
- Unclear, however, how well these findings extrapolate to more general cases

- In the following, a QCNN is applied to implement both \hat{U}_A and \hat{U}_Ψ for the problem studied in Hayes 2023:

$$\tilde{A}(f) = f^{-7/6}, \quad (8)$$

$$\Psi(f) = c_0 + c_1 f + c_2 f^{-1/3} + c_3 f^{-2/3} + c_4 f^{-1} + c_5 f^{-5/3}, \quad (9)$$

with $40 \text{ Hz} \leq f \leq 168 \text{ Hz}$

- In the paper, \hat{U}_A is implemented via a quantum generative adversarial network (**QGAN**) as well as the Grover-Rudolph (**GR**) algorithm while **LPFs** are used for \hat{U}_Ψ
- Hayes 2023 uses $n = 6$ as well as 22 ancilla qubits

Encoding the Amplitude

- The **QCNN** outperforms the **QGAN** and **nearly reaches GR** w.r.t. mismatch
- $L = 3$ with mismatch as loss function and 600 epochs of training was used

Method	CX	Mismatch
QGAN	100	8.6×10^{-3}
GR ⁵	23,796	5.7×10^{-4}
QCNN	72	6.5×10^{-4}

Table 1: Comparison of \hat{U}_A implementations

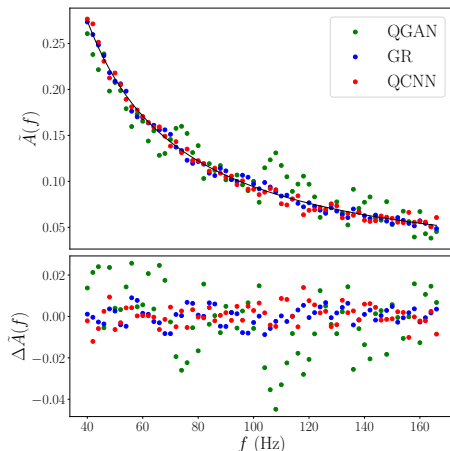


Figure 2: Reconstruction of $\tilde{A}(f)$ from different methods. Target in black.

⁵Hayes 2023 reports a mismatch of 4.1×10^{-4}

Encoding the Amplitude

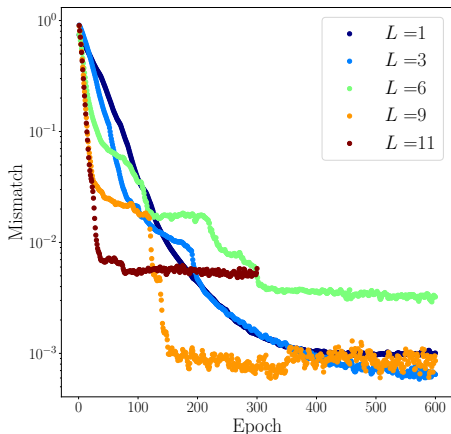


Figure 3: QGAN training for different circuit depths.

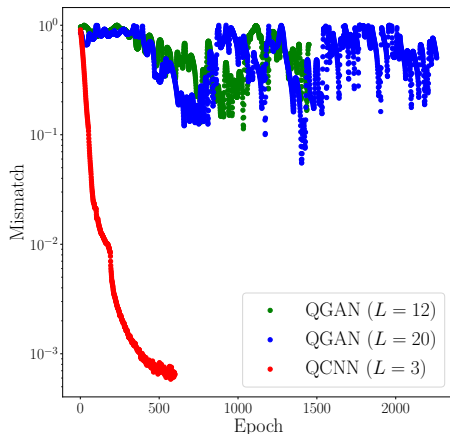


Figure 4: Comparison of QGAN and QCNN training. Note: QGAN results do not match Hayes 2023!

Encoding the Phase

- The implementation of \hat{U}_Ψ is not the only factor affecting the encoding of $\Psi(f)$
- The size, m , of the target register limits the available precision due to rounding to $\sim 2^{-m}$
- A meaningful representation of $\Psi(f)$ requires $m \gtrsim 6$
- In combination with $n = 6$ this gives a state-space of size $\gtrsim 2^{12} = 4,096$ resulting in long training times

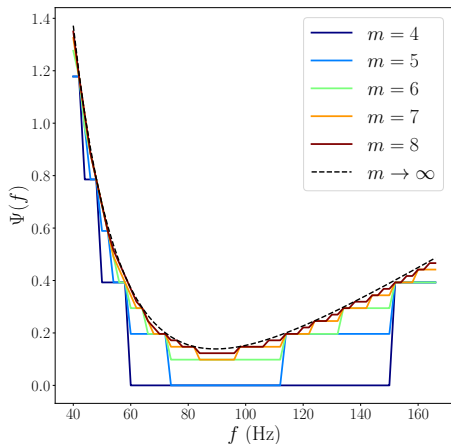


Figure 5: Attainable precision due to rounding for different target register sizes

Encoding the Phase

- ... USE L1 TS r L3 ???
-

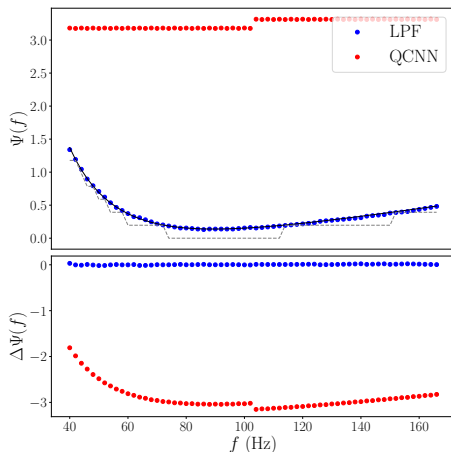


Figure 6: Encoding of $\Psi(f)$ using LPFs versus a QCNN