PQC Function Evaluation

Carnegie Vacation Scholarship

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Week 4
(22/07/2024 - 26/07/2024)

Aims for the Week

The following aims were set at the last meeting (22/07/2024):

1. Change Input Layer Structure

Improve the connectivity of input layers. Each input qubit should ideally control each target qubit at some point in the network.

2. Fix Parameters

Add the option to keep parameters fixed for each type of network layer.

3. Improve Loss Function

Develop a distance measure taking into account digital encoding. Either incorporate this into weights for an existing loss function or define a new loss function on this basis.

Glossary

Acronym	Meaning
CL	convolutional layer
AA-CL	all-to-all convolutional layer
NN-CL	neighbour-to-neighbour convolutional layer
IL	input layer
SAM	sign-adjusted mismatch

Table 1: Acronyms and short-hands used in the following.

Variable	Meaning
n	input register size
m	target register size
L	number of network layers

Table 2: Variables used in the following.

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Changing Input Layer Structure

- Previously, the jth input qubit controlled an operation on the jth target qubit (with wrap-around for $n > m$)
- An optional shift parameter, s, has now been added so that the jth input qubit controls an operation on the $j + s$th target qubit
- This shift parameter is incremented for each successive IL
- The QCNN is padded with additional ILs to ensure that the number of ILs is $\geq m$
- Thus, each input qubit now controls an operation on each target qubit at some point in the QCNN
- Note that ILs still alternate between control states 0 and 1

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Changing Input Layer Structure

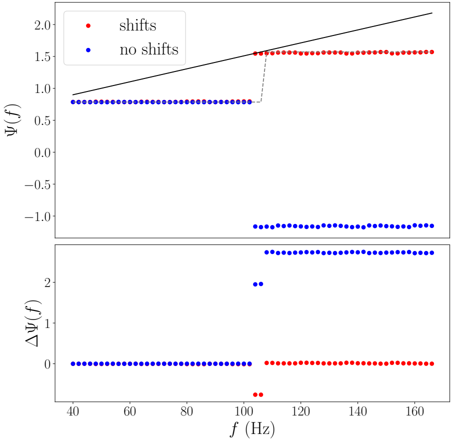

Figure 1: Effects of shifted ILs for $\Psi(f) \sim f$ and $m = 3$ ($L = 6$, 600 epochs, SAM)

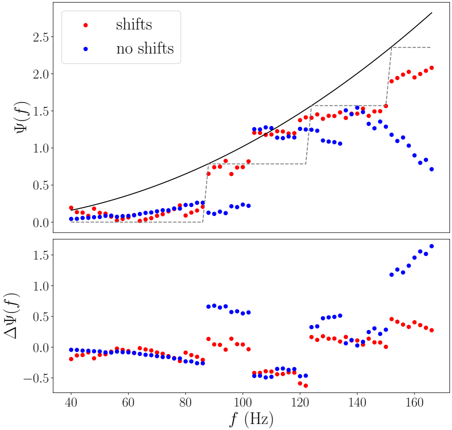

Figure 2: Effects of shifted ILs for $\Psi(f) \sim f^2$ and $m = 3$ ($L = 6$, 600 epochs, SAM)

Changing Input Layer Structure

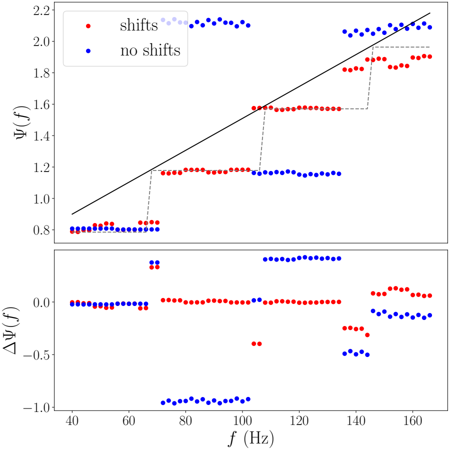

Figure 3: Effects of shifted ILs for $\Psi(f) \sim f$ and $m = 4$ ($L = 6$, 600 epochs, SAM)

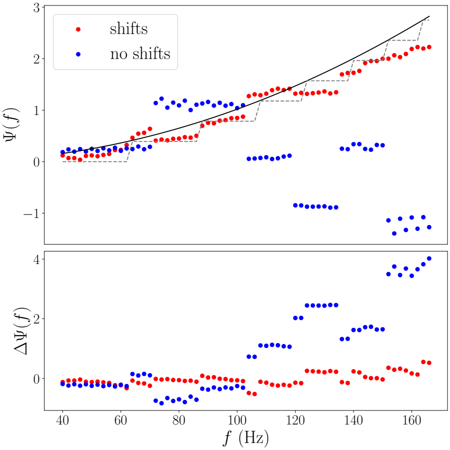

Figure 4: Effects of shifted ILs for $\Psi(f) \sim f^2$ and $m = 4$ ($L = 6$, 600 epochs, SAM)

Changing Input Layer Structure

- The data for 'no shifts' was obtained by setting $s = 0$ for all ILs instead of incrementing s
- This should be equivalent to last week's circuit structure
- However, the 'no shifts' results are significantly worse than the results shown last week (???)
- Thus, the improvements due to the new IL structure are somewhat exaggerated
- Nonetheless, increased IL connectivity clearly leads to improved performance

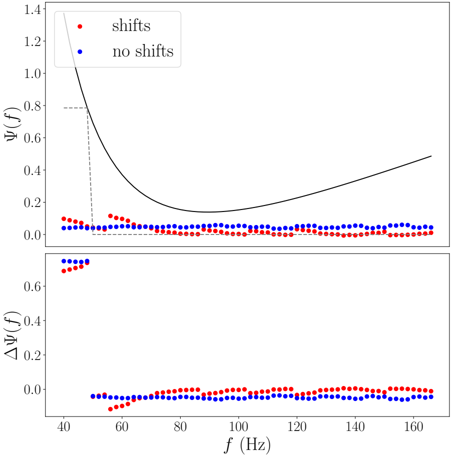

Figure 5: Effects of shifted ILs for $\Psi_{\text{Hayes2023}}$ and $m = 3$ ($L = 6$, 600 epochs, SAM)

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Fixing Parameters

- Implemented the option to fix parameters for each layer type
- This means that each instance of a layer type (IL, AA-CL, NN-CL) uses the same set of parameters
- This significantly reduces the number of trainable parameters at large L
- Surprisingly, reducing the parameter space produces no noticeable speed-up (so-called qiskit primitives, i.e. the sampler, take up roughly 95% of the computational time)

Fixing Parameters

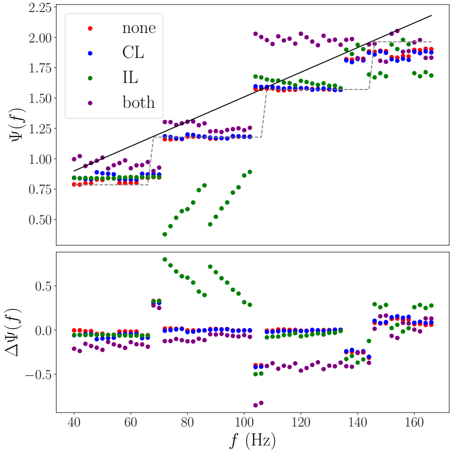

Figure 6: Effects of fixing parameters ILs for $\Psi(f) \sim f$ and $m = 4$ ($L = 6$, 600 epochs, SAM)

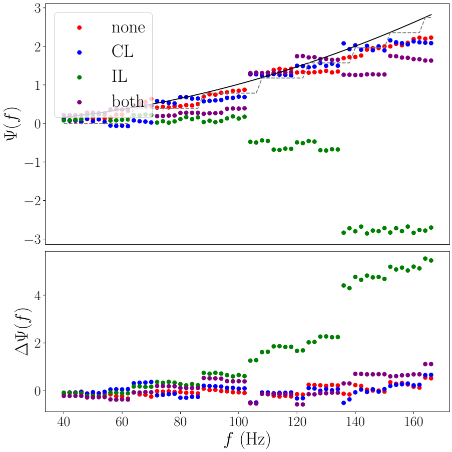

Figure 7: Effects of fixing parameters for $\Psi(f) \sim f^2$ and $m = 4$ ($L = 6$, 600 epochs, SAM)

Fixing Parameters

- Legend for the plots on the previous slide:
  - '*none*' : no parameters fixed
  - '*CL*': only CL parameters fixed
  - '*IL*': only IL parameters fixed
  - '*both*': all parameters fixed
- Evidently, keeping parameters fixed leads to (slightly, for '*CL*', or drastically, for '*IL*' and '*both*') worse performance
- This is likely due to a reduction of the search space
- Note that '*IL*' as well as '*both*' lead to incomplete phase extraction (formalise this concept!) and hence somewhat meaningless results
- Thus, especially taking into account the equivalent computational times, not keeping parameters fixed yields better results
- These results suggest a particular importance of ILs compared to CLs

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Preliminary Definitions

- Consider a computational basis state, $|j\rangle$, in a p-qubit register:

$$|j\rangle = \bigotimes_{\alpha=0}^{p-1} |j_\alpha\rangle\,, \qquad |j_\alpha\rangle \in \{|0\rangle\,, |1\rangle\} \tag{1}$$

- Define

$$j_\alpha \equiv \begin{cases} 0 & \text{if } |j_\alpha\rangle = |0\rangle \\ 1 & \text{if } |j_\alpha\rangle = |1\rangle \end{cases} \tag{2}$$

- Two digitally encoded binary numbers can be associated with $|j\rangle$:

$$j \equiv \sum_{\alpha=0}^{p-1} j_\alpha 2^\alpha \qquad (0 \le j \le 2^p - 1), \tag{3}$$

$$j' \equiv \sum_{\alpha=0}^{p-1} j_\alpha 2^{\alpha-p} \qquad (0 \le j' \le 1) \tag{4}$$

Preliminary Definitions

- Consider an n-qubit input register and an m-qubit target register, denoted with subscripts i and t, respectively
- A computational basis state of the combined system, $|k\rangle_{i+t}$, can be decomposed into

$$|k\rangle_{i+t} = |j\rangle_i \otimes |l\rangle_t \,, \tag{5}$$

  for computational basis states $|j\rangle_i$, $|l\rangle_t$ of the two registers
- Define

$$\text{input}(|k\rangle_{i+t}) \equiv |j\rangle_i \tag{6}$$

$$\text{target}(|k\rangle_{i+t}) \equiv |l\rangle_t \tag{7}$$

$$\tag{8}$$

- A general state of the two-register system is then

$$|z\rangle = \sum_{k=0}^{2^{n+m}-1} z_k \, |k\rangle_{i+t} \tag{9}$$

Preliminary Definitions

- For training in superposition, the two-register target state is

$$|y\rangle = \sum_{j=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |j\rangle_i |\Psi'(j)\rangle_t \equiv \sum_{k=0}^{2^{n+m}-1} y_k |k\rangle_{i+t}, \tag{10}$$

with $y_k = 0$ if target($|k\rangle_{i+t}$)

Preliminary Definitions

- FIND GOOD NOTATION FOR ALL THIS !!!
- THINK ABOUT THIS ALL MORE DEEPLY! MAYBE HAVE TO SWITCH TO WEIGHTED L1/2 LOSS INSTEAD? (WILL)

Improving WIM

- Recall the definition of WIM (WeIghted Mismatch):

$$\text{WIM}(x, y) = \left| 1 - \sum_{k=0}^{2^{n+m}-1} \tilde{w}_k x_k y_k \right|, \tag{11}$$

  where
  - $|x\rangle = \sum_{k=0}^{2^{n+m}-1} x_k |k\rangle$ is the output state produced by the QCNN,
  - $|y\rangle = \sum_{k=0}^{2^{n+m}-1} y_k |k\rangle$ is the target state,
  - $\tilde{w}_k \in \mathbb{R}_+$ are weighting factors
- .... calculate w_k
- $\tilde{w}$

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Results

Show phase encoding with improved methods show the full waveform [new frame]

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Next Steps

- ...