

# PQC Function Evaluation

## Carnegie Vacation Scholarship

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# Aims

- Continue work on WIM (develop a distance measure taking into account  $\Psi$ , e.g.  $w_j \sim L_\Psi(x, y) = \sum_j 2^{j-m} |x_j - y_j|$ )
- Change IL structure (all-to-all...)
- Keep parameters fixed for each layer

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① Aims

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# Preliminary Definitions

- Consider a **computational basis state**,  $|j\rangle$ , in a  $p$ -qubit register:

$$|j\rangle = \bigotimes_{\alpha=0}^{p-1} |j_{\alpha}\rangle, \quad |j_{\alpha}\rangle \in \{|0\rangle, |1\rangle\} \quad (1)$$

- Define

$$j_{\alpha} \equiv \begin{cases} 0 & \text{if } |j_{\alpha}\rangle = |0\rangle \\ 1 & \text{if } |j_{\alpha}\rangle = |1\rangle \end{cases} \quad (2)$$

- Two **digitally encoded binary numbers** can be associated with  $|j\rangle$ :

$$j \equiv \sum_{\alpha=0}^{p-1} j_{\alpha} 2^{\alpha} \quad (0 \leq j \leq 2^p - 1), \quad (3)$$

$$j' \equiv \sum_{\alpha=0}^{p-1} j_{\alpha} 2^{\alpha-p} \quad (0 \leq j' \leq 1) \quad (4)$$

# Preliminary Definitions

- Consider an  $n$ -qubit **input register** and an  $m$ -qubit **target register**, denoted with subscripts  $i$  and  $t$ , respectively
- A computational basis state of the combined system,  $|k\rangle_{i+t}$ , can be decomposed into

$$|k\rangle_{i+t} = |j\rangle_i \otimes |l\rangle_t, \quad (5)$$

for computational basis states  $|j\rangle_i$ ,  $|l\rangle_t$  of the two registers

- Define

$$\text{input}(|k\rangle_{i+t}) \equiv |j\rangle_i \quad (6)$$

$$\text{target}(|k\rangle_{i+t}) \equiv |l\rangle_t \quad (7)$$

$$(8)$$

- A **general state of the two-register system** is then

$$|z\rangle = \sum_{k=0}^{2^{n+m}-1} z_k |k\rangle_{i+t} \quad (9)$$

# Preliminary Definitions

- For training in superposition, the two-register target state is

$$|y\rangle = \sum_{j=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |j\rangle_i |\Psi'(j)\rangle_t \equiv \sum_{k=0}^{2^{n+m}-1} y_k |k\rangle_{i+t}, \quad (10)$$

with  $y_k = 0$  if  $\text{target}(|k\rangle_{i+t})$

# Preliminary Definitions

- FIND GOOD NOTATION FOR ALL THIS !!!



# Improving WIM

- Recall the definition of **WIM** (**W**eighted **M**ismatch):

$$\text{WIM}(x, y) = \left| 1 - \sum_{k=0}^{2^{n+m}-1} \tilde{w}_k x_k y_k \right|, \quad (11)$$

where

- $|x\rangle = \sum_{k=0}^{2^{n+m}-1} x_k |k\rangle$  is the output state produced by the QCNN,
- $|y\rangle = \sum_{k=0}^{2^{n+m}-1} y_k |k\rangle$  is the target state,
- $\tilde{w}_k \in \mathbb{R}_+$  are weighting factors
- .... calculate  $w_k$
- $\tilde{w}$