Field Properties *What are the five fundamental properties that define a field?*	Supremum Definition *What does it mean for* $\beta = \sup S$*?*
Mathematical Induction *State the Principle of Mathematical Induction.*	Sequence Convergence *Define convergence of a sequence* $\{s_n\}$.
Limit Superior and Inferior *Define* $\limsup_{n\to\infty} s_n$ *and* $\liminf_{n\to\infty} s_n$.	Function Limit *Define* $\lim_{x\to x_0} f(x) = L$.
Continuity *Define continuity of* f *at* x_0.	Intermediate Value Theorem *State the Intermediate Value Theorem.*
Derivative Definition *Define the derivative* $f'(x_0)$.	Mean Value Theorem *State the Mean Value Theorem.*

$\beta = \sup S$ if and only if: (1) $x \leq \beta$ for all $x \in S$ (β is an upper bound) (2) If $\gamma < \beta$, then $\exists x \in S$ such that $x > \gamma$ (no smaller upper bound exists)	(A) $a + b = b + a$ and $ab = ba$ (commutative laws) (B) $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$ (associative laws) (C) $a(b + c) = ab + ac$ (distributive law) (D) $\exists 0, 1 \in \mathbb{R}$ such that $a + 0 = a$ and $a \cdot 1 = a$ for all a (E) For each a, $\exists(-a)$ such that $a + (-a) = 0$, and if $a \neq 0$, $\exists a^{-1}$ such that $a \cdot a^{-1} = 1$
$\lim_{n\to\infty} s_n = s$ if and only if: $\forall \epsilon > 0, \exists N \in \mathbb{N}$ such that $\|s_n - s\| < \epsilon$ whenever $n \geq N$	Let $P_1, P_2, \ldots, P_n, \ldots$ be propositions. If: (a) P_1 is true (b) For each positive integer n, $P_n \implies P_{n+1}$ Then P_n is true for each positive integer n.
$\lim_{x\to x_0} f(x) = L$ if and only if: $\forall \epsilon > 0, \exists \delta > 0$ such that $0 < \|x - x_0\| < \delta \implies \|f(x) - L\| < \epsilon$	$M_k = \sup_{n\geq k} s_n$, then $\limsup_{n\to\infty} s_n = \lim_{k\to\infty} M_k$ $m_k = \inf_{n\geq k} s_n$, then $\liminf_{n\to\infty} s_n = \lim_{k\to\infty} m_k$
If f is continuous on $[a, b]$, $f(a) \neq f(b)$, and μ is between $f(a)$ and $f(b)$, then $\exists c \in (a, b)$ such that $f(c) = \mu$.	f is continuous at x_0 if: $\lim_{x\to x_0} f(x) = f(x_0)$ Equivalently: $\forall \epsilon > 0, \exists \delta > 0$ such that $\|x - x_0\| < \delta \implies \|f(x) - f(x_0)\| < \epsilon$
If f is continuous on $[a, b]$ and differentiable on (a, b), then $\exists c \in (a, b)$ such that: $f'(c) = \frac{f(b)-f(a)}{b-a}$	$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$ provided this limit exists.

SERIES CONVERGENCE *Define convergence of the series $\sum_{n=1}^{\infty} a_n$.*	ABSOLUTE CONVERGENCE *Define absolute and conditional convergence.*
UNIFORM CONVERGENCE *Define uniform convergence of $\{f_n\}$ to f on S.*	POWER SERIES *Define a power series and its radius of convergence.*
TAYLOR SERIES *Define the Taylor series of f about x_0.*	CAUCHY CONVERGENCE CRITERION *State the Cauchy convergence criterion for sequences.*
MONOTONIC SEQUENCES *State the convergence theorem for bounded monotonic sequences.*	COMPLETENESS AXIOM *State the Completeness Axiom for $\mathbb{R}$.*
COMPARISON TEST *State the Comparison Test for series.*	RATIO TEST *State the Ratio Test.*

$\sum a_n$ converges absolutely if $\sum |a_n|$ converges. $\sum a_n$ converges conditionally if $\sum a_n$ converges but $\sum |a_n|$ diverges.

$\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n\to\infty} S_n$ exists and is finite, where $S_n = \sum_{k=1}^{n} a_k$ are the partial sums.

A power series is $\sum_{n=0}^{\infty} a_n(x-x_0)^n$. Radius of convergence: $\frac{1}{R} = \limsup_{n\to\infty} |a_n|^{1/n}$

$\{f_n\}$ converges uniformly to f on S if: $\forall \epsilon > 0, \exists N$ such that $n \geq N \implies |f_n(x) - f(x)| < \epsilon$ for all $x \in S$

$\{s_n\}$ converges if and only if: $\forall \epsilon > 0, \exists N$ such that $m, n \geq N \implies |s_m - s_n| < \epsilon$

$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$ where $f^{(n)}(x_0)$ is the n-th derivative of f evaluated at x_0.

Every nonempty set $S \subset \mathbb{R}$ that is bounded above has a supremum in $\mathbb{R}$. Equivalently: $\sup S \in \mathbb{R}$ whenever $S \neq \emptyset$ and S is bounded above.

If $\{s_n\}$ is increasing and bounded above, then $\lim_{n\to\infty} s_n = \sup\{s_n\}$. If $\{s_n\}$ is decreasing and bounded below, then $\lim_{n\to\infty} s_n = \inf\{s_n\}$.

Let $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$. - If $L < 1$, then $\sum a_n$ converges absolutely - If $L > 1$, then $\sum a_n$ diverges - If $L = 1$, the test is inconclusive

If $0 \leq a_n \leq b_n$ for all $n \geq N$, then: - If $\sum b_n$ converges, then $\sum a_n$ converges - If $\sum a_n$ diverges, then $\sum b_n$ diverges

HEINE-BOREL THEOREM *State the Heine-Borel Theorem.*	BOLZANO-WEIERSTRASS THEOREM *State the Bolzano-Weierstrass Theorem.*
EXTENDED REAL NUMBERS *Define the extended real number system* $\overline{\mathbb{R}}$.	INDETERMINATE FORMS *List the seven indeterminate forms.*
OPEN AND CLOSED SETS *Define open and closed sets in* $\mathbb{R}$.	L'HÔPITAL'S RULE *State L'Hôpital's Rule.*
ONE-SIDED LIMITS *Define left-hand and right-hand limits.*	ONE-SIDED DERIVATIVES *Define left-hand and right-hand derivatives.*
ROLLE'S THEOREM *State Rolle's Theorem.*	TAYLOR'S THEOREM *State Taylor's Theorem with Lagrange remainder.*

Every bounded sequence $\{s_n\}$ in $\mathbb{R}$ has a convergent subsequence.	A subset $K \subset \mathbb{R}$ is compact if and only if K is closed and bounded. Equivalently: Every open cover of K has a finite subcover.
$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, $\infty - \infty$, 0^0, 1^∞, ∞^0	$\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$ With arithmetic: $a + \infty = \infty$, $a \cdot \infty = \infty$ (if $a > 0$), $\frac{a}{\infty} = 0$ Undefined: $\infty - \infty$, $0 \cdot \infty$, $\frac{\infty}{\infty}$, $\frac{0}{0}$
If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$, and $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists, then: $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$	U is open if $\forall x \in U, \exists \delta > 0$ such that $(x-\delta, x+\delta) \subset U$ F is closed if $\mathbb{R} \setminus F$ is open Equivalently: F is closed if it contains all its limit points
$f'_-(x_0) = \lim_{h\to 0^-} \frac{f(x_0+h)-f(x_0)}{h}$ $f'_+(x_0) = \lim_{h\to 0^+} \frac{f(x_0+h)-f(x_0)}{h}$	$\lim_{x\to x_0^-} f(x) = L$ if $\forall \epsilon > 0, \exists \delta > 0$ such that $x_0 - \delta < x < x_0 \implies \lvert f(x) - L\rvert < \epsilon$ $\lim_{x\to x_0^+} f(x) = L$ if $\forall \epsilon > 0, \exists \delta > 0$ such that $x_0 < x < x_0 + \delta \implies \lvert f(x) - L\rvert < \epsilon$
If f is $(n+1)$ times differentiable on (a,b) containing x_0, then: $f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$ for some ξ between x_0 and x.	If f is continuous on $[a,b]$, differentiable on (a,b), and $f(a) = f(b)$, then $\exists c \in (a,b)$ such that $f'(c) = 0$.

TAYLOR POLYNOMIALS *Define the n-th Taylor polynomial of f about x_0.*	UNIFORM CONTINUITY *Define uniform continuity of f on S.*
SUBSEQUENCES *Define a subsequence of $\{s_n\}$.*	WEIERSTRASS TEST *State the Weierstrass M-Test.*
DIRICHLET'S TEST *State Dirichlet's Test for series.*	ABEL'S THEOREM *State Abel's Theorem for power series.*
INTEGRATION OF POWER SERIES *State the theorem on term-by-term integration of power series.*	DIFFERENTIATION OF POWER SERIES *State the theorem on term-by-term differentiation of power series.*
REARRANGEMENT OF SERIES *State Riemann's Rearrangement Theorem.*	INTEGRAL TEST *State the Integral Test.*

f is uniformly continuous on S if: $\forall \epsilon > 0, \exists \delta > 0$ such that $x, y \in S$ and $|x-y| < \delta \implies |f(x) - f(y)| < \epsilon$

$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$

If $|f_n(x)| \leq M_n$ for all $x \in S$ and $\sum M_n$ converges, then $\sum f_n$ converges absolutely and uniformly on S.

$\{s_{n_k}\}$ is a subsequence of $\{s_n\}$ if $n_1 < n_2 < n_3 < \cdots$ are positive integers. If $\lim_{n\to\infty} s_n = s$, then $\lim_{k\to\infty} s_{n_k} = s$.

If $\sum a_n r^n$ converges for some $r > 0$, then $\sum a_n x^n$ converges uniformly on $[0, r]$.

If $\{a_n\}$ is monotonic with $\lim_{n\to\infty} a_n = 0$ and the partial sums $\sum_{k=1}^{n} b_k$ are bounded, then $\sum a_n b_n$ converges.

If $f(x) = \sum_{n=0}^{\infty} a_n(x - x_0)^n$ has radius of convergence $R > 0$, then:
$f'(x) = \sum_{n=1}^{\infty} n a_n (x - x_0)^{n-1}$ for $|x - x_0| < R$

If $\sum a_n(x - x_0)^n$ has radius of convergence $R > 0$, then:
$\int_a^b \sum_{n=0}^{\infty} a_n(x - x_0)^n dx = \sum_{n=0}^{\infty} a_n \int_a^b (x - x_0)^n dx$ for $[a, b] \subset (x_0 - R, x_0 + R)$

If f is positive, continuous, and decreasing on $[1, \infty)$, then: $\sum_{n=1}^{\infty} f(n)$ and $\int_1^{\infty} f(x)dx$ both converge or both diverge.

If $\sum a_n$ converges conditionally, then for any $L \in \mathbb{R} \cup \{\pm\infty\}$, there exists a rearrangement $\sum a_{\sigma(n)}$ that converges to L. If $\sum a_n$ converges absolutely, then every rearrangement converges to the same sum.

ROOT TEST *State the Root Test (Cauchy's Test).*	RAABE'S TEST *State Raabe's Test.*
COMPACTNESS *Define compactness in* $\mathbb{R}$.	POINTWISE VS UNIFORM CONVERGENCE *State the key difference between pointwise and uniform convergence.*

Let $L = \lim_{n\to\infty} n\left(1 - \left|\frac{a_{n+1}}{a_n}\right|\right)$. - If $L > 1$, then $\sum a_n$ converges absolutely - If $L < 1$, then $\sum a_n$ diverges - If $L = 1$, the test is inconclusive

Let $L = \limsup_{n\to\infty} |a_n|^{1/n}$. - If $L < 1$, then $\sum a_n$ converges absolutely - If $L > 1$, then $\sum a_n$ diverges - If $L = 1$, the test is inconclusive

Pointwise: $\forall x \in S, \forall \epsilon > 0, \exists N(x,\epsilon)$ such that $n \geq N \implies |f_n(x) - f(x)| < \epsilon$ Uniform: $\forall \epsilon > 0, \exists N(\epsilon)$ such that $n \geq N \implies |f_n(x) - f(x)| < \epsilon$ for all $x \in S$

$K \subset \mathbb{R}$ is compact if every open cover $\{U_\alpha\}$ of K has a finite subcover. Equivalently in $\mathbb{R}$: K is compact $\iff$ K is closed and bounded.