Peckness of Edge Posets

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Definition of the Edge Poset

Definition

For P a finite graded poset, it's *edge poset* $\mathcal{E}(P)$ is the finite graded poset defined as follows.

- Elements of $\mathcal{E}(P)$ are ordered pairs $(x, y) \in P \times P$ where $x \lessdot y$
- Define $(x, y) \lessdot_{\mathcal{E}} (x', y')$ if $x \lessdot_P x'$ and $y \lessdot_P y'$
- Define $\leq_{\mathcal{E}}$ to be the transitive closure of $\lessdot_{\mathcal{E}}$
- Define $\mathrm{rk}_{\mathcal{E}}(x, y) = \mathrm{rk}_P(x)$.

A Conjecture on the Peckness of Edge Posets

Definition

The *boolean algebra of rank n* is the poset whose elements are subsets of $[n]$ with order given by containment, i.e. for $A, B \in B_n$, $A \leq B$ if $A \subseteq B$.

Conjecture (Hemminger, Landesman, and Yao 2014)

Let $G \subseteq \mathrm{Aut}(B_n)$. *Then* $\mathcal{E}(B_n/G)$ *is Peck.*

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Statement of Theorem

Definition

A group action of G on P is *cover transitive* if whenever $x, y, z \in P$ such that $x \lessdot z$, $y \lessdot z$, and $y \in Gx$, there exists some $g \in \mathrm{Stab}_G(z)$ such that $g \cdot x = y$.

Theorem (Hemminger, Landesman, and Yao 2014)

If a group action of G on B_n is cover transitive, then $\mathcal{E}(B_n/G)$ is Peck.

Definition

Given a group action of G on P, we define a group action of G on $\mathcal{E}(P)$ by letting $g \cdot (x, y) = (g \cdot x, g \cdot y)$ for all $g \in G$.

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Proposition

The map $q \colon \mathcal{E}(P)/G \to \mathcal{E}(P/G)$ defined by $q(G(x, y)) = (Gx, Gy)$ is a surjective morphism. Furthermore, q is also injective if and only if the action of G on P is cover transitive.

Lemma

If $f : P \to Q$ is a bijective morphism and P is Peck then Q is Peck.

It then suffices to show that $\mathcal{E}(P)/G$ is Peck. The following theorem will be useful.

Theorem (Stanley, 1984)

If P is unitary Peck and $G \subseteq \mathrm{Aut}(P)$, then P/G is Peck.