

LIST OF TODOS

It's not immediately obvious which relations analogous to the original R3 relations we want to choose. Here we try out several possible relations, in each case trying to prove an analogous lemma to Lemma 4.1 in Barot and Marsh's paper.

1. ATTEMPTS

1.1. Here we test out the set of relations R3' where if $i_{a_0}, i_{a_1}, \dots, i_{a_{d-1}}, i_{a_0}$ is a chordless cycle with all weights 1, then we add in the relation

$$s_0 s_1^{-1} s_2 s_3^{-1} \dots s_{d-2}^{(-1)^{d-2}} s_{d-1} s_{d-2}^{(-1)^{d-1}} \dots s_4^{-1} s_3 s_2^{-1} s_1 = s_1^{-1} s_2 s_3^{-1} \dots s_{d-2}^{(-1)^{d-2}} s_{d-1} s_{d-2}^{(-1)^{d-1}} \dots s_4^{-1} s_3 s_2^{-1} s_1 s_0$$

We then have

$$\begin{aligned} & s_{d-1} s_0^{-1} s_1 s_2^{-1} s_3 \dots s_{d-3}^{(-1)^{d-2}} s_{d-2} s_{d-3}^{(-1)^{d-3}} \dots s_4 s_3^{-1} s_2 s_1^{-1} s_0 \\ &= s_0^{-1} s_0 s_{d-1} s_0^{-1} s_1 s_2^{-1} s_3 \dots s_{d-3}^{(-1)^{d-2}} s_{d-2} s_{d-3}^{(-1)^{d-3}} \dots s_4 s_3^{-1} s_2 s_1^{-1} s_{d-1} s_{d-1}^{-1} s_0 \\ &= s_0^{-1} s_{d-1}^{-1} s_0 s_{d-1} s_1 s_2^{-1} s_3 \dots s_{d-3}^{(-1)^{d-2}} s_{d-2} s_{d-3}^{(-1)^{d-3}} \dots s_4 s_3^{-1} s_2 s_1^{-1} s_{d-1} s_{d-1}^{-1} s_0 \end{aligned}$$

...so I don't think this direction is going to work from here, simply because I don't see a way to flip all of the generators in the middle to their inverses.