List of Todos

**Lemma 0.1.** *[Proposition 5.2 Analog] The elements $t_i$, for $i$ a vertex of $\Gamma$, satisfy the relations $(R2)$ and $(R3)$.*

After Lemma **??** we have left to check the relations $(R2)$ when both $i$ and $j$ are connected to $k$ and the relations $(R3)$. Beginning with the relations $(R2)$, and following cases a-f from Corollary 2.3 in Barot and Marsh:

a) i)

$$t_i t_j = s_k s_i s_k^{-1} s_k s_j s_k^{-1} = s_k s_i s_j s_k^{-1} = s_k s_j s_i s_k^{-1} = t_j t_i$$

ii)

$$t_i t_j = s_i s_j = s_j s_i = t_j t_i$$

b) i)

$$\begin{aligned} t_i t_j t_i &= s_k s_i s_k^{-1} s_j s_k s_i s_k^{-1} \\ &= s_k s_i s_j s_k s_j^{-1} s_i s_k^{-1} \\ &= s_k s_j s_i s_k s_i s_j^{-1} s_k^{-1} \\ &= s_k s_j s_k s_i s_k s_j^{-1} s_k^{-1} \\ &= s_j s_k s_j s_i s_j^{-1} s_k^{-1} s_j \\ &= s_j s_k s_i s_k^{-1} s_j \\ &= t_i t_j t_i \end{aligned}$$

$$\begin{aligned} t_j t_k^{-1} t_i t_k &= s_j s_k^{-1} s_k s_i s_k^{-1} s_k \\ &= s_j s_i \\ &= s_i s_j \\ &= t_k^{-1} t_i t_k t_j \end{aligned}$$

ii)

$$t_i t_j = s_i s_k s_j s_k^{-1} = s_i s_j^{-1} s_k s_j = s_j^{-1} s_k s_j s_i = s_k s_j s_k^{-1} s_i = t_j t_i$$

c) i)

$$t_i t_j = s_k s_i s_k^{-1} s_k s_j s_k^{-1} = s_k s_i s_j s_k^{-1} = s_k s_j s_i s_k^{-1} = t_j t_i$$

ii)

$$t_i t_j = s_i s_j = s_j s_i = t_j t_i$$

d) i)

$$\begin{aligned} t_it_jt_it_jt_i^{-1}t_j^{-1}t_i^{-1}t_j^{-1} &= s_ks_is_k^{-1}s_js_ks_is_k^{-1}s_js_ks_i^{-1}s_k^{-1}s_j^{-1}s_ks_i^{-1}s_k^{-1}s_j^{-1} \\ &= s_ks_is_k^{-1}s_js_ks_is_js_ks_j^{-1}s_i^{-1}s_js_k^{-1}s_j^{-1}s_i^{-1}s_k^{-1}s_j^{-1} \\ &= s_ks_is_k^{-1}s_ks_js_ks_is_ks_i^{-1}s_k^{-1}s_j^{-1}s_i^{-1}s_k^{-1}s_j^{-1} \\ &= s_ks_js_is_ks_is_ks_i^{-1}s_k^{-1}s_i^{-1}s_k^{-1}s_j^{-1}s_k^{-1} \\ &= e \end{aligned}$$

We also have

$$t_jt_k^{-1}t_it_k = s_js_k^{-1}s_ks_is_k^{-1}s_k = s_is_j = t_k^{-1}t_it_kt_j$$

ii)

$$\begin{aligned} t_it_j &= s_is_ks_js_k^{-1} \\ &= s_is_j^{-1}s_ks_j \\ &= s_j^{-1}s_ks_js_i \\ &= s_ks_js_k^{-1}s_i \\ &= t_jt_i \end{aligned}$$

e) i)

$$\begin{aligned} t_it_jt_it_jt_i^{-1}t_j^{-1}t_i^{-1}t_j^{-1} &= s_ks_is_k^{-1}s_js_ks_is_k^{-1}s_js_ks_i^{-1}s_k^{-1}s_j^{-1}s_ks_i^{-1}s_k^{-1}s_j^{-1} \\ &= s_i^{-1}s_ks_is_js_i^{-1}s_ks_is_js_i^{-1}s_k^{-1}s_is_j^{-1}s_i^{-1}s_k^{-1}s_is_j^{-1} \\ &= s_i^{-1}s_ks_js_ks_js_k^{-1}s_j^{-1}s_k^{-1}s_j^{-1}s_i \\ &= e \end{aligned}$$

We also have

$$t_jt_k^{-1}t_it_k = s_js_k^{-1}s_ks_is_k^{-1}s_k = s_js_i = s_is_j = t_it_j$$

ii)

$$\begin{aligned} s_k^{-1}t_it_jt_i^{-1}t_j^{-1}s_k &= s_k^{-1}s_is_ks_js_k^{-1}s_i^{-1}s_ks_j^{-1} \\ &= s_is_ks_i^{-1}s_js_is_k^{-1}s_i^{-1}s_j^{-1} \\ &= e \end{aligned}$$

f) i)

$$\begin{aligned} s_k^{-1}t_it_jt_it_j^{-1}t_i^{-1}t_j^{-1} &= s_is_k^{-1}s_js_ks_is_k^{-1}s_j^{-1}s_ks_i^{-1}s_k^{-1}s_j^{-1}s_k \\ &= e \end{aligned}$$

$$
\begin{aligned}
t_i t_j^{-1} t_k t_j t_i^{-1} t_j^{-1} s_k^{-1} t_j &= s_k s_i s_k^{-1} s_j^{-1} s_k s_j s_k s_i^{-1} s_k^{-1} s_j^{-1} s_k^{-1} s_j \\
&= s_k s_i s_j s_k s_j^{-1} s_k^{-1} s_k s_i^{-1} s_k^{-1} s_j^{-1} s_k^{-1} s_j \\
&= s_k s_i s_j s_k s_j^{-1} s_i^{-1} s_k^{-1} s_j^{-1} s_k^{-1} s_j \\
&= s_k s_i s_j s_k s_j^{-1} s_i^{-1} s_j s_k^{-1} s_j^{-1} s_k^{-1} \\
&= s_k s_j s_k s_j^{-1} s_i s_i^{-1} s_j s_k^{-1} s_j^{-1} s_k \\
&= e
\end{aligned}
$$

ii) This follows from part (i) by symmetry

,