Kinds of Ensembles

Tested on apple quality dataset

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Apple quality dataset

Variables

- `Size`
- `Weight`
- `Sweetness`
- `Crunchiness`
- `Juiciness`
- `Ripeness`
- `Acidity`

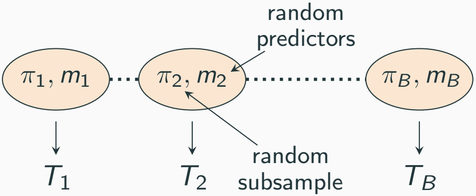
Binary classification task

Class distribution: 0.49 – 0.51

Methods

- kNN, Decision tree, Logistic regression
- Random forest
- AdaBoost
- Super learner

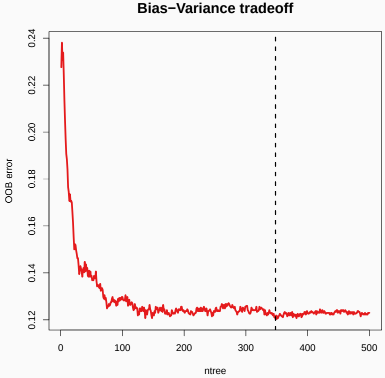
Random forest



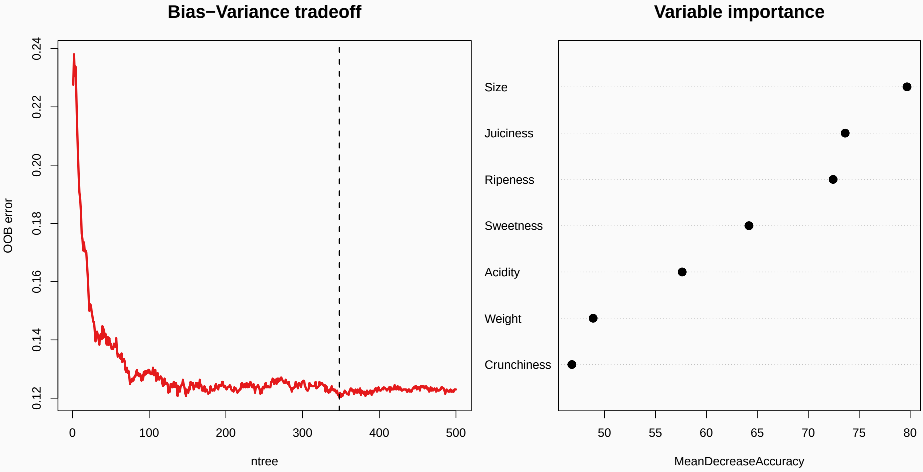

```
randomForest(x, y,
  importance=TRUE,
  ntree=500,
  ntree=B.oob ← B*)
```

$$G(x) = \arg\max_k \sum_{b=1}^{B} \mathbb{I}(T_b(x) = k)$$

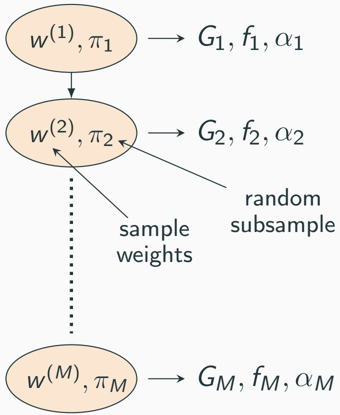
$$\mathtt{mtry} = \lfloor \sqrt{p} \rfloor$$



Random forest tuning



AdaBoost algorithm



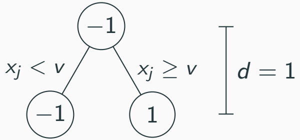
$$f_m(x) = f_{m-1}(x) + \lambda \alpha_m G_m(x)$$

$$G(x) = \text{sign}(f_M(x))$$

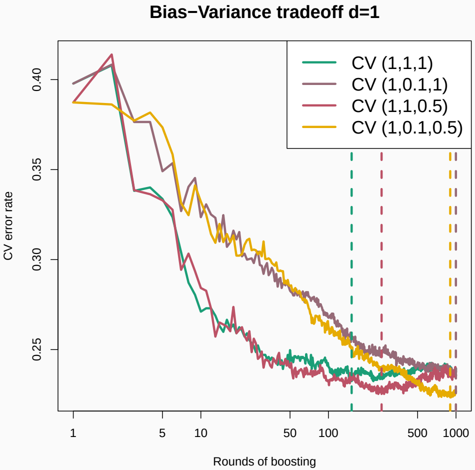
$$L(y, f(x)) = \exp(-yf(x))$$

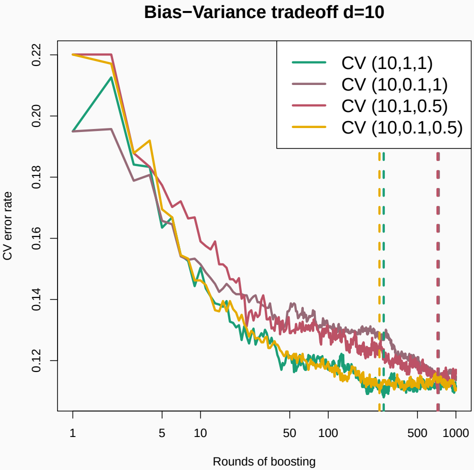
Encoding $\mathcal{Y} \in \{-1, 1\}$

```
ada::ada(x, y,
  loss="exponential",
  type="discrete",
  iter ← M*, nu ← λ*,
  bag.frac ← π*,
  control=base.learner)
```

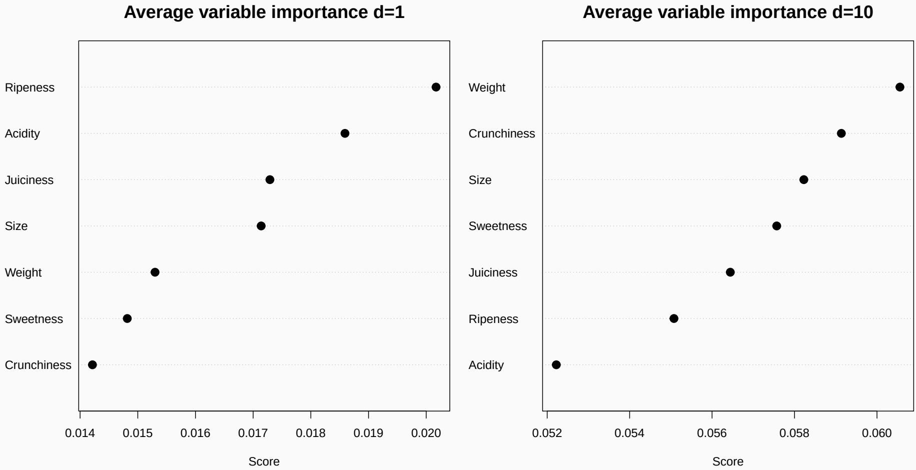


AdaBoost tuning





AdaBoost variable importance



Super Learner flow diagram

$$\hat{\alpha} = \arg\min_{\alpha} \sum_{i=1}^{N} L(Y_i, m(z_i|\alpha))$$

$$m(z|\alpha) = \sum_{k=1}^{K} \alpha_k \hat{\varphi}_{k,T(\nu)}(\boldsymbol{X}_{V(\nu)})$$

strong learners
on full $\mathcal{D}$, $\hat{\varphi}_k(\boldsymbol{X})$

$$\boxed{\hat{\varphi}_1, \hat{\varphi}_2, \ldots, \hat{\varphi}_K}$$

learners
prediction

$$\mathcal{D} = \{(x_i, y_i)\}_1^N$$

$$\hat{\varphi}_{\mathrm{SL}} = \sum_{k=1}^{K} \hat{\alpha}_k \hat{\varphi}_k$$

prediction
weights

cross-validation
$\hat{\varphi}_{k,T(\nu)}(\boldsymbol{X}_{V(\nu)})$

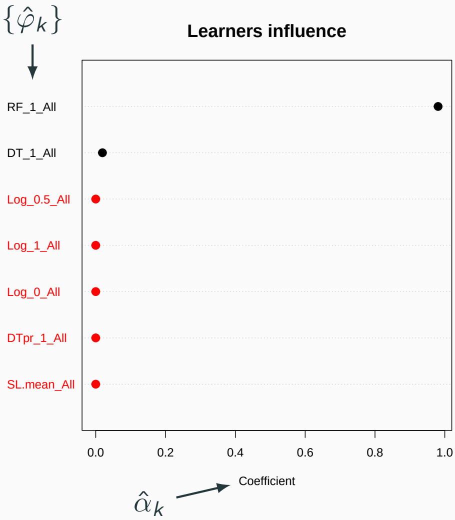
$\hat{\varphi}_1$	$\hat{\varphi}_2$	$\ldots$	$\hat{\varphi}_K$	Y
1	1	$\ldots$	1	1
2	2	$\ldots$	2	2
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
V	V	$\ldots$	V	V

Super Learner in practice

```
SuperLearner(Y, X,
   cluster,
   SL.library ← {φ_k},
   cvControl=list(
     V=10,shuffle=FALSE))
```

What's in the ensemble?

- Response variable mean $\bar{y}$
- Logistic Regression with $\alpha = 0,\ 1$ and 0.5
- Grown and pruned Decision Tree
- Random Forest



Performance

Model	Train score	Test score
CART	0.0000	0.0000
Random forest	0.0000	0.0000
AdaBoost	0.0000	0.0000
Super learner	0.0000	0.0000

References i

T. Hastie, R. Tibshirani, and J. H. Friedman
The Elements of Statistical Learning
Springer, 2009.

E. C. Polley, and M. J. van der Laan
Super Learner in Prediction
U.C. Berkeley Division of Biostatistics Working Paper Series.
Working Paper 266, 2010

M. Culp, K. Johnson and G. Michailidis
`ada`: **The R Package Ada for Stochastic Boosting**
Journal of Statistical Software, 17(2), 1–27, 2006

Discrete AdaBoost algorithm

Discrete AdaBoost with shrinkage and out-of-bag, as an additive model with prediction function $f_m(x)$

Input: M, $\{(x_i, y_i)\}_1^N$, $x_i \in \mathbb{R}^p$

Initialize $f_0(x) = 0$;

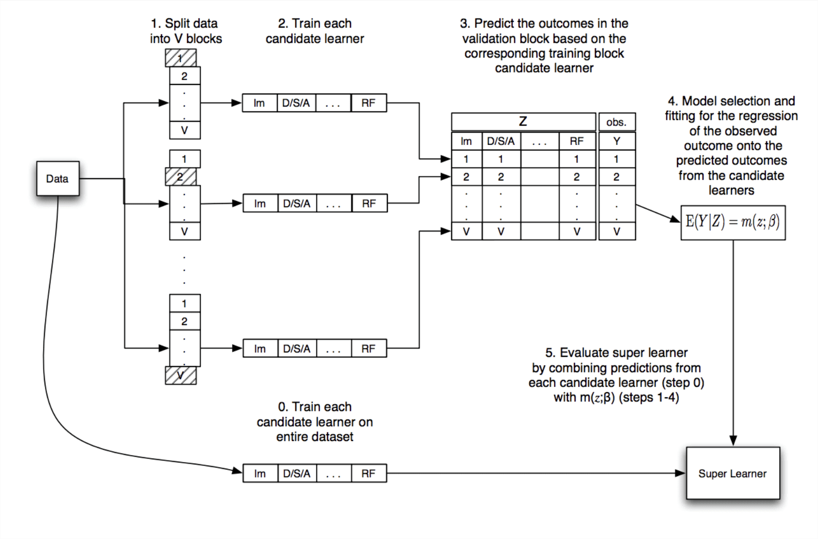
for $m = 1$ **to** M **do**

- Set $w_i^{(m)} = -\frac{\partial L(y,g)}{\partial g}\Big|_{g=f_m(x)}$ s.t. $\sum_{i=1}^N w_i^{(m)} = 1$;
- Fit classifier $G_m(x)$ using $w_i^{(m)}$ with samples from π_m;
- Weighted error rate $\mathrm{err}_m = \sum_{i=1}^N w_i^{(m)} \mathbb{I}(y_i \neq G(x_i))$;
- Set $\alpha_m = \frac{1}{2} \log\left(\frac{1-\mathrm{err}_m}{\mathrm{err}_m}\right)$;
- Update $f_m(x) \leftarrow f_{m-1}(x) + \lambda \alpha_m G_m(x)$;

end

Output: $G(x) = \mathrm{sign}(f_M(x))$

Super Learner algorithm flow diagram



Input: $\mathcal{D} = \{(x_i, y_i)\}_1^N$, $\mathcal{L} = \{\varphi_k(X)\}_{k=1}^K$

foreach *strong learner in* $\mathcal{L}$ **do**

  Fit φ_k on $\mathcal{D} \Rightarrow \hat{\varphi}_k(\boldsymbol{X}) \to \hat{\mathcal{L}} = \{\hat{\varphi}_k\}_{k=1}^K$;

end

for $\nu = 1, 2, \ldots, V$ **do**

  foreach *strong learner in* $\mathcal{L}$ **do**

    Fit φ_k on $T(\nu)$, predict $\hat{\varphi}_{k,T(\nu)}(X_i \in V(\nu))$;

  end

end

Stack output in an $N \times K$ matrix $Z = \left\{\hat{\varphi}_{k,T(\nu)}\left(X_{V(\nu)}\right)\right\}$;

Propose a family of weighted combinations

$$m(z|\alpha) = \sum_{k=1}^{K} \alpha_k \hat{\varphi}_{k,T(\nu)}\left(\boldsymbol{X}_{V(\nu)}\right) \to \hat{\alpha} = \arg\min_{\alpha} \sum_{i=1}^{N} L(Y_i, m(z_i|\alpha))$$

of size N s.t. $\alpha_k \geq 0$, $\sum_k \alpha_k = 1$ and minimizes $\sum_k \alpha_k \hat{\varphi}_k$;

Combine $\hat{\alpha}$ with the library $\hat{\mathcal{L}} \to \hat{\varphi}_{\mathrm{SL}}(\boldsymbol{X}) = \sum_{k=1}^K \hat{\alpha}_k \hat{\varphi}_k(\boldsymbol{X})$;

Output: $\hat{\varphi}_{\mathrm{SL}}$