Stochastic Gradient Descent with Momentum and Line Searches

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Abstract

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1 Introduction

1.1 Optimization problem

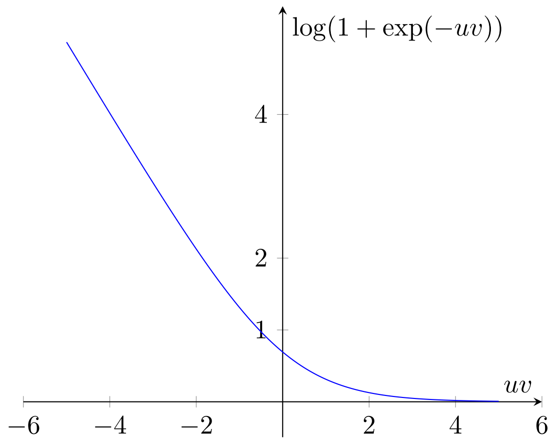
$$\min_{w \in \mathbb{R}^p} f(w) = L(w) + \lambda \Omega(w)$$

$$\min \sum_{i=1}^{N} \log\bigl(1 + \exp(-y^{(i)} w^T x^{(i)})\bigr) + \lambda \|w\|^2$$

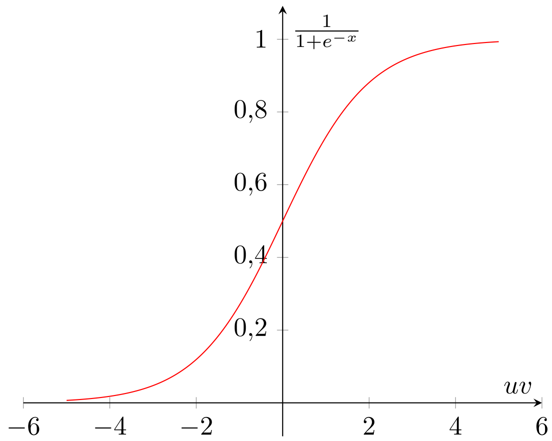
where $i = \ldots, N$ are the dataset indices, $y^i \in \{-1, 1\}$ is the response variable corresponding to the negative or positive class, $x^i \in \mathbb{R}^p$ are dataset examples.

$$\nabla f(w) = X^T r + 2\lambda w, \quad r = -y^{(i)} \sigma(-y^{(i)} w^T x^{(i)})$$

$$\nabla^2 f(w) = X^T D X + 2\lambda I, \quad d_{ii} = \sigma(y^{(i)} w^T x^{(i)}) \sigma(-y^{(i)} w^T x^{(i)})$$



(a) *Log-loss*



(b) *Sigmoid function*

Figure 1: Log-loss and sigmoid function plots

- $uv \gg 0$: the example is labelled correctly
- $uv \ll 0$: the class assigned to the example is the wrong one
- the hessian matrix is positive defined $\forall w$, this means that the objective function, which is quadratic, is coercive and for the continuity that function admits global minimum
- the hessian matrix being positive defined implies also that the objective function is strictly convex (on the other hand the loss function is just convex, due to its hessian matrix being positive semi-defined), this implies that if the global minimum exists, that solution is unique
- a global minimum is a point that satisfy $\nabla f(w^*) = 0$, which is a sufficient condition implied by the convexity of the problem, see figure 1a on the preceding page
- the ℓ_2 regularization implies that the objective function is strongly convex, this speeds up the convergence

2 Mini-batch gradient descent variants

2.1 Fixed step-size

Mini-batch Gradient Descent with fixed step-size

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dati $w^0 \in \mathbb{R}^n$, $f(w) = \sum_{i=1}^{N} \log\big(1 + \exp(-y^{(i)} w^T x^{(i)})\big) + \lambda \|w\|^2$, $k = 0$ e $\{\alpha_k\} \mid \alpha_k = \alpha$
while ($\|\nabla f(w^k)\| > \varepsilon$)
  shuffle $\{1, \dots, N\}$ in $N/M$ blocchi $B_1, \dots, B_{N/M}$ di dimensione $1 < |B_t| = M \ll N$
  $y_0 = w^k$
  for $t = 1, \dots, N/M$
    $y_t = y_{t-1} - \alpha_k \frac{1}{M} \sum_{j \in B_t} \nabla f_j(y_{t-1})$
  end for
  $w^{k+1} = y_{N/M}$
  $k = k + 1$ fine epoca
end while
```