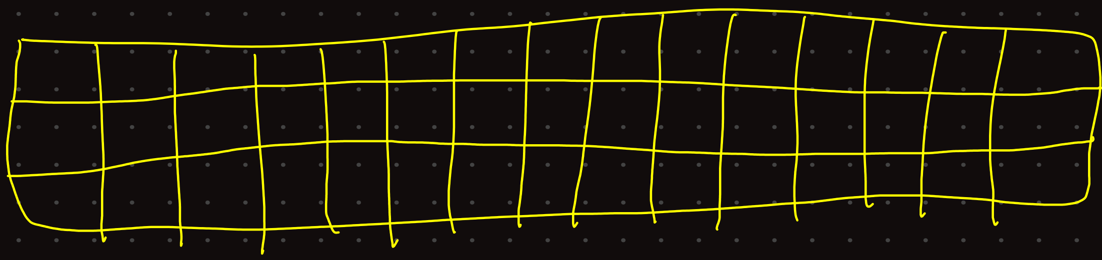
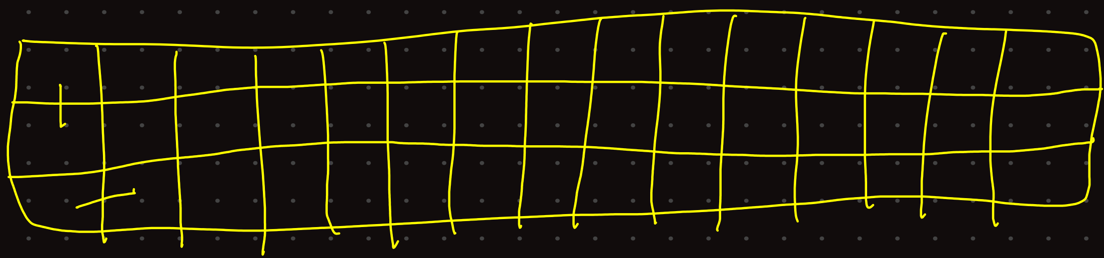
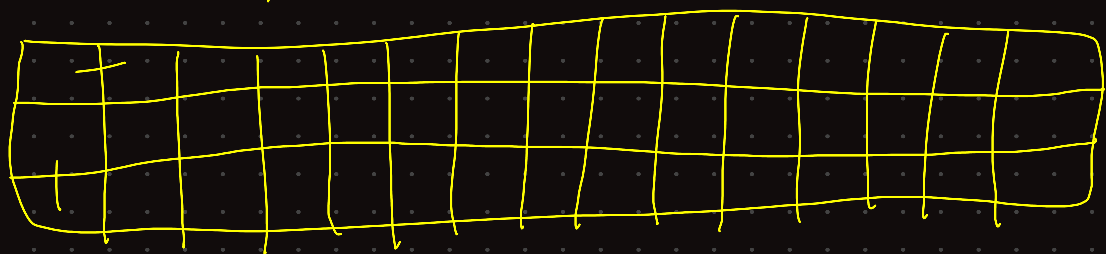
dp tiling $3 \times N$ with 2×1 tile

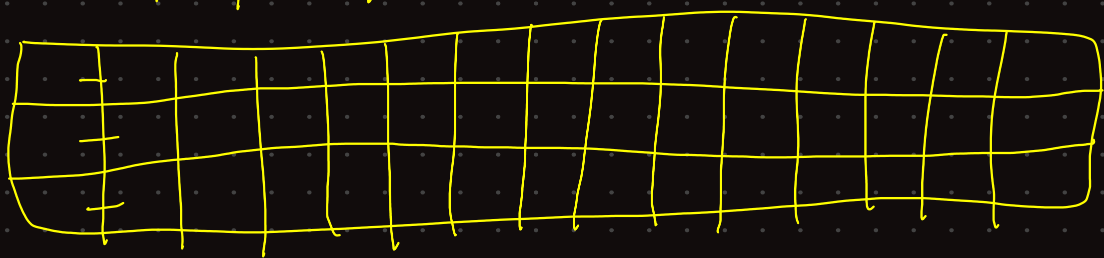


PRIORITIZE FILLING UP THE FIRST COL.

* $f(n)$ = # of ways to fill $3 \times N$

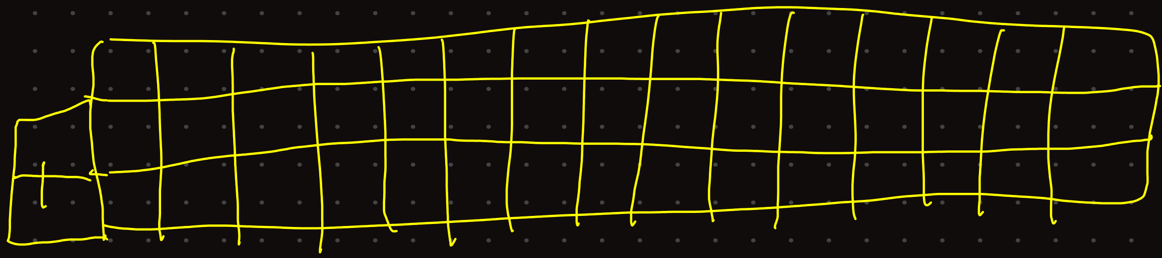


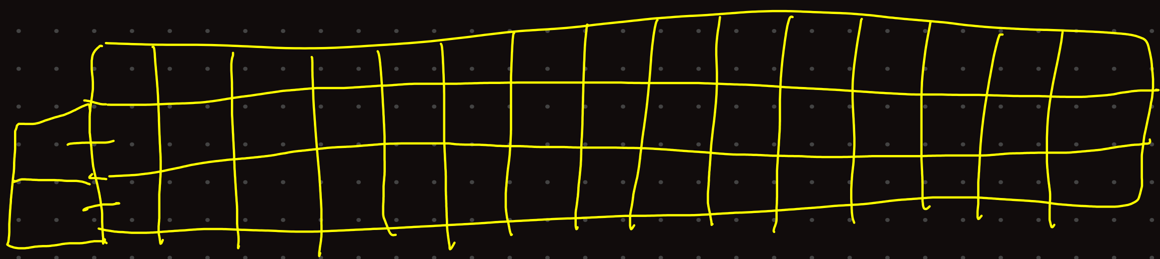




$$f(n) = 2g(n-2) + f(n-2) \quad (i)$$

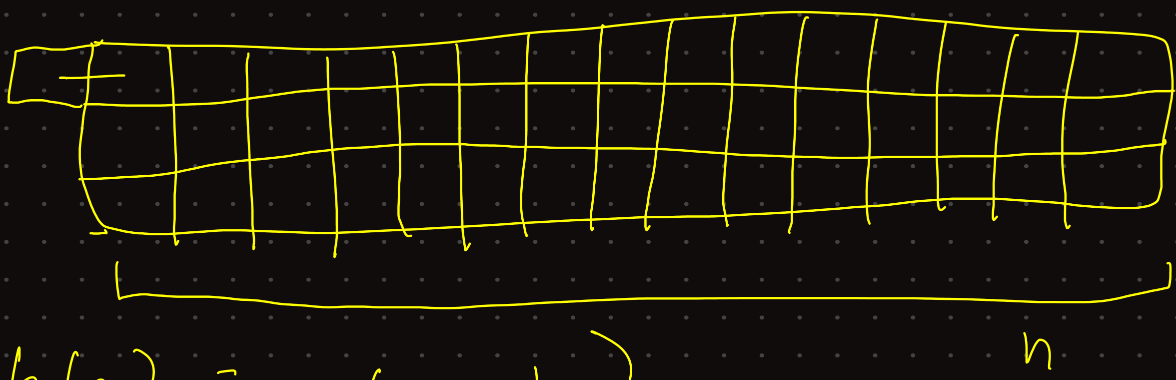
$g(n)$ = # of ways to fill $3 \times N$ with 2 extra cell





$$g(n) = f(n) + h(n-1)$$

$h(n)$ = # of ways to fill $3 \times N$ with 1 extra cell



$$h(n) = g(n-1)$$

hence, $g(n) = f(n) + g(n-2)$ (ii)

$$\boxed{f(n) = 2g(n-2) + f(n-2)} \qquad \boxed{g(n) = f(n) + g(n-2)}$$

$$g(n-2) = \frac{f(n) - f(n-2)}{2} \qquad f(n) = g(n) - g(n-2)$$

$$g(n) = \frac{f(n+2) - f(n)}{2} \qquad f(n) = \frac{f(n+2) - f(n)}{2} - \frac{f(n) - f(n-2)}{2}$$

$$f(n) = \frac{1}{2}\left(f(n+2) - f(n) - f(n) + f(n-2)\right)$$

$$2f(n) = \frac{1}{2} f(n+2) + \frac{1}{2} f(n-2)$$

$$4f(n) - f(n-2) = f(n+2)$$

$$f(n) = 4f(n-2) - f(n-4)$$

$$\begin{bmatrix} f(n) \\ ? \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} f(n-2) \\ f(n-4) \end{bmatrix}$$

$$\begin{bmatrix} f(n) \\ f(n-2) \end{bmatrix} = \begin{bmatrix} a \cdot f(n-2) + b \cdot f(n-4) \\ c \cdot f(n-2) + d \cdot f(n-4) \end{bmatrix}$$

$$= \begin{matrix} a = 4 & b = -1 \\ c = 1 & d = 0 \end{matrix}$$

$$\begin{bmatrix} f(n) \\ f(n-2) \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} f(n-2) \\ f(n-4) \end{bmatrix}$$