Week 3 - dsolve() with Initial Conditions

MAT330: Differential Equations

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Problems:

Problem 1: Use the MATLAB dsolve() function to solve the differential equation $y'' - y' = 2t^2 - t - 5$ with initial conditions $y(1) = 3, y'(0) = 0$. Plot the your solution on a figure as a single solid curve for time $t = -2$ to $2.$ Make sure to label both axes, title your figure, and turn on the plotting legend. Set the y-axis limits to [-15 10].

```
% Problem 1 Code Here
syms y(t) t;

Dy(t) = diff(y(t));
Dy2(t) = diff(y(t),2);

ode = Dy2(t) - Dy(t) == 2*t^2 - t - 5
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ode =

$$\frac{\partial^2}{\partial t^2}\ y(t) - \frac{\partial}{\partial t}\ y(t) = 2\,t^2 - t - 5$$

```
cond1 = y(1) == 3;
cond2 = Dy(0) == 0;
cond = [cond1; cond2];

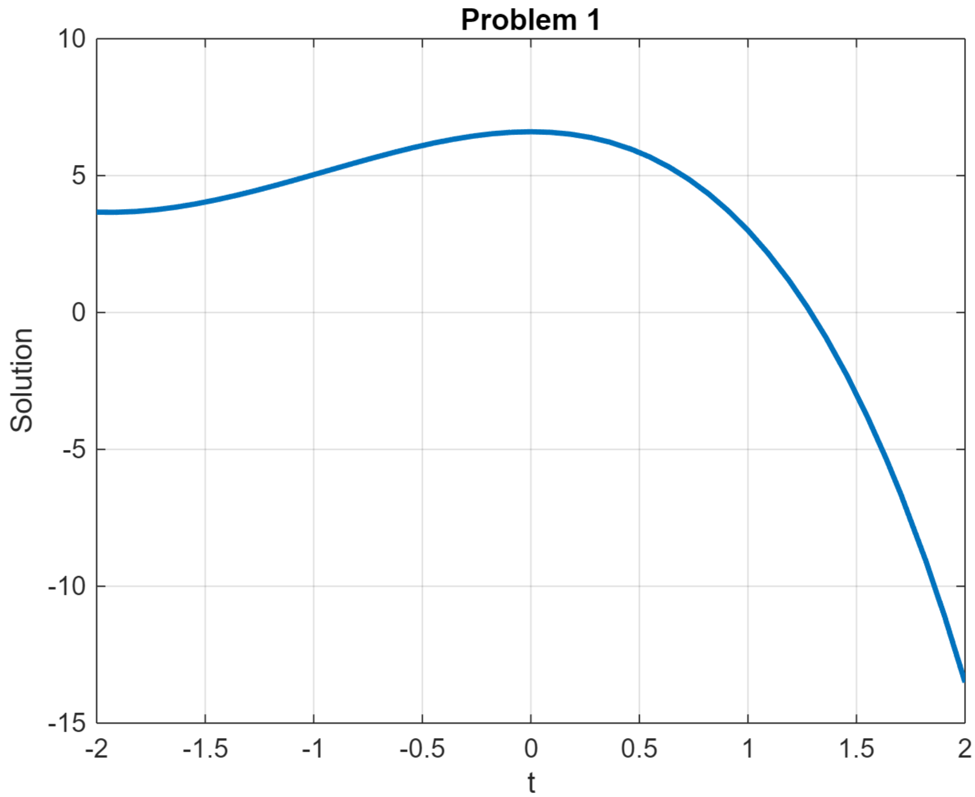
ySolnA(t) = dsolve(ode,cond)
```

ySolnA(t) =

$$2\,t + 2\,\mathrm{e} - 2\,\mathrm{e}^t - \frac{3\,t^2}{2} - \frac{2\,t^3}{3} + \frac{19}{6}$$

```
figure;
fplot(ySolnA, [-2, 2], '-', 'linewidth',2);
xlabel('t');
ylabel('Solution');
title('Problem 1');
grid on;
```

```
ylim([-15 10]);
```



Problem 2: Analyze your solution from Problem 1 to check that the initial conditions are satisfied. Compute $y(1)$ and $y'(0)$ and by hand to verify their values. Make sure to show your work.

Find General Solution:

$$y(t) = y_1(t) + y_2(t) = -\frac{2t^3}{3} - \frac{3t^2}{2} + 2t + c_1 + c_2e^t$$

Solve for Unknown Constants:

$$\frac{\mathrm{dy}(t)}{\mathrm{dt}} = \frac{d}{\mathrm{dt}}\left(-\frac{2t^3}{3} - \frac{3t^2}{2} + 2t + c_1 + c_2e^t\right) = -2t^2 - 3t + c_2e^t + 2$$

Substitute Initial Condition $y(1) = 3$:

$$c_1 + c_2e - \frac{1}{6} = 3$$

Substitute Initial Condition $y'(0) = 0$:

$$c_2 + 2 = 0$$

Solve the System:

$c_1 = \frac{1}{6}(19 + 12e) \quad c_2 = -2$

Substitute c_1 and c_2:

$$y(t) = -2e^t - \frac{2t^3}{3} - \frac{3t^2}{2} + 2t + \frac{19}{6} + 2e$$

$y(1) = -2e - \frac{2}{3} - \frac{3}{2} + 2 + \frac{19}{6} + 2e = 3$ Condition is Satisfied.

$$y'(t) = -2t^2 - 3t - 2e^t + 2$$

$y'(0) = 0 - 0 - 2 + 2 = 0$ Condition is Satisfied.

Problem 3: Use the MATLAB dsolve() function to solve the differential equation $8y''' = \cos(20t) + \sin(2t)$ for initial conditions $y(10) = 50, y'(0) = 0, y''(0) = 0$ and $y(10) = 15, y'(0) = 0, y''(0) = 3$. Plot the your solutions on a single figure as a solid curve for the first set initial conditions and a dotted curve for the second set of initial conditions for time $t = -10$ to 20. Make sure to label both axes and title your figure, and turn on the plotting legend. Set the y-axis limits to [-150 200].

```
% Problem 3 Code Here
syms y(t) t;

Dy(t) = diff(y(t));
Dy2(t) = diff(y(t),2);
Dy3(t) = diff(y(t),3);

ode = 8*Dy3(t) == cos(20*t) + sin(2*t)
```

ode =

$$8\frac{\partial^3}{\partial t^3}\, y(t) = \cos(20\,t) + \sin(2\,t)$$

```
cond1 = y(10) == 50;
cond2 = Dy(0) == 0;
cond3 = Dy2(0) == 0;
cond = [cond1; cond2; cond3];

ySolnA(t) = dsolve(ode,cond)
```

ySolnA(t) =

$$\frac{t}{3200} + \frac{\cos(2\,t)}{64} - \frac{\sin(20\,t)}{64000} - \frac{\cos(20)}{64} + \frac{\sin(200)}{64000} + \frac{t^2}{32} + \frac{14999}{320}$$

```
cond1 = y(10) == 15;
cond2 = Dy(0) == 0;
```

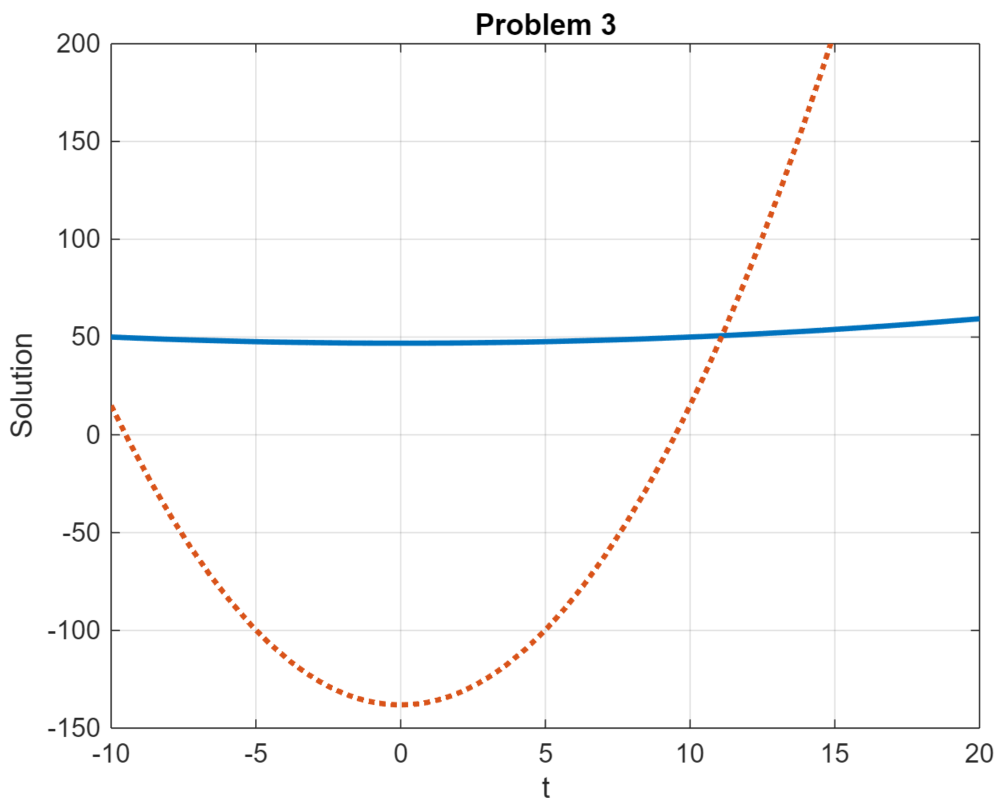
```
cond3 = Dy2(0) == 3;
cond = [cond1; cond2; cond3];

ySolnB(t) = dsolve(ode,cond)
```

ySolnB(t) =

$$\frac{t}{3200}+\frac{\cos(2\,t)}{64}-\frac{\sin(20\,t)}{64000}-\frac{\cos(20)}{64}+\frac{\sin(200)}{64000}+\frac{49\,t^2}{32}-\frac{44201}{320}$$

```
figure;
fplot(ySolnA,[-10 20],'-','linewidth',2);
hold on;
fplot(ySolnB,[-10 20],':','linewidth',2);
xlabel('t');
ylabel('Solution');
title('Problem 3');
grid on;
ylim([-150 200]);
```



Problem 4: Analyze your solution from Problem 3. How did changing the initial conditions change the solution? Which terms changed when the initial conditions changed? Explain.

By changing the initial conditions we observed a deeper curve in the parabola shape of the function, where in the first set of conditions it has a bit more of a linear shape. The terms that change within this function are the terms for the y-value as the graph progresses. It can be observed that the values for y decrease as they approach (0, -150) and then start increasing again as the function approaches infinity.