Module Five - Multiple Integration

MAT325: Calculus III: Multivariable Calculus

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February 8th, 2025

Problems:

Problem 1: Use MATLAB and the integral2() function to evaluate the integral of the function $f(x, y) = x^3 - 2y^3$ over the rectangular region $R = [1, 2] \times [2, 4]$.

```
clear all;
syms x y;

f = @(x,y) (x.^3-2.*y.^3);

xMin = 1;
xMax = 2;
yMin = 2;
yMax = 4;

format long;
integral2(f,xMin,xMax,yMin,yMax)
```

```
ans =
    -1.125000000000074e+02
```

Problem 2: Use MATLAB and the integral2() function to evaluate the integral $\int\int_R (x + y)\, dA$ over the region $R = \{(x, y) | 2 \leq x^2 + y^2 \leq 4, y \geq 0\}$.

Convert Cartesian to Polar coordinates so $x = \text{rcos}(\theta)$ and $y = \text{rsin}(\theta)$

The region now becomes $\sqrt{2} \leq r \leq 2$ and $0 \leq \theta \leq \pi$

$\int_0^{\pi} \int_{\sqrt{2}}^{2} (r^2\text{cos}(\theta) + r^2\text{sin}(\theta))\, \text{dr}\, d\theta$ where $r = y$ and $\theta = x$

```
clear all;
syms x y;

f = @(x,y) (y.^2.*cos(x)+y.^2.*sin(x));

xMin = 0;
xMax = pi;
yMin = sqrt(2);
```

```
yMax = 2;

format long;
integral2(f,xMin,xMax,yMin,yMax)
```

```
ans =
   3.447715250099210
```

Problem 3: Use MATLAB and the integral3() function to evaluate the integral

$$\int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{16-r^2}} r^2\,dz\,dr\,d\theta.$$

$r = y$

```
clear all;
syms y;
f = @(x,y,z) (y.^2);

xMin = 0;
xMax = 2*pi;
yMin = 0;
yMax = 2;
zMin = 0;
zMax = @(x,y) sqrt(16-y.^2);

format long;
integral3(f,xMin,xMax,yMin,yMax,zMin,zMax)
```

```
ans =
  61.744595536591504
```