Module Two - Vector-Valued Functions

MAT325: Calculus III: Multivariable Calculus

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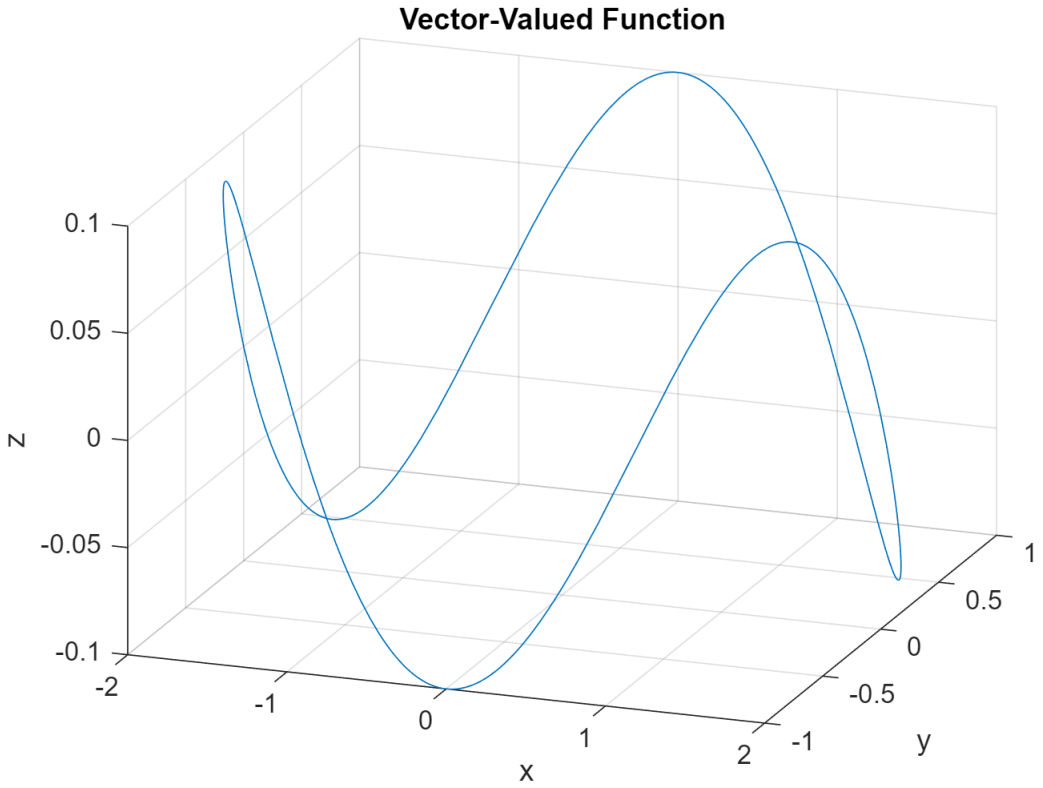
Problems:

Problem 1: Use MATLAB to plot the vector-valued function $\mathbf{r}(t) = 2\cos(t)\mathbf{i} - \sin(t)\mathbf{j} + 0.1\sin(3t)\mathbf{k}$ on the interval $0 \leq t \leq 2\pi$. Choose an appropriate view to best visualize the curve, label axes appropriately, and title the figure.

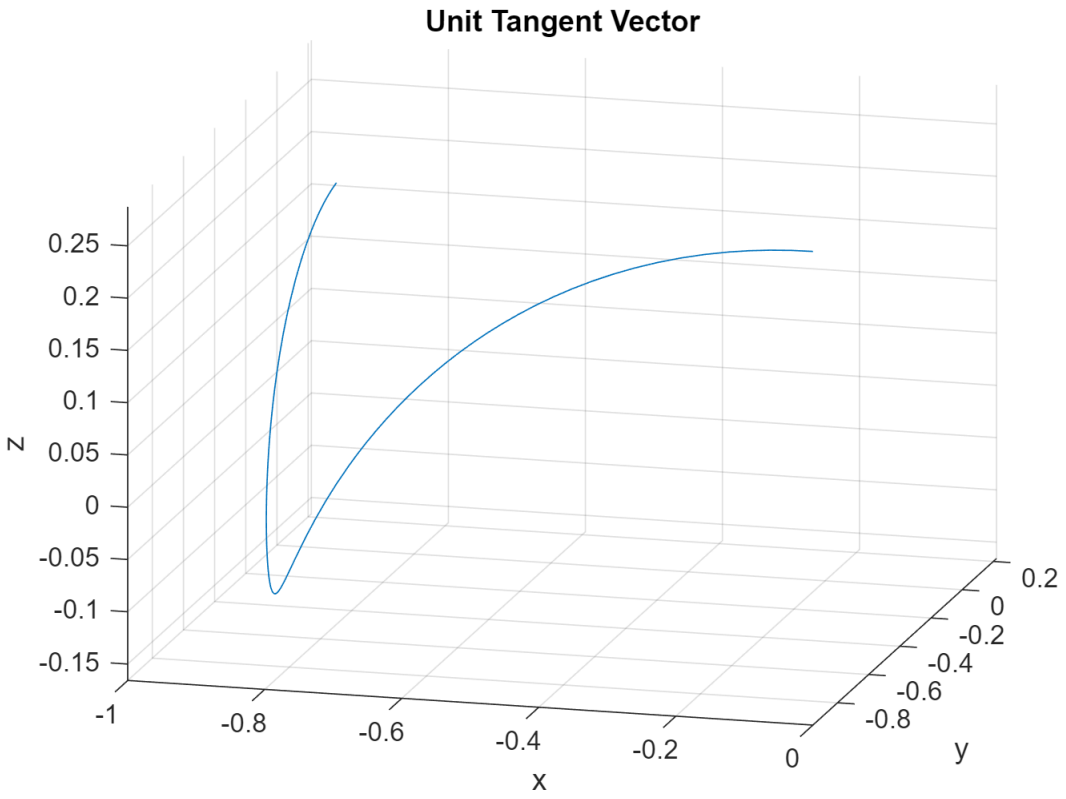
```
% Problem 1 Code Here
clear all;
syms t;

x(t) = 2*cos(t);
y(t) = -1*sin(t);
z(t) = 0.1*sin(3*t);

figure;
fplot3(x(t),y(t),z(t),[0,2*pi]);
xlabel('x');
ylabel('y');
zlabel('z');
title('Vector-Valued Function');
view([20 25]);
```



Problem 2: Consider the again the vector-valued function $\mathbf{r}(t) = 2\cos(t)\mathbf{i} - \sin(t)\mathbf{j} + 0.1\sin(3t)\mathbf{k}$ on the interval $0 \leq t \leq 2\pi$. Compute the unit tangent vector $\mathbf{T}(t)$ and plot for $0 \leq t \leq 2\pi$. Choose an appropriate view to best visualize the curve, label axes appropriately, and title the figure.

```
% Problem 2 Code Here
clear all;
syms t;

rPrimeX(t) = -2*sin(t);
rPrimeY(t) = -1*cos(t);
rPrimeZ(t) = 0.3*cos(3*t);

rNorm(t) = sqrt(rPrimeX(t).^2 + rPrimeY(t).^2 + rPrimeZ(t).^2);

TX(t) = rPrimeX(t)./rNorm(t);
TY(t) = rPrimeY(t)./rNorm(t);
TZ(t) = rPrimeZ(t)./rNorm(t);

figure;
fplot3(TX(t),TY(t),TZ(t),[0,2]);
xlabel('x');
ylabel('y');
zlabel('z');
```

```
title('Unit Tangent Vector');
view([15 20]);
```



Problem 3: Consider the again the vector-valued function $\mathbf{r}(t) = 2\cos(t)\mathbf{i} - \sin(t)\mathbf{j} + 0.1\sin(3t)\mathbf{k}$ on the interval $0 \leq t \leq 2\pi$. Compute an expression for $||\mathbf{r}'(t)||$, then use the MATLAB functions int() and vpa() to compute its arc length on the interval $0 \leq t \leq 2\pi$.

```
% Problem 3 Code Here
syms t;

rNorm(t) = sqrt(0.9 + 4*t^2);
symbolicAnswer = int(rNorm,0,2*pi)
```

```
symbolicAnswer =
```

$$\frac{9\,\mathrm{asinh}\left(\frac{4\pi\sqrt{10}}{3}\right)}{40} + \frac{\pi\sqrt{40}\sqrt{160\pi^2+9}}{20}$$

```
numericalAnswer = vpa(symbolicAnswer)
```

```
numericalAnswer = 40.328369298992037023340343274698
```