Module Three - Multivariable Functions and Limits

MAT325: Calculus III: Multivariable Calculus

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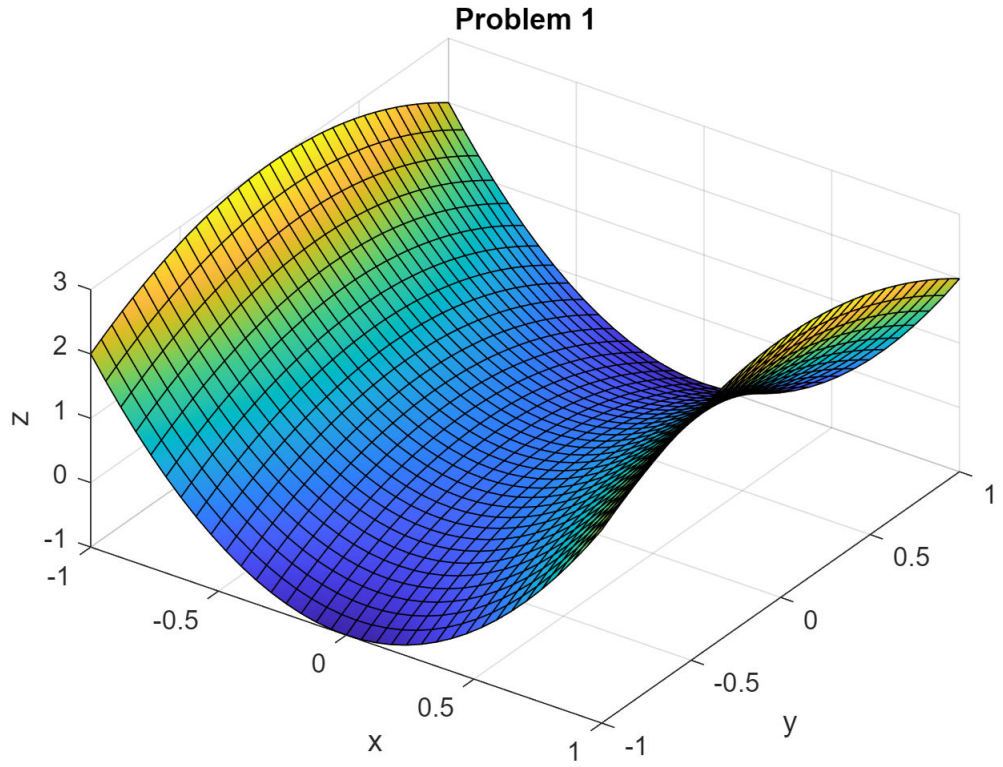
Problems:

Problem 1: Use MATLAB to plot the multi-variable function $z(x, y) = 3x^2 - y^2$ over the region $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. Choose an appropriate view to best visualize the surface, label axes appropriately, and title the figure.

```
% Problem 1 Code Here
syms x y;

z(x,y) = 3*x^2 - y^2;

figure;
fsurf(z(x,y),[-1,1,-1,1]);
xlabel('x');
ylabel('y');
zlabel('z');
title('Problem 1');
view([35 50]);
```

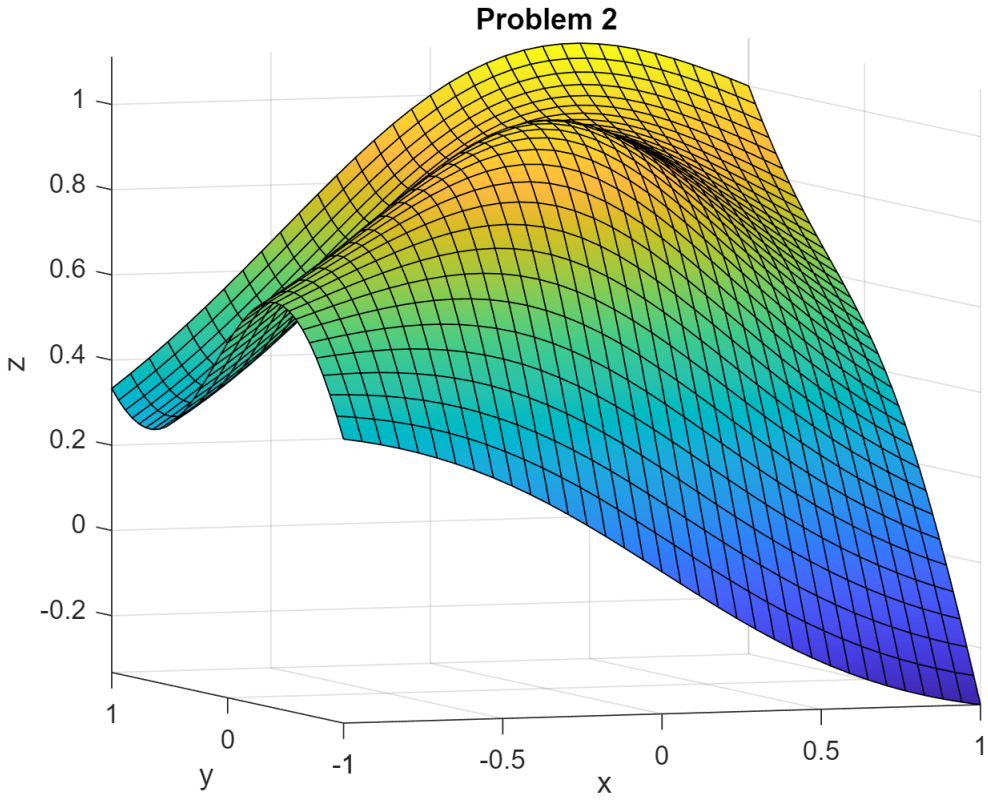


Problem 2: Use MATLAB to plot the multi-variable function $z(x, y) = \dfrac{xy + y^3 + 1}{x^2 + y^2 + 1}$ over the region $-1 \le x \le 1$, $-1 \le y \le 1$. Choose an appropriate view to best visualize the surface, label axes appropriately, and title the figure. From your examination of the figure does the limit $(x, y) \to (0, 0)$ of the function exist? Yes or no? Explain.

```
% Problem 2 Code Here
clear all;
syms x y;

z(x,y) = (x*y + y^3 +1)./(x^2 + y^2 + 1);

figure;
fsurf(z(x,y),[-1,1,-1,1]);
xlabel('x');
ylabel('y');
zlabel('z');
title('Problem 2');
view([340 5]);
```



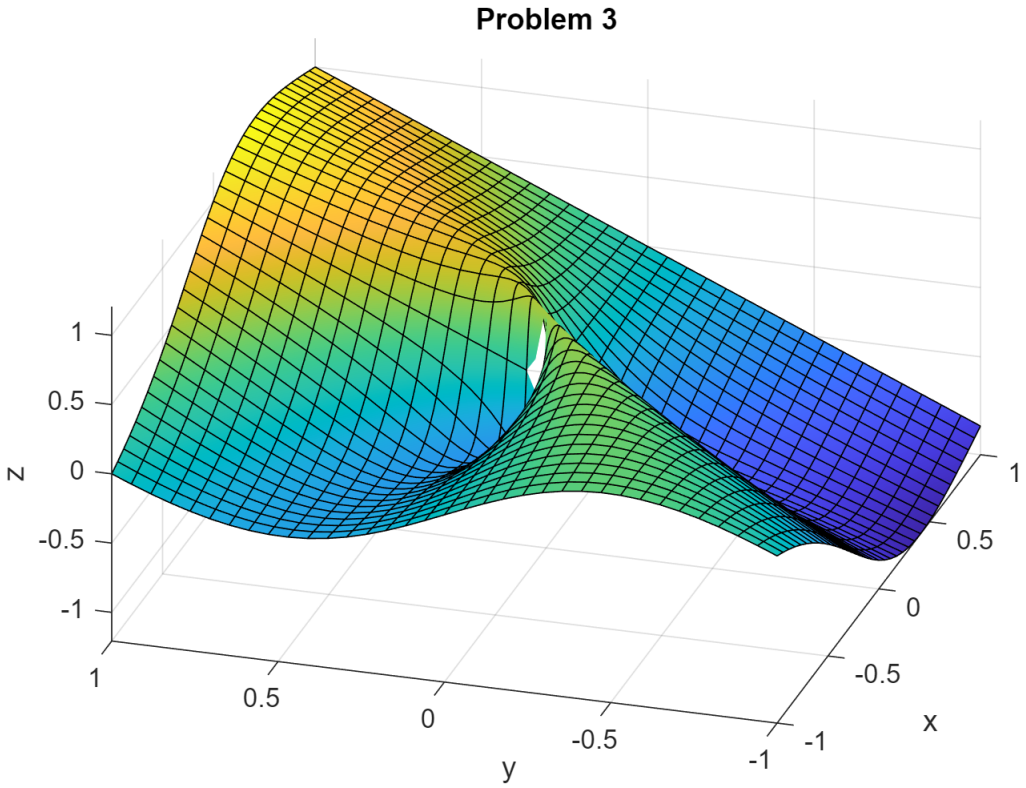

```
% By examining the surface, approaching the point (0,0) from any
% path appears to result in a value of 1. If we plug (0,0) into our
% function, we geet a value of 1. Therefore, I would say that there
% exists a limit of 1 at the point (0,0).
```

Problem 3: Use MATLAB to plot the multi-variable function $z(x, y) = \frac{xy + y^3}{x^2 + y^2}$ over the region $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. Choose an appropriate view to best visualize the surface, label axes appropriately, and title the figure. From your examination of the figure does the limit $(x, y) \to (0, 0)$ of the function exist? Yes or no? Explain.

```
% Problem 3 Code Here
clear all;
syms x y;

z(x,y) = (x*y + y^3)./(x^2 + y^2);

figure;
fsurf(z(x,y),[-1,1,-1,1]);
xlabel('x');
ylabel('y');
zlabel('z');
title('Problem 3');
view([287 40]);
```




```
% By examining the surface, approaching the point (0,0) from any
% path appears to result in different values. Therefore, I would
% say that no limit exists at (0,0). Additionally, if we plug (0,0)
% into our equation, the denominator would be zero. This also shows
% us that no limit exists at this point.
```

Problem 4: Consider the multi-variable function $f(x, y) = \cos(xy + 3x)$. Use MATLAB and the diff() function to compute the partial derivatives $\frac{\partial}{\partial x} f(x, y)$ and $\frac{\partial}{\partial y} f(x, y)$.

```
% Problem 4 Code Here
clear all;
syms x y;

f(x,y) = cos(x*y + 3*x);

diff(f(x,y),x)
```

ans = $-\sin(3\,x + x\,y)\,(y + 3)$

```
diff(f(x,y),y)
```

ans = $-x\sin(3\,x + x\,y)$