MAT 330: Differential Equations

Module Two Template

Complete this template by replacing the bracketed text with the relevant information.

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Problems:

Problem 1: Use the MATLAB dsolve() function to solve the differential equation $y'' - y' = 2t^2 - t - 5$. Plot the family of solutions on a single plot for $C_1 = C_2 = 0$ as a solid curve, $C_1 = C_2 = -1$ as a dotted curve, and $C_1 = C_2 = 3$ as a dashed curve. Make sure to label both axes, title your figure, and turn on the plotting legend. Set the y-axis limits to [-15 25].

```
syms y(t) t;
Dy = diff(y(t));
Dy2 = diff(y(t),2);

ode = Dy2 - Dy == 2*t^2 - t - 5
```

ode =

$$\frac{\partial^2}{\partial t^2}\, y(t) - \frac{\partial}{\partial t}\, y(t) = 2\,t^2 - t - 5$$

```
ySoln(t) = dsolve(ode)
```

ySoln(t) =

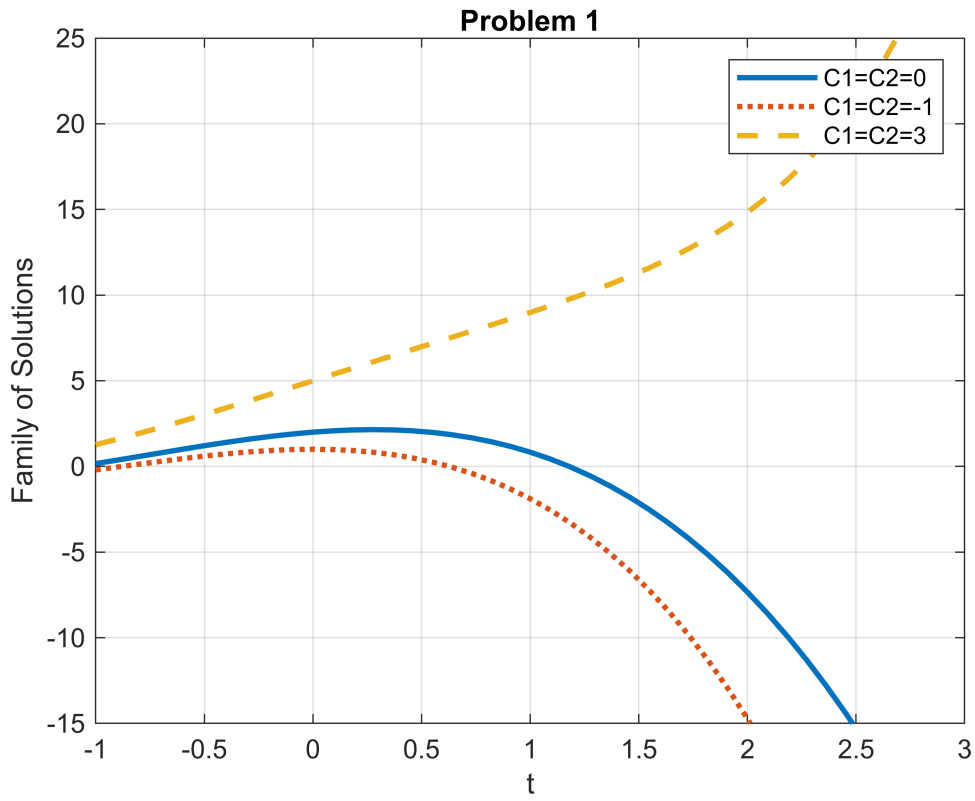
$$C_1 + 2\,t + C_2\,\mathrm{e}^{t} - \frac{3\,t^2}{2} - \frac{2\,t^3}{3} + 2$$

```
C1 = 0;
C2 = 0;
ySoln1 = C1 + 2&t + C2*exp(t) - 3*t^2/2 - 2*t^3/3 + 2;

C1 = -1;
C2 = -1;
ySoln2 = C1 + 2&t + C2*exp(t) - 3*t^2/2 - 2*t^3/3 + 2;

C1 = 3;
C2 = 3;
ySoln3 = C1 + 2&t + C2*exp(t) - 3*t^2/2 - 2*t^3/3 + 2;
```

```
figure;
fplot(ySoln1,[-1,3],'-','linewidth',2);
hold on;
fplot(ySoln2,[-1,3],':','linewidth',2);
fplot(ySoln3,[-1,3],'--','linewidth',2);
xlabel('t');
ylabel('Family of Solutions');
title('Problem 1');
grid on;
ylim([-15,25]);
legend('C1=C2=0','C1=C2=-1','C1=C2=3');
```



Problem 2: Use the MATLAB dsolve() function to solve the differential equation $y'' - y' - 12y = 500\cos(10t)$. Plot the family of solutions on a single plot for $C_1 = C_2 = 0$ as a solid curve, $C_1 = 0.1, C_2 = 0$ as a dotted curve, and $C_1 = C_2 = 0.1$ as a dashed curve. Make sure to label both axes, title your figure, and turn on the plotting legend. Set the y-axis limits to [-10 50].

```
syms y(t) t;
Dy = diff(y(t));
Dy2 = diff(y(t),2);

ode = Dy2 - Dy - 12*y  == 500*cos(10*t)
```

```
ode(t) =
```

$$\frac{\partial^2}{\partial t^2}\, y(t) - \frac{\partial}{\partial t}\, y(t) - 12\, y(t) = 500\cos(10\, t)$$

```
ySoln(t) = dsolve(ode)
```

ySoln(t) =

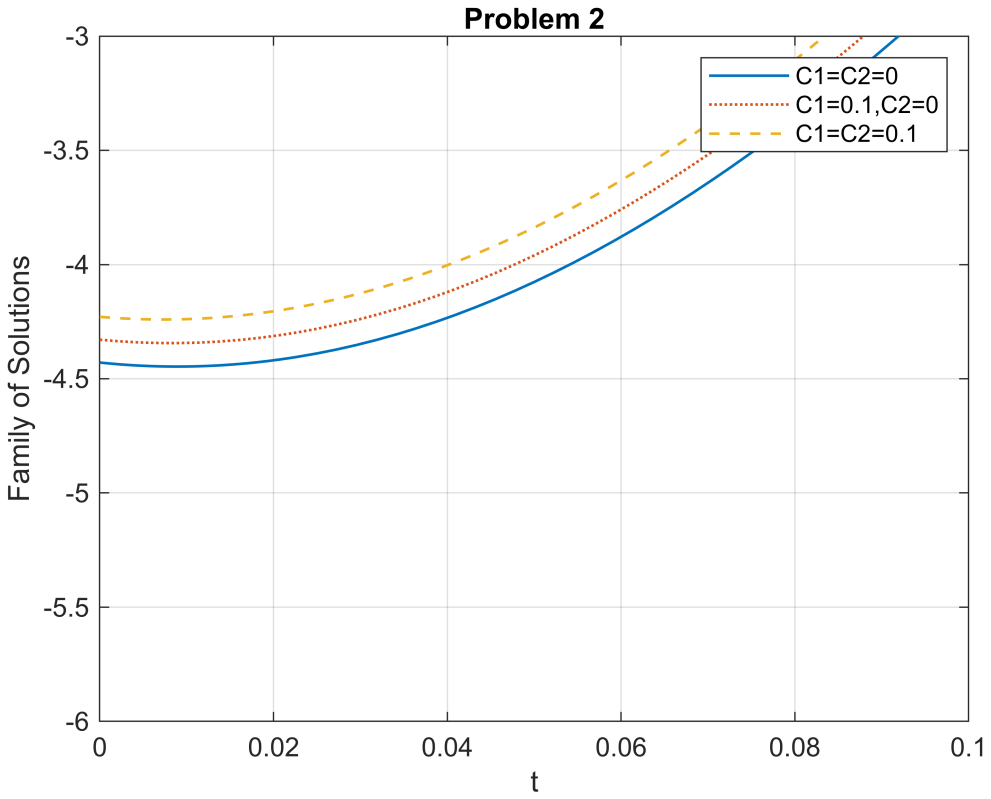
$$C_1\, \mathrm{e}^{-3\,t} - \frac{250\ \sqrt{3161}\ \cos\left(10\, t - \operatorname{atan}\left(\frac{5}{56}\right)\right)}{3161} + C_2\, \mathrm{e}^{4\,t}$$

```
C1 = 0;
C2 = 0;
ySoln1 = C1*exp(3*t) - (250*sqrt(3161)*cos(10*t - atan(5/56)))/3161 + C2*exp(4*t);

C1 = 0.1;
C2 = 0;
ySoln2 = C1*exp(3*t) - (250*sqrt(3161)*cos(10*t - atan(5/56)))/3161 + C2*exp(4*t);

C1 = 0.1;
C2 = 0.1;
ySoln3 = C1*exp(3*t) - (250*sqrt(3161)*cos(10*t - atan(5/56)))/3161 + C2*exp(4*t);

figure;
fplot(ySoln1,[0,0.1],'-','linewidth',1);
hold on;
fplot(ySoln2,[0,0.1],':','linewidth',1);
fplot(ySoln3,[0,0.1],'--','linewidth',1);
xlabel('t');
ylabel('Family of Solutions');
title('Problem 2');
grid on;
ylim([-6,-3]);
legend('C1=C2=0','C1=0.1,C2=0','C1=C2=0.1');
```



Problem 3: For the differential equation solved in Problem 2, will any specific values of C_1 and C_2 cause the solution to converge to zero as $t \to \infty$? Yes or no? Explain.

Due to the linear nature of the equation that was solved in problem two coupled with the fact that it has a positive curve upwards, I will say no. There exists no specific values for C1 and C2 to cause the solution to converge to zero as *t* approaches infinity.