Module Six - Vector Calculus

MAT325: Calculus III: Multivariable Calculus

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Problems:

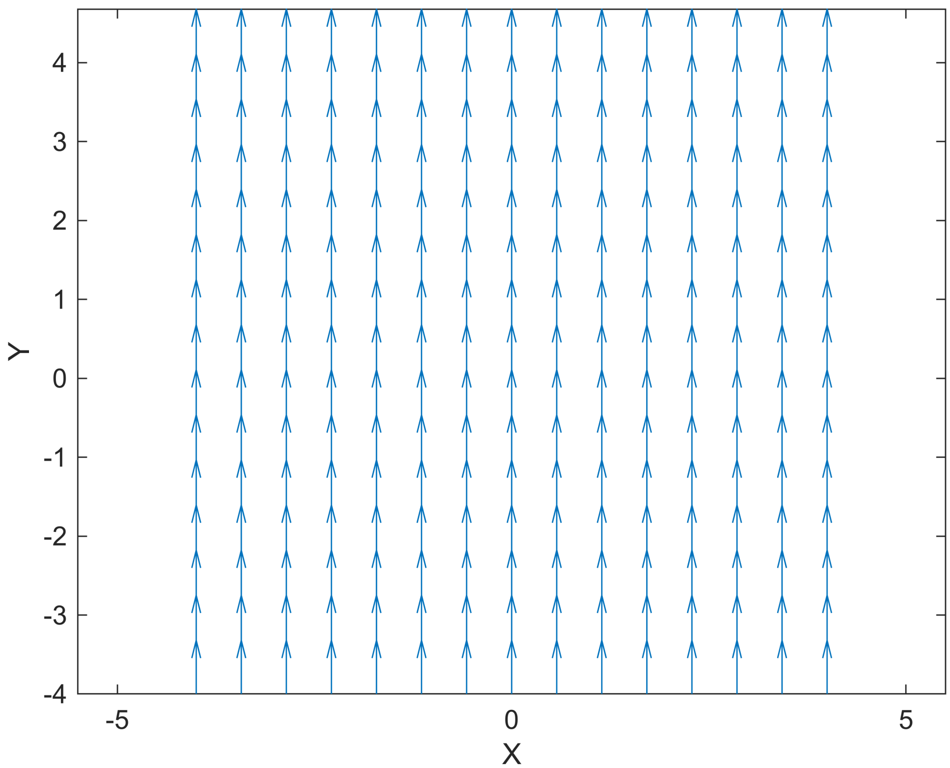
Problem 1: Use MATLAB and the quiver() function to plot the vector field $\mathbf{F}(x, y) = 2x^2\mathbf{i} + xy\mathbf{j}$. Label axes appropriately and title the figure.

```
close all;
clear all;
clc;

x = linspace(-4,4,15);
y = linspace(-4,4,15);
[X, Y] = meshgrid(x,y);

Fi = 2*X^2;
Fj = X*Y;

quiver(X,Y,Fi, Fj);
axis equal;
xlabel('X');
ylabel('Y');
```



Problem 2: Consider the line integral $\int_C (2x + 4y^2)\, ds$ where C is the curve parameterized by $x = t$ and $y = 3t$ for $0 \le t \le 2$. Find a simplified expression for the line integral and then use the MATLAB integral() function to compute its value.

$$\frac{\mathrm{dx}}{\mathrm{dt}} = 1 \quad \frac{\mathrm{dy}}{\mathrm{dt}} = 3 \quad \mathrm{ds} = \sqrt{1^2 + 3^2} = \sqrt{10}\,\mathrm{dt}$$

$$2x + 4y^2 = 2t + 4(3t)^2 = 2t + 36t^2$$

$$\int_0^2 (2t + 36t^2)\, \sqrt{10}\,\mathrm{dt}$$

```
syms t;

f = @(t) (2*t+36*t.^2)*sqrt(10);
integral(f,0,2)
```

```
ans =
     3.162277660168379e+02
```

Problem 3: Consider the line integral $\oint_C (2x^2 + y)\, dx + (xy^2 - 4)\, dy$ where C is the rectangular region with vertices $(-1, 0)$, $(3, 0)$, $(3, 6)$, $(-1, 6)$, oriented in the counterclockwise direction. Use Green's Theorem to find a simplified expression for the line integral and then use the MATLAB integral2() function to compute its value.

$F(x, y) = \langle 2x^2 + y, \mathrm{xy}^2 - 4 \rangle \;\; Q_x = 4x \;\; P_y = 2\mathrm{yx}$

$$\oint_C (2x^2 + y)\mathrm{dx} + (\mathrm{xy}^2 - 4)\mathrm{dy} = \int \int_D (4x - 2\mathrm{yx})\mathrm{dA}$$

$$\int_0^6 \int_{-1}^3 (4x - 2\mathrm{yx})\mathrm{dxdy}$$

```
syms x y;
clear all;

f = @(x,y) (4*x - 2*y.*x);
xmin = -1;
xmax = 3;
ymin = 0;
ymax = 6;

format long;
integral2(f,xmin,xmax,ymin,ymax,'Method','tiled')
```

```
ans =
 -47.999999999999993
```