Week 5 - Direction Fields and Phase Portraits

MAT330: Differential Equations

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February 8th, 2025

Problems:

Problem 1: Consider the system of differential equations $x_1' = -5x_1 + x_2$ and $x_2' = 4x_1 - 2x_2$. Use MATLAB to plot the eigenvectors and direction field of this system on a single plot. Make sure to label both axes and title your figure. Generate your plots for $-1 \le x_1 \le 1$ and $-1 \le x_2 \le 1$ and set both x_1 and x_2 axes to limits of [-1 1].

```
%x1'=-5x1+x2
%x2'=4x1-2x2

A = [-5 1; 4 -2];
[V,D] = eig(A);

v1 = V(:,1)./min(V(:,1))
```

```
v1 = 2×1
     1
    -1
```

```
v2 = V(:,2)./min(V(:,2))
```

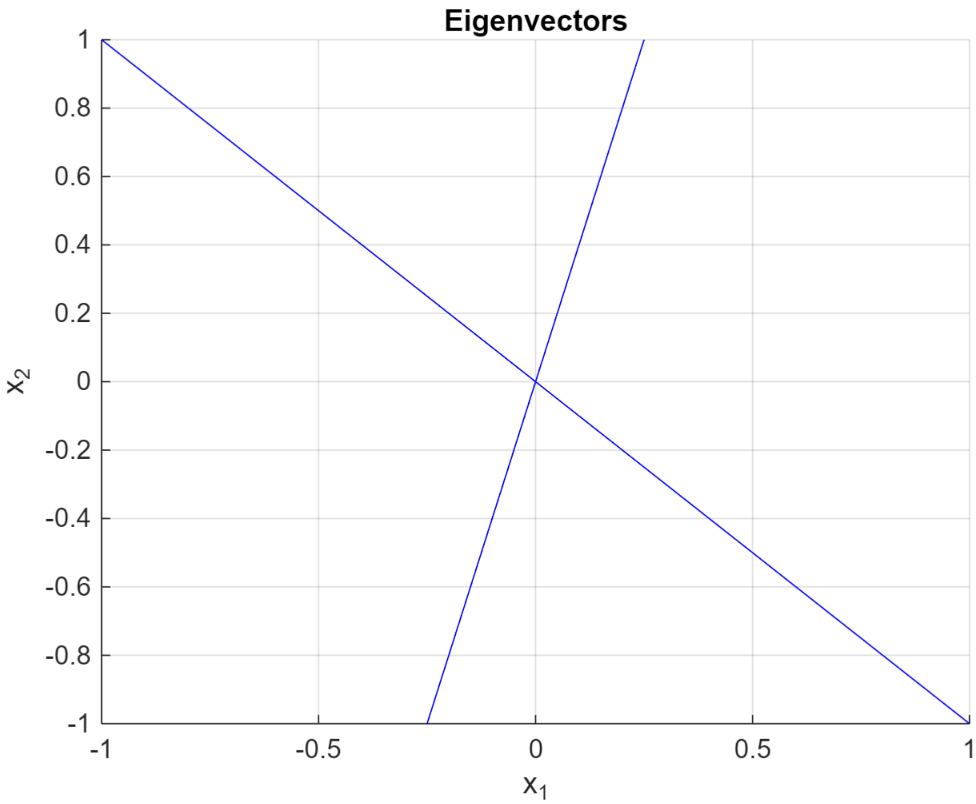
```
v2 = 2×1
    0.2500
    1.0000
```

```
x1Vec = -1:0.05:1;
x2Vec = -1:0.05:1;

vec1 = -1*x1Vec;
vec2 = 4*x2Vec;

figure;
hold on;
plot(x1Vec,vec1,'b');
plot(x2Vec,vec2,'b');
xlabel('x_{1}');
ylabel('x_{2}');
grid on;
```

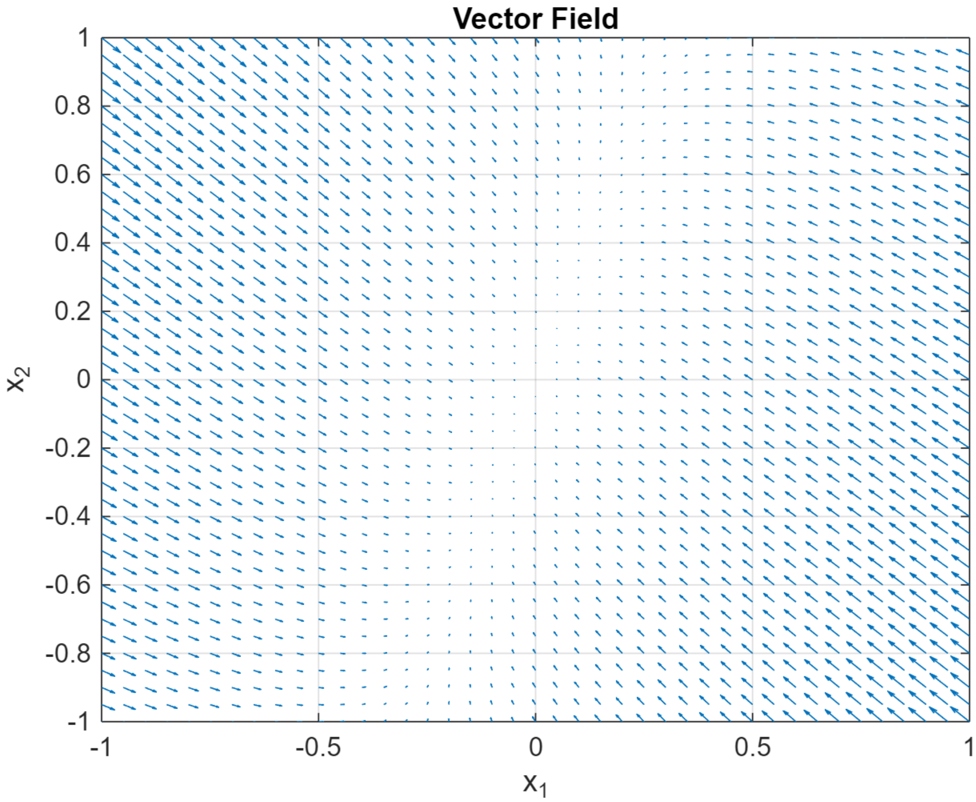
```
title('Eigenvectors');
xlim([-1 1]);
ylim([-1 1]);
```



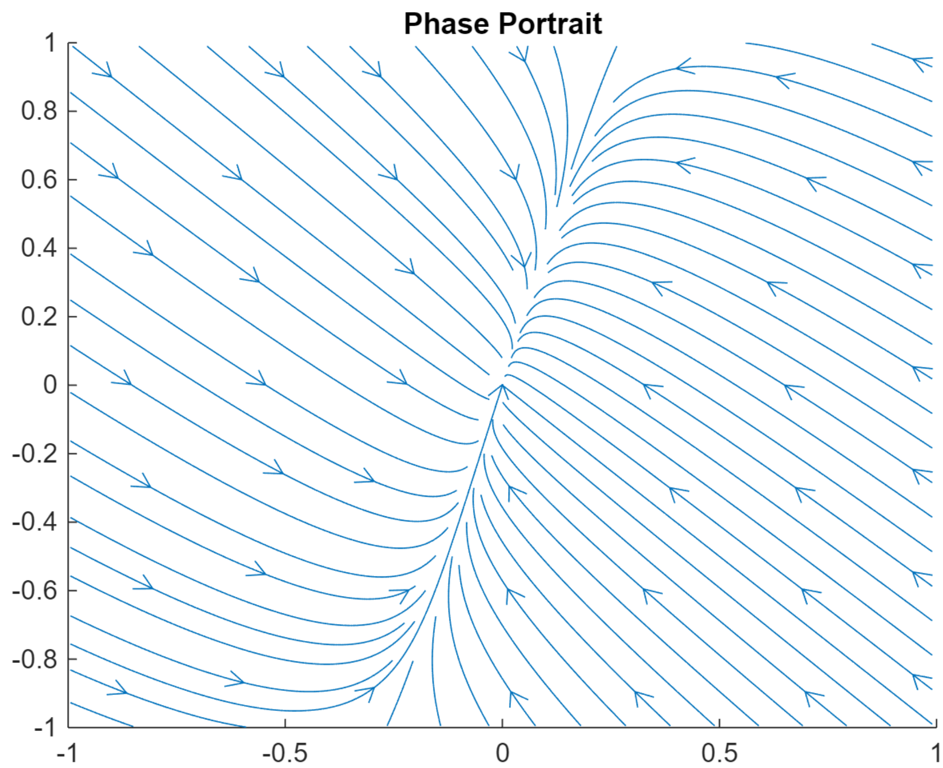
Problem 2: Consider the same system of differential equations in Problem 1. Use MATLAB to plot the phase portrait of this system of differential equations. Make sure to label both axes and title your figure. Use the same x1-axis and x2-axis limits as in Problem 1.

```
[x1, x2] = meshgrid(x1Vec,x2Vec);
x1dot = -5*x1 + x2;
x2dot = 4*x1 - 2*x2;

figure;
quiver(x1,x2,x1dot,x2dot);
xlabel('x_{1}');
ylabel('x_{2}');
grid on;
title('Vector Field');
xlim([-1 1]);
ylim([-1 1]);
```




```
figure;
 streamslice(x1Vec,x2Vec,x1dot,x2dot);
 title('Phase Portrait');
```



Problem 3: Analyze the phase portrait from Problem 2 for the initial conditions of $x_1(0) = 0.5$ and $x_2(0) = 0.3$. As $t \to \infty$ to what values do $x_1(t)$ and $x_2(t)$ converge to?

As $t \longrightarrow \infty$ both $x_1(t)$ and $x_2(t)$ converge towards the origin of $(0, 0)$. This suggests that the origin is the stable equilibrium point for these two linear systems. The phase portrait provides a more clear depiciton of the contours and direction of the arrows contained in the vector field.