Week 6 - Introduction to ode45()

MAT330: Differential Equations

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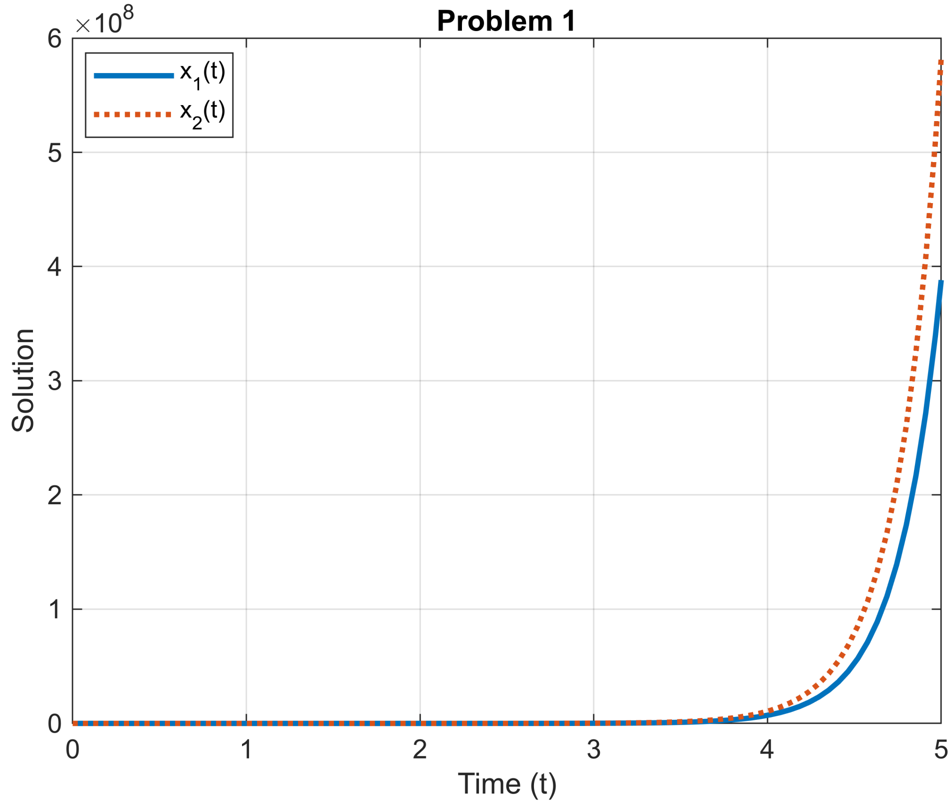
Problems:

Problem 1: Consider the system of differential equations $x_1{}' = x_1 + 2x_2$, $x_2{}' = 3x_1 + 2x_2$ with initial conditions $x_1(0) = 1, x_2(0) = 1$. Use the MATLAB ode45() function to solve this system of differential equations and then plot the solution on a single plot for time $t = 0$ to $t = 5$. Plot $x_1(t)$ as a solid line, $x_2(t)$ as a dotted line, and turn on the plotting grid and legend. Make sure to label your axes. Explain what happens to the solutions as $t \to \infty$. Does this answer change if the initial conditions change?

```
clear all;

function dXdt = problem01ODEFunction(t,X)
    x1 = X(1);
    x2 = X(2);

    dx1dt = x1 + 2*x2;
    dx2dt = 3*x1 + 2*x2;

    dXdt = [dx1dt; dx2dt];
end

opts = odeset('RelTol',1e-6,'AbsTol',1e-8);

soln = ode45(@problem01ODEFunction,[0 5],[1 1],opts);

figure;
plot(soln.x,soln.y(1,:),'-','linewidth',2);
hold on;
plot(soln.x,soln.y(2,:),':','linewidth',2);
grid on;
title('Problem 1');
xlabel('Time (t)');
ylabel('Solution');
legend('x_{1}(t)','x_{2}(t)','location','best');
```

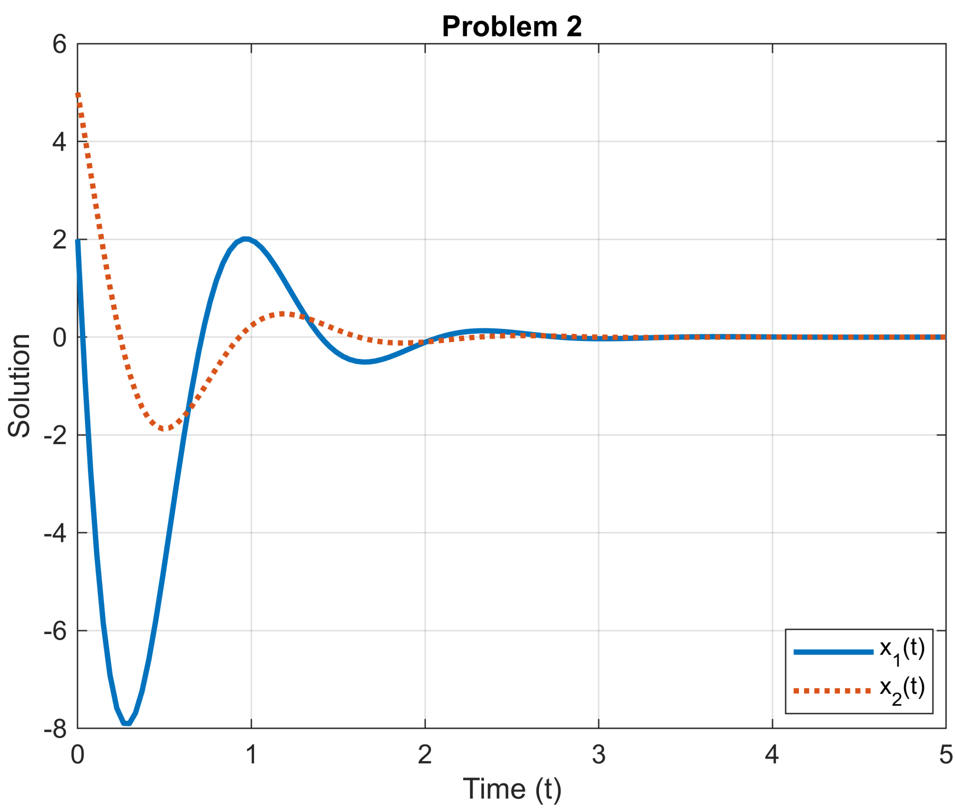


As $t \to \infty$ so does the function. Both systems of equations increase positively towards infinity. By changing the initial conditions in MATLAB, I observed no change in the behavior of the system of differential equations.

Problem 2: Consider the system of differential equations $x_1{}' = x_1 - 15x_2$, $x_2{}' = 2x_1 - 5x_2$ with initial conditions $x_1(0) = 2, x_2(0) = 5$. Use the MATLAB ode45() function to solve this system of differential equations and then plot the solution on a single plot for time $t = 0$ to $t = 5$. Plot $x_1(t)$ as a solid line, $x_2(t)$ as a dotted line, and turn on the plotting grid and legend. Make sure to label your axes. Explain what happens to the solutions as $t \to \infty$.

```
clear all;

function dXdt = problem02ODEFunction(t,X)
    x1 = X(1);
    x2 = X(2);

    dx1dt = x1 - 15*x2;
    dx2dt = 2*x1 - 5*x2;

    dXdt = [dx1dt; dx2dt];
end
```

```
opts = odeset('RelTol',1e-6,'AbsTol',1e-8);

soln1 = ode45(@problem02ODEFunction,[0 5],[2 5],opts);

figure;
plot(soln1.x,soln1.y(1,:),'-','linewidth',2);
hold on;
plot(soln1.x,soln1.y(2,:),':','linewidth',2);
grid on;
title('Problem 2');
xlabel('Time (t)');
ylabel('Solution');
legend('x_{1}(t)','x_{2}(t)','location','best');
```

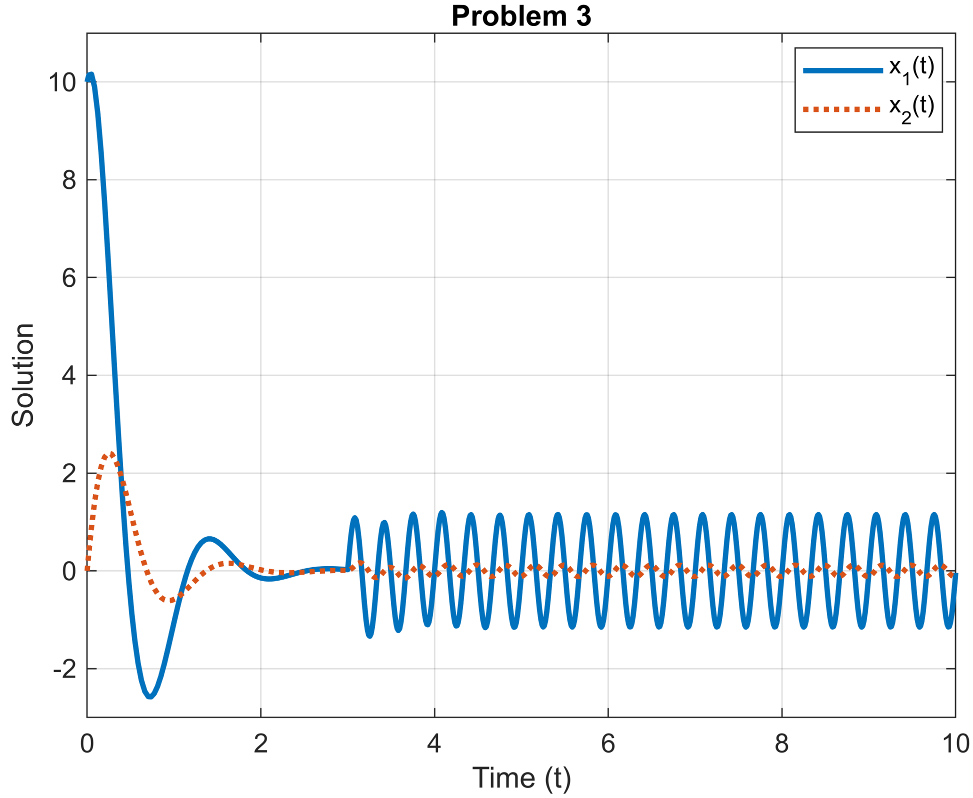


As $t \to \infty$ the solutions to the system of differential equations approach zero. They fluctuate rising above zero and falling below zero for a bit but the slowly increase positively towards zero from the negative half of the y-axis.

Problem 3: Consider the system of differential equations $x_1{}' = x_1 - 15x_2 + 20\cos(6\pi t)u(t-3)$, $x_2{}' = 2x_1 - 5x_2$ **with initial conditions** $x_1(0) = 10, x_2(0) = 0$**. Use the MATLAB ode45() function to solve this system of differential equations and then plot the solution on a single plot for time** $t = 0$ **to**

$t = 10$. **Plot $x_1(t)$ as a solid line, $x_2(t)$ as a dotted line, and turn on the plotting grid and legend. Make sure to label your axes. Explain what happens to the solutions as $t \to \infty$.**

```
clear all;

function dXdt = problem03ODEFunction(t,X)
    x1 = X(1);
    x2 = X(2);

    dx1dt = x1 - 15*x2 + 20*cos(6*pi*t)*heaviside(t-3);
    dx2dt = 2*x1 - 5*x2;

    dXdt = [dx1dt; dx2dt];
end

opts = odeset('RelTol',1e-6,'AbsTol',1e-8);

soln2 = ode45(@problem03ODEFunction,[0 10],[10 0],opts);

figure;
plot(soln2.x,soln2.y(1,:),'-','linewidth',2);
hold on;
plot(soln2.x,soln2.y(2,:),':','linewidth',2);
grid on;
title('Problem 3');
xlabel('Time (t)');
ylabel('Solution');
legend('x_{1}(t)','x_{2}(t)','location','best');
ylim([-3,11]);
```

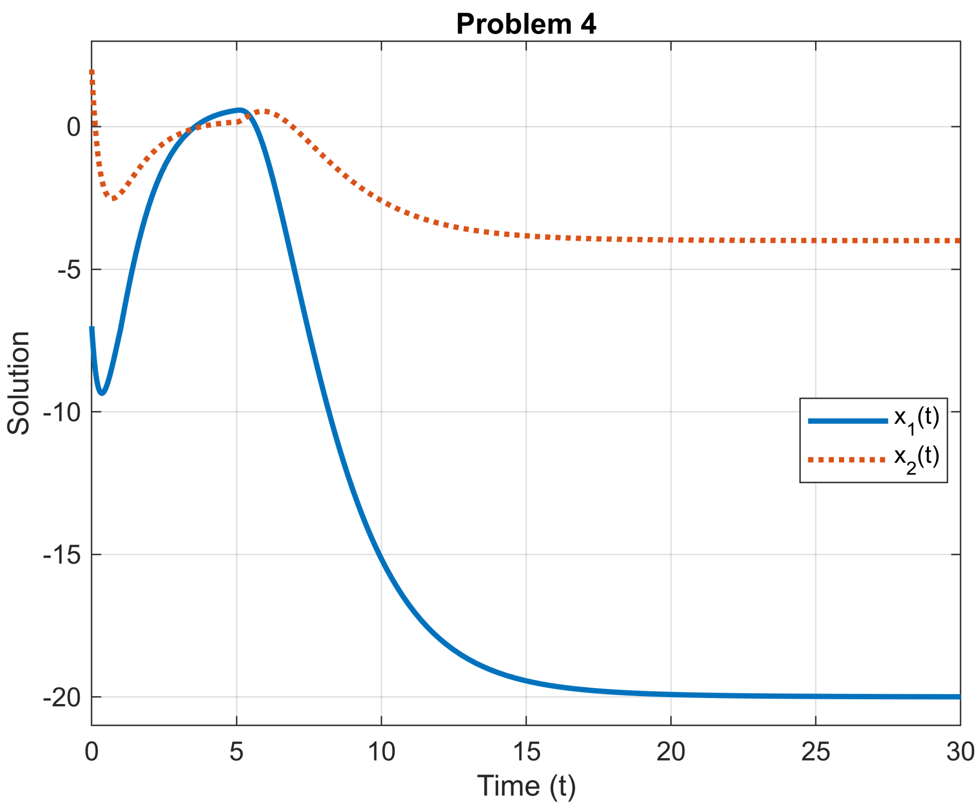


As $t \to \infty$ the solutions the each function has a different outcome. The solution to the function x_1 oscillates between approximately -1 and 1. By zooming in on our plot produced by MATLAB, we can obtain the information that the solution for x_2 oscillates between approximately -0.1 and 0.1.

Problem 4: Consider the system of differential equations $x_1' = x_1 - 5x_2 + e^{-0.2t}u(t-1)$, $x_2' = 2x_1 - 5x_2 - 0.5x_1' + 10(1 - e^{-0.5(t-5)})u(t-5)$ with initial conditions $x_1(0) = -7, x_2(0) = 2$. Use the MATLAB ode45() function to solve this system of differential equations and then plot the solution on a single plot for time $t = 0$ to $t = 30$. Plot $x_1(t)$ as a solid line, $x_2(t)$ as a dotted line, and turn on the plotting grid and legend. Make sure to label your axes. As $t \to \infty$, what do $\frac{dx_1}{dt}$ and $\frac{dx_2}{dt}$ converge to? Explain.

```
clear all;

function dXdt = problem04ODEFunction(t,X)
    x1 = X(1);
    x2 = X(2);
```

```
    dx1dt = x1 - 5*x2 + exp(-0.2*t)*heaviside(t-1);
    dx2dt = 2*x1 - 5*x2 - 0.5*x1' +10*(1 - exp(-0.5*(t-5)))*heaviside(t-5);

    dXdt = [dx1dt; dx2dt];
end

opts = odeset('RelTol',1e-6,'AbsTol',1e-8);

soln3 = ode45(@problem04ODEFunction,[0 30],[-7 2],opts);

figure;
plot(soln3.x,soln3.y(1,:),'-','linewidth',2);
hold on;
plot(soln3.x,soln3.y(2,:),':','linewidth',2);
grid on;
title('Problem 4');
xlabel('Time (t)');
ylabel('Solution');
legend('x_{1}(t)','x_{2}(t)','location','best');
ylim([-21,3])
```



For x_1, as $t \to \infty$, the solution approaches -20. For x_2, as $t \to \infty$, the solution approaches -4. Since both x_1 and x_2 approach constant values, their derivatives will approach zero. Therefore, $\frac{dx_1}{dt}$ and $\frac{dx_2}{dt}$ converge to zero.