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Effects of delay and probability combinations on discounting in humans

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ABSTRACT

To determine discount rates, researchers typically adjust the amount of an immediate or certain option relative to a delayed or uncertain option. Because this adjusting amount method can be relatively time consuming, researchers have developed more efficient procedures. One such procedure is a 5-trial adjusting delay procedure, which measures the delay at which an amount of money loses half of its value (e.g., $1000 is valued at $500 with a 10-year delay to its receipt). Experiment 1 (n = 212) used 5-trial adjusting delay or probability tasks to measure delay discounting of losses, probabilistic gains, and probabilistic losses. Experiment 2 (n = 98) assessed combined probabilistic and delayed alternatives. In both experiments, we compared results from 5-trial adjusting delay or probability tasks to traditional adjusting amount procedures. Results suggest both procedures produced similar rates of probability and delay discounting in six out of seven comparisons. A magnitude effect consistent with previous research was observed for probabilistic gains and losses, but not for delayed losses. Results also suggest that delay and probability interact to determine the value of money. Five-trial methods may allow researchers to assess discounting more efficiently as well as study more complex choice scenarios.

1. Introduction

Humans often choose between outcomes that differ in delay, probability, and amount. For example, smoking cigarettes may result in immediate relief from withdrawal and delayed, yet uncertain, adverse health outcomes. Abstaining from smoking may result in immediate discomfort and delayed and uncertain positive health outcomes. Similar differences in delay, probability, and amount of adverse or beneficial outcomes may occur for a number of health-related behaviors such as physical exercise, balanced nutrition, substance use, and risky sexual behavior.

Researchers have been primarily interested in the effects of manipulating one or two of these dimensions on choice (e.g., Ritschel et al., 2015; McKerchar et al., 2009; Bruce et al., 2015; McKerchar and Renda, 2012). For example, researchers have studied how delay to receiving a commodity affects its current value (see Odum, 2011; for review). The tendency for the current value of a commodity to decrease as a function of delay to receipt is termed delay discounting. The rate at which a commodity loses value can be described mathematically by a hyperbolic equation (Mazur, 1987):

$$V = \frac{A}{(1 + k\mathrm{D})}. \tag{1}$$

In this equation, V is the current value of a delayed commodity, A is the undiscounted value of the commodity, D is the delay to receipt of that commodity, and k is a parameter representing rate of delay discounting. Other researchers have assessed how the probability of obtaining a commodity affects its current value (see McKerchar and Renda, 2012; for review). A reduction in the current value as a function of the odds against receiving the commodity is termed probability discounting. The same hyperbolic equation used to describe delay discounting can be extended to probability discounting:

$$V = \frac{A}{(1 + \mathrm{h}\theta)}. \tag{2}$$

Odds against (Θ) is substituted for delay and is calculated as (1-p)/p, where p is the probability of receiving the commodity. V and A are the same as Eq. (1) and h is a parameter representing rate of probability discounting (i.e., how the value of a commodity reduces as a function of increasing uncertainty).

Researchers have observed several patterns when studying delay and probability discounting. When the outcomes are gains,

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the magnitude of the outcome alters the rate of discounting (i.e., a magnitude effect). Delay discounting decreases as magnitude increases (e.g., Green et al., 1997). In contrast, probability discounting increases as magnitude increases (e.g., Green et al., 1999; Yi and Bickel, 2005). Unlike outcomes involving gains, however, researchers have not found a magnitude effect in delay or probability discounting when the outcomes are described as losses (Estle et al., 2006; Green et al., 2014). Although an integrated, theoretical account of these patterns has not been widely accepted, they nevertheless provide benchmarks to evaluate the validity of new methods to measure discounting.

A challenge in assessing the influence of multiple outcomes on choice is that traditional methods involve relatively lengthy procedures when choice alternatives become more complex. For example, Du et al. (2002) assessed delay discounting using a common adjusting amount procedure at seven delays. A series of six choices were presented at each of seven delays resulting in 42 total response trials for each participant. Using a similar adjusting amount method, the total number of responses increases to 125 when five choices are presented for alternatives that are both delayed and probabilistic (five delays x five probabilities x five response trials per combination of delay and probability; e.g., Vanderveldt et al., 2015). If one is interested in studying the effects of two outcomes, where both alternatives are delayed and probabilistic, the total number of responses by each participant expands to 3125 (125 trials for delayed and probabilistic first outcome x five delays of the second outcome x five probabilities of the second outcome). The duration of task administration for more complex discounting scenarios may limit practicality for researchers and result in participant fatigue, which may influence the quality of responses. Therefore, a more efficient method is needed to measure the effects of multiple outcomes on choice behavior.

Koffarnus and Bickel (2014) described a procedure to determine a delay at which a commodity loses half of its value (i.e., effective delay of 50% or ED_{50}). In this procedure, the immediate alternative is fixed at half the delayed alternative (e.g., \$500 if the delayed alternative is \$1000). The delay to the larger amount adjusts over five trials based on participant responses. The final delay (i.e., ED_{50}) following these adjustments provides the discounting parameter for that participant (Yoon and Higgins, 2008). Koffarnus and Bickel (2014) demonstrated that this 5-trial adjusting delay task provides similar *k*s compared to traditional adjusting amount tasks. In addition, their results replicated several other effects from the discounting literature (Green et al., 1997; Bickel et al., 2008; Yi et al., 2006; Estle et al., 2007; Jimura et al., 2009; Magen et al., 2008; Radu et al., 2011), including the magnitude effect, which suggests that this method is valid for delayed gains (see Koffarnus and Bickel, 2014 for full explanation).

Experiment 1 sought to extend 5-trial adjusting delay and probability tasks to delayed losses, probabilistic gains, and probabilistic losses. Experiment 2 extended 5-trial tasks to examine the combined effects of delayed and probabilistic gains, and delayed and probabilistic losses. We compared discounting rates obtained from 5-trial tasks to traditional adjusting amount method. In addition, we assessed whether 5-trial adjusting delay and probability tasks would result in magnitude effects similar to traditional measures of discounting.

2. Experiment 1

Koffarnus and Bickel (2014) demonstrated that traditional adjusting amount and 5-trial adjusting tasks produced similar rates of discounting for delayed gains. The authors also found evidence for the magnitude effect. Experiment 1 sought to extend the 5-trial adjusting delay procedure to probabilistic gains, probabilistic losses, and delayed losses. Specifically, we compared 5-trial adjusting delay and probability procedures to traditional adjusting amount procedures. In addition, we manipulated amount using the 5-trial adjusting tasks to evaluate the magnitude effect.

2.1. Method

2.1.1. Participants

Two hundred and twelve participants were recruited from the Psychology participant pool from a large public university in the southeast United States. The average age of participants was 19.09 (range 18–22) and 68% were female. Participants were randomly assigned to receive hypothetical monetary outcomes that involved delayed loss, probabilistic loss, or probabilistic gain.

2.1.2. Delayed loss

Each participant assigned to the delayed loss condition completed three discounting tasks. This included a traditional adjusting amount task with the larger-later value of \$1000, a 5-trial adjusting delay task with a larger-later amount of \$1000, and a 5-trial adjusting delay task with a larger-later amount of \$10. Individual trials were presented by asking the participant, "Would you prefer losing \$(*amount*) immediately or \$1000 in (*delay*)?"

The traditional adjusting amount procedure was completed for seven different delays. Delays assessed were 1-week, 1-month, 4-months, 8-months, 12-months, 5-years, and 10-years. The first trial always presented the immediate amount at half the value of the delayed amount. The amount of the delayed option stayed the same for all trials (i.e., \$1000). The amount of the immediate alternative adjusted following each choice made by a participant and was rounded to the nearest dollar for ease of presentation. Specifically, the immediate amount increased if the immediate option was chosen or decreased if the delayed option was chosen. The amount of the immediate alternative adjusted by 25% of the larger amount following the first trial, by 12.5% of the larger amount following the second trial, and by 6.25%, and 3.125% of the larger amount following the third and fourth trials. The amount of the immediate option adjusted by 1.5625% of the larger amount following the fifth trial and the resulting value was selected as the indifference point for the participant at that particular delay.

An identical version of the 5-trial adjusting delay task from Koffarnus and Bickel (2014) was used for participants assigned to the delay loss condition in Experiment 1. Table 1 contains the 31 potential delays (i.e., Indices 1–31) and the potential trial number that each delay could be presented to a participant. The immediate amount remained fixed at half of the larger delayed amount for all trials (i.e., \$500 vs. \$1000, or \$5 vs. \$10). The participant was first presented with a choice between a smaller immediate amount and a larger amount at a 3-week delay (i.e., index 16, trial number 1). If the participant selected the immediate alternative, the delay to the larger amount increased by 8 indices (i.e., to a delay of 2-years at index 24, trial number 2). If the participant selected the delayed alternative, the delay to the larger amount decreased by 8 indices (i.e., to 1-day, index 8, trial number 2). Following the choice made on the second, third, and fourth trial, the delay to the larger alternative increased if the immediate alternative was chosen and decreased if the delayed alternative was chosen. Delays adjusted by 4, 2, and 1 indices, following the second, third, and fourth trials respectively. Fig. 1 shows an example of how choice alternatives would change based on the pattern of responses made by a hypothetical participant. The left side displays hypothetical participant responding for the 5-trial adjusting delay task.

The choice alternative selected for the fifth trial was used to determine *k* as outlined in Table 1. For immediate choices at fifth trial indices, *k* was calculated by dividing 1 by the geometric mean of the delay for the fifth trial and the delay at the index immediately

Table 1
The referencing index number, delay presented with the specific choice, the trial number a given delay could be presented at, and the resulting delays with corresponding k parameter. *Note.* ED_{50} = Effective Delay 50%.

			ED_{50} (days) if last choice is:		k if last choice is:	
Index	Delay Choice	Trial No.	Immediate	Delayed	Immediate	Delayed
1	1 h	5	0.05893	0.04167	17.0	24.0
2	2 h	4				
3	3 h	5	0.1444	0.1021	6.93	9.79
4	4 h	3				
5	6 h	5	0.3062	0.2041	3.27	4.90
6	9 h	4				
7	12 h	5	0.7071	0.4330	1.41	2.31
8	1 day	2				
9	1.5 days	5	1.732	1.225	0.577	0.816
10	2 days	4				
11	3 days	5	3.464	2.450	0.289	0.408
12	4 days	3				
13	1 week	5	8.573	5.292	0.117	0.189
14	1.5 weeks	4				
15	2 weeks	5	17.15	12.12	0.0583	0.0825
16	3 weeks	1				
17	1 month	5	43.05	25.28	0.0232	0.0396
18	2 months	4				
19	3 months	5	105.4	74.56	0.00949	0.0134
20	4 months	3				
21	6 months	5	210.9	149.1	0.004741	0.00671
22	8 months	4				
23	1 year	5	516.5	298.2	0.00194	0.00335
24	2 years	2				
25	3 years	5	1265	894.7	0.000791	0.00112
26	4 years	4				
27	5 years	5	2310	1633	0.000433	0.000612
28	8 years	3				
29	12 years	5	5368	3579	0.000186	0.000279
30	18 years	4				
31	25 years	5	9131	7748	0.000110	0.000129

Would you prefer…

Trial Number	Index Number	Losing…	or	Losing…
1	16	$500 Immediately		$1000 in 3 Weeks
2	24	$500 Immediately		$1000 in 2 Years
3	20	$500 Immediately		$1000 in 4 months
4	22	$500 Immediately		$1000 in 8 months
5	21	$500 Immediately		$1000 in 6 months
	Final k Parameter	0.004741		

Index Number	Losing…	or	Losing…
16	100% Chance of $500		50% Chance of $1000
8	100% Chance of $500		76% Chance of $1000
12	100% Chance of $500		63% Chance of $1000
14	100% Chance of $500		57% Chance of $1000
13	100% Chance of $500		60% Chance of $1000
Final h Parameter	1.410096		

Fig. 1. Example adjustments of choice alternatives in the 5-trail adjusting delay loss (left) and 5-trial adjusting probability loss (right) tasks. The trial number represents the order of choice presentation. The index number corresponds to the index listing in Table 1 (delay discounting) or in Table 2 (probability discounting). Circled alternatives represent the choice made by a hypothetical participant leading to the presented choice index in the following trial. The final parameter represents the rate at which the value of $1000 reduces as a function of increasing delay (k) or increasing odds against (h).

below it. For example, if a participant selected the immediate alternative at index 21, the delay at the final index would be 6 months and the delay at the index immediately below is 8 months. The geometric mean of 183 and 243 is 211.74. Dividing 1 by 211.74 equals 0.0047 which is the value listed in the "k if last choice is: immediate" column in Table 1. If the delayed alternative was selected on the fifth trial, k was calculated by dividing 1 by the geometric mean of the delay of the fifth trial and the delay at the index immediately above it. The above adjusting delay task allows for obtaining 1 of 32 potential indifference points for each participant. These indifference points are spread approximately logarithmically between 1 h and 25 years. Five-trial adjusting delay tasks were completed with amounts of $1000 and $10.

2.1.3. Probabilistic loss

Each participant assigned to the probabilistic loss condition completed three discounting tasks. This included a traditional adjusting amount task with the larger-uncertain value of $1000, a 5-trial adjusting delay task with a larger-uncertain amount of $1000, and a 5-trial adjusting delay task with a larger-uncertain amount of $10.

The traditional adjusting amount procedure was completed for seven different probabilities and a larger-later amount of $1000. Probabilities assessed were 95%, 80%, 50%, 25%, 10%, 5%, and 1%. The first trial always presented the certain amount at half the value of the uncertain amount. The smaller-certain amount adjusted up or down following each choice made by a participant. The smaller-certain amount changed by 25%, 12.5%, 6.25%, 3.125% and 1.5625% of the larger amount following the first through fifth trials, respectively. The smaller-certain amount following adjustment after the fifth trial was used as the indifference point for the participant at that probability.

A modified version of the adjusting delay 5-trial adjusting delay tasks from Koffarnus and Bickel (2014) was used for participants assigned to the probability loss condition in Experiment 1. Table 2 contains the 31 potential probabilities (i.e., Indices 1–31) and the potential trial number that each probability could be presented to a participant. Adjustment following each choice occurred in a similar manner to the 5-trial adjusting delay task described above. That is, the probability of the uncertain alternative increased if the certain alternative was chosen, and decreased if the uncertain alternative was chosen. In addition, h was calculated in the same manner as outlined above for each participant based on their choice of the certain or uncertain alternative for the fifth trial. The right side of Fig. 1 shows an example of how choice alternatives would change based on the pattern of responses made by a hypothetical participant on the 5-trial adjusting probability task.

2.1.4. Probabilistic gain

Participants randomly assigned to the probabilistic gain condition completed a traditional adjusting amount task and two 5-trial adjusting probability tasks. The tasks and manner of calculating h for the probabilistic gain group were identical to the probabilistic loss group described above with one exception. The only difference for this group was that adjustment following each choice occurred in the opposite direction than the probabilistic loss group. Selecting the certain alternative in the traditional adjusting amount task decreased the amount of the certain alternative for the next trial. Selecting the certain alternative in the 5-trial adjusting probability tasks increased the probability of the uncertain alternative for the next trial.

2.1.5. Ordering of tasks

All traditional tasks were presented in both an ascending and descending order with the average indifference value used for subsequent fitting of discounting models. The ordering of traditional tasks and 5-trial adjusting delay or probability tasks were presented randomly for each participant.

2.1.6. Data exclusion criteria

Each participant's data from the traditional adjusting amount task were analyzed to determine if it met criteria proposed by Johnson and Bickel (2008). If the criteria were met, all data sets from that participant were removed from the analysis. This algorithm for eliminating nonsystematic delay discounting responses was chosen based on recent findings the algorithm excludes fewer cases and is uncorrelated with log k values (White et al., 2015). The above data exclusion criteria reduced the total number of response sets from 212 to 172.

2.1.7. Data analysis

Discounting parameters were estimated for each participant using Microsoft Excel Solver Add-In for Microsoft Excel 2013. Parameters were log transformed prior to statistical comparisons (Yoon and Higgins, 2008). Results from the adjusting amount task and the 5-trial adjusting tasks were compared using a Pearson's product-moment correlation coefficient. Comparisons between amounts were assessed using paired student's t-tests.

2.2. Results and discussion

Averaged parameter estimates for each task are presented in Table 3. We found significant correlations between discounting parameters derived from the traditional task and the 5-trial adjusting delay and probability tasks for delayed losses at \$1000 ($r = 0.84$, $p < 0.0001$), probabilistic losses of \$1000 ($r = 0.35$, $p = 0.003$), and probabilistic gains of \$1000 ($r = 0.51$, $p < 0.001$). Koffarnus and Bickel found a correlation of 0.67 ($p < 0.001$) for delayed gains between the two tasks (2014). Differences in mean ks and hs were observed between the adjusting amount and 5-trial adjusting delay or probability tasks. Although the absolute parameter values differed, the relative difference in values remained consistent. Thus, assessing effects of pharmacological or behavioral interventions using 5-trial tasks is likely to yield similar outcomes to those using adjusting amount tasks.

To validate further the 5-trial adjusting task, we assessed the effects of magnitude for delayed and probabilistic losses. In contrast to previous research (e.g., Green et al., 2014), for delayed losses we found a significantly greater k as magnitude increased ($p = 0.001$, $t_{62} = 3.35$). However, consistent with previous research (Green et al., 2014), we did not observe a magnitude effect between probabilistic losses of \$1000 and \$10 ($p = 0.12$, $t_{69} = 1.56$). Finally, and also consistent with previous research (e.g., Estle et al., 2006), we observed significantly greater hs for gains of \$1000 compared to \$10 ($p < 0.001$, $t_{38} = 3.76$).

Of the three magnitude comparisons in Experiment 1, only delayed losses were inconsistent with previous research. Green et al. (2014), however, examined a much wider, parametric range of magnitudes and found no systematic change in discounting. Although the ks differed at magnitudes similar to those used in the present study (i.e., \$20 and \$3000), there was no functional relation across the full range of magnitudes. Thus, the magnitude effect in the present study may be a spurious outcome of using a narrow range of magnitudes.

The ultimate goal of these experiments was to determine if 5-trial adjusting delay or probability tasks would provide an efficient method to analyze more complex choice scenarios. The results from Experiment 1 suggest that 5-trial adjusting probability tasks are useful toward this end. Combined with the results from Koffarnus and Bickel (2014), 5-trial adjusting delay tasks likely provide an accurate and efficient method for incorporating delay discounting into more complex scenarios. Thus, Experiment 2 extended 5-trial adjusting delay and probability tasks to monetary gains and losses that are delayed and probabilistic.

3. Experiment 2

Researchers assessing delayed and probabilistic alternatives have observed three patterns. First, probability seems to affect the current value of a commodity to a greater extent than delay (Vanderveldt et al., 2015; Weatherly et al., 2015; cf. Bialaszek et al., 2015). That is, the current value of money remains relatively unchanged across increasing delays when choices are both delayed and probabilistic. Second, increases in delay and decreases in probability may reduce the current value of money multiplicatively as opposed to additively (Vanderveldt et al., 2015). In other words, the change in value of a reinforcer that is delayed and probabilistic is best described when the denominator from Eq. (1) is multiplied by the denominator in Eq. (2), as opposed to subtracting delay from probability or probability from delay. Finally, there is no magnitude effect for delayed and probabilistic gains (Vanderveldt et al., 2015).

To compare results from Experiment 2 to previous literature, we assessed delay discounting at each probability and probability discounting at each delay. Hyperbola-like discounting functions for delay (Eq. (3)) and probability (Eq. (4)) were used to complete this analysis (Green et al., 1994). These equations can be written as:

$$V = \frac{A}{(1 + k\mathrm{D})^{s}d}, \tag{3}$$

Table 2
The referencing index number, probability presented with the specific choice, corresponding odds against, the trial number a given probability could be presented at, and the resulting certainties with corresponding *h* parameter. *Note.* EP_{50} = Effective Probability 50%.

				EP_{50} (%) for Loss Group if last choice is:		*h* if last choice is:	
Index	Uncertain Choice	Odds Against	Trial No.	Certain	Uncertain	Certain	Uncertain
1	99%	0.0101	5	0.020515248	0.010050378	48.744230	99.498744
2	96%	0.0417	4				
3	92%	0.0870	5	0.103669838	0.060192927	9.646007	16.613248
4	89%	0.1236	3				
5	86%	0.1628	5	0.182599762	0.141845686	5.476458	7.049915
6	83%	0.2048	4				
7	79%	0.2658	5	0.289730974	0.233335875	3.451478	4.285668
8	76%	0.3158	2				
9	73%	0.3699	5	0.39813656	0.341758462	2.511701	2.926043
10	70%	0.4286	4				
11	66%	0.5152	5	0.550044819	0.469871494	1.818034	2.128241
12	63%	0.5873	3				
13	60%	0.6667	5	0.709171331	0.62572709	1.410096	1.598141
14	57%	0.7544	4				
15	53%	0.8868	5	0.941696582	0.817914287	1.061913	1.222622
16	50%	1.0000	1				
17	47%	1.1277	5	1.222621999	1.061913167	0.817914	0.941697
18	43%	1.3256	4				
19	40%	1.5000	5	1.598140812	1.410096484	0.625727	0.709171
20	37%	1.7027	3				
21	34%	1.9412	5	2.128241472	1.818033669	0.469871	0.550045
22	30%	2.3333	4				
23	27%	2.7037	5	2.926043129	2.511701012	0.341758	0.398137
24	24%	3.1667	2				
25	21%	3.7619	5	4.2856676	3.451477714	0.233336	0.289731
26	17%	4.8824	4				
27	14%	6.1429	5	7.049914801	5.476458403	0.141846	0.182600
28	11%	8.0909	3				
29	8%	11.5000	5	16.61324773	9.646007181	0.060193	0.103670
30	4%	24.0000	4				
31	1%	99.0000	5	99.00000000	48.74423043	0.010101	0.020515

Table 3
Mean and 95% confidence interval of discount parameters for each task. Pearson correlations and paired student's *t*-test values for comparison between discounting tasks.

Task		Mean *k* or *h* (95% CI)	Pearson Correlation	Student's *t*
Delayed	Adjusting Amount \$1000	0.50 (0.33)	0.84***	–
Losses	5-Trial Adjusting Delay \$1000	0.30 (0.31)		3.35*
	5-Trial Adjusting Delay \$10	0.15 (0.07)	–	
Probabilistic	Adjusting Amount \$1000	1.65 (0.55)	0.35**	–
Losses	5-Trial Adjusting Delay \$1000	2.54 (1.35)		1.56
	5-Trial Adjusting Delay \$10	1.22 (3.71)	–	
Probabilistic	Adjusting Amount \$1000	3.09 (0.97)	0.51***	–
Gains	5-Trial Adjusting Delay \$1000	2.41 (0.58)		3.76***
	5-Trial Adjusting Delay \$10	1.53 (0.61)	–	

Asterisks indicate a significant difference between tasks.
* $p < 0.05$.
** $p < 0.01$.
*** $p < 0.001$.

$$V = \frac{A}{\left(1 + h\theta\right)^{s}p}. \quad (4)$$

The differences from the hyperbolic equations in Experiment 1 are the added *s* parameters – s_d for delay and s_p for probability. The *s* parameter is often added to discounting models describing human choice as the current value of commodities at long delays or high odds against tend to level off. The *s* parameter reflects nonlinear scaling of amount and delay based on power laws from psychophysical research (see Vanderveldt et al., 2016 for overview). Values of *s* less than 1.0 account for such flattening. Values of 1.0 for *s* cause the hyperbola-like equation to reduce to the hyperbolic discounting functions used in Experiment 1.

We also evaluated the hyperbolic additive (Eq. (5)) and hyperbolic multiplicative (Eq. (6)) discounting models from Vanderveldt et al. (2015). The additive and multiplicative models can, respectively, be written as:

$$V = A - A\left(1 - \frac{1}{(1 + kD)^{s_d}}\right) - A\left(1 - \frac{1}{\left(1 + h\theta\right)^{s_p}}\right), \quad (5)$$

$$V = \frac{A}{\left[(1 + kD)^{s_d} * \left(1 + h\theta\right)^{s_p}\right]}. \quad (6)$$

In these models, *V*, *A*, *D*, Θ, *k*, *h*, s_d, and s_p represent the same parameters of discounting noted above. The primary implication of each model is whether the influence of delay and probability depend on the level of the other dimension or if the influence of delay and probability are independent. If the additive model describes data more accurately, delay and probability could be argued as influencing choice behavior independently. If the multiplicative model describes the data more accurately, then the influence of delay would depend on the probability of the outcome occurring and the influence of probability would depend on the delay to the outcome.

3.1. Method

3.1.1. Participants

Ninety-eight participants were recruited from the Psychology participant pool at a University in southeast United States. The average age of participants was 18.78 (range 18–22) and 73.58% were female. Fifty-four participants completed the delayed and probabilistic gain tasks and 44 participants completed the tasks involving delayed and probabilistic losses.

3.1.2. Delayed and probabilistic gains

Participants assigned to this condition completed three tasks. The first task was a traditional adjusting amount task in which each alternative was both delayed and probabilistic and the larger alternative was $1000. The other two were 5-trial adjusting delay and probability tasks with either a larger amount of $1000 or $10.

The traditional discounting task results in the researcher obtaining 25 indifference points from combinations of delay and probability. Five delays and five probabilities were used. Delays were 0-months (i.e., immediate), 1-month, 6-months, 2-years, and 5-years. Probabilities were 100%, 80%, 40%, 25%, and 10%. The method of adjustment of the immediate amount was the same as the gains condition from Experiment 1 and identical to adjustments by Vanderveldt et al. (2015). A single traditional adjusting amount task was completed for each participant at the larger amount of $1000.

Experiment 2 combined the 5-trial adjusting delay and probability tasks from Experiment 1. The tasks adjusted in similar fashion to the 5-trial adjusting probability tasks with monetary gains described above. The main difference was each choice alternative involved both a delay and a probability of gaining hypothetical money.

Half the participants were exposed to delay adjusting before probability and the other half were exposed to probability adjusting before delay. For example, half of the participants were asked first if they would prefer a 100% chance of getting $500 immediately OR a 100% chance of getting $1000 in 3 weeks. The delay for the larger amount then adjusted over 5 total trials while keeping the probability of each alternative at 100%. The final delay from the fifth trial was then used as the delay for the 5-trials in which probability adjusted. For example, if the fifth trial involved a delay of 1 week, the first trial for the adjusting probability portion would ask the participant to choose between a 100% chance of getting $500 in 1 week OR a 50% chance of getting $1000 in 1 week. The probability of occurrence for the larger amount then adjusted for 5 total trials while the delay remained fixed. The other half of the participants received the adjusting probability portion first with delay fixed at immediate. The probability of occurrence on the fifth trial was then used for both alternatives for the adjusting delay portion.

3.1.3. Delayed and probabilistic losses

Participants assigned to this condition completed the same three tasks as the delayed and probabilistic gains group. The only difference was that the direction of the adjustment was in the opposite direction following each choice made by a participant. That is, selection of the immediate and certain alternative for the traditional adjusting amount task would result in the amount of that alternative increasing on the next trial. Selection of the delayed and uncertain option would result in the amount of the immediate and certain alternative decreasing on the subsequent trial. For the 5-trial adjusting delay and probability tasks, selection of the immediate and certain alternative would result in the delay to the larger alternative increasing or the probability of occurrence of the larger alternative decreasing on the next trial. Selection of the larger delayed and uncertain alternative resulted in the delay to the larger alternative decreasing or probability of occurrence of the larger alternative increasing on the subsequent trial.

3.1.4. Ordering of tasks

The 25 combinations of delay and probability from the adjusting amount task were randomly presented to the participants. The 5-trial adjusting delay and probability task with a larger amount of $1000 and the 5-trial adjusting delay and probability task with a larger amount of $10 were randomly interspersed among the 25 combinations of delay and probability from the adjusting amount task.

3.1.5. Data analysis

Discounting parameters for the adjusting amount tasks were estimated using the Solver Add-In for Microsoft Excel 2013. Discounting parameters for the 5-trial adjusting delay and probability tasks were obtained as described in Sections 3.1.2 and 3.1.3 above. Parameters were estimated for each participant individually, and for the median indifference points (Vanderveldt et al., 2015). To permit comparisons across studies, we did not omit data from Experiment 2 because Vanderveldt et al. did not omit data. Differences between k and h distributions from the 5-trial adjusting delay and probability tasks were analyzed for statistical significance using paired student's t-tests conducted on log-transformed k and h values. We compared h and k parameters from the 5-trial task to the adjusting amount task using Pearson's correlation coefficients: Because the 5-trial task does not allow for estimation of a sensitivity parameter, h and k parameters were estimated using Eq. (6) with the sensitivity parameters fixed at 1.0.

3.2. Results and discussion

Fig. 2 shows the fits of Eqs. (3) and (4) to the median indifference points. The left panels show the hyperbola-like delay discounting function plotted at each probability (Eq. (3)). The right panels show the hyperbola-like probability discounting function plotted at each delay (Eq. (4)). The results for gains are similar to Vanderveldt et al. (2015) and Weatherly et al. (2015). Specifically, rates of probability discounting were steep across all delays, and delay discounting was observed at high probabilities with little or no effect of delay at low probabilities. Overall, the results of the current study replicate findings that probability has greater influence than delay on the current value of monetary gains. Results also extend the greater relative influence of probability to monetary losses that are both delayed and probabilistic.

Fig. 3 shows the results of fitting the multiplicative and additive models. The left panels show the multiplicative equation (Eq. (6)). The multiplicative model accounted for 99% of the variance for both monetary gains (top left panel) and monetary losses (bottom left panel). The right panels show the additive model (Eq. (5)). The additive model accounted for 91% of the variance for monetary gains and 96% of the variance for monetary losses. In addition, the additive model predicted a negative subjective value for long delays and greater odds against for gains. This would be interpreted as an individual preferring to pay money rather than potentially gaining $1000 at a long delay and high odds against. This seems to make little logical sense, and calls into question the accuracy of the additive model.

Wilcoxon Signed-Rank Tests were used to compare how well the multiplicative and additive models described the data in Experiment 2. The results of model comparisons for monetary gains were similar to those observed by Vanderveldt et al. (2015) in that the multiplicative model accounted for more variance in the data than the additive model ($z = 5.17$, $p < 0.001$). A greater amount of vari-

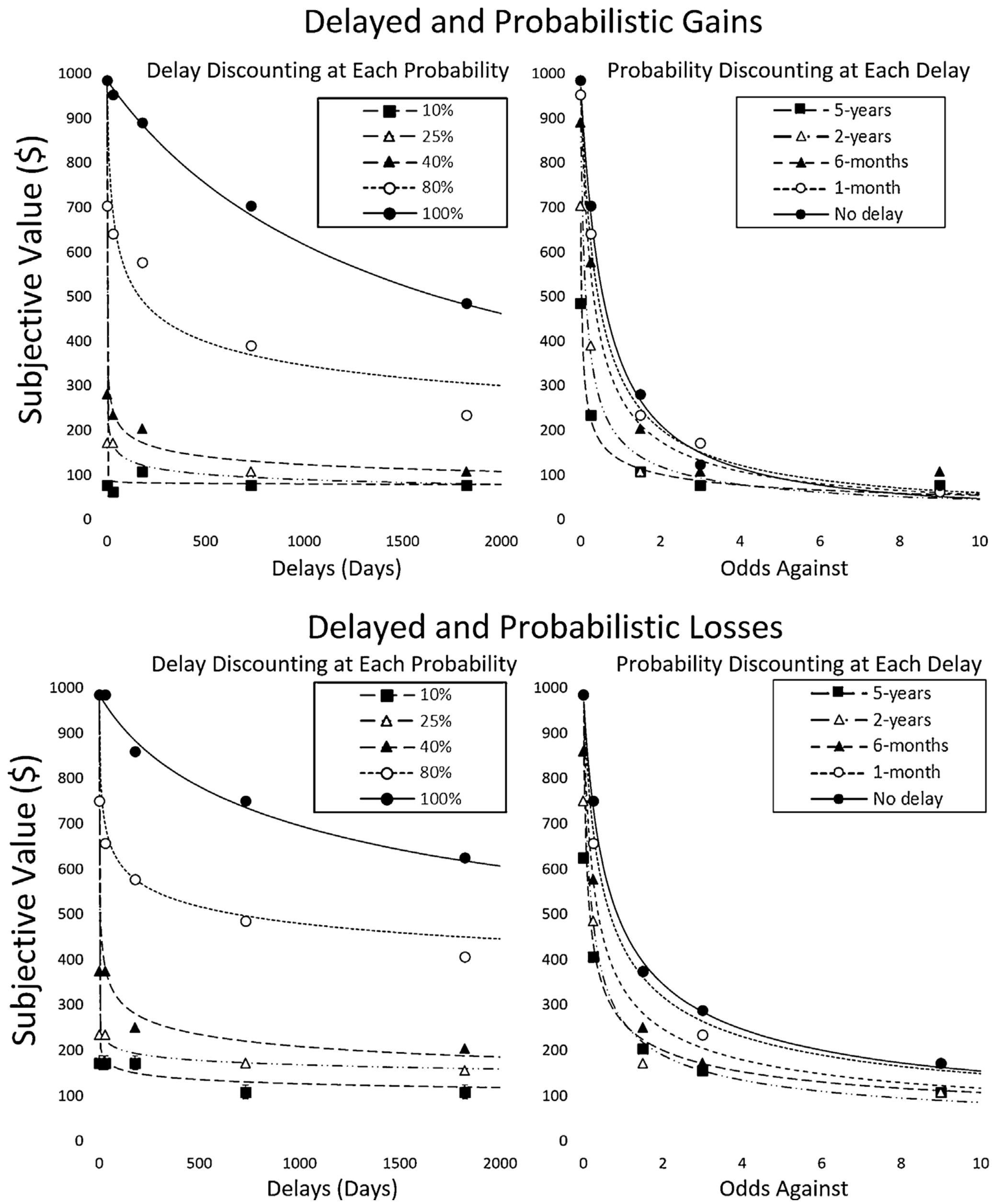

Fig. 2. Median indifference points and best fitting curves from Eq. (3) (left panels) and Eq. (4) (right panels) for Experiment 2. The upper panels are delayed and probabilistic gains. The lower panels are delayed and probabilistic losses.

ance was accounted for by the multiplicative model for delayed and probabilistic monetary losses as well ($z = 5.73$, $p < 0.001$).

Averaged parameter estimates for each task in Experiment 2 are presented in Table 4. Significant correlations between discounting parameters derived from the traditional task and the 5-trial adjusting delay and probability tasks were observed with h parameters for monetary gains ($r = 0.49$, $p = 0.0001$) as well as with k and h parameters for monetary losses (k: $r = 0.54$, $p = 0.008$; h: $r = 0.40$, $p = 0.0002$). A statistically significant correlation was not observed for ks for delayed and probabilistic monetary gains ($r = 0.16$, $p = 0.25$). It is unclear why no correlation was observed for only this comparison.

Overall, the results from Experiment 2 replicate previous findings that probability influences choice more so than delay when alternatives are both delayed and probabilistic (Weatherly et al., 2015; Vanderveldt et al., 2015). Experiment 2 also extends this finding to monetary losses. The results provide further support that a multiplicative model describes discounting of choices that are delayed and probabilistic. Finally, 5-trial adjusting delay and probability tasks provided similar measures of discounting in 115 fewer trials when alternatives are both delayed and probabilistic.

4. General discussion

The results of Experiment 1 indicate adjusting amount tasks and 5-trial adjusting probability and delay tasks provide similar measures of probability discounting for monetary gains and losses. In addition, magnitude effects consistent with previous research were obtained for gains and losses using 5-trial adjusting probability tasks. Inconsistent with previous research (Green et al., 2014), a magnitude effect was observed using 5-trial adjusting delay tasks involving monetary losses. However, this may have been an artifact of the smaller range of magnitudes assessed in Experiment 1 compared to previous research (Green et al., 2014).

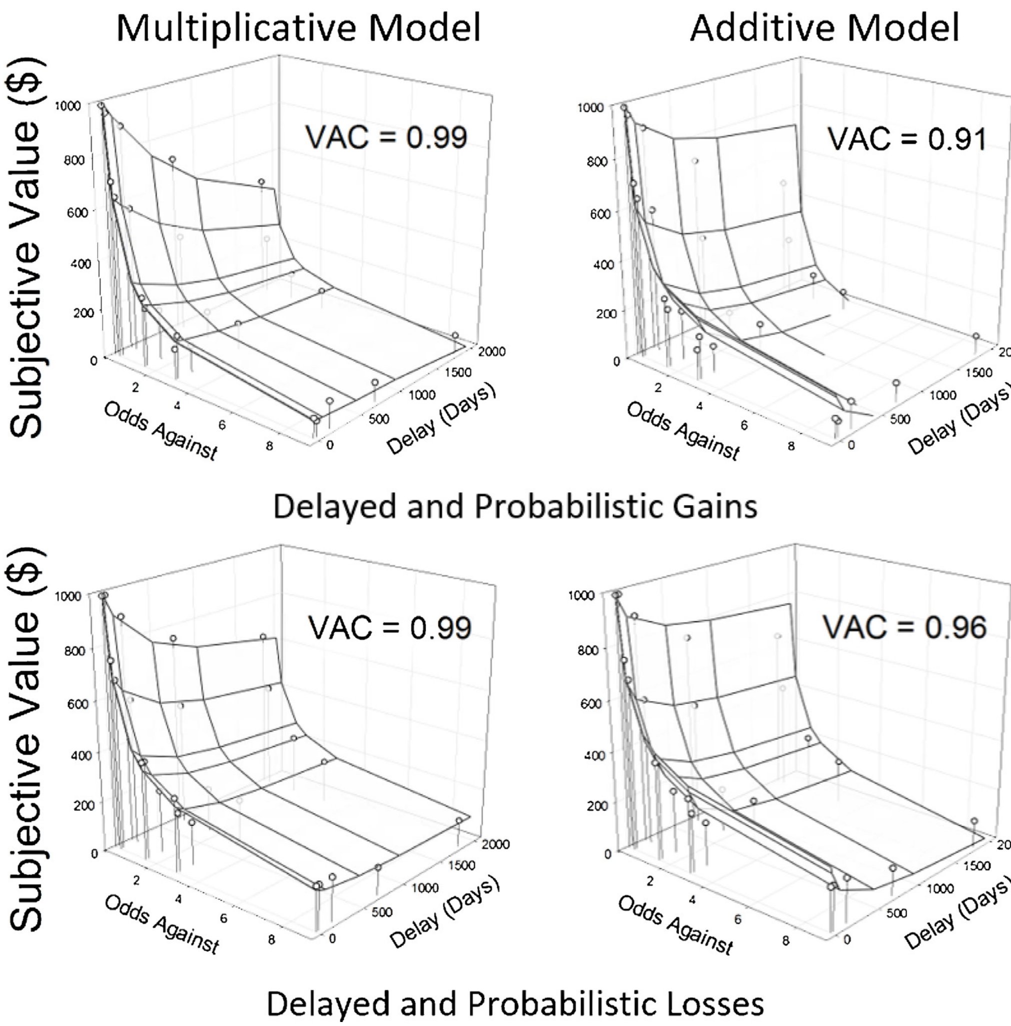

Fig. 3. Median indifference points and best fitting curves from Eq. (6) (left panels) and Eq. (5) (right panels) for Experiment 2. The upper panels are delayed and probabilistic gains. The lower panels are delayed and probabilistic losses.

Table 4
Mean and 95% confidence interval of discount parameters for each task. Pearson correlations for comparison between discounting tasks.

Task		Mean k or h (95% CI)	Pearson Correlation
Delayed & Probabilistic Gains	Adjusting Amount $1000 k	0.04 (0.03)	0.16
	5-Trial Adjusting Delay $1000 k	2.93 (1.98)	
	Adjusting Amount $1000 h	3.27 (0.66)	0.49^{***}
	5-Trial Adjusting Delay $1000 h	2.72 (0.52)	
Delayed & Probabilistic Losses	Adjusting Amount $1000 k	0.001 (0.21)	0.54^{**}
	5-Trial Adjusting Delay $1000 k	0.002 (0.001)	
	Adjusting Amount $1000 h	1.04 (0.21)	0.40^{***}
	5-Trial Adjusting Delay $1000 h	1.01 (0.21)	

Asterisks indicate a significant difference between tasks.
$^{*}p < 0.05$.
$^{**}p < 0.01$.
$^{***}p < 0.001$.

Experiment 2 provides novel evidence that probability is more salient than delay for monetary losses, and it replicates this finding for monetary gains (Vanderveldt et al., 2015; Weatherly et al., 2015; cf. Bialaszek et al., 2015). That is, when outcomes were both delayed and probabilistic and the probability of an outcome decreased, delay discounting became nearly flat. In contrast, rate of probability discounting remained steep across all delays. As reported by Vanderveldt et al. (2015), it is possible that the increased salience of probability was an artifact of the range of amounts, delays, and probabilities assessed. Vanderveldt et al. (2015) suggested that one could test these hypotheses by using a range of delays and probabilities where the decrease in subjective value of an immediate reward at the lowest probability equals the decrease in subjective value of a certain reward at the longest delay. Evidence would be provided for the former hypothesis if probability continued to have a greater effect under this arrangement.

Several researchers have suggested that there are three types of discounting: discounting of delayed gains, discounting of probabilistic gains, and discounting of losses (Green et al., 2014). Support for this position comes from the differential influence of amount on three types of discounting. Experiment 1 provides further support that discounting of probabilistic gains and discounting of losses are distinct: increasing the magnitude resulted in steeper discounting of probabilistic gains but no change in discounting of probabilistic losses. Similarly, humans tend to discount gains more steeply than losses regardless of whether the outcome is delayed or probabilistic (e.g., Mitchell and Wilson, 2010). In Experiment 1, steeper discounting of probabilistic gains was observed compared to probabilistic losses. In Experiment 2, gains continued to be discounted more steeply than losses when the choices were both delayed and probabilistic. In contrast to gain/loss patterns, the effects of amount on discounting become minimal when choices are both delayed and probabilistic (Blackburn and El-Deredy, 2013; Vanderveldt et al., 2015; Yi et al., 2006). Five-trial adjusting delay and probability tasks could allow researchers to examine further these patterns across delayed, probabilistic, and delayed and probabilistic alternatives using a within-subject study design. The resulting data would help researchers understand whether different "impulsivities" exist even for complex choices.

Future research could focus on assessing the influence of multiple outcomes on choices. that are delayed and probabilistic. Initial research could extend these tasks by adding a second outcome of the same commodity. For example, researchers could assess rate of delay and probability discounting of some monetary amount as a function of a second monetary outcome that occurs at varying delays, probabilities, and amounts. Research could also assess various combinations of gains and losses across the two outcomes. That is, how does one delayed and probabilistic gain influence the value of a second delayed and probabilistic gain? Relatedly, how does a loss influence a gain, a gain influence a loss, and a loss influence a loss when both outcomes occur at some delay and some probability? Future research could also focus across specific commodities as well as across consumable versus non-consumable commodities. Many day-to-day choices involve more than one outcome, each with some probability of occurrence, at differing delays, and across different commodities. The comparisons would allow researchers to test and refine quantitative models that describe choice in complex settings.

5. Conclusion

These experiments extended research on 5-trial adjusting delay tasks (Koffarnus and Bickel, 2014) from delayed monetary gains to probability discounting of monetary gains, probability discounting of monetary losses, delay discounting of monetary losses, delayed and probabilistic monetary gains, and delayed and probabilistic monetary losses. The 5-trial adjusting tasks resulted in hs and ks that correlated with hs and ks from adjusting amount tasks. We also found magnitude effects that were generally consistent with previous literature. For scenarios involving choices that were delayed and probabilistic, 5-trial adjusting delay and probability tasks resulted in estimation of h and k discounting parameters in 115 fewer trials than an adjusting amount task. These parameters correlated with those from adjusting amount tasks with the exception of ks for delayed and probabilistic gains. The multiplicative model of discounting (Vanderveldt et al., 2015) described choice for both gains and losses that were delayed and probabilistic. Delay and probability therefore interact to influence the value of money.

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