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Influence of second outcome on monetary discounting

David J. Cox*, Jesse Dallery

University of Florida, Department of Psychology, United States

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ABSTRACT

The rates that an outcome (e.g., money) loses value as delay increases or probability decreases are called delay and probability discounting, respectively. Discounting is typically studied by asking participants to make choices between two options that vary in amount and delay (e.g., \$50 now vs. \$100 in 3 months) or probability (e.g., 100% chance of \$50 vs. 60% chance of \$100). Little is known about how more complex options affect discounting. We asked participants ($N = 56$) to choose between two options that each resulted in two outcomes (e.g., getting \$50 now and losing \$1000 in 6 months vs. getting \$100 in 3 months and losing \$500 now). The second outcome varied across a range of delays and probabilities. Results indicated the probability of the second outcome had a greater influence on rates of discounting compared to the delay to the second outcome. Increasing the probability of the loss decreased rates of discounting the first outcome (i.e., increased preference for the larger-later alternative). Finally, a multiplicative model best described discounting of two, delayed-and-probabilistic outcomes. This suggests the value of two outcomes interact to influence rates of discounting.

1. Introduction

Human behavior often results in more than one outcome, each occurring at different delays and probabilities. For example, smoking a cigarette may result in both immediate relief from withdrawal as well as delayed adverse health outcomes. Similarly, abstaining from smoking may result in delayed positive health outcomes but immediate discomfort. Multiple outcomes resulting from a single choice may occur following a variety of health-related behaviors such as food consumption, risky sexual behavior, physical exercise, and substance use.

Researchers often use a discounting framework to study the influence of delayed and probabilistic outcomes on choice. Delay discounting refers to the rate at which a commodity (e.g., \$1000, a pack of cigarettes) decreases as a function of delay to receipt (see McKerchar and Renda, 2012, for a review). In these procedures, participants typically make choices between an immediate, small amount of money and a larger, delayed amount of money, and the delay to the larger amount is increased across a range of delays. The rate at which the value of the larger outcome decreases can be described by a hyperbolic equation (Mazur, 1987):

$$V = \frac{A}{(1 + k\mathrm{D})} \tag{1}$$

Here, V is the subjective value of a delayed commodity, A is the undiscounted value of the commodity, D is the delay to receipt of that commodity, and k is a parameter that represents rate of delay discounting. Probability discounting describes how the value of a commodity decreases as a function of the odds against receiving the commodity (McKerchar and Renda, 2012). The same hyperbolic equation used to describe delay discounting can be used to describe probability discounting:

$$V = \frac{A}{(1 + h\theta)} \tag{2}$$

Here, odds against (θ) is substituted for delay and is calculated as $(1\text{-}p)/p$, where p is the probability of receiving the commodity. V and A are the same as in Eq. (1) and h is a parameter representing rate of probability discounting.

Recently, researchers have assessed more complex choice scenarios where the outcomes are both delayed and probabilistic (Cox and Dallery, 2016; Vanderveldt et al., 2015; Weatherly et al., 2015). For example, in Vanderveldt et al. (2015) participants made choices between smaller rewards that were both immediate and certain and larger rewards that were both delayed and probabilistic. These studies suggest that probability influences the value of an outcome more than delay. Vanderveldt et al. also found that a multiplicative equation described choice relative to monetary gains, and these findings have subsequently been extended to monetary losses (Cox and Dallery, 2016). Specifically, Eqs. (1) and (2) combine to yield:

$$V = \frac{A}{(1 + k\mathrm{D})(1 + h\theta)} \tag{3}$$

* Corresponding author at: University of Florida, Department of Psychology, 945 Center Drive, Gainesville, FL, 32611-2250, United States.
E-mail addresses: david.j.cox@ufl.edu (D.J. Cox), dallery@ufl.edu (J. Dallery).

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Trial Number	Index Number	Option A	OR	Option B
1	16	100% Chance of LOSING \$500 Immediately		100% Chance of LOSING \$1000 in 3 weeks
2	8	100% Chance of LOSING \$500 Immediately		100% Chance of LOSING \$1000 in 1 day
3	12	100% Chance of LOSING \$500 Immediately		100% Chance of LOSING \$1000 in 4 days
4	14	100% Chance of LOSING \$500 Immediately		100% Chance of LOSING \$1000 in 1.5 weeks
5	13	100% Chance of LOSING \$500 Immediately		100% Chance of LOSING \$1000 in 1 week

Final k parameter = $1/ED_{50} = 1/8.57 = 0.117 = k$ *Delay of "1 week" used for adjusting probability*

Trial Number	Index Number	Option A	OR	Option B
6	16	100% Chance of LOSING \$500 in 1 Week		50% Chance of LOSING \$1000 in 1 week
7	24	100% Chance of LOSING \$500 in 1 Week		24% Chance of LOSING \$1000 in 1 week
8	20	100% Chance of LOSING \$500 in 1 Week		37% Chance of LOSING \$1000 in 1 week
9	18	100% Chance of LOSING \$500 in 1 Week		43% Chance of LOSING \$1000 in 1 week
10	19	100% Chance of LOSING \$500 in 1 Week		40% Chance of LOSING \$1000 in 1 week

Final h parameter = $1/EP_{50} = 1/1.598 = 0.626 = h$

Fig. 1. Example adjustments of choice alternatives in the discrete choice task with one outcome per option. The trial number represents the order of choice presentation. The index number corresponds to the index listing in Appendices A and B. Circled options represent the choice made by a hypothetical participant leading to the presented choice index in the following trial. The final parameter represents the rate at which the value of losing \$1000 reduced as a function of increasing delay (k) and increasing odds against (h).

This equation suggests delay and probability interact to influence the value of an outcome as opposed to independently influencing value.

Research has not assessed discounting in even more complex situations in which each option in a discrete choice procedure results in more than one outcome. For example, consider a regular smoker who chooses between smoking or abstaining. Smoking will result in (1) immediate and certain effects of nicotine and (2) delayed and uncertain negative health impacts. Abstaining from smoking will result in (1) immediate and certain withdrawal symptoms and (2) delayed and uncertain positive health impacts. In other words, both choices result in one immediate and certain outcome and one delayed and probabilistic outcome, and both choices result in gains and losses. To manipulate these variables in an efficient manner, we used a method devised by Koffarnus and Bickel (2014). The method, described in the methods section below, yielded estimates of delay (k) and probability (h) discounting as a function of parametric changes in the probability and delay of a second outcome.

Another goal of the present study was to characterize choice quantitatively. Eq. (3) describes the reduction in value of *one* delayed and probabilistic outcome for both gains and losses (Cox and Dallery, 2016; Vanderveldt et al., 2015). Quantitatively describing discounting across *two* delayed and probabilistic losses should therefore involve a combination of two Eq. (3)s – one for each outcome. This assumes the influence of one outcome on choice depends on the influence of the other outcome. Combining the amount of each outcome and maintaining the interactive effects of delay and probability could result in:

$$V_1 = \frac{A_1}{\left(1 + \frac{(k_1 D_1)}{\left(\frac{1}{A_2} + k_2 D_2\right)}\right)\left(1 + \frac{(h_1 \theta_1)}{\left(\frac{1}{A_2} + h_2 \theta_2\right)}\right)} \tag{4}$$

Here, all the parameters are the same as above and the subscripts *1* and *2* refer to the first and second outcomes, respectively. This equation involves increasing or decreasing the value of the first outcome based on its relative value to the second outcome. It does this by dividing the influence of delay and probability for the first outcome (i.e., k_1D_1 and $h_1\theta_1$ – Eq. (3)) by the value of the second outcome – $\left(\frac{1}{A_2} + k_2D_2\right)$ and $\left(\frac{1}{A_2} + h_2\theta_2\right)$.

Thus, the present experiment assessed changes in k and h for the first outcome across changes in delay and probability to a second outcome. This was completed for outcome combinations where the first outcome was a gain and the second one a loss. Finally, we evaluated how well Eq. (4) described the data.

2. Method

2.1. Participants

Fifty-six participants were recruited from the Psychology Department participant pool from a large public university in the southeast United States. The average age of participants was 18.94 years old (range 18–22) and 68% self-identified as female. Participants first completed a discrete choice procedure where each option resulted in one outcome, and then completed a discrete choice procedure where each option resulted in two outcomes.

2.2. Discrete choice with one outcome per option

The goal of this task was to generate direct measures of k and h for losing an amount of \$1000. These measures of k and h were later used to fit Eq. (4). In the discrete choice task with one outcome per option, participants made repeated choices between a small amount of money and a larger amount of money. The smaller amount was fixed at half of the larger amount for all trials. For example, participants were asked to choose between:

(a) 100% chance of losing \$500 immediately.
OR
(b) 100% chance of losing \$1000 in 3 weeks.

Here, Option A results in *one outcome* of \$500, with a probability of 100%, and a delay of zero. Option B results in *one outcome* of \$1000, with a probability of 100%, and a delay of 3 weeks. We first adjusted the delay of Option B for the first 5 trials based on participant responding. If the participant selected Option A (100% chance of losing \$500 immediately), the delay to the larger amount would increase (e.g., to 2 years). If the participant selected Option B (100% chance of losing \$1000 in 3 weeks), the delay to the larger amount would decrease (e.g., to 1 day). This adjustment to the delay of Option B continued for 5 trials. The top panel in Fig. 1 provides an example of how choices might be presented to a participant and the adjustments to delay following each choice. The choice on the 5th trial was used to calculate k. The calculation methods are described below in the section titled "Data analysis."

We next adjusted the probability of Option B to assess probability discounting. The delay on the 5th trial was used as the delay for both options during the 5-trials where probability adjusted. For example, if the 5th trial was a delay of 1 week, the first trial for the adjusting probability portion would ask the participant to choose between:

(a) 100% chance of losing $500 in 1 week.
OR
(b) 50% chance of losing $1000 in 1 week.

The probability of Option B then adjusted for trials 6–10 based on participant responding. If Option A was chosen, the probability of the larger amount decreased (e.g., to 24%). If the participant chose Option B, the probability of the larger amount increased (e.g., to 76%). This adjustment to the probability of Option B continued for 5 total trials. The bottom panel of Fig. 1 provides an example of how choices might be presented to a participant and the adjustments to probability following each choice. Again, the choice on the 5th trial was used to calculate *h*. The calculation methods are described below in the section titled "Data analysis."

We adjusted the delay and probability following each choice in an identical manner to previous research using adjusting delay and probability tasks (Cox and Dallery, 2016; Koffarnus and Bickel, 2014). Appendix A shows the 31 potential delays (i.e., indices 1–31) and the trial number each delay might be presented to a participant. Participants always started at delay index 16 (i.e., $1000 delayed by 3 weeks; trials 1–5). As described above, the delay was increased or decreased by 8 indices following the participant's first choice (i.e., to a delay of 1 day at index 8; or to a delay of 2 years at index 24). The delay then adjusted by 4, 2, and 1 indices following participant choice on the 2nd, 3rd, and 4th trial, respectively. This same process was used to adjust probability during trials 6–10. Appendix B shows the 31 potential probabilities and trial number each might be presented to a participant. The probability of the larger amount always started at probability index 16 (i.e., 50% chance of $1000). The probability changed by 8, 4, 2, and 1 indices following the 6th–9th trials, respectively.

2.3. Discrete choice with two outcomes per option

The two-outcome tasks were structured to be analogous to many problematic behaviors, as explained in the Introduction. For example, participants were asked to choose between:

(a) (100% chance of getting $50 immediately) + (75% chance of losing $1000 in 6 months).
OR
(b) (50% chance of getting $100 in 3 weeks) + (100% chance of losing $500 immediately).

The first outcome for Option A was always $50 and the second outcome for Option A was always $1000. The first outcome for Option B was always $100 and the second outcome for Option B was always $500. Note, the first outcomes for Options A and B are similar to the one-outcome tasks presented above. A second outcome has just been added to the choice frame for each option.

We adjusted the delay to the first outcome in Option B (50% chance of getting $100 in 3 weeks) in an identical manner to the discrete choice tasks with one-outcome per option. That is, the first five trials involved adjusting the delay based on participant responding. Then, the delay from the fifth trial was again used as the delay for trials 6–10, and the probability of the first outcome of Option B adjusted following each choice. Finally, we calculated delay and probability discounting parameters in the same manner as outlined above for one-outcome tasks. Fig. 2 provides an example of how choices might be presented to a participant and the adjustments to the delay or probability following each choice by a hypothetical participant.

We parametrically varied the delay and probability of the second outcome for "Option A" across nine combinations. This allowed us to determine how the delay and probability of the second outcome (losing $1000) influenced discounting of the first outcome. Delays were: immediate, 6 months, and 2 years; probabilities were: 100%, 75%, and 45%. The second outcome for the second alternative (i.e., "Option B") was always a 100% chance of losing $500 immediately. The order in which these nine combinations were presented to each participant was randomly determined.

2.4. Data analysis

We first calculated *k* and *h* for the discrete choice with one outcome per option. If the participant chose the immediate option on the fifth trial, *k* was calculated by dividing 1 by the geometric mean of the delay for the fifth trial and the delay at the index immediately below it (Appendix A). If the participant chose the delayed option on the fifth trial, *k* was calculated by dividing 1 by the geometric mean of the delay for the fifth trial and the delay at the index immediately above it. We used a similar process to calculate the participant's individual probability discounting parameter (*h*) based on the alternative selected on the tenth overall trial (i.e., the fifth adjusting probability trial) as outlined in Appendix B.

We calculated *k* and *h* for the discrete choice with two outcomes per option task in the same way as when there was one outcome per option. That is, the discrete choice task with two outcomes per option identified the delay and probability the value of the first outcome decreased by 50% – just like the one outcome per option task. This allowed us to use the choice on the fifth trial to estimate *k* when there were two outcomes per option using Appendix A. And, we used the choice on the tenth trial to estimate *h* when there were two outcomes per option using Appendix B.

The estimates of *k* and *h* following the fifth and tenth trials provided a measure for how participants discounted a delayed and probabilistic outcome when there were two outcomes per option. We calculated nine *k*s and nine *h*s using this method – one for each second outcome combination of delay and probability. However, this method only informs us how the first outcome is discounted. This method does not inform us how the first and second outcome combine to influence choice. To understand the interaction of two outcomes on choice, we fit Eq. (4).

Fitting Eq. (4) proceeded in three steps. First, we inserted individual *k* and *h* parameters from the tasks with one outcome per option as k_2 and h_2 in Eq. (4). For example, the delayed value of the second outcome in Eq. (4) is represented by,

$$\frac{1}{A_2} + k_2 D_2. \tag{5}$$

If the amount of the second outcome (A_2) was $1000, the participant discounted losing $1000 at a rate $k = 0.117$, and the delay to the second outcome was 6 months, then Eq. (5) would equate to $\left[\left(\frac{1}{1000}\right) + (0.117)(6)\right] = 0.703$. Next, the same process was used to calculate the probability value of the second outcome in Eq. (4). The final step involved fitting Eq. (4) to the observed indifference points (i.e., ED_{50} & EP_{50}) from the discrete choice tasks with two outcomes per option. Eq. (4) was fit to indifference points by estimating the two remaining parameters in Eq. (4) – k_1 and h_1 – using Microsoft Excel Solver Add-In for Microsoft Excel 2013. We repeated this process for each of the nine delay and probability combinations for the second outcome.

Lastly, we used IBM SPSS Statistics 24 for all statistical analyses.

3. Results

Delay (*k*s) and probability (*h*s) discounting parameters were obtained from the discrete choice with one outcome per option. The average (standard deviation) rates that participants discounted losing a delayed and probabilistic $1000 were *k*s of 0.03 (0.06) and *h*s of 3.55 (12.57).

Fig. 3 shows the average *k*s (top panel) and *h*s (bottom panel) from the nine discrete choice tasks with two outcomes per option. These *k* and *h* parameters were obtained in the same manner as the discrete choice with one outcome per option tasks (i.e., using Appendices A and

Trial Number	Index Number	Option A	OR	Option B
1	16	100% Chance of GETTING \$50 Immediately WITH A 75% Chance of LOSING \$1000 in 6 months		50% Chance of GETTING \$100 in 3 weeks WITH A 100% Chance of LOSING \$500 Immediately
2	8	100% Chance of GETTING \$50 Immediately WITH A 75% Chance of LOSING \$1000 in 6 months		50% Chance of GETTING \$100 in 2 years WITH A 100% Chance of LOSING \$500 Immediately
3	12	100% Chance of GETTING \$50 Immediately WITH A 75% Chance of LOSING \$1000 in 6 months		50% Chance of GETTING \$100 in 4 months WITH A 100% Chance of LOSING \$500 Immediately
4	14	100% Chance of GETTING \$50 Immediately WITH A 75% Chance of LOSING \$1000 in 6 months		50% Chance of GETTING \$100 in 8 months WITH A 100% Chance of LOSING \$500 Immediately
5	13	100% Chance of GETTING \$50 Immediately WITH A 75% Chance of LOSING \$1000 in 6 months		50% Chance of GETTING \$100 in 1 year WITH A 100% Chance of LOSING \$500 Immediately

Final k parameter = 1/ED$_{50}$ = 1/298.2 = 0.003 = k *Delay of "1 year" used for adjusting probability*

Trial Number	Index Number	Option A	OR	Option B
6	16	100% Chance of GETTING \$50 in 1 year WITH A 75% Chance of LOSING \$1000 in 6 months		50% Chance of GETTING \$100 in 1 year WITH A 100% Chance of LOSING \$500 Immediately
7	24	100% Chance of GETTING \$50 in 1 year WITH A 75% Chance of LOSING \$1000 in 6 months		76% Chance of GETTING \$100 in 1 year WITH A 100% Chance of LOSING \$500 Immediately
8	20	100% Chance of GETTING \$50 in 1 year WITH A 75% Chance of LOSING \$1000 in 6 months		63% Chance of GETTING \$100 in 1 year WITH A 100% Chance of LOSING \$500 Immediately
9	18	100% Chance of GETTING \$50 in 1 year WITH A 75% Chance of LOSING \$1000 in 6 months		70% Chance of GETTING \$100 in 1 year WITH A 100% Chance of LOSING \$500 Immediately
10	19	100% Chance of GETTING \$50 in 1 year WITH A 75% Chance of LOSING \$1000 in 6 months		73% Chance of GETTING \$100 in 1 year WITH A 100% Chance of LOSING \$500 Immediately

Final h parameter = 1/EP$_{50}$ = 1/0.342 = 2.926 = h

Fig. 2. Example adjustments of choice alternatives in the discrete choice task with two outcomes per option. The trial number represents the order of choice presentation. The index number corresponds to the index listing in Appendices A and B. Circled options represent the choice made by a hypothetical participant leading to the presented choice index in the following trial. The final parameter represents the rate at which the value of gaining \$100 reduces as a function of increasing delay (k) or increasing odds against (h).

B). Each panel shows nine discounting parameters – one from each of the nine delay and probability combinations for the second outcome (x-axis).

We first conducted a 3 (delay) x 3 (probability) repeated measures ANOVA on k and h values. Rates of delay discounting (k) increased significantly as a function of decreasing probabilities (F(2, 99.38) = 43.82, $p < 0.001$, $\eta^2 = 0.44$) but not as a function of changing delays (F(2, 110) = 0.91, $p = 0.41$, $\eta^2 = 0.02$). Pairwise comparisons with Bonferroni corrections indicated rates of delay discounting increased as the probability of the second outcome decreased ($p \le 0.008$ for all pairwise comparisons). That is, rates of delay discounting were influenced by the probability of the second outcome, but not the delay to the second outcome.

Rates of probability discounting (h) showed similar patterns to rates of delay discounting (k). Overall, changing the probability of the second outcome led to increased rates of probability discounting (F(2, 95.43) = 37.27, $p < 0.001$, $\eta^2 = 0.40$). But, changing the delay to the second outcome did not result in changes in probability discounting (F(2, 98.53) = 0.40, $p = 0.65$, $\eta^2 = 0.01$). Post-hoc pairwise comparisons indicated rates of probability discounting increased across all comparisons of probability of the second outcome ($p \le 0.05$ for all comparisons). That is, rates of probability discounting were also influenced by the probability of the second outcome, but not the delay to the second outcome.

As shown in Table 1, Eq. (4) described individual participant indifference points well for all delay and probability combinations to the second outcome. Specifically, Eq. (4) resulted in median VACs near 1.0 for all delay and probability combinations except when the second outcome was immediate and certain (median VAC = 0.79). These results suggest the values of two outcomes per option will interact to influence choice. That is, the influence of any one outcome on choice is relative to the value of other outcomes resulting from the same option.

Fig. 4 depicts how delay and probability to the second outcome interact to influence discounting of the first outcome. Each panel in Fig. 4 shows the median rate that participants discounted gaining \$100 (i.e., the first outcome in each choice option). To generate these figures, we calculated the median k and h values obtained in each discrete choice task with two options. The median k and h parameters were entered into Eq. (3) and values were estimated across a range of delays (0–2 years) and a range of odds against (0–9). The delay and probability listed in each panel is for the second outcome – losing \$1000.

Moving left-to-right across rows shows that reducing the probability to the second outcome increased rates of discounting while the delay remained fixed. Moving top-to-bottom down columns shows how increasing the delay to the second outcome impacted rates of discounting while the probability remained fixed. Discounting increased only

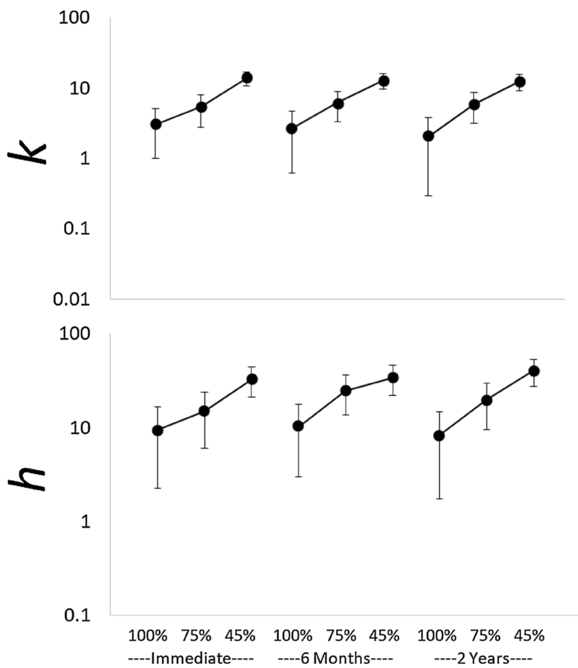

Fig. 3. Average delay (top panel) and probability (bottom panel) discounting parameters for the discrete choice task with two outcomes per option estimated using Appendices A and B. The listed delay and probability refer to the delay and probability of the second outcome in "Option A". Error bars represent 95% confidence intervals.

Table 1
R^2 values for Eq. (4) when fit to individual participant indifferent points for the nine discrete choice tasks with two outcomes per option.

Probability, Delay of Second Outcome	Individual VAC Median (Min, Max)
100%, Immediate	0.79 (0.11, 1.00)
75% Immediate	0.99 (-0.69, 1.00)
45%, Immediate	0.99 (-1.98, 1.00)
100%, 6 Months	0.99 (0.80, 1.00)
75%, 6 Months	1.00 (-0.70, 1.00)
45%, 6 Months	1.00 (-3.08, 1.00)
100%, 2 Years	0.99 (0.94, 1.00)
75%, 2 Years	1.00 (-0.78, 1.00)
45%, 2 Years	1.00 (-3.23, 1.00)

between the two comparisons involving delays of immediate and 6 months, and probabilities of 100% and 75% (i.e., the upper-left and middle-left panels; and upper-middle and middle-middle panels). Furthermore, these data suggest that the influence of a larger-later-uncertain loss (e.g., lung cancer) paired with a smaller-sooner-certain gain (e.g., smoking) remains relatively unchanged once the larger loss is longer than 6 months and occurs with a probability less than 75%.

4. Discussion

Human behavior often results in more than one outcome, where each outcome occurs at different delays and probabilities. The purpose of this experiment was to determine how rates of probability and delay discounting for a smaller, immediate, and certain gain would change as a function of a second larger, delayed, and uncertain loss. Finally, we sought to evaluate a quantitative description of the data.

This experiment was the first study to examine rates of discounting in a discrete choice task with more than one outcome per option. We assessed situations where the second outcome was a loss that was 10 times greater than the first outcome. We found that increasing the probability of the second outcome (loss) decreased preference for the immediate-certain gain and increased preference for the delayed-uncertain gain. This makes sense as the immediate and certain gain was now paired with a large and certain loss. Thus, the guaranteed large loss shifted preference away from that option and toward the option with a larger-later-uncertain gain and smaller-sooner-certain loss. But, as the large loss decreased in probability, preference shifted toward the smaller-sooner-certain gain.

This experiment provides novel evidence that probability is more influential than delay for two-outcome monetary combinations. That is, rates of both delay and probability discounting increased significantly across decreasing probabilities of a second outcome. However, we did not observe different rates of delay or probability discounting as the delay to a second outcome increased. This extends findings by previous researchers who observed the probability of one outcome is more influential than delay for monetary gains and monetary losses (Cox and Dallery, 2016; Vanderveldt et al., 2015; Weatherly et al., 2015; c.f., Blackburn and El-Deredy, 2013; Yi et al., 2006). Together, these findings suggest interventions aimed at increasing or decreasing preference for smaller-sooner-certain options may have greater impact by manipulating the probability of outcomes occurring, rather than delay.

The fact that multiple distinct characteristics of a context impact choice is not a novel finding. Nevertheless, the present experiment contributes to the discounting literature in several ways. First, the good fits of Eq. (4) suggest that two outcomes resulting from a single choice interact to influence value. We also evaluated an alternative, additive model (results not reported; Killeen, 2009; Vanderveldt et al., 2015). When extended to tasks with two outcomes per option, the assumption would be that one outcome influences the overall value independently of the second outcome. The fits of the additive model were poor with negative R^2 values for the majority of participants. These results are similar to Vanderveldt et al. (2015) and Cox and Dallery (2016) where a multiplicative model described discounting better than an additive model. Thus, two outcomes resulting from a single choice do not independently influence the value of a target outcome.

Second, Eq. (4) is a novel conceptualization of gain-loss interactions. Quantitative models from various research domains have incorporated gains and losses in the same equation by denoting these outcomes as positive and negative, respectively (e.g., prospect theory – Kahneman and Tversky, 1979; matching law – de Villiers, 1980; Klapes et al., 2018). When multiple outcomes follow a response, the outcomes sum to an overall value. Experienced losses or punishers reduce or suppress the value (i.e., subtraction), and experienced gains or reinforcers increase or strengthen the value (i.e., addition). However, these 'direct-suppression' models can produce negative values if the amount or rate of losses/punishers is greater than the amount or rate of gains/reinforcers. As noted by Klapes et al. (2018), producing negative values is a problem when predicting behavior. Negative value leads to predictions of negative behavior (which is illogical) or removal of those data from the quantitative analysis (which reduces the scope of the model).

The structure of Eq. (4) conceptualizes gain-loss interactions differently. Rather than outcome value being defined with a positive or negative valence, outcome value can be defined by its magnitude of influence on behavior. More or less behavior can be allocated to a response option. Eq. (4) can never predict a negative value for an option because gains and losses are in a ratio – rather than subtracting one from the other. In this ratio form, as the amount or rate of losses/punishers becomes increasingly greater than the amount or rate of gains/reinforcers, the value of the outcome becomes increasingly closer to zero. Future research will have to determine if the present quantitative conceptualization of gain-loss interaction is useful in the prediction and control of behavior.

5. Conclusion

These experiments extended research on discounting of one delayed

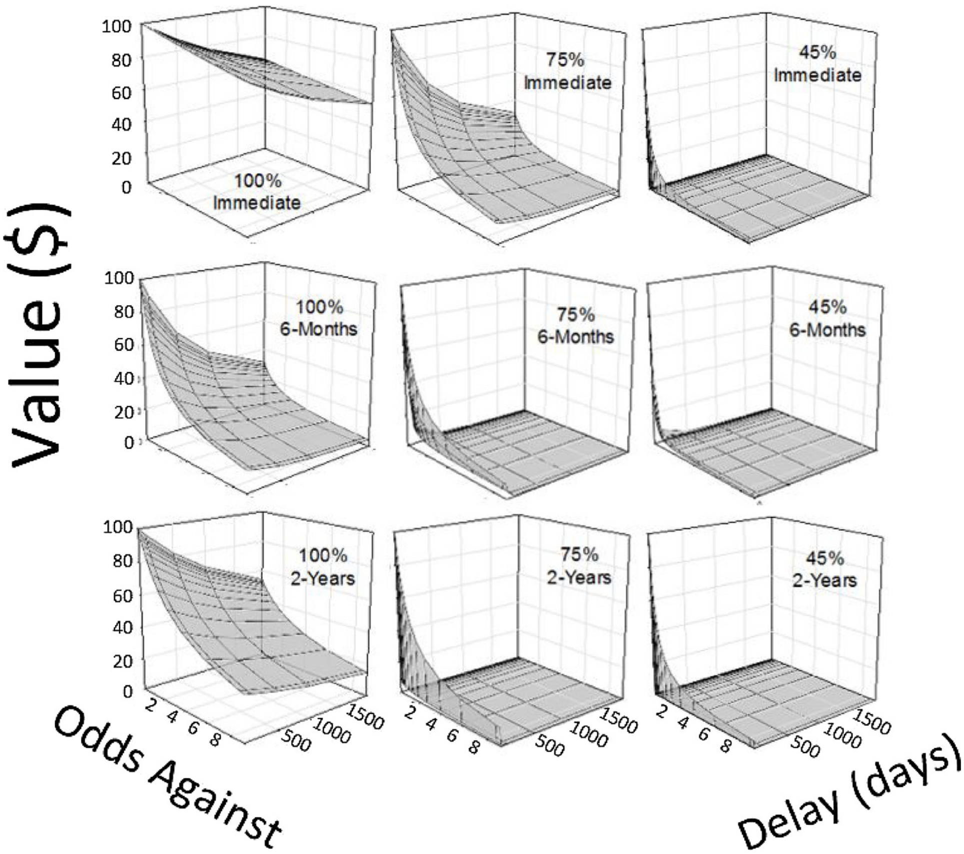

Fig. 4. Curves obtained from using median group k and h parameters from the two outcomes per option task in Eq. (3). The probability and delay in each plot are the probability and delay to the second outcome.

and probabilistic outcome to discounting of two delayed and probabilistic outcomes (Cox and Dallery, 2016; Vanderveldt et al., 2015). Specifically, we examined situations where the first outcome was a smaller gain and the second outcome was a larger loss. Rates of delay and probability discounting increased across decreasing probability of the second outcome. However, rates of discounting did not change as the delay to the second outcome increased. A multiplicative model of discounting extended from Vanderveldt et al. (2015) described individual patterns of choice. The value of two delayed and probabilistic outcomes interact to influence the value of hypothetical money.

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Appendix A

			ED_{50} (days) for Loss If last choice is:		k if last choice is:	
Index	Delay Choice	Trial No.	Immediate	Delayed	Immediate	Delayed
1	1 hour	5	0.05893	0.04167	17.000	24.000
2	2 hours	4				
3	3 hours	5	0.1444	0.1021	6.930	9.790
4	4 hours	3				
5	6 hours	5	0.3062	0.2041	3.270	4.900
6	9 hours	4				
7	12 hours	5	0.7071	0.4330	1.410	2.310
8	1 day	2				
9	1.5 days	5	1.732	1.225	0.577	0.816
10	2 days	4				
11	3 days	5	3.464	2.450	0.289	0.408
12	4 days	3				
13	1 week	5	8.573	5.292	0.117	0.189
14	1.5 weeks	4				

15	2 weeks	5	17.15	12.12	0.058	0.083
16	3 weeks	1				
17	1 month	5	43.05	25.28	0.023	0.040
18	2 months	4				
19	3 months	5	105.4	74.56	0.009	0.013
20	4 months	3				
21	6 months	5	210.9	149.1	0.005	0.007
22	8 months	4				
23	1 year	5	516.5	298.2	0.005	0.003
24	2 years	2				
25	3 years	5	1265	894.7	0.001	0.0012
26	4 years	4				
27	5 years	5	2310	1633	0.0004	0.0006
28	8 years	3				
29	12 years	5	5368	3579	0.0002	0.0003
30	18 years	4				
31	25 years	5	9131	7748	0.00011	0.00013

The referencing index number, delay presented with the specific choice, the trial number a given delay could be presented. *Note.* ED_{50} = Effective Delay 50% and the “immediate” and “delayed” column values reverse between gains and losses.

Appendix B

				EP_{50} (θ) for Loss if last choice is:		h if last choice is:	
Index	Uncertain Choice	Odds Against	Trial No.	Uncertain	Certain	Uncertain	Certain
1	99%	0.0101	5	0.010	0.021	99.499	48.744
2	96%	0.0417	4				
3	92%	0.0870	5	0.060	0.104	16.613	9.646
4	89%	0.1236	3				
5	86%	0.1628	5	0.142	0.183	7.050	5.476
6	83%	0.2048	4				
7	79%	0.2658	5	0.233	0.290	4.286	3.451
8	76%	0.3158	2				
9	73%	0.3699	5	0.342	0.398	2.926	2.512
10	70%	0.4286	4				
11	66%	0.5152	5	0.470	0.550	2.128	1.818
12	63%	0.5873	3				
13	60%	0.6667	5	0.626	0.709	1.598	1.410
14	57%	0.7544	4				
15	53%	0.8868	5	0.818	0.942	1.223	1.061
16	50%	1.0000	1				
17	47%	1.1277	5	1.062	1.223	0.942	0.818
18	43%	1.3256	4				
19	40%	1.5000	5	1.410	1.598	0.709	0.626
20	37%	1.7027	3				
21	34%	1.9412	5	1.818	2.128	0.550	0.470
22	30%	2.3333	4				
23	27%	2.7037	5	2.512	2.926	0.398	0.342
24	24%	3.1667	2				
25	21%	3.7619	5	3.451	4.286	0.290	0.233
26	17%	4.8824	4				
27	14%	6.1429	5	5.476	7.050	0.183	0.142
28	11%	8.0909	3				
29	8%	11.5000	5	9.646	16.613	0.104	0.060
30	4%	24.0000	4				
31	1%	99.0000	5	48.744	99.000	0.021	0.010

The referencing index number, probability presented with the specific choice, corresponding odds against, the trial number a given probability could be presented. *Note.* EP_{50} = Effective Probability 50% and the “certain” and “uncertain” column values reverse between gains and losses.

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