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Discounting and the Portfolio of Desires

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The additive utility theory of discounting is extended to probability and commodity discounting. Because the utility of a good and the disutility of its delay combine additively, increases in the utility of a good offset the disutility of its delay: Increasing the former slows the apparent discount even with the latter, time-disutility, remaining invariant, giving the magnitude effect. Conjoint measurement showed the subjective value of money to be a logarithmic function of its amount, and subjective probability—the probability weighting function—to be Prelec's (1998). This general theory of discounting (GTD) explains why large amounts are probability discounted more quickly, giving the negative magnitude effect. Whatever enhances the value of a delayed asset, such as its ability to satisfy diverse desires, offsets its delay and reduces discounting. Money's liquidity permits optimization of the portfolio of desired goods, providing added value that accounts for its shallow temporal discount gradient. GTD predicts diversification effects for delay but none for probability discounting. Operations such as episodic future thinking that increase the larder of potential expenditures—the portfolio of desirable goods—increase the value of the asset, flattening the discount gradient. States that decrease the larder, such as stress, depression, and overweening focus on a single substance like a drug, constrict the portfolio, decreasing its utility and thereby steepening the gradient. GTD provides a unified account of delay, probability, and cross-commodity discounting. It explains the effects of motivational states, dispositions, and cognitive manipulations on discount gradients.

Keywords: additive utility theory, general theory of discounting, probability and delay discounting, portfolio diversification, liquidity

When the receipt of a good is delayed money is discounted much more gradually than other goods such as food, entertainment, alcohol, drugs, sex, and so forth. But when the receipt of money is probabilistic, it is discounted at about the same rate as other goods. These are just two of the many perplexities in the extensive discounting literature. Another layer of complexity is that large magnitudes are discounted over delays in their receipt more slowly than small ones—the positive magnitude effect—but are discounted more quickly in probability discounting—the negative magnitude effect. Figure 1, after Estle et al. (2007), offers a paradigmatic *representation* of these central issues. Their experimental procedure, typical of discounting tasks, was:

> Participants were told that on each trial, two amounts of a hypothetical reward (money, beer, candy, or soda) would appear on the screen. For the temporal-discounting task, they were instructed that one amount could be received right now, whereas the other amount could be received after some specified period of time. For the probability-discounting task, they were instructed that one amount could be received for sure, whereas the other amount could be received with some specified probability. (p. 59)

After being asked to choose, the immediate or sure thing was adjusted to achieve indifference between it and the delayed or probabilistic one. The relative discount, the value of the immediate sure thing divided by the value of the delayed or probabilistic one, is then plotted as a discount function (e.g., Figure 1).

Why are money delay functions shallower than commodity functions, why are there magnitude effects, and why are those for delay and probability discordant? This article resolves these questions with an existing theoretical model (additive utility theory [AUT]; Killeen, 2009, 2023). AUT is then extended to probability discounting and to the many interesting effects of cognitive and emotional manipulations and demographic correlates on the rate of delay discounting, and why these variables play the roles that they do. To set the stage, it is necessary to reconstruct the AUT of delay discounting, as it forms the necessary foundation for this solution.

A General Frame

The utility of a package of goods, $U(G)$, is given by some concatenation of the variables considered in this article:

$$U(G_{a,d,p,n}) = F[f_a(a),\ f_d(d),\ f_p(p),\ f_n(n)]. \tag{1}$$

Here, a is its amount, d is the delay of its receipt, p is the probability of receiving it, and n is the diversification it affords. The functions within the brackets, f_x, are psychophysical value functions (Stevens, 1986/1975), transforming the numbers on which they operate into their psychological counterparts. $F[]$ concatenates those attributes and converts that combination into the quantity on the left-hand side (*lhs*), the utility of the package. This utility adds to that of the existing portfolio of goods, which constitutes their reference level (Thaler, 1999). For the experiments covered in this article, it is not necessary to determine the form of the utility transformation F.

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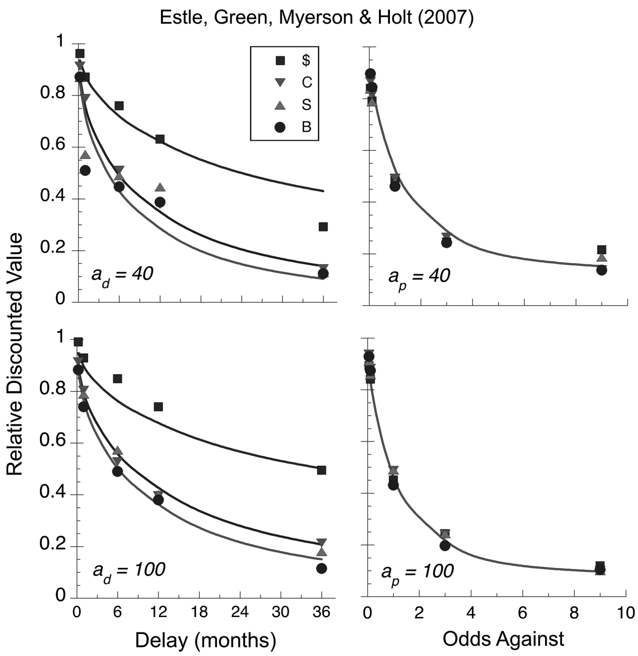
Figure 1

Delay and Probability Discounting Based on Estle et al.'s (2007) Figure 1



Note. The left column is delay discounting, and the right is probability discounting. Odds against is $(1-p)/p$. The top row is for a small amount, the bottom is for a large amount. The curves are from Equation 18, reduced to Equation 20 for the left column and Equation 12 for the right column. The data are for money, candy, soda, and beer. The delay discount curves for the sweets were equivalent and are drawn with the same function. See the text for parameters.

Delay Discounting

The AUT of delay discounting (Killeen, 2009) took the utility of a good to be a concatenation of separable power functions of its amount and its delay. For these variables, Equation 1 becomes:

$$U(G_{a,d}) = F\left[(k_a a)^{\alpha} \dotplus (k_d d)^{\beta}\right], \tag{2}$$

where $\dotplus$ symbolizes concatenation—either addition (the plus) or multiplication (its dot). The k coefficients have the inverse dimension of the variables, rendering the parentheticals dimensionless.

AUT first explored the simplest aggregation function, addition ($\dotplus \equiv +$):

$$U(G_{a,d}) = F[(k_a a)^{\alpha} - (k_d d)^{\beta}]. \tag{3}$$

The utility of a good is a function of the sum of the amount and the delay, each raised to its own power, their sum taking a minimum of 0. The delay to a positive good constitutes a disutility, thus the negative sign. ~~Equation 3~~. Equation 3 entails that the effect of delay is separable from the nature of the good that is delayed. Frederick et al. (2002, p. 358) emphasized that "This feature [invariance across all forms of consumption] is crucial to the notion of time preference," as did Paglieri et al. (2015, p. 204). Later, Frederick et al. (2002) allowed that "the evidence in favor of a single construct of time preference is hardly compelling" (p. 391). Their analyses concerned discounted utility (DU) models,[1] the conventional multiplicative concatenation of a utility function with probability, or later with a probability weight. It is the thesis of the present article that such models do not concatenate the controlling variables properly, and when that is corrected, time invariance is restored. In this respect, AUT is a radical departure from traditional DU models.

In a typical discounting experiment, subjects are asked to indicate their preference between a fixed good at a delay of d, a_d, and an immediate good at the delay of zero, a_0, the latter having utility

$$U(G_{a,d=0}) = F[(k_a a_0)^{\alpha}]. \tag{3'}$$

The amount a_0 is then adjusted to converge on the amount isohedonic with the delayed amount, a_d, so that the utilities are equal, and:

$$F[(k_a a_0)^{\alpha}] = F\left[(k_a a_d)^{\alpha} - (k_d d)^{\beta}\right]$$
$$(k_a a_0)^{\alpha} = (k_a a_d)^{\alpha} - (k_d d)^{\beta}. \tag{4}$$

Take the inverse of the utility function $F[x]$ on each side of the first line to remove it, leaving the second line (Killeen, 2009, p. 604). The process of setting utilities equal, so that the conversion from values to utilities need not be specified, exemplifies a Brindley "Class A" psychophysical procedure (Chirimuuta, 2014; Gescheider, 1997). Solve Equation 4 for the discounted value a_0:

$$a_0 = \left(a_d^{\alpha} - \lambda d^{\beta}\right)^{1/\alpha}. \tag{5}$$

Equation 5 gives the equilibrium amount of the immediate good, a_0, that is isohedonic with the package of amount a_d at a delay d (Killeen, 2009, Equation 6). The rate constant λ (lambda) is a ratio of the k coefficients raised to their powers. Because it is only their ratio that matters, its use saves a parameter and avoids the problem of their collinearity. These key equations[2] are summarized in the Appendix.

Terminology

The frame set is now useful to clarify terms. Setting $d = 0$ in Equation 5 gives $a_0 = a_d$: The immediate worth a_0 of a good a_d delayed 0 s is simply a_d, its dimensions dollars or pounds or counts some other metric dimension. This is not a utility; those are given by $F[x]$ in Equations 1–4. The functions $f(x)$ inside the brackets of Equation 1 are psychological *value functions*. They rescale the metric dimensions into psychological ones. This is in the tradition of rescaling sensory dimensions into their psychological counterparts

[1] Most aggregation functions in the literature involving delay and probability are multiplicative, including the DU framework and the popular exponential and hyperbolic discount functions. Ballard and Knutson (2009) provided evidence for processing of magnitude and delay in separate regions of the brain during temporal discounting tasks, consistent with separable representations.

[2] A Maclaurin expansion of (a^{α}) shows that it curves it down into an approximately logarithmic function of amount as α approaches 0 (see, e.g., Figure 1 of Killeen, 2015a). Equation 5 then reduces it to the exponential power function of Ebert and Prelec (2007). A series expansion of that function in turn further reduces Equation 5 to Rachlin's (2006) hyperboloid (as shown in Killeen, 2015a, Equations 5 and 6). Equation 5 thus nests other strong forms of the discount function. The parenthetical term is bounded from below by 0.

(Shepard et al., 1972). Those, after establishing the rules for concatenating them, provided the basis for a universal law of generalization (Shepard, 1987). The utility function $F[x]$ acts on the concatenated value functions inside the brackets of Equation 1, transferring them to a single utility dimension. $F[x]$ may be an identity function or a power function or a logarithmic function, or something else. In the matching experiment represented by Equation 4, these functions, whatever they are, are mathematically removed from its top line by taking their inverses on each side, giving the bottom line. Physiologically, they are also removed by the matching operation: Consider a perception experiment with a participant named ElGreco, whose astigmatism caused him to see vertical dimensions more elongated than horizontal ones. Yet, if he were asked to sketch a model, the sketch would not be elongated: However, aberrant in his perception, the sketch would match the model when it reached the same veridical proportion because his image of the sketch would also be warped, canceling the effect. It is only when operations on the subjective representations are required that we may infer transfer functions. If we asked ElGreco the ratio of height to width, his answer would reveal his strange transfer function.

The Class A matching experiments are a kind of "outer psychophysics," whereas those that engage the internal values are a kind of "inner psychophysics" (although this was not Fechner's use of those terms). There is a long history of efforts to understand both the concatenation problem ("ratios or differences?" e.g., Grace et al., 2018; Heller, 2021; Masin, 2014) and how those interact with the transfer functions $F[]$ (e.g., Ellermeier & Faulhammer, 2000). AUT comprises a part of that venerable literature. In particular, Shepard found that concatenation of "separable" sensory dimensions (e.g., size and color of a circle) required the "city block" metric (the L1 norm), wherein psychological distances are additive, just as they are in AUT (Equation 3). Shepard's proximities transferred onto a logarithmic axis, yielding exponential generalization gradients. The concatenated value functions might also be mapped onto a decision axis by a logarithmic utility function (Walsh, 2003), but that does not enter the analyses of this article, focused on Class A experiments.

Relative Discount

The discounted amount a_0 is often normed by the nondiscounted amount a_d to give the relative discount. Dividing Equation 5 through by a_d gives:

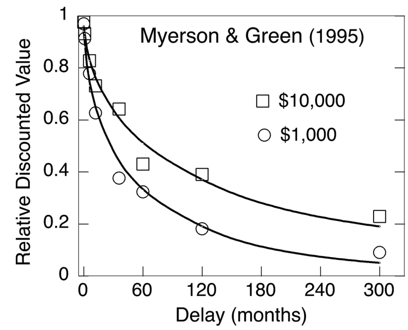
$$\frac{a_0}{a_d} = \left(1 - \frac{\lambda}{a_d^{\alpha}} d^{\beta}\right)^{1/\alpha}, a_d, \alpha > 0. \qquad (6)$$

Notice that a^{α} divides the rate constant λ. This is what causes a magnitude effect, and does so intrinsically, without parameter adjustment, with large amounts discounted more slowly than smaller ones. Equation 6 takes a floor of 0.

Figure 2 shows that AUT provides a parsimonious account of some relevant data. Similar parameters ($\alpha = .09$, $\beta = 0.65$, and $\lambda = 0.08$) also provided an excellent fit to the larger data set of Green et al. (1997) involving four different amounts. The coefficient of determination (*CD*, the proportion of variance accounted for) was 0.96 in that analysis.

Figure 2

Discounting Data From Myerson and Green (1995), Curves From Equation 6



Note. The parameters are $\alpha = .196$, $\beta = 0.508$, $\lambda = 0.096$ for both curves.

This formulation reconciles the desire for "invariance of time preference across all forms of consumption" (Frederick et al., 2002; the right addend in Equation 4), with the dependence of the empirical discount function on the magnitude and nature of the good (Equation 6; also see Kirby, 1997). This occurs for two reasons: (a) the concatenation is addition and (b) the functions on the amount are nonlinear, in particular[3]: $\alpha < 1$. To return to discounted value, the package is raised to the power $1/\alpha$. This nonlinear interaction has been noticed before by Pitts et al. (2016) and by Frederick et al. (2002, p. 381):

> The standard approach to estimating discount rates assumes that the utility function is linear in the magnitude of the choice objects .... If, instead, the utility function for the good in question is concave, estimates of time preference will be biased upward .... This confound is rarely discussed.

Equations 5 and 6 resolve that confound.

Probability Discounting

How can Equation 1 be developed for probability discounting? What function on probability will permit it to be combined with what function on the amount? How will those functions be combined? Myerson et al. (2011) crossed five probabilities with nine amounts, providing the database necessary to answer to those questions. Reducing Equation 1 to:

$$U(G_{a,p}) = F[f(a) \dotplus w(p)], \qquad (7)$$

conjoint measurement (Luce & Tukey, 1964) then determined the values of the functions on the marginals of the $a \times p$ matrix that were required to predict the body of the matrix. Analysis (Killeen, 2023) started with: (a) The working hypothesis that the concatenation was multiplicative: $\dotplus \equiv \times$. (b) Proceeded to find the numerical values that must be assigned to each of the amounts and probabilities to generate the Myerson et al. (2011) data matrix, treating the

[3] A magnitude effect is predicted for any positive value of α; in this range of delays, however, it is amplified by small values of that exponent. The average difference in the curves in Figure 2 is 0.11; with α increased to 0.5 and then to 0.9, the difference is reduced to 0.07 and 0.003. At much longer delays, a robust magnitude effect is recovered for larger values of α.

marginals as free parameters and adjusting them to maximize the goodness of fit to the data. (c) Determined the value functions *f* and *w* that would represent those 14 free parameters more parsimoniously. Figure 3 shows the results of this analysis.

The ordinates in the first two panels are the fit parameters for amount and probability, the subjective marginals for the matrix. The impressive regression in the first panel shows that *f*(*a*) is a logarithmic function of amount (Portugal & Svaiter, 2011; Walsh, 2003).

There were several good candidates for the weighting function on probability, but the best was clearly Prelec's (Gonzalez & Wu, 1999; Luce, 2001; Prelec, 1998)—Equation 8—which grounded the fit parameters in the second panel.

$$w(p) = e^{-\kappa(-\ln(p))^{\gamma}}. \tag{8}$$

We may now flesh out Equation 7, multiplying the value of the amount by the probability weight:

$$\begin{aligned} U(G_{a,p}) &= F\{\alpha \log(k_a a_p) \cdot w(p)\} \\ &= F\{\log[(k_a a_p)^{\alpha w(p)}]\}. \end{aligned} \tag{9}$$

When $p = 1$, $w(p) = 1$, giving the utility of a sure thing as:

$$U(G_{a,P=1}) = F\{\log[(k_a a_{p=1})^{\alpha}]\}. \tag{9$'$}$$

That utility is isohedonic with the probabilistic utility when, setting them equal as in the matching paradigm, and taking the inverse of the (unspecified) utility function *F*:

$$\log{(k_a a_{p=1})^{\alpha}} = \log{(k_a a_p)^{\alpha w(p)}}. \tag{10}$$

Exponentiate:

$$(k_a a_{p=1})^{\alpha} = (k_a a_p)^{\alpha w(p)}, \tag{11}$$

and take the α root of each side:

$$k_a a_{p=1} = (k_a a_p)^{w(p)}. \tag{11$'$}$$

For the relative discount, divide each side of Equation 11′ by $k_a a_p$:

$$\frac{a_{p=1}}{a_p} = (k_a a_p)^{w(p)-1}, \tag{12}$$

with a_p the undiscounted amount of the good. Equations 12 and 8 provide an excellent account of Myerson and associates' 45 data, as shown in the right panel of Figure 3. Equation 12 acquitted itself equally well with other data sets (Killeen, 2023).

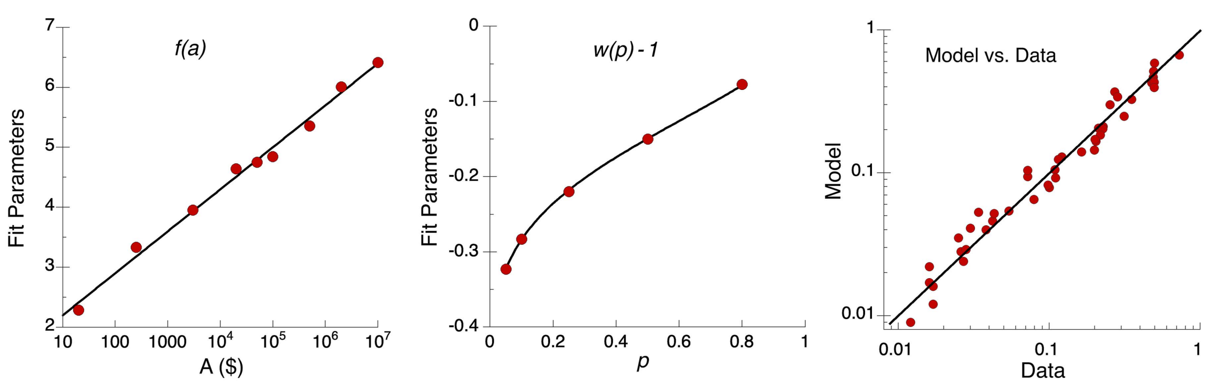
For traditional lotteries (Erev et al., 2017), evaluating the sums of Equation 9 for all possible outcomes of the alternatives, and embedding those into a standard logistic choice model (Luce, 1959), provided good conformity with data. Because the current theory involves both additive (delay) and multiplicative (probability) factors, it will be referred to as a general theory of discounting (GTD).

Delay *and* Probability Discounting

How should Equation 1 be developed when both probability and delay are varied? One possibility is to subtract the delay from the right-hand side of Equation 10. But then exponentiating to return to the metric world yields a multiplicative model of delay discounting: a DU model with the probabilistic amount multiplying an exponentially decreasing function of delay. This makes time preference depend on the amount, whereas we have seen that they are separable dimensions of a good. The considered model does not reduce to Equation 5 from AUT, which has proven superior to such multiplicative models (Killeen, 2009, 2015a, 2015b, 2019; McKerchar & Mazur, 2023).

Figure 3

Results of the Conjoint Analyses of Amount and Probability



Note. The ordinates of the first two panels are the free parameters corresponding to the value function on the amount, and the probability weights, necessary to fit the observed data of Myerson et al. (2011); the third demonstrates the goodness of fit of the resulting model (Equations 8 and 12). The regression in the first panel shows that the value of the gamble is a logarithmic function of its amount, here drawn by 0.30 ln(a) + 1.5. The curve in the second panel is drawn by *w(p)* − 1, with *w(p)* given by Equation 8 using parameters $\kappa = 0.33$ and $\gamma = 0.60$. The unit subtrahend is due to the exponent's use in estimating relative discount rates. The insertion of Equation 8 into Equation 12 gives the general model governing the third panel, with $k_a = 138/\$$, $\kappa = 0.14$, and $\gamma = 0.59$. See the online article for the color version of this figure.

A model that maintains separability may be constructed from Equation 4: Subtracting the value function on delay from the probabilistically discounted function on the amount given by Equation 11:

$$\begin{aligned}(k_a a_{d=0,p=1})^{\alpha} &= (k_a a_{d,p})^{\alpha w(p)} - (k_d d)^{\beta}\\ k_a a_{d=0,p=1} &= \left[(k_a a_{d,p})^{\alpha w(p)} - (k_d d)^{\beta}\right]^{1/\alpha}. \end{aligned} \tag{13}$$

This simply raises the first term of Equation 4 to the power *w(p)*. When $p = 1$, $w(p) = 1$, restoring Equation 13 to its original form (Equation 4). When $d = 0$, we are returned to Equations 11; there, we can see that $(k_a a_p)^{w(p)}$ devalues the amount a_p to predict the certain immediate equivalent (CIE) amount, $a_{d=0,p=1}$. The CIE is not a value function on amount: That is its logarithmic transform. Equation 13 has the important implication that participants evaluate the two parts of the package—the CIE and the value of delay—independently, and then add.

Why should probability modify the psychological value of the amount (Equation 9) but delay subtract from its CIE (Equation 3)? Probabilities are qualifications of the good; they are adjectives, "probabilities *of*" something. A year, however, is not about anything else; it is a noun. Computing the subjective expected value of a good involves mental arithmetic—inner psychophysics—just as does addition and subtraction of subjective magnitudes (Ellermeier & Faulhammer, 2000). It makes sense that participants would simplify multidimensional choice problems by first cashing out the CIE of the probabilistic good and from that subtracting the value of the delay. There are other ways to construe the mental operations, as sketched in the first paragraph of this section; but they do not fare so well when confronted by data.

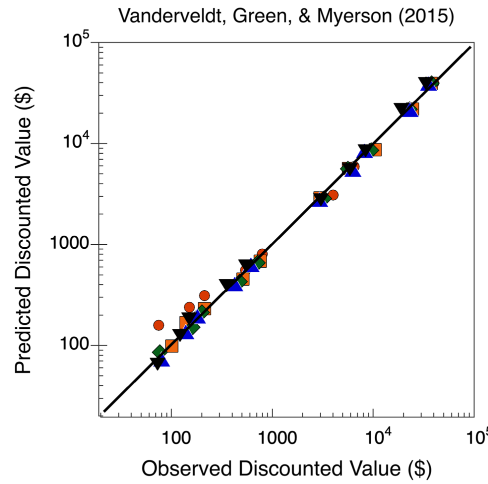
With the addition of delay the exponent α will not cancel from Equation 13, as it did for Equation 11. Solving for the CIE of the package, $a_{d=0,p=1}$:

$$\begin{aligned}a_{d=0,p=1} &= \left(k_{\mathrm{a}}^{\alpha(w(p)-1)} a_{d,p}^{\alpha w(p)} - \lambda d^{\beta}\right)^{1/\alpha}\\ &\approx \left(a_{d,p}^{\alpha w(p)} - \lambda d^{\beta}\right)^{1/\alpha}. \end{aligned} \tag{14}$$

In the cluttered first line of Equation 14, the coefficient of $a, k_{\mathrm{a}}^{\alpha(w(p)-1)}$ goes to 1 with p ($w(1) = 1$, driving the exponent of the coefficient to zero, reducing the coefficient to 1). k_a is multicollinear with the other parameters, creating an unstable model. It may be fixed at 1, letting λ pick up the slack, which has a negligible effect on the goodness of fit to data.[4] Although the first line is the correct form, the use of the second line approximation adds stability and parsimony by deleting a somewhat redundant parameter when fitting data. When $p = 1$, Equation 14 reduces to Equation 5.

Again, thanks to the excellent work of the Washington University laboratory, there are strong data to test this synthesis. With both variables in play, Equation 14 describes the results of Vanderveldt et al. (2015), as seen in Figure 4. Retention of the k coefficient scarcely improved the consilience of data and theory (Footnote 4).

Figure 4
Discounting of Both Delayed and Probabilistic Goods



Note. The predictions are from Equation 14 on logarithmic axes, with *w(p)* given by Equation 8, and α = .28, β = 0.40, λ = 0.14, κ = 0.17, and γ = 0.81. The symbols are for 50 different amount/probability/delay combinations. From "Variations on a Theme by Rachlin: Probability Discounting," by P. R. Killeen, 2023, *Journal of the Experimental Analysis of Behavior*, *119*(1), pp. 140–155 (https://doi.org/10.1002/jeab.817). Copyright 2022 by Society for the Experimental Analysis of Behavior. Reprinted with permission. See the online article for the color version of this figure.

Losses

How can GTD deal with the situation in which the "goods" are debts? Rewrite Equation 4 for losses with their negative values:

$$-(k_a a_0)^{\alpha} = -(k_a a_d)^{\alpha} + (k_d d)^{\beta}. \tag{15}$$

Here the delay has a positive psychological value: Savings can gain interest in the market, inflation is working in your favor, and your creditor may forget or die. Multiply through by −1 to return to Equation 4, whose predictions follow *en suite*[5]

Estle et al. (2006) conducted delay and probability discounting tasks with gains and losses. The procedure for gains is familiar (see the procedural excerpt at the beginning of this article). For losses, participants had to pay an amount now, or a larger amount at some delay, and the immediate debt was titrated until indifference was achieved. The resulting discount functions were of familiar hyperboloid shape, but the loss function lay significantly above the gain function in delay discounting. Why were the loss functions shallower than the gain functions? The answer is found in the critical Equation 5, where the delay is multiplied by $\lambda = k_d^{\beta}/k_a^{a}$. The coefficient for the amount is in the denominator. k's for losses are larger than those for gains—30% larger on average—classic loss aversion (Schmidt & Zank, 2005). This predicts a slope for losses on the order of 1/1.3 = 77% that of the slope for gains, consistent with the results of Estle et al. (2006).

[4] Using the top line of Equation 15 with an optimal value of $k_a = 2.7 \times 10^{12}$, and with compensatory changes in the other parameters, increases the goodness of fit for the data in Figure 4 from a CD of 0.994–0.998. Whereas this fuller model is approved by the Akaike information criterion, its primary contribution is to draw in the outlying disks at the bottom of the figure.

[5] If we were to construe the losses as decrements from the value of some bank account, B, that value would subtract out from each side of Equation 15.

If we assign negative psychological values to both certain and probabilistic losses in Equation 11, Equation 12 gives the predictions for probability discounting. The exponent of the right-hand side is negative, putting k in the denominator, and thus, again, predicting the shallower gradient for losses than gains (Estle et al., 2006, Figures 2 and 7). The difference in slopes was smaller than for temporal discounting because the coefficient in Equation 12 is raised to the subunitary power $w(p)-1$.

The literature on magnitude effects in the discounting of losses often yields inconsistent results. Unlike Estle et al. (2006), Green et al. (2014) found no magnitude effect with delayed or probabilistic losses; Cox and Dallery (2016) found magnitude effects for delayed gains and losses and probabilistic gains, but not for probabilistic losses. Inconsistent results may be due to inhomogeneity in how subjects construe their situation, with average results a mélange of those frames. Equation 15 treats loss as negative gain and holds for individuals who are loss averse, as most people are. The loss gradient is not always decreasing, however: Furrebøe (2020) found that some participants had flat loss discount functions pegged at 1.0. Such people are eager to settle debt: They pay off mortgages and credit cards before they are due, even though that money could have earned greater income if invested, evincing a kind of financial "precrastination" (Rosenbaum et al., 2019). Myerson et al. (2017) found that most of their participants conformed to Equation 15, characterizing them as "Loss Averse." Over 25% of their participants, however, had inverted discount functions, and they were characterized as "Debt Averse." Their choices are captured by subtracting the value of delay in Equation 15: The longer a debt is held, the more onerous it becomes to the Debt Averse (also see Furrebøe, 2022; Yeh et al., 2020).

A change in sign for the value of delay also appears in some situations involving gains, where individuals might prefer to delay a kiss or other positive good in order to savor it before the fact (also see Albers et al., 2000). Much of the pleasure in prose, and poetry, and humor—is found in the delay of its denouement—and the titillation that affords. In the next section, it is shown that the steepness of the delay gradient depends on the diversity of the items that the good can purchase. When that is minimal, delay gradients are steepest. With debt, there is only one thing that you can purchase, and that is relief.

Liquidity, Diversification, and the Portfolio of Desires

Liquidity, a term introduced by Keynes, is the ability to quickly convert an asset to cash—to liquidate it. Money is the most liquid of assets: its market value is in hand; it is fungible, and easily exchanged for other assets. Interest on certificates of deposit is the rent that the bank pays for borrowing that liquidity. A remote cabin, however valuable to you, must find an interested buyer and is therefore less liquid. Unless you are a novelty trader, candy bars are less liquid than dollars or cabins. Consider the contrast between the immediate value to you of $1,000 delayed for a year and of 1,000 candy bars delayed for a year. You would have to eat the latter, give them away, or convince your local banker to accept them as legal tender for your debts. Against 1,000 later, you might happily settle for 10 candy bars now, but never for $10 now. It is not that the intrinsic rate of discounting changes, but the utility of a big pile of candy, is less than the utility of a big pile of money. The liquidity of money, and its ability to help diversify the goods in the package it will buy, is key to solving the question posed at the start of this article: Why delayed money is apparently discounted more slowly than other goods? The rest of this section works through the implications of that insight, where it is shown that the intrinsic rate of time discounting does not change for money or other commodities but rather it is offset by changes in the utility of the thing discounted.

I assume, and I think that you will grant, that most subjects in delay and probability discounting tasks consider assets such as candy illiquid: They wonder what they will do with them, not what they can barter them for, or where. A problem with large amounts of any good is its decreasing marginal utility.[6] The thousandth candy bar is less valuable than the first and may even have negative utility ("Now where the hell am I supposed to put *that* one?"). This is noted in Paglieri et al.'s (2015) motivational account: "People's intertemporal choices with nonmonetary rewards would not reveal steeper delay discounting, but rather *a weaker motivation to maximize those rewards in the first place*" (p. 202). In the terminology of GTD, the value function for nonmonetary rewards is shallower than that for monetary ones. Equation 6 shows why the rate of discounting of delayed goods appears to vary with the magnitude of the delayed goods: Their value is a divisor of the rate of decay. This is why discount functions vary for various goods and for various people. Because of the decreasing marginal utility of goods, liquidity, as we shall see, has a profound effect on the psychological value of goods and thus on the apparent rate of discounting.

To the extent that candy is illiquid, you are stuck with it. This is not the case with money. The aspect of liquidity that is central to this story is the diversification it enables: How many different types of goods of interest to you are readily available given the experimenter's promissory note for the money, present or future? Money enables the greatest diversification and can be traded for any goods in your market. Soda is a less diverse set of goods, colas less so, and Pepsi–Cola less so again. It is the variety within that category of goods, as well as the variety of categories, that matters. Because money is fungible, except for the convenience of carry its variety does not matter: 100 singles, 20 fives, 5 twenties, 2 fifties, and 1 hundred will all buy you exactly the same amount of candy. The variety it can buy, however, does matter.

A fundamental assumption of consumer theory is that purchasers attempt to maximize overall utility. This entails that they will not spend their last dollar on the thousandth candy bar. Instead, they will distribute their expenditures to goods that will provide, ensemble, the greatest pleasure. Just how to do that is called the *utility maximization problem*. Under general assumptions, the problem is solved when the marginal utility of the purchases divided by their prices are all equal. This is because if one marginal utility per cost is greater than another, you may improve the utility of your portfolio by taking some from the latter and redistributing it to the former, to achieve "more bang for the buck." Because any item's marginal utility continues to decrease with additions of it to the portfolio, redistributions will proceed until equilibrium. That equilibrium is only temporary, of course, as consumers' desires evolve. If price is measured as the time required to acquire the good, it follows that the allocation of resources should match the rate of reward afforded by them (Rachlin et al., 1980). Because marginal utilities tend to be

[6] For the logarithmic utility function given by Equation 9′, the marginal utility of amount a is $d[U(G_a)]/da = \alpha/a$, demonstrating the inverse relation between marginal utility and amount. More is better, but not so much.

greatest for those goods that one has the least of—they are then at the steepest part of their utility functions—that is where the cash will flow. Perhaps some candy, some popcorn, a movie, and a good dinner afterward; not 100 candy bars. Let us call such a set of potential assets your *portfolio of desires* and develop GTD to account for the maximization of the utility of that portfolio. The portfolio is the frame of prospects that affect the current decision (Tversky & Kahneman, 1981). It is the beneficent tunnel vision that lets externals—your current income, marital satisfaction, job prospects, and so forth fade into the background for this choice.

Given you have won a lottery of a dollars, how should you allocate that new resource? Assume you have a number n of goods in mind that you could buy and that they all have similar costs and utilities as given by Equation 3′. You can then trade your money for n assets of average magnitude a/n, $n \geq 1$, increasing the utility of the portfolio by:

$$\begin{aligned} U(P) &= F\left[n\left(k\frac{a}{n}\right)^{\alpha}\right] \\ &= F[n^{1-\alpha}(ka)^{\alpha}]. \end{aligned} \tag{16}$$

The second line consolidates by taking n outside the parenthesis.

For $n = 1$, you will invest the windfall a all in one asset. For $n = 2$, you will purchase two assets, each for half the amount of the windfall. How large should n be—how much you should diversify? As long as the exponent of n is less than 1, which is typical, then the change in value is a concave function of the number of assets in the portfolio. As a consequence of Jensen's inequality—the sum of concave functions of a variable is greater than the concave function of their sum—you should increase n as much as possible to maximize your utility. Lots of stuff rather than lots of candy to take advantage of each new asset at the steepest part of its utility function.

The only things that keep you from putting an infinite number of minuscule assets in your portfolio are (a) your imagination; (b) their availability (Shah et al., 2015); (c) the fact that most assets come in minimal useable sizes (you cannot easily acquire, or much enjoy, one hundredth of a candy bar, a crumb of a cookie, or a fender of a Ferrari)[7]; (d) some of the potential assets, such as candy and cookie, might be partial substitutes so that having one will lessen the utility of the other; (e) there are diminishing returns to such diversification, as the marginal utility of n decreases as n increases. In addition, if there are costs[8] to acquiring the desired goods (most of which are themselves illiquid, requiring some effort and delay to acquire), a fairly modest portfolio could be optimal.

In light of Equation 16, develop Equation 13 for diversity[9] to arrive at:

$$n^{1-\alpha}(k_a a_{d=0,p=1})^{\alpha} = n^{1-\alpha}(k_a a_{d,p})^{\alpha w(p)} - (k_d d)^{\beta}. \tag{17}$$

Divide through by the diversity index,[10] $n' = n^{1\text{-}\alpha}$, and develop Equation 17 in the same steps as was done for Equation 14:

$$\begin{aligned} a_{d=0,p=1} &= \left(k_{\mathrm{a}}^{\alpha(w(p)-1)} a_{d,p}^{\alpha w(p)} - \frac{\lambda d^{\beta}}{n'}\right)^{1/\alpha} \\ &\approx \left(a_{d,p}^{\alpha w(p)} - \frac{\lambda d^{\beta}}{n'}\right)^{1/\alpha}. \end{aligned} \tag{18}$$

As before, for typical exponents, the coefficient $k_{\mathrm{a}}^{\alpha(w(p)-1)}$ is close to 1, and in respect of parsimony, its work can be offloaded to a slightly inflated value for λ. Obtain the relative discounted value by dividing by the undiscounted amount, $a_{d,p}$:

$$\frac{a_{d=0,p=1}}{a_{d,p}} = \left(a_{d,p}^{\alpha(w(p)-1)} - \frac{\lambda}{n' a_{d,p}^{\alpha}} d^{\beta}\right)^{1/\alpha}. \tag{19}$$

Equations 18 and 19 reduce to their familiar forms when $d = 0$, $p = 1$, or $n' = 1$. For an illiquid good, the variety of goods it can be exchanged for, n, is close to 1: It can only be traded for similar items (fungible, or near-substitute, goods), as children do with candy from their Halloween plunder, bartering to diversify, instinctively maximizing the utility of their portfolio of desirable candies.

It is important to remember that although n' divides the coefficient for delay λ, it is introduced in Equation 17 as an amplifier of the value of the amount of payoff; a is more attractive for all the things you can spend it on. Its mode of action is to increase the value of the amounts in Equations 18 and 19, which causes it, in this additive model, to offset or overshadow the disutility of delay, functionally decreasing its apparent discount rate. It was the process of solving Equation 17 for the relative value of the sure immediate amount that landed both n' and $a_{d,p}$ in the basement of the delay addend in 18 and 19.

Econometricians measure the diversity of a portfolio by its entropy (Jacquemin & Berry, 1979; Palepu, 1985). For a portfolio of size n with an equal number of items of each kind in it, entropy equals the logarithm of n. If the utility function $F[x] = \log[x]$, then Equations 16 may be written as:

$$\begin{aligned} U(P) &= \log[n^{1-a}(ka)^{\alpha}] \\ U(P) &= \log(n^{1-a}) + \log[(ka)^{\alpha}] \\ U(P) &= (1-\alpha)\log(n) + \alpha\log(ka). \end{aligned} \tag{16′}$$

The utility of the portfolio is a weighted average of its entropy ($\log[n]$) and the value of the added amount. When n equals its minimum of 1, we are returned to the basic model. In the case of a linear utility function with α at its maximum of 1, Jensen's inequality no longer applies, diversification no longer adds value, and we are returned to the basic model. Equation 16′ is another way of illustrating how diversity adds to the utility of a portfolio, up to the point where n approaches the maximum useful division of goods. It reminds us that the intrinsic value function for the amount of money is logarithmic (cf. the first panel of Figure 3) and that other

[7] For nondivisible goods such as machines and animals, the utility function resembles a staircase starting on a tread, with each subsequent riser being of shorter height. A continuous version of the staircase is an s-shaped utility function that accelerates until a viable unit is acquired and decelerates thereafter. It is sometimes called a Markowitz utility function. See Sawicki and Markiewicz (2016) and Locey et al. (2023) for further discussion of the nondivisibility-of-goods problem.

[8] Sometimes such acquisition (viz., shopping) constitutes a good in itself, rather than a cost, leading to an overdiversity in many closets (https://shorturl.at/cgqGK).

[9] Because the immediate sure amount is smaller than the delayed probabilistic one, its diversity index may perforce be smaller. Furthermore, if there is a minimal unit size of some goods—that is, if they have Markowitz utility functions—it may not be possible to spread a small a around evenly. The more parsimonious assumption of equality suffices for the moment; the issue will be revisited below.

[10] This substitution is made for simplicity, as that index may be treated as a single free parameter, without raising it to a power.

discounting functions may owe their curvature to the degree of diversity—the entropy of the portfolio—that the discounted good affords—the first term in the *lhs* of Equation 16′.

Application to Data

Setting $p = 1$, whence $w(p) = 1$, reduces Equation 19 for simple delay discounting to

$$\frac{a_{d=0,p=1}}{a_{d,p}} = \left(1 - \frac{\lambda}{n' a_{d,p}^{\alpha}} d^{\beta}\right)^{1/\alpha}. \tag{20}$$

Equation 20 directed the curves in the left column of Figure 1, with parameters $\alpha = .205$, $\beta = 0.490$, and $\lambda = 0.59$ for all curves. The values for n' are relative; so, set the liquidity for money, the gold standard, to $n' = 10$. For typical values of α, this is equivalent to $n \approx 7$ items, about the number of simple things we can keep in mind at once (Shiffrin & Nosofsky, 1994). Keeping all other parameters invariant, then adjust n' for the other goods. The discount functions for the sweets were not significantly different from each other, so a common diversity index, $n' = 4.8$, sufficed for candy and soda. For beer, $n' = 4.0$. The model projects the relative discount functions at 36 months for the large amounts in Figure 1 to lie 7 points above those for the small amounts, demonstrating a positive magnitude effect.

For probability discounting, reduce Equation 19 by setting $d = 0$. This returns us to Equation 12, reprinted here for convenience:

$$\frac{a_{p=1}}{a_p} = (k_a a_p)^{w(p)-1}. \tag{12}$$

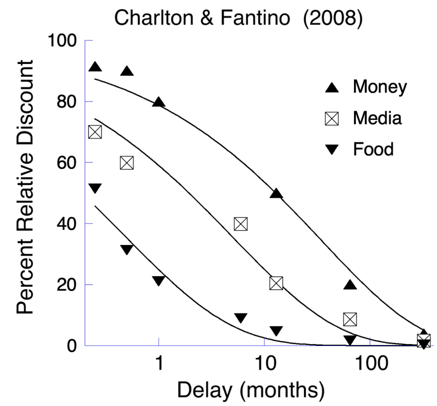
The diversity factor has dropped out with *d*. Liquidity should matter for delay discounting, but GTD predicts that it will not matter for probability discounting: Cash and commodities should follow the same discount curve. This is because diversity is a factor of amount on both sides of Equation 17, and absent the additive disutility of delay, it can cancel out (assuming it is the same for the immediate and delayed goods, viz., the amount in both cases suffices to meet their preferred diversification). The exponent α also cancels out.

Equation 12 provided an excellent fit to the data (see the right column of Figure 1). To avoid multicollinearity, the coefficient was moved outside the parentheses[11] and is called k_a'. So configured, Equation 12 directed the curves through the data in the right column of Figure 1, with parameters $k_a' = 0.94$, $\kappa = 0.26$, and $\gamma = 1.12$. The model projects the relative discount functions at odds of 9/1 for the large amounts to lie 5.5 points *below* those for the small amounts, demonstrating a negative magnitude effect. This occurs because the exponent in Equation 12 is negative, placing a_p in the denominator. Increases in that amount thus decrease the height of the discount function. Anderson and Dallery (2021), among others (e.g., R. M. Green & Lawyer, 2014, Figure 13.7), also reported such a negative magnitude effect, which wass ~~a~~ forced prediction of GTD, just as a positive magnitude effect for the delay wass intrinsic to the model.

An experiment by Charlton and Fantino (2008) throws further light on GTD and portfolio diversification. The authors contrasted discount functions for money, music CDs, movie DVDs, books, and food. Figure 5 displays their results. The functions for the media did not differ significantly and are averaged.

Figure 5

Discounting of Various Goods as a Function of Their Delay



Note. The *x*-axis is log-transformed to make the data at short delays discernible. The predictions are from Equation 20, with $\alpha = .097$, $\beta = 0.405$, $\lambda = 0.36$, and $n' = 10$, 4.7, and 1.8 for money, media, and food. ~~See the online article for the color version of this figure.~~

In this study, the diversity of the media was half that of money. There are many different books, sounds, and sights that media afford. The diversity factor for food was quite low—18%, much less than for the comestibles of Estle and associates. But those authors did not constrain the brands or types of those edibles. Charlton and Fantino (2008) identified the favorite food (typically pizza for college students) and specified that as the food. Fewer varieties of pizza than of "food" and thus a lower *n*.

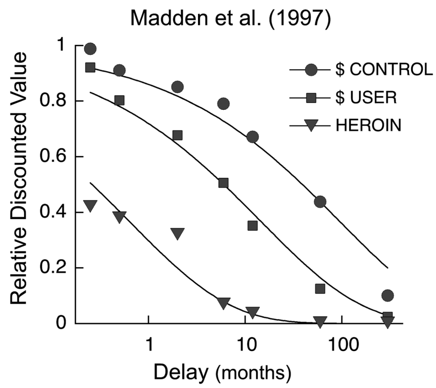
Special Populations

It is well known that drug abusers discount money more steeply than nonabusers (e.g., Bickel & Marsch, 2001; Kirby & Petry, 2004; MacKillop et al., 2011; Reynolds, 2006). Figure 6 gives one example (Madden et al., 1997).

There is an interesting similarity to Figure 5: The discount function at the top of each is shallow for money, and for substances at the bottom quite similar, with a diversity index of $n' = 1.8$ for pizza in Figure 5 and 1.2 for heroin in Figure 6. Pizza, in its varieties, is slightly more diversified than heroin. The remarkable result, here and in many other similar studies, is that drug users discount money much more steeply than nonusers. Why should that be? Is it a character flaw? A mark of Cain that enables their addiction? Possibly. Bickel et al. (1999) found discount functions for money to be the same for never- and ex-smokers and much steeper for current smokers. This militates against a trait explanation. A simpler

[11] k_a was multicollinear with κ and γ (Equation 8), so that a wide range of values for the first, correlated with a range of values for the latter, provided equally good fits to the data. This problem was solved by moving the coefficient outside the parenthesis. This stabilized the parameters at unique values, while producing a negligible decrease in goodness of fit (from a CD of 0.944–0.941). It is then possible to raise this external coefficient, k_a' to the power $1/(w(p_{\text{med}}) - 1)$, where p_{med} is the median probability investigated, and restore it to inside the parenthesis.

Figure 6
Discounting of Money and Heroin as a Function of Their Delay for Controls and Heroin Users



Note. The *x*-axis is log-transformed to make the data at short delays discernible. The predictions are from Equation 20, with $\alpha = .010$, $\beta = 0.415$, $\lambda = 0.02$, and $n' = 10$ for money (control participants), 4.6 for money (heroin users), and 1.2 for heroin.

explanation is that heroin users will have a very unbalanced (lower entropy) portfolio of desires: Their diversity factor for money is half that of controls. It is not hard to intuit what the drug users plan to spend their money on: Most on their drug of choice. L. Moody et al. (2016) studied participants who smoked, drank, or used cocaine. All discounted more steeply than community controls, and dual substance users discounted more steeply than mono-substance users. There was still, apparently, some ability to benefit from diversification into two substances—but not three, according to their data. But this should have made the discount functions shallower, contra GTD's prediction. An ad hoc explanation might be that single substance users—say cigarette smokers—would not spend all their resources on cigarettes and still had diversity in their portfolio. With a second substance in the mix, such as cocaine, all the resources were corralled by these two substances. Cocaine, whether alone or with other substances, had the steepest rates of discounting (Figure 1, adjusted means). Most of their assets would, apparently, go up their nose.

Cross-Commodity Discounting

Because the immediate sure amount is smaller than the delayed probabilistic one, its diversity index may perforce be smaller. If there is a minimal unit size of some goods—if they have Markowitz utility functions—it may not be possible to spread a small a_0 around evenly (Locey et al., 2023). In cross-commodity discounting, the diversity factors are also likely to differ. In both cases, the factor on the lhs of Equation 17 becomes n'_0 and that on the right, $n'_{\mathrm{d,p}}$, leading to:

$$a_{d=0,p=1} \approx \left(\frac{n'_{d,p}}{n'_0} a_{d,p}^{\alpha w(p)} - \frac{\lambda}{n'_0} d^{\beta}\right)^{1/\alpha} n'_0 > 0, \qquad (18')$$

with analogous changes in Equation 19 (divide each side of Equation 18′ by $a_{\mathrm{d,p}}$). When the diversity indices n' are equal, Equations 18 and 19 are recovered.

For cross-commodity discounting involving money (M) and some other commodity (C) such as candy or drugs which have smaller diversity indices, consider the pairs (M, M), (C, C), (M, C), and (C, M), where the first good is the immediate outcome and the second good is the delayed/probabilistic one. For the first two pairs, the factor of the left addend in Equation 18′ reduces to 1, returning us to Equation 18. Because C has a smaller diversity index n'_0 than does M, and it divides λ, discounting will be greater for the pairs in this order: (C, C) > (M, M).

In the case of (M, C), the fraction multiplying the amount is $n'_{\mathrm{C}}/n'_{\mathrm{M}}$, < 1, which decreases the value of the delayed commodity and thereby increases discounting (there is less of the first addend to offset the disutility of delay). In the case of (C, M), the situation is reversed: The fraction multiplying the amount is now $n'_{\mathrm{M}}/n'_{\mathrm{C}}$, > 1, which amplifies the value of the delayed commodity and thereby decreases discounting. The presence of the diversity factor for the immediate good under λ works in the opposite direction, but not so strongly. The predicted order of steepness of the discount functions is (M, C) > (C, C) > (M, M) > (C, M). Two studies of cross-commodity discounting validate this ordinal prediction (Jarmolowicz et al., 2014, C = sex; L. N. Moody et al., 2017, C = alcohol). A third (Bickel et al., 2011, C = cocaine) does, except for a nonsignificant reversal of the (M, M) > (C, M) prediction.

L. N. Moody et al. (2017) reported the area under the discount curves (AUCs) that essentially averaged the relative discounts at each delay. The reported AUCs for the pairs, in the order listed in the predictive inequality above, were 0.19, 0.49, 0.59, and 0.77. Take the parameters from the Madden et al.'s (1997; Figure 6 above) study for α, β, and n'_{M}, and leave λ and n'_{C} as adjustable parameters, the same for all pairs. The model's AUCs were 0.23, 0.50, 0.53, and 0.80, showing a correlation of 0.983 with the data. Odum et al. (2020) reviewed other theories of cross-commodity discounting, in the course of providing an invaluable review of that literature.

Qualitative Effects

Motivational Manipulations

GTD predicts that decreasing diversification of the portfolio of desires steepens the apparent temporal discount function (Equation 19). It finds support in the results of Giordano et al. (2002), who showed that opioid deprivation steepened temporal discounting for both money and heroin in opioid-dependent patients. The former is predicted by GTD. The latter makes sense if the portfolio of desires plays a role in such decisions: Undeprived users might have dreams of other goods to acquire; deprived, only one dream.

Other studies have found a similar effect of motivational state. Field et al. (2006) compared money and cigarette discounting by deprived and nondeprived smokers. Cigarettes were discounted more steeply than money. Thirteen or more hours of deprivation led to significant (and similar) steepening of both discount functions. In a subsequent behavioral economics task, the authors showed that increasing the price of cigarettes would increase the amount spent on them and decrease the amount spent on other goods (viz., clothing, travel, leisure, alcohol, and household goods), thus decreasing the entropy of the portfolio. Mitchell (2004) also found a steepening of the discount function for cigarettes for deprived smokers, but no spillover effect to the gradient of money discounting.

Under one of the more intriguing titles in the reference section, Van den Bergh et al. (2008) found that upon exposure to sexually arousing stimuli (e.g., pictures of beautiful women, lingerie), their male participants discounted both money and edibles more steeply than controls. Sex on the mind narrows the portfolio of desires. Wang and Dvorak (2010) had participants drink soda with natural or artificial sweeteners and found that changes in blood sugar level predicted the steepness of the discount functions for money, with higher levels predicting shallower discounting: Hunger focuses desire, and when blood sugar was higher, food was no longer a dominant expenditure, and the entropy of their portfolios increased. Finally, Skrynka and Vincent (2019) tested six different models of how the steepening might occur (e.g., domain-specific, only to domain and money, partially "spilling over" into other domains). They tested hungry and deprived participants on a delay discounting task for candy bars, music, or money. Functions were steepest for candy, closely followed by music, and flattest for money. The effect of hunger was to greatly steepen the gradient for food, and—to a lesser and equal extent (25% of the log(k) for candy)—the gradients for money and music. Their data and analyses strongly confirmed their spillover model. (In this, and other studies like it, exposure to conditions is randomized. So, some of the spillover effects might be due to hysteresis in the evaluation of delayed goods; Robles et al., 2009; Robles & Vargas, 2007; Rung, Frye, et al., 2019; Yi et al., 2017). Loewenstein (1996) considered such "visceral influences" on economic behavior in general; as do (Downey et al., 2022; Paglieri et al., 2015), with Downey and colleagues reviewing the sometimes mixed results in deprivation studies.

Cognitive Operations

There are other ways to manipulate the diversity of a portfolio. Just having a person think about a variety of events in the future may do. Peters and Büchel (2010) had participants contemplate events set on the day of future payoffs (e.g., vacation in Paris, their birthday) and found a substantial decrease in the discounting of money promised for that day. "60 days from now" surely calls to mind fewer affordances for spending money than a Paris vacation 60 days from now. This manipulation is called *episodic future thinking* (*EFT*), and its effect on discounting has been thoroughly replicated (Radu et al., 2011; Rösch et al., 2022). It is consistent with Rachlin's teleological behaviorism: "To achieve self-control, addicts and non-addicts alike must avoid making each decision on an apparently rational case-by-case basis and learn to make particular decisions in conformity with optimal molar patterns" (Rachlin, 2007, p. S98).

Benoit et al. (2011) found that imagining spending $42 in a pub in 90 days had a bigger emotional impact, and a bigger effect on the discount function, than merely estimating how much the money could buy. The former adds more variety to the portfolio. O'Donnell et al. (2017) used two kinds of EFT primes in which participants listed future events that they were looking forward to. In the first group, the events were related to their financial goals and planning (e.g., "In 2 weeks, I am purchasing a new computer"). In the second group, the events were not related to spending (e.g., "In 2 weeks, I am going home for the weekend"). The first group showed substantially shallower discount functions. It is not, therefore, just about looking into the future that causes this effect; it is about managing assets for the delayed payoff. Mac or PC? Laptop or tablet? Gray or steel blue? Rung and Madden (2018) and Smith et al. (2019) provided reviews and meta-analyses of studies that had manipulated the steepness of the discount functions through motivational, cognitive, and clinical techniques.

Demographic Correlates

Reimers et al. (2009) found that steeper discount functions were associated with younger age, lower income, and lower education; as they are also with lower IQ (Shamosh & Gray, 2008). We can speculate how these conditions may have narrowed the participants' perspectives, and their ability to contemplate the possibility of, or of realizing, a diverse portfolio of desires. Bickel et al. (2014) found that obese individuals had steeper delay discounting functions than nonobese, but the same probability discounting functions. If obese individuals think more about food than about the acquisition of other goods, then both of these results are predicted by GTD (Equations 20 and 12).

Sheffer and colleagues (Sheffer et al., 2014; also see Stanger et al., 2012) found steep delay discounting functions to be a good predictor of smoking-cessation relapse. Perhaps the subjects had increasingly one thing on their mind. They surmised that "Adding an intervention designed to decrease discounting rates to a comprehensive treatment for tobacco dependence has the potential to decrease relapse rates" (p. 1682). On a similar note, Acuff et al.'s (2017) results "provide support for efforts to enhance future time orientation as part of alcohol harm-reduction approaches" (p. 412). Such hopeful goals have driven—and funded—much of the research cited in this article (see, e.g., Rung, Peck, et al., 2019), occasioning a recent interchange (Bailey et al., 2021, 2023; Stein et al., 2023). Yoon et al. (2007) reported similar results and also showed that (a) youth, (b) no more than high school education, and (c) depression were correlated with steeper discount functions. Youth, limited education, and depression are all suggestive of a constricted view of possibilities, limited horizons, and a narrowed portfolio of desires. The authors listed an impressive range of studies showing that depression is associated with "impulsive" behaviors (e.g., suicide, gambling, substance abuse, and compulsive buying; p. 184). Perhaps it is not impulsivity to blame, as much as a low entropy of portfolios: Brooding thoughts and desires that concentrate it too much. The same may be said of the positive correlation between measures of stress and steepness of the discount function (Fields et al., 2014). Acuff et al. (2018) found that lack of access to environmental reward mediated the association between posttraumatic stress symptoms and alcohol problems and craving and suggested that treatments "should attempt to reduce barriers to accessing natural rewards" (p. 177). That is consistent with the work of Berry et al. (2014) who found that exposure to nature scenes substantially decreased delay discounting rates compared to urban scenes and geometric figures and suggested stress reduction as a possible mechanism (see Nukarinen et al., 2022, for a review). Consistent with that hypothesis is the work of Repke et al. (2018) who found that nature accessibility and exposure from home were significantly correlated with reduced scores on a depression, anxiety, and stress scale (cf. Barnes et al., 2019).

Based on correlations of discount rates across domains, Odum (2011) concluded that delay discounting was a trait (cf. Odum et al., 2020). But were those correlations indicative of a trait of impulsivity or of a state of impoverished imagination, which, through poverty or other causes, may itself be heritable?

Contrary Evidence

GTD proposes that the steepness of the discount function for goods—money in particular—depends on the entropy of the portfolio of (achievable) desires. Manipulations that reduce that entropy steepen the delay gradients (as narrowed expenditures move one farther out on one or a few utility functions, incurring decreased marginal utility for them). "Food" has a greater diversity than "candy," and money is greater than either. We, therefore, predict that money would be discounted less than a particular expenditure on the same date in the future. Stuppy-Sullivan et al. (2016) conducted a laborious experiment that belies that prediction. They first conducted a monetary delay discounting task with two amounts. They then had participants say what they would do with each of the immediate and delayed monetary amounts, given their values and delays. They were told to be specific. A week later, they were engaged in a commodity discounting task, using the goods that they had identified as their desire from the previous week. Thus, one option might be "Would you rather have: dinner with your friends (valued at $50) today, or … ." Unlike most commodity discounting tasks, in the large magnitude condition, the rate of decay for these goods was not significantly different from that for money. This might be explained by the participants' defaulting to the dollar value of the outcomes, mindless of its limited contractual liquidity. That explanation will not work for the small magnitudes, however, where nonmonetary goods were discounted at a substantially *lower* rate than money.

Many of the goods that would be purchased have Markowitz utility functions. The utility of the dinner is at its maximal value—half a dinner with a friend is not half as good as a whole dinner, and yet, a second whole meal on that date with hir might have negative utility. The point of liquidity is to enable the acquisition of the most desirable assets. Once they are committed to, liquidity becomes irrelevant. The diversity of a menu is irrelevant once you have ordered. Indeed, surveying one's options for expenditures may itself increase the entropy of the portfolio, making that dinner possibly more attractive even than its costs. Of course, all hypotheses about what is happening in this study are ad hoc.

Summary and Discussion

This article has reviewed the additive utility theory of discounting and extended it to probability and commodity discounting. This more general theory is called the general theory of discounting (GTD). Equations 3–6 show that amount and delay are separable aspects of a good. Equation 3 is virtually identical to Dai and Busemeyer's (2014) core "attribute-wise" model of intertemporal choice, which dominated three other core models (and which received further support in Cheng & González-Vallejo, 2016). In both their model and AUT, the immediate value of the good is computed and added to the value of its delay. Despite all the ways of manipulating discounting functions, the value of k_d in Equations 3 and 4 remains invariant, as does λ in Equations 5 and 6, and as does time's exponent β, which often ranges around or just below 0.5. In this additive model, larger values of the good can offset larger values of longer delays, thus causing a positive magnitude effect. That is not the case for probabilistic discounting, which is an integral dimension of the good. The subjective probability of a good multiplies its intrinsic utility, which is a logarithmic function of amount. When returned to the metric world, the value of the package is the amount raised to a power, the Prelec probability weighting function (Equation 8). This results in a negative magnitude effect for relative probability discounting.

GTD answers the question of why discount functions for money are so much shallower than those of other goods: It lies in the liquidity of money, which can readily be exchanged for other goods, and the diversity of the goods that it can be exchanged for. This is not the first article to suggest these to be important. Holt et al. (2016) compared discounting of diversified versus nondiversified goods (e.g., clothing gift card vs. jeans; grocery gift card vs. candy) and found shallower discount functions for the former than the latter. An exception was a visa gift card versus cash, where the former was discounted more slowly than the latter. Perhaps the gift card helped conjure the various goods it could/must be spent on, whereas cash could go to only paying college debt or to savings, cloistered pleasures at best. The issue of liquidity is reviewed by Odum et al. (2020).

The liquidity of an asset permits the individual to distribute it to maximize the overall utility of the package of goods it can be exchanged for. This is accomplished by selecting goods with the greatest marginal utility per cost. It sufficed for this article to assume that the amount of money would be divided among *n* goods and showed that the utility of that package increased as a concave function of *n*. If the candidate goods for exchange are limited by impoverished conditions (a desert island) or imagination (an impoverished home or narrow focus on a few goods, such as drugs, sex, or power), the diversity index decreases, carrying with it the utility of future possibilities. In AUT, the utility of the good and the disutility of the delay are additive. Anything that increases the value of the good, such as diversification, permits the individual to tolerate a longer delay until its receipt: it decreases delay discounting. For probability discounting, the diversity factor cancels out of the prediction (assuming diversity is the same for discounted and nondiscounted goods). Figure 1 validated these predictions.

In cross-commodity discounting, there are two diversity indices to consider: those of the immediate and the delayed goods. A liquid-delayed good such as money has enhanced utility; an illiquid delayed good such as candy has diminished utility. When this is contrasted with the liquidity of the immediate good, GTD predicts the rank order of the delay gradients for all four combinations of immediate and delayed goods.

It is perhaps not surprising that special populations such as substance users have different discount functions for their substance of choice: That exemplifies the basic commodity effect. It is surprising, however, that they also have steeper discount functions for money. One hypothesis is that steep discounting is a trait, one which might also cause substance use. GTD proposes an alternative explanation: Substance users have a lowered entropy of their portfolio of desires, which is dominated by their drug of choice. Rather than benefit from a distribution of assets to goods still at their highest marginal rate of return, the user goes all in for the drug, incurring substantially decreased marginal utility for larger amounts of it, which fail to balance long delays to receipt of that, or other goods.

There are many ways to change the slope of the discount functions. Increasing the drive for the good of choice is one. That motivation operation will decrease the entropy of the portfolio, focusing it on the deprived substance, decreasing *n*, and increasing the discount of future goods. Commodities that decrease the value of other goods, as drugs often do, will also steepen the discount function (Rachlin, 2000). An important meta-analysis indicated that steep discount functions are

> robustly associated with severity and [quantity and frequency] of addictive behaviors. Importantly, the magnitude of this relation did not significantly differ across the types of addictive behavior examined, offering further support for steep [delay discounting] as a trans-diagnostic process in addiction. (Amlung et al., 2017, p. 59)

Cognitive manipulations such as EFT can enlarge and diversify the portfolio, thereby decreasing discounting. The different gradients found as a function of participants' demographics are consistent with this theory. In their critique of the literature relating delay discounting to addictive behavior, Bailey et al. (2023, p. 1) did not question that association, but rather complained that "there is no clear theoretical formulation that accounts for these associations." GTD provides one.

Econometricians measure the diversity of a portfolio by its entropy. When all goods are equally represented in the portfolio, entropy increases as the logarithm of *n*, the number of items in the portfolio. Entropy decreases when some goods are overrepresented. Overinvestment in a single good means those resources could have been better spent on alternate goods with a greater marginal utility. This may well not matter to a dedicated substance user, who has moved from the "liking" to the "wanting" phase of addiction (Berridge & Robinson, 2016), one who no longer is trying to maximize the utility of the package as a whole, but one in which family, profession, and other goods do not have much standing ("You become a narcotics addict because you do not have strong motivations in any other direction. Junk wins by default." Burroughs, 2012). Or, perhaps because junk has won, you no longer have strong motivations in any other direction.

The entropy formulation permits the structuring of investment into multiple portfolios and then computing the entropy within portfolios and the entropy of those portfolios overall (Hoopes, 1999). This is reminiscent of the way that people naturally structure their assets into portfolios: Expenditures are often considered within category so that a lost $100 theater ticket might not be bought again (it would stress your "entertainment portfolio" to do so), but $100 lost from a general account might not deter the purchase (Kahneman & Tversky, 1984). So too it may be with portfolios of desire. A liquid asset like money has command of all the subportfolios, whereas "food" is only that of one subportfolio. But the diversity—both across and within subportfolios—may not be appreciated until stimulated, as done by EFT and other interventions that encourage the mind to wander toward new prospects.

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Appendix

Summary of Key Equations

$$U(G_{a,d,p,n}) = F[f_a(a), f_d(d), f_p(p), f_n(n)]. \tag{1}$$

General framework giving the utility of a good of amount a, delay d, probability p, and liquidity n. The constituent $f(x)$ are value functions: logarithmic and power functions in the case of amount and delay, a Prelec weighting function (Equation 8) in the case of probability, and an identity function in the case of the diversity index n. The function $F[x]$ indicates both how they are concatenated (addition or multiplication) and translated into a utility function (e.g., as a logarithmic transformation; although that does not need to be evaluated in this article).

$$U(G_{a,d}) = F\left[(k_a a)^{\alpha} - (k_d d)^{\beta}\right]. \tag{3}$$

For delay discounting, utility is reduced to a function of an additive concatenation of power functions.

$$a_0 = \left(a_d^{\alpha} - \lambda d^{\beta}\right)^{1/\alpha}. \tag{5}$$

For matching experiments, the utility function F cancels and the prediction of the immediate equivalent of a delay-discounted good is delivered.

$$\frac{a_0}{a_d} = \left(1 - \frac{\lambda}{a_d^{\alpha}} d^{\beta}\right)^{1/\alpha}, \; a_d, \alpha > 0. \tag{6}$$

The current monetary value is relative to the nominal delayed monetary value. Because the subjective value of the delayed amount

(*Appendix continues*)

divides the discount parameter in the right-hand side, this predicts a positive magnitude effect.

$$w(p) = e^{-\kappa(-\ln(p))^{\gamma}}. \tag{8}$$

The Prelec probability weighting function,

$$\frac{a_{p=1}}{a_p} = (k_a a_p)^{w(p)-1}. \tag{12}$$

In probability discounting matching experiments, this is the relative monetary value of a good a delivered with a probability p.

$$a_{d=0,p=1} \approx \left(a_{d,p}^{\alpha w(p)} - \lambda d^{\beta}\right)^{1/\alpha}. \tag{14}$$

Prediction for experiments in which both probability and delay are varied.

$$U(P) = F[n^{1-\alpha}(ka)^{\alpha}]. \tag{16}$$

Increment in the utility of a portfolio of goods with an amount a, distributed to n different constituent goods.

$$\begin{aligned} U(P) &= \log[n^{1-a}(ka)^{\alpha}] \\ &= (1-\alpha)\log(n) + \alpha\log(ka). \end{aligned} \tag{16′}$$

An expansion of Equation 16, demonstrating how diversity adds to the utility of a portfolio, with *F[x] = log(x)*; log(n) is the entropy of the portfolio.

$$a_{d=0,p=1} \approx \left(a_{d,p}^{\alpha w(p)} - \frac{\lambda}{n'}d^{\beta}\right)^{1/\alpha}. \tag{18}$$

A version of Equation 14 that includes the diversity factor $n' = n^{(1-\alpha)}$.

$$a_{d=0,p=1} \approx \left(\frac{n'_{d,p}}{n'_0}a_{d,p}^{\alpha w(p)} - \frac{\lambda}{n'_0}d^{\beta}\right)^{1/\alpha}. \tag{18′}$$

Equation 18 was developed for cross-modality discounting.

$$\frac{a_{d=0,p=1}}{a_{d,p}} = \left(a_{d,p}^{\alpha w(p)-1} - \frac{\lambda}{n' a_{d,p}^{\alpha}}d^{\beta}\right)^{1/\alpha}. \tag{19}$$

The full expression of Equation 1: The relative amount predicted when the parameters *d, p,* and $n' = n^{1-\alpha}$ are in play

$$\frac{a_{d=0,p=1}}{a_{d,p}} = \left(1 - \frac{\lambda}{n' a_{d,p}^{\alpha}}d^{\beta}\right)^{1/\alpha}. \tag{20}$$

For $p = 1$, Equation 19 reduces to deterministic delay discounting.

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