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From data through discount rates to the area under the curve

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Abstract

The rate of discounting future goods is a crucial factor in intertemporal trade-offs, upon which depends not only individual well-being but also that of our planet: How much privation now for a temperate future for our grandchildren? What is the best way to measure how the value of future goods decreases with its delay? The most accurate discount functions involve several covarying parameters, making interpretation equivocal. A universal and robust measure is the area under the discount curve, the *AuC*. The *AuC* of a hyperbolic discount function is a logarithmic function of the discount rate, k. The same integral also approximates the area under a hyperboloid function. A simple technique converts each datum into estimates of the discount rate, eliminating rogue data points in the process. These trimmed estimates are converted into areas and tested against data, where they succeed at predicting the *AuC* and its relation to $\log(k)$.

KEYWORDS

area under the Curve (*AuC*), area under the data (*AuD*), discounting, hyperbolae

INTRODUCTION

\$x now or \$1,000 later? "How much later?" you ask. How much you will accept now depends on how long the delay to the larger good is. This is the story of delay discounting, the intertemporal trade-offs we all make: Mortgages or convertible debentures; a kiss now or a marriage later? The human condition. Can our behavior science make sense of this? Yes, to a certain extent. It can describe it, as in the following equations, but why the equations take the form they do is not known (although Killeen, 2023, speculates). Much effort has nonetheless gone into the precise fitting of models whose parameters are then often used to characterize how discounting varies as the nature or magnitude of the delayed good is varied. Scores of studies have examined the rate of discounting in different groups, distinguished by age, income, a pattern of substance abuse, or other moderating variables (see, e.g., Madden & Bickel, 2010). The differences in the discount rates in those different populations make this task both important and intriguing. Similar equations describe probability discounting: \$x for sure or \$1,000 with a probability of ½?

The most common form for the discounting of delayed or probabilistic goods is the inverse linear function of the independent variable that is typically called "hyperbolic" (Mazur, 1987, 2001):

$$v_x = \frac{v_0}{1+kx}. \tag{1}$$

Here v_x is the current sure value of a good v_0, which is delayed or probabilistic. The independent variable x is the delay or improbability of the deferred outcome v_0: $x = d$ in the case of delay discounting, and $x = O$, odds against, in the case of probability discounting, with $O = (1 - p)/p$, $p > 0$, is the probability of the risky alternative. The coefficient k is the rate at which delay or improbability reduces the value of v_0. Equation 1 is often rearranged by dividing by the nominal value, v_0, yielding proportional discount as the dependent variable and Equation 2 as the simplest model of the discount function:

$$\frac{v_x}{v_0} = \frac{1}{1+kx}. \tag{2}$$

Equations 1 and 2 fit the data from nonverbal animals very well and provide a decent approximation to the discounting of humans. In the latter case, however, the power function of Equation 3 (Green et al., 1994; Loewenstein & Prelec, 1992), often called a hyperboloid, typically provides a better fit to the data.

$$\frac{v_x}{v_0} = \frac{1}{(1+kx)^s}. \tag{3}$$

Equation 2 is "nested" in Equation 3, which it becomes when $s = 1$. Given its additional parameter, Equation 3 can do no worse in terms of coefficient of determination than Equation 2, although it might in a model-comparison approach (Newland, 2019).

Given the variety of potential discount functions (Killeen, 2009; Luckman et al., 2020; Takahashi et al., 2007) and the interest of many investigators in having a simple measure of discounting, not a theoretical exercise, Myerson et al. (2001) proposed that the degree of discounting be measured by the atheoretical "area under the curve" (*AuC*; Gilroy and Hantula 2018, call it a *point-based AuC*). For concision, I call it *AuD*, area under the data, as no curves were involved). This is a standard approach in pharmacokinetics and psychophysics. They and Smith and Hantula (2008) gave many reasons for liking the AuD as a general measure of discounting. The *AuD* provides a single measure of discounting, whereas Equation 3's two free parameters do not univocally address the rate of discounting. Myerson and colleagues first demonstrated the calculation of the area with Figure 1, the data of one subject in a delay-discounting task.

The *AuD* is highly correlated (−.80 to −.95) with the logarithm of the rate parameter k in experiments that report both (e.g., Franck et al., 2015; Wan et al., 2023; Yoon et al., 2017). This article shows why that is the case and provides more direct routes from data to the *AuC*. It also explains the deviation from linearity in plots of log (k) against *AuD* noted by Yoon and associates. Gilroy and Hantula (2018) demonstrated the use of numerical integration of the best of competing discount functions to compute the *AuC*, providing a program to automate that

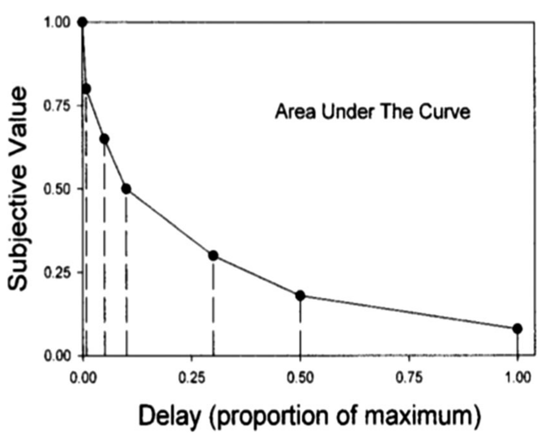

FIGURE 1 Copied from Myerson et al. (2001) demonstrating the calculation of the area under the data. Data from Subject P-24. *AuD* is computed by summing the area of each of the trapezoids. Those are computed by multiplying the extent on the abscissae ($x_{i+1} - x_i$) by the average heights of the ordinates ($y_{i+1} + y_i$)/2. Here the *AuD* is 0.258. Reprinted with permission of Wiley.

process (Gilroy, 2017, 2019). Franck et al. (2015) provide another elegant approach to model selection for discounting data. The present article gives another, simpler approach to calculating the *AuC*.

SIMPLE HYPERBOLA

In many cases Equation 2, the simple hyperbola, provides at least a good approximation to the data. It is the form assumed in the Kirby method (Kirby, 2009) which directly hones in on discount rates through successive questionnaire prompts, under the assumption that discounting is hyperbolic. The *AuC* for the hyperbola is

$$\int_0^{x_{\max}} \frac{1}{1+kx} dx = \frac{\ln(1+kx_{\max})}{k}, \quad k > 0, \tag{4}$$

where *ln*() is the natural logarithmic function, k is the rate parameter, and $x_{\max}$ is the maximum value of the independent variable. The equation holds whether $x_{\max}$ is measured in terms of duration or odds, whether it is an ordinal rescaling of the independent variable (Borges et al., 2016), or is set to 1 with the other values of x rescaled appropriately (Myerson et al., 2001).

Equation 4 gives the area when the x-axis is not normalized, but the *AuD* is always reported for abscissae that have been normalized to range from 0 to 1. In that case the area becomes

$$AuC = \frac{\ln(1+kx_{\max})}{kx_{\max}} \; kx_{\max} > 0. \tag{5}$$

It is clear from this derivation why the *AuC* is correlated with the logarithm of k: the *AuC* for the hyperbola involves a logarithmic function of k. Unlike a simple logarithmic transformation of k, however, Equation 4 is an *AuC* that is logically derived and measured in the same units as the empirical *AuD*. For Equation 5 the units are dimensionless. A closer look at the relation between k and the areas is given in Figure 4.

HYPERBOLOID

The area under the hyperboloid (Equation 3) is

$$\int_0^{x_{\max}} (1+kx)^{-s} dx = \frac{(1+kx_{\max})^{1-s} - 1}{k(1-s)} s \neq 1, k \neq 0. \tag{6}$$

The advantage of using Equation 6 is that it delivers a single measure of discounting (the *AuC*) rather than two parameters, k and s, the effect of whose interactions on overall discounting may be hard to construe. Recovered values of k and s are often correlated (e.g., Franck et al., 2015, Table 3), which means the information they convey is not independent. They may in some cases be near collinear, meaning that different sets of values for them may yield essentially the same discount curve with the same *AuD* (Franck et al., 2015, Figure 3). A series expansion of Equation 6 around $s = 1$ gives Equation 4 as its first term, plus an addend with the factor $(1\text{-}s)/k$, indicating that if s is close to 1 or k is large, the addend is small and Equation 4 will be a good approximation to Equation 6. As with Equation 4, Equation 6 must be divided by $x_{\max}$ to rescale to the normalized abscissae typically used for the *AuD*.

Note that both the *AuD* and the *AuC* take the y intercept to be 1.0—that is, that the subjects are indifferent between a dollar in hand and a dollar given with 0 delay (the dollars are fungible). That is not always true. People sometimes value the good in hand more or less than the one given immediately (Killeen, 2009, 2019). Yoon et al. (2017) have shown that such misalignment will have little effect on the computed area.

How well are data that are generated by Equation 3, with random variations in the parameter and data, treated by a method such as the Kirby procedure that imputes a simple hyperbola (Equation 3)? To answer that question, simulate 25 discount data sets from Equation 3, with random variations in k and some jiggle in the abscissae. Then fit Equation 2 to each of them and compare the *AuC* predicted from Equation 5 with the standard *AuD* derived directly from the simulated data. The delays/odds were 1, 2, 4, 8, 16, and 32 rescaled to an $x_{\max}$ of 1 so that the normed x-values were 1/32, 2/32, etc. These were the independent variables used for the discount functions. The mean value of k was selected to yield a relative discount of around 0.9 at the smallest value of the independent variable and a discount of .10 to .50 at the longest. Noise was added to the value of k to generate different discount functions, and noise (SD = .06) jiggled the value of x. For a value of $s = 1$, the simple hyperbola, the fits to the data were excellent and the correlations between the *AuD*s and the *AuC*s were $r = .997$. The correlation between log(k) and the *AuC* was almost as good ($r = -.992$). These high correlations are surprising until one recognizes the noise injected into the x-values affected the empirical *AuD* and Equation 5 in a similar manner.

For fixed values of s between 0.2 and 2.0, the fits of the simple hyperbola to the data degenerated as s deviated from 1.0. Nonetheless, the correlation between Equation 5 and the empirical *AuD*s was never less than .98. How can this be? Both Equation 5 and the *AuD* are summary statistics that reflect average discounts over the range. Equation 2 must adjust to minimize deviations around each point, with those above balancing those below, so on the average it comes close to the bulk of the data; that is, it conforms to the area below it. This resolves the paradoxical result: whereas Equation 2 and its parameter k may give a relatively poor description of data points generated by Equation 3, it is very accurate in what they have to say on the average about the degree of discounting.

Every datum tells the story

Finally, there is a simple procedure that avoids nonlinear curve fitting, makes efficient use of the data, and is robust to outliers: Use each of the data to estimate k individually. Solve Equation 2 for k:

$$k_i = \frac{v_0 - v_i}{x_i v_i}. \tag{7}$$

Here v_i is the immediate/sure value of the outcome v_0 at the delay/odds of x_i. Evaluate Equation 7 to estimate k_i for each datum, giving n estimates, $k_1, k_2, \ldots k_n$. Compute the *trimmed mean* of those estimates: discard the largest and smallest estimates and average those remaining (Wilcox, 1998; see the Appendix). Then insert the trimmed mean into Equation 5 to convert it into an area. (In Equation 5, $x_{\max}$ is the largest value of the independent variable regardless of whether it is included in the data that survive the trimming).

APPLICATION

Does this last analysis work for real data from individual subjects? Equation 7 and then Equation 5 predict the *AuC* in Figure 1 to be .232, close to the empirical *AuD* of .258. The slight overestimation of the area by the *AuD* is explained below. Takahashi et al. (2008) used the method of limits to study delay discounting of large and small amounts of money and dried rice in Thailand at a time when severe inflation devalued the future prospects of money more than those of rice. They reported data for 98 discount functions, 67 of which supported the use of Equation 2. Figure 2 is a scattergram of those reported *AuD*s against the *AuC*s from Equations 5 and 7. The rendering is respectable, with $r = .95$; Gilroy and Hantula's (2018) more precise analyses of these data returned a correlation of $r = .96$. Going from the authors' values of k directly to *AuC* via Equation 5 yields an $r = .93$. The *AuD*s are larger than the *AuC*s. This is because the *AuD* is computed by interpolating straight lines between proximal data points. For convex curves such as discount functions, the true locus will always lie below the interpolated points, especially around areas of .50. This is called *Jensen's inequality* and difference between the function and the interpolation *Jensen's gap*. It becomes negligible as number of delay values increases.

Takahashi and associates deleted 29% of the data that could not return a positive coefficient of determination from Equation 2. When those are included with Equations 5

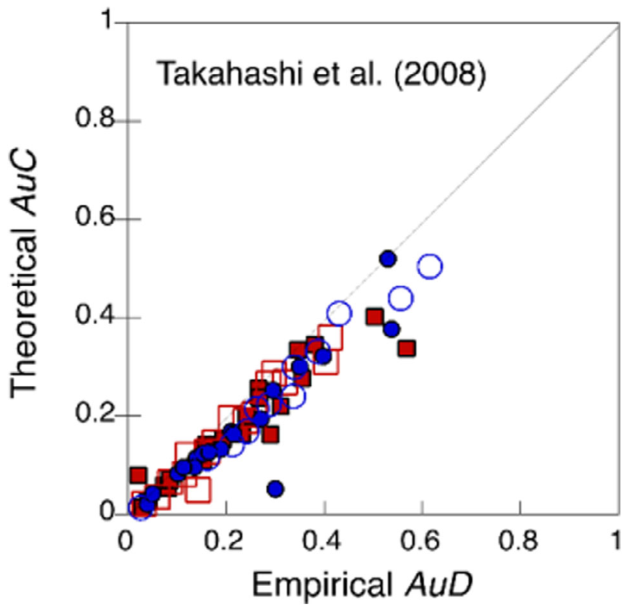

FIGURE 2 *AuC*s computed from Equation 5 and 7 are plotted against the *AuD*s reported by Takahashi and associates (2008). Circles are for rice, and squares are for money; large empty symbols represent large amounts, and small filled ones indicate small amounts. The outlier at bottom comes from a condition where relative value increased for the first three delays, then fell to near zero.

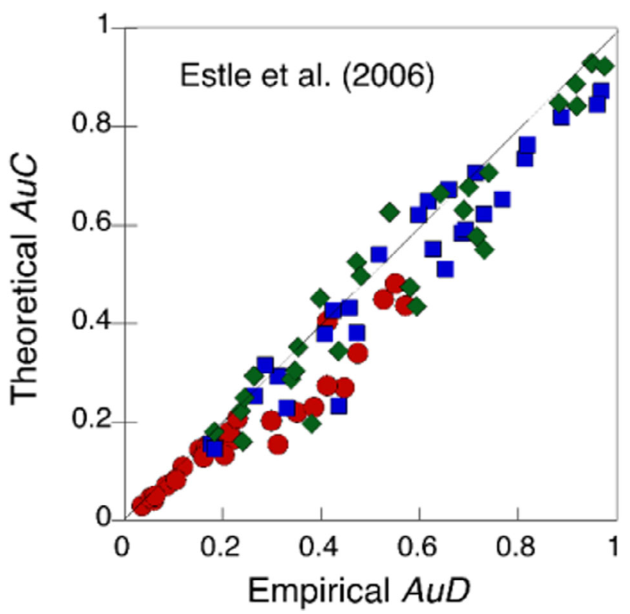

FIGURE 3 *AuC*s computed from Equations 5 and 7 are plotted against the *AuD*s for data reported by Estle et al. (2006). The data are from the authors' Experiment 3. Symbols: Disks v0 = \$100; squares, v0 = \$20,000; diamonds, v0 = \$60,000. The correlations of *AuC* and *AuD* for these ranged from $r = .95$ to .97. Their other experiments (with fewer subjects and conditions) yielded correlations of around .95 as well. Appreciation to the authors for sharing their data.

and 7, the coefficient of determination decreases to .92; the data base is thus expanded by 29% with only a small loss of concordance between *AuD* and *AuC*.

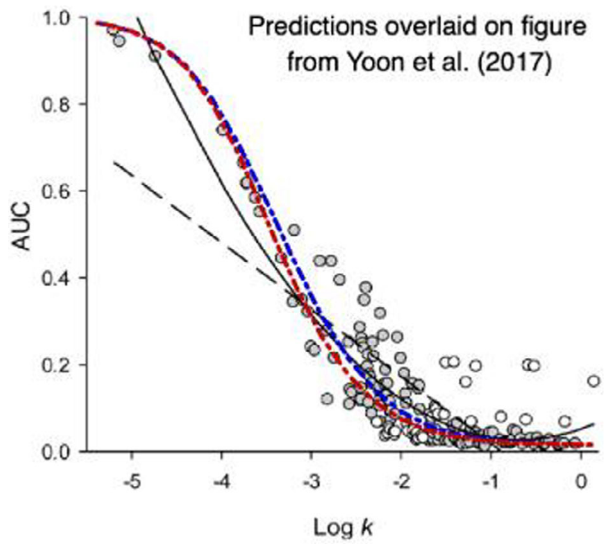

FIGURE 4 *AuC*s computed from Equation 5 are plotted against a range of values for log(*k*). The stippled ogives are the conventional *AuD*s on top, and just below them are the *AuC*s from Equation 5. Figure used with permission.

Charlton and Fantino (2008) reported *k*s and *AuD*s for six different types of goods from 110 students. The correlation between *AuD* and *AuC* was $r = .98$, yet once again there was a noticeable gap for these steep discount functions. The theoretical *AuC* closes the gap, and in that respect it is more accurate than the atheoretical *AuD*. Data from St. Louis (Estle et al., 2006) may be enlisted for a further test of the correlation of *AuC* and *AuD* (Figure 3). These authors had 27 participants choose between delayed amounts (see note to Figure 3) and an immediate amount that was adjusted to achieve indifference. Note, once again, the gap between the measures.

Borges et al. (2016) observed that the standard procedure for calculating *AuD* gives unequal weight to the data points. For the data to univocally address the discount rate, they should be given equal weights. To do this Borges et al. (2016) suggested ordinally rescaling the abscissae so that each datum is given unit weight. Doing so provides a more efficient use of data and more precise estimates of discounting, although the resulting index is not the area under the discount function. This procedure is equivalent to averaging the discount points. That average was very highly correlated with the *AuD* ($r \approx .98$ over all conditions), but because of the differential weighting of the data by the *AuD*, differed slightly. If comparing groups with those averages, it is essential that the same values of the independent variables be used in each, which is less of a consideration when using Equation 5 or the empirical *AuD*. Because most researchers use a quasi-geometric progression for the abscissae, another technique suggested by Borges and associates is using the logarithms of the delays as the abscissae in the trapezoid formula. This yields valid *AuD*s, and plotting the data on a logarithmic *x*-axis has the additional advantage of making the results more visually apprehensible.

Area vs. log (*k*)

Equation 5 explains why there is a high correlation between these variables, but it does not predict a simple linear one. What is the nature of that relation? Figure 4 gives the picture. Figure 4 is based on a figure from Yoon and associates (2017), who showed that a linear relation (dashed line) did not do the justice to the correlation. The authors also fit a quadratic function (continuous ski jump). I used the delay values from their study and computed hyperbolic discount functions for various values of *k* with Equation 5. I also computed the standard *AuD*s. These are shown as the stippled ogives, with *AuD* lying above *AuC*, as expected. No free parameters, no mysteries; the analysis is working as it should.

The robustness of this approach, which automatically discards data generating the least reliable estimates of *k*, permits it to be applied where estimates of *k* are not available or some of the discounts are significantly nonmonotonic. Takahashi et al. (2008) found 28 conditions in which they could not converge on a fit for the hyperbola, constituting 29% of their data. Similar percentages of failed convergence are not uncommon: Equation 1 failed Odum and Rainaud (2003) for 30% of their participants, and Equation 3 failed Franck et al. (2015) 26% of the time. Applying the above approach (trimming, etc.) for Takahashi et al.'s data including the rejectees in the data set reduced the areal correlation only slightly, from $r = .949$ to .922. Thus, although the trimming discards the interval information from some data (but not as much as the median does, which is a maximal trim), it may permit the investigators to include data sets that they would otherwise discard and decreases the role of ad hoc criteria for data inclusion. It also increases the generality of the results, as eliminating up to 1/3 of the participants makes a data set nonrepresentative; perhaps those misfits will drive interesting distinctions between groups or manipulations.

The use of Equation 7 for each datum provides independent estimates of *k*. Unlike a fit hyperbola, whether those are changing monotonically or not with the independent variable does not matter. An outlier (a "rogue data point") will distort the hyperbola by the square of its distance from its "true" location, but only linearly in the average *k* and not at all if it is trimmed. The *k*'s may be used generate several estimates of the *AuC* for the same curve and thence to test the significance of difference between individual functions. This permits one to ask, for instance, whether a manipulation had a significant effect on the *AuC* of an individual subject (although the small number of data mean that failure to find significant differences may simply be due to lack of power). If using trimmed means, consult Anderson (2001) or Wilcox (2011) for appropriate tests or use permutation techniques (see, e.g., Jacobs, 2019).

DISCUSSION AND SUMMARY

The empirical *AuD* provides a bulk measure of discounting that is of utility in comparing groups (Myerson et al., 2014; but see Smith & Hantula, 2008). The theoretical *AuC* may be derived from the discount rate, *k*, estimated by fitting a discount function via Equation 2 (the hyperbola) or Equation 3 (the hyperboloid) and transforming *k* by Equation 5 or 6. There are more powerful ways of computing the *AuC* (e.g., Gilroy & Hantula, 2018), but the present simple approach may suffice for many purposes. Although redundant with *k*, the *AuC* is more intuitive and statistically better composed than the discount rate, and in the case of the hyperboloid Equation 3, its area (Equation 6) provides a single index of discounting rather than two parameters that interact in nonlinear ways. Values for *k* may also be derived for both verbal and nonverbal animals by the use of an efficient procedure such as the Kirby questionnaire (Kirby, 2009), which assumes a hyperbolic discount function (Equation 1). It was shown that even though the individual data points generated by the hyperboloid Equation 3 may be ill-fit by Equation 2, the derived value of *k* as interpreted by Equation 5 can provide an excellent predictor of the *AuC*. This is because Equation 6 minimizes the deviation from the data *ensemble*, and that is just what the *AuC* based on Equation 2 (viz. Equation 5) does. Thus, the reality of Equation 3 as a better description of much human discounting should not dissuade one from the use of the Kirby questionnaire or similar technique if what one wants is a summative measure, perhaps with Equation 5 as its interpreter.

A problem with taking the areas at face value is that they can give the same result for curves that look quite different or are even increasing rather than decreasing with delay (Gilroy & Hantula, 2017). These authors found such an increase for one subject in the experiment from which Figure 2 was drawn. This will probably be uncommon for positive goods (although in some cases people would rather delay a good so they can enjoy anticipating its consummation). In the case of losses, however, a substantial minority of individuals would rather take care of them immediately rather than have them hanging over them through the delay (Myerson et al., 2017). Thus, the strengths of the area measures—their simplicity and interpretability—can also be a weakness if taken at face value, without inspecting the data.

This is not the first proposal to rectify the hyperbola. Killeen (2019, 2020) inverted Equation 1 and fit linear regressions to the results, treating them as predicting the amount by which a remote good would need to be incremented to generate parity with the more proximal one. Berk et al. (2021) inverted Equation 2 and rearranged it so that the ratio of the value lost due to its delay to the value preserved despite its delay, which they labeled the "immediacy premium," was on one side. On the other was the product of the decay rate k and the delay (viz. Equation 7 with both sides multiplied by x). This "regression through the origin" can be readily solved for k (Eisenhauer, 2003). In both cases, the average of linear functions gives more meaningful results than the average of nonlinear ones (Estes, 1956).

Another approach is to use the delay at which the discount has fallen to half the nondiscounted value (The ED50: Franck et al., 2015; Yoon & Higgins, 2008). For the simple hyperbola, ED50 = $1/k$, (and for Equation 3, ED50 = $(2^s - 1) / k$). This point can be empirically estimated by an efficient titration procedure: the reciprocal of Equation 7 estimates the ED50, so after estimating that on one trial, the next observation could be made at that estimated value and so on. Koffarnus and Bickel (2014) have devised another quickly converging technique. The distribution of ED50 is likely to be positively skewed (variability on the y-axis will carry the corresponding value on the x-axis farther to the right than to the left (Franck et al., 2015, Figure A1); when averaged it should therefore be with the geometric mean. ED50 may be converted to the AuC by inserting its inverse into Equation 5.

With data for a full discount function that is well fit by Equation 3, one may compute the AuC by integration (Equation 6). Alternatively, a simple, robust, and efficient approach is to have each datum provide an estimate of the AuC via Equations 7 and then 5. If these are trimmed, they will be robust to rogue data and permit the inclusion of data sets or individuals that traditional analyses would discard, making the inferences more representative of the population at large.

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APPENDIX A

To compute *AuC* from Equations 5 and 7, place v_0, in the first column of a spreadsheet, the x values in a row above, and the v_x, in rows beneath the x values. Rows are subjects. Alongside that compute Equation 7 for all the data and delays/probabilities (xs), giving the values of k_i. Then add a column that computes a trimmed mean of those, omitting the major outliers above and below: $k_{est} = [\text{sum}(k_1{:}k_n) - \max(k_1{:}k_n) - \min(k_1{:}k_n)] \,/\, (n - 2)$. Alternatively, you can use the TRIMMEAN function in EXCEL. Then insert these k_{est} into Equation 5 for each subject and condition as the final column. Contact the author for a spreadsheet.