THEORETICAL ARTICLE

Theory of reinforcement schedules

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Abstract

The three principles of reinforcement are (1) events such as incentives and reinforcers increase the activity of an organism; (2) that activity is bounded by competition from other responses; and (3) animals approach incentives and their signs, guided by their temporal and physical conditions, together called the "contingencies of reinforcement." Mathematical models of each of these principles comprised *mathematical principles of reinforcement* (MPR; Killeen, 1994). Over the ensuing decades, MPR was extended to new experimental contexts. This article reviews the basic theory and its extensions to satiation, warm-up, extinction, sign tracking, pausing, and sequential control in progressive-ratio and multiple schedules. In the latter cases, a single equation balancing target and competing responses governs behavioral contrast and behavioral momentum. Momentum is intrinsic in the fundamental equations, as behavior unspools more slowly from highly aroused responses conditioned by higher rates of incitement than it does from responses from leaner contexts. Habits are responses that have accrued substantial behavioral momentum. Operant responses, being predictors of reinforcement, are approached by making them: The sight and feel of a paw on a lever is approached by placing paw on lever, as attempted for any sign of reinforcement. Behavior in concurrent schedules is governed by approach to momentarily richer patches (melioration). Applications of MPR in behavioral pharmacology and delay discounting are noted.

KEYWORDS
actions, contrast, momentum, MPR, principles of reinforcement, reinforcement schedules

Few protocols in psychology produce data so orderly as do schedules of reinforcement (Ferster & Skinner, 1957). Orderly data invite orderly theories to explain them. Construction of such a theory has been my preoccupation since graduate school. The best theories have well-defined principles to constrain model development. Such principles are general, distilled by more specific models. The three principles of the *Mathematical Principles of Reinforcement* (MPR; Killeen, 1994) concern (1) the excitement of behavior generated by some events such as reinforcers; (2) constraints on the emission of that behavior, due to limited time and competition for that time by other responses; and (3) the direction of those responses onto measurable targets—keys and levers—by the contingencies of reinforcement.

The observations that led to the first of the principles emerged when my students and I placed pigeons in a chamber, fed them once, measured their general activity for 20 min, and then removed them. Their activity declined as exponential decay functions—perfect straight lines on semi-log coordinates (Killeen et al., 1978, [KHO], Figure 5). Because the birds had no history that would ever lead them to expect a second feeding in that context, these were aftereffects of the single one that they got. What would happen if they were then fed periodically? Their activity levels increased following a concave function (KHO's Figure 7) that was predicted by the same "first-order kinetics" that governed the accumulation of a drug having an exponential dissipation (KHO's Figure 6).

This orderly instigation of behavior and its cumulative invigoration, together with earlier work on contingencies of reinforcement (Killeen, 1969), inspired my students and me to collect data and develop models for a general quantitative treatment of schedules of reinforcement. Those were assembled in 1994 into MPR, based on (1) arousal/induction cumulation, (2) constraint/competition, and (3) direction/approach. This article

reviews those principles, how they were realized in mathematical models, and how they and the models that articulated them have evolved and been implemented. The article is divided into sections devoted to those principles and then to their evolution in light of data. The ideas in those sections are previewed here.

Preview

1. The first principle is rederived, amplified by models for conditions where satiation or habituation slows the cumulation of arousal, and related to other similar observations and theories.
2. Responses, whether of the same operant class or of different ones, reduce the time available for additional responding. The second principle guides the development of models for the emission of responses under such constraint and competition. The original account dealt primarily with time constraints involving competition for expression of the same target response with others of that class. That predicted a hyperbolic increase in responding with increases in rate of reinforcement (Equations 2 below). It is extended to concurrent reinforcement of different classes of responding, providing a model for behavioral contrast and for behavioral momentum.
3. The third principle is now grounded in proximities to goals and the approach that they foster. Whenever the memory of an event is "close" to that of a reinforcer or other important event, they may be associated—the event may be reinforced. Closeness—proximity—along any of several dimensions facilitates that association. We typically think of responses as the events that are reinforced, but stimuli may also be strengthened as they become conditioned stimuli, CSs, and discriminative stimuli, S^Ds. CSs control the class of responses that preceded them, and S^Ds control the classes of responses that follow them. A stimulus may have both properties to varying degrees. The primary effect of such reinforcement is for animals to approach sources and signs of those events. If the event is a response, the animal approaches it by making it again. If it is a stimulus, the animal tracks it. If it is a different type of response (like chasing a prey), any responses that gain access to the affordances for that behavior (such as searching) will be reinforced, and subsequent responses (e.g., capturing the prey) will be directed.

Just as the time available for responding is bounded, so also is associative strength. Attention paid to one event leaves less time for attention to others; a response that is reinforced leaves less associative strength available for those that, coming before, stand in its shadow. The delay of reinforcement gradient is a manifestation of this fact. Events—stimuli or responses—that are reinforced

occlude memory for prior events, blocking their control by the reinforcer, due to such limits on associability.

MPR takes response rate as its primary dependent variable. This choice is justified by studies showing that various possible dependent variables—overall rate, running rate, and latency—are highly correlated and that the variable with the highest factor loading over various manipulations of motivation was overall response rate.

The principles are put into play by developing mathematical models of reinforcement schedules. The models developed in MPR are rederived with clarifications. It is shown, for instance, that the original model of fixed-ratio (FR) performance governs both overall response rate and running rate. That model is elaborated into one for progressive ratio performances. The basic model of associative strengthening is applied to concurrent control by different CSs as found in autoshaping experiments. It provides a model for the postreinforcement pause in terms of the concurrent strengthening of interim responding versus approach to the operant response state, in which the target response is the dominant behavior. The uses of MPR in behavioral neuroscience are noted, as are the commonalities with Baum's molar behaviorism and Catania's reflex reserve. Derivations too disruptive for the text are relegated to appendices. Concerns about the different character of principles and models are first allayed.

Principles versus models

Principles are not laws, which describe empirical regularities (e.g., Ohm's law, the matching law). They are structures that guide model development.[1] Models are pliable, principles are more abstract and inviolate. Principles are generalizations from observed data or shared intuitions. Models, organized to respect them, make more intimate contact with data. Models can fail, no fault; principles had better not. Principles are intrinsically general; they are the bones of a theory, its stable structure, providing anchors for its muscles—its models—which do the heavy lifting. Because principles are general, versions of them will be found in various theoretical systems. Arousal empowers induction (Baum, 2012; Segal, 1972; Sheffield & Campbell, 1954), for instance. It empowers adjunctive responses (Killeen, 1975, Figures 9 & 10; Killeen & Pellón, 2013), the often species-specific responses that are elicited in the context of arousal (Falk, 1971; Jenkins et al., 1978). As the potential for responding (given the appropriate context), arousal comprises the contents of the primary reserve in Skinner's reflex reserve. Unmotivated—unaroused—animals tend

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[1] "The role of fundamental principles is to serve as the guiding principles of a theory, telling us 'what the theory is about' … By doing so, they point us towards the relevant factors to consider when explaining a specific class of phenomena. Empirical applications of the theory require more specific [models] that limit the scope of the fundamental principle" (Borgstede & Luque, 2021, p. 10)

to be quiescent. Arousal is the thing on which motivating operations operate (Edwards et al., 2019; Killeen & Jacobs, 2017b), and when released by affordances in the environment, is called incentive motivation (Bindra, 1978; Killeen & Jacobs, 2017a; Logan, 1960, 1979). It is the accelerator on the vehicle of behavior (Braitenberg, 1986). Models such as Equations 1 and 2 put a fine point on the broad brush of their principles. They describe and predict and can be broken, and it is these abilities that make them the most valuable part of a theoretical system. Yet, disjoint from the principles that they instance, they would be just curve fitting.

MATHEMATICAL PRINCIPLES OF REINFORCEMENT

Arousal

The model for the cumulation of arousal in KHO (1978) predicted that the asymptotic level of activity would be proportional to reinforcement rate. But there were problems: Right after a feeding, activity was low: Pigeons engaged in "area-restricted search," aka looking for residual grains. And, toward the end of the interval, activity subsided. We had instituted contingencies that would not let the birds be fed if they moved within 5 s of when a feeding was due. Our floor panels picked up neither of these classes of behavior. We took out these effects with a model of general activity (borrowed from Killeen (1975) and implemented in KHO's Figure 6). An asymptote A is approached by a fast exponential decay of area-restricted search and retreated from by a slower exponential decay due both to the DRO contingency and to targeted search, resulting in an Erlang (gamma) response distribution. This gives both an accurate description of the distribution of adjunctive responses, (Killeen, 1975, and KHO, Figures 1 & 2), and yields an uncontaminated asymptote (A), that is a perfect linear function of rate of reinforcement.

This work grounded the first principle:

I. *Arousal (the potential for action, A) is proportional to the rate of incitement (r):*

$$A = ar. \qquad (1)$$

The constant of proportionality, a—the slope in Figure 1–is called *specific activation*. Its value depends on situational variables such as the level of deprivation and the magnitude of the incentive; its dimension is the inverse of the temporal unit for r (e.g., s when r is measured per second). It gives the number of seconds of behavior strengthened by the reinforcer. Arousal (A) is the same construct as Skinner's *Reflex Reserve* (Killeen, 1988). His precise verbal descriptions of the properties of the reserve give rise to a cumulative-
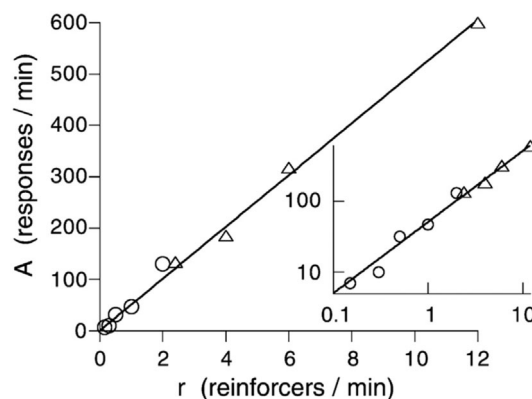

FIGURE 1 The increase in asymptotic arousal (A) as a function of rate of reinforcement (r). The insert presents the same data on double-log coordinates to enhance visibility. From Killeen and Sitomer (2003, Figure 7), as permitted by Elsevier's Author's Rights.

exponential function of increments in the reserve, common to both his account (Killeen, 1988, Equation 7) and MPR, with similar asymptotic properties. There were many astute aspects to Skinner's reserve, whose creation and ramification constituted the core of his *Behavior of Organisms* (1938). After describing the reserve in detail, I opined that his theory was "of contemporary relevance, both for substantive scientific reasons and for procedural, meta-scientific ones. His theory anticipated many contemporary models" (p. 329). Catania (2005) redeemed this opinion with a modern version of the reserve, one having much in common with MPR.

There is good physiological evidence for an arousal mechanism (Pfaff, 2006; Pfaff et al., 2008), and historical precedent: "Each instinct involves an element of appetite, or aversion, or both. An appetite … is a state of agitation which continues so long as a certain stimulus … is absent. When the appeted stimulus is at length received it stimulates a consummatory reaction. [Appetite] is accompanied by a readiness to act" (Craig, 1918, p. 91).

Extensions

The original model of how arousal cumulates did not take satiation and habituation to the reinforcer into account. Estêvão Bittar and colleagues (Bittar et al., 2012) recognized that to be problematic, collected relevant data, and generated an effective model of satiation/habituation. They stipulated that each reinforcer had slightly less energizing efficacy (A_i) than the one before, decreasing geometrically with each successive reinforcer. This generated a treatment coherent with the original formulation, but with the ability to accurately describe 25 experimental results: their own, and those of Cannon & McSweeney (1995), DeMarse et al. (1999), and Melville et al. (1996). Although the resulting model for the change in arousal was not simple, there was

substantial parameter invariance, both within and across experiments (for instance, the time constant for the cumulation and decay of arousal was the same as that found by KHO—6 min—in all conditions; the basic jolt of arousal, A_0, was the same within those experiments in which the magnitude of reinforcement was constant; etc.). By stipulating that A has a maximum of 1, they were able to write the fundamental equation for behavior as $b = AC/\delta$, a beautiful synopsis, with A given by their Equation 12, C the relevant coupling coefficient (see below), and δ the response duration. In terms of the Akaike information criterion (AIC) their model strongly dominated the models of McSweeney et al. (1996) and Killeen (1995) in the 25 conditions they analyzed.

When satiation and habituation to the reinforcer are not at issue, the authors' Equation 11 reduces to the original treatment. Because that is less complicated, and usually satiation and habituation are not so extreme as in the conditions Bittar and colleagues analyzed, the original treatment is used here, relying on an average level of arousal, A. In experiments where satiation is likely to be an issue, such as when drugs are used as reinforcers, their analysis is preferred. For averaged data it could be engaged by computing its predictions on a reinforcer-by-reinforcer basis and averaging those, just as the data are typically averaged across the session.

Killeen (1998) extended the first principle to warm-up and extinction under both positive and negative reinforcement, to the partial reinforcement extinction effect, and to probability of responding as a function of arousal level. This treatment did not address the effects of contingencies or competition.

Competition

Incentives (reinforcers, unconditioned stimuli (USs), etc.) empower behavior. Figure 1 and Equation 1 imply, however, that arousal is unbounded as rate of incitement increases (cf. Bittar et al., 2012). What are the brakes on the vehicle? What causes ceilings on response rates, and how are they modeled? The emission of a target response faces competition from both other target and nontarget responses. In the original MPR (Killeen, 1994) it was reasoned that rate of elicitation of responses was proportional to A but that the time base for emission of those rates was the *time available for responding* (that is, the time left after other responses in the interval had been made). This resulted in the same treatment as that of Staddon (1977) and also the same as that for a blocked counter, such as a Geiger counter, whose responses cannot keep up with the rate of incoming pulses [response emissions, A], given the finite time it takes to process each impulse [response durations, δ] (Bharucha-Reid, 1960). Killeen and Sitomer (2003, Figure 8) provided a visual treatment, but the original is perhaps the simplest:

II. *Responding is the product of arousal level (A) and the time available for responding:*

$$\delta b = A(1 - \delta b), \tag{2a}$$

where δb is the total amount of behavior in a unit of time, with b measured in, say responses per second, and δ the average response duration measured in seconds. Responding is evoked with an intensity of A (given by Equation 1); the amount of time left available for responses to be emitted is $1 - \delta b$. The total amount of responding is given by Equation 2a; solving for b gives the rate of emission of responses:

$$b_{\text{Total}} = \frac{1}{\delta}\left(\frac{A}{1+A}\right), \quad \delta > 0. \tag{2b}$$

Here b is subscripted to remind us that the total amount of behavior induced by the incentive includes both target and competing nontarget responses. As A grows large, the parenthesis approaches 1, and $b_{\text{Total}} \rightarrow 1/\delta = k$. Substituting aR for A (Equation 1), Equation 2b may be rearranged to

$$b_{\text{Total}} = \frac{kr}{r + 1/a}. \tag{2c}$$

Setting $1/a = r_0$ further reveals its mathematical isomorphism with Herrnstein's (1970) hyperbola. But these are nonetheless different models. Equations 2 do not predict target response rates; they predict the total amount of behavior induced by the incentive, and that includes adjunctive and off-operandum responses.[2] Herrnstein's interpretation of r_0 has proven incorrect, (see, e.g., Dallery & Soto, 2004, for an overview). The next section shows how this model of constrained induction, Equations 2, is translated into a model of target responding.

Directions to the goal

Incentives empower behavior; competition from other responses limit it. Arousal is the accelerator of the vehicle of behavior, and competition the brakes on it; but what steers it? The nature of the drive (e.g., hunger, affiliation, sex, defense; Smith, 1984) and the affordances in the environment (Killeen, 1975; Sheffield & Campbell, 1954) play an important role in which "adjunctive" behaviors are induced (Killeen & Jacobs, 2017a, 2017b); yet those are not precisely directed. The third principle directs them:

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[2]That was also true of Herrnstein's model, which allowed for nontarget responses, his B_0, a fact sometimes overlooked by those who criticize the model for nonconstancy of asymptotic response rates across experimental manipulations. His model did not, however, include a factor that directed behavior on or off target, which MPR does, as described in Section 3.

III. *Animals approach signs and sources of reinforcement.*

This common knowledge serves as the third principle. "Adience [movement toward goals; see (Holt & Brown, 1931)] and abience [away from antigoals] are so obvious in animal behavior that they cannot be overlooked" (Hull, 1943, p. 349; as cited in Mowrer, 1960, p. 333). "Approach and withdrawal are the only empirical objective terms applicable to all motivated behavior in all animals" (Schneirla, 1959, p. 1). Thorndike (1911) took approach and withdrawal as the operational definitions of goals/reinforcers ("satisfiers") and antigoals/punishers ("annoyers"). "All of the diverse positively motivated behaviors exhibited by animals (e.g., thermoregulation, feeding, drinking, salt-appetite, hoarding, predation, sexuality, maternal behavior, shelter seeking) seem to be effected, to a substantial extent, by a common emotive system. The command impulse for all these goal-directed behaviors appears to arise from the primal tendency for an animal to move from where it is to where it must be to acquire materials" (Panksepp, 1989, p. 8).

In modern parlance this is sign and goal tracking. "Goal" is a generic term, one that does not entail conscious striving toward but rather includes unconditioned stimuli (US), incentives (Lajoie & Bindra, 1976; Logan, 1960), phylogenetically important events (PIE's; Baum, 2018a), and reinforcers. Learning is greatly impeded in tasks where animals must move away from the goal (the "detour" problem. See, e.g., Hershberger, 1986; Rashotte, 1987, for an overview). Ignoring nuances in meaning, these words will be used interchangeably in this paper as stimuli that animals approach or avoid.

When goals are absent, signs of them are typically good predictors of their appearance. Approaching those will usually bring organisms closer to the goal. Think of a predator's scent-tracking. Approach to signs of goals (or retreat from antigoals) is exploited in the conditioned place preference paradigm (Bardo & Bevins, 2000). In the laboratory, proximities are typically arranged by *schedules of reinforcement*, which specify the stimuli and responses that occasion reinforcement, arrangements that are called the *contingencies of reinforcement*. The same forces, less marshalled, exist outside the laboratory.

This treatise primarily concerns adience, the tendency to approach a situation. There is a complementary story to be told of abience, the tendency to avoid a situation. Fanselow and colleagues have applied a behavior systems approach to identifying a series of behavioral modules in which different defensive responses are primed as a function of predator immanence (Fanselow, 1994, 1997; Fanselow et al., 2019; Fanselow & Lester, 1988), just as Timberlake did for adient responses (Bowers et al., 2019). Denny (1971, 1976) has shown that the ability to approach a safe environment, and the amount of time rats are allowed to relax in it after escape or avoidance of shock, significantly affects their performance, consistent with Anger's (1963) theory of avoidance (approach to cues of safety). Hineline (1976) showed that the ability to enter a safer environment (lever retracted with up to 18 s of shock-free time) maintained avoidance behavior despite constancy in overall shock frequency. This immediate safe time maintained robust responding despite there being no correlation between overall shock rate and the behavior of the rats. Eubanks et al. (1975) were able to maintain performance on a VI schedule of response-contingent shock as long as the shock was followed by a signaled timeout. As Denny found, response rate covaried with duration of the timeout. It is notable that the route of approach to the safety signal was through a response-produced shock. The negative reinforcement of the timeout was more attractive than the positive punishment of the response that brought it about was repulsive. Due to the paucity of systematic data involving aversive control, however, MPR has not been applied in that arena.

Proximity

Various *proximities* make an event a good predictor of a goal:

a. Proximity in *time*—The importance of this proximity is attested in the voluminous literature on Pavlovian conditioning, such as the crucial role of CS–US proximity relative to US–US proximity in acquisition of sign tracking.[3] The proximity of response to reinforcer—the delay of reinforcement gradient—has played a central role in the analysis of behavior in our field (Skinnerian conditioning, see, e.g., Killeen, 2011).

b. Proximity in physical *space* facilitates conditioning (Mazur & Sanabria, 2011; Rescorla & Cunningham, 1979; Silva et al., 1992)—as does directed motion through it (positively or negatively correlated with goal direction: Cabrera et al., 2009). Spatial proximities between both stimulus and response (Hommel, 2011), stimulus and reinforcer (e.g., Williams, 1999), and response and reinforcer (Farwell & Ayres, 1979; Reilly et al., 2012; Williams et al., 1990) all play important roles in conditioning.

c. Proximity in *stimulus* space—The greater the similarity of a stimulus or a response to the goal, the quicker conditioning occurs (Rescorla, 1988; Rescorla & Furrow, 1977; Rescorla & Gillan, 1980; Steinhauer, 1982) and the greater the generalization of behavior to it. Proximity in stimulus space summates with proximity in time (Blough, 1972; see, e.g., his Figure 3).

d. Proximity in *semantic* space facilitates conditioning (Feather, 1965; Maltzman, 1977). For instance

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[3]Originally known as autoshaping, it is the approach to and contact with signals predicting reinforcement (see, e.g., Gibbon & Balsam, 1981).

Dunsmoor and associates (Dunsmoor et al., 2012) found that reinforcing one exemplar of a category (e.g., tools) generalized much more to other stimuli in that category than to stimuli in a different category (e.g., animals). Roediger and McDermott (1995) showed that when words in a list (e.g., legs, seat, table, …) were close in semantic space to another non-present word (e.g., chair), that absent word would often be "remembered," and thus available for learning (Pashler et al., 2005). Dymond et al. (2015) review existing behavioral research on the perceptual, conceptual, and symbolic generalization of fear and avoidance in humans; see Dymond and Rehfeldt (2000) and Sidman (1994) for general reviews of transfer of conditioning across semantically-related stimuli.

Repeated exposure to these proximities causes conditioning: That is, it causes stimuli to become signs of reinforcement, eliciting approach and manipulation of the signs. These proximities and repetitions accord with Hume's (1888/ 1939) criteria for inferring a causal relation between events. Animals have thus evolved "proximity filters" to discriminate what events in their environment or in their behavior predict the goals toward which they are driven (Killeen, 1981b).

"Proximity" means "closeness." How is that measured? Theoreticians have speculated: In space we find Hull's (1932) goal gradients and Miller's (1944) approach/avoidance gradients; over space, time, and responses, Killeen's traces (2005, 2011); more abstractly, Lewin's field theory (1951), Kantor's interbehavioral field theory (1958), Townsend and Busemeyer's (1989) dynamic field theory, Teigen's (2005) probabilistic proximity, and Killeen's (1991, 1992) behavioral mechanics in which animals follow trajectories through the multidimensional space along paths that maximize proximity to the goal (viz., are geodesics).

Blocking direction to the goal

MPR posits that a reinforcer strengthens all reinforceable events (Shettleworth, 1972) that precede it. It does this by associating the memory of those events—their "traces"—with a reinforcer (Killeen, 2005, 2011). As each subsequent response occurs, it dims memory for those preceding it, partially displacing, or blocking, their memory, and decreasing the strengthening available for them. This assumption is a temporal dynamic "error-correction" model. It is similar to that found in the Rescorla–Wagner model of conditioning in which predictive stimuli share the limited associative strength afforded by a US (Miller et al., 1995). In a crucial experiment Williams showed that responses proximal to reinforcement blocked a reinforcer's ability to strengthen more distal responses and concluded that the results supported "blocking as a fundamental principle of behavior, providing evidence

against the principle of reinforcement being stated solely in terms of temporal proximity between response and reinforcer" (Williams, 1975, p. 215).

Blocking is not all or none but a matter of degree. Assume that each response absorbs a proportion β (beta, for "block") of memory, diminishing that available for those that come before it. This will leave $(1 - \beta)$ for the prior response, which, absorbing $\beta(1 - \beta)$, leaves in turn $(1 - \beta)^2$ for those prior to it, etc. In general the directional effect of a reinforcer on each response will decrease geometrically as $\beta(1 - \beta)^{n-1}$; this effect may also interpreted as the memory strengthening of the nth response at the time of reinforcement. The sum of this series gives the total strengthening for that class of responses: $1 - (1 - \beta)^n$. Bizo and Killeen (1997) took this as their model of the coupling coefficient on FR schedules. That coefficient measures the proportion of the arousal-induced behavior that the schedule will direct into target responding.

Responses of different topographies will support different delay of reinforcement gradients (Starin, 1989). Non-target responses, such as adventitious behaviors which are common on interval schedules, will also occupy memory, partially blocking prior target responses, decreasing coupling to those, and causing lower rates of target responding (demonstrated by Killeen, 1969). Any event—stimuli, responses, reinforcers—can block the reach of a reinforcer to earlier target responses (Bizo et al., 2001; Pellón & Killeen, 2015). The longer or more articulated the response, the larger the value of β (Killeen & Sitomer, 2003).

Given a sequence of brief colored key lights as S^Ds, the control exerted by observation of a light n removes from a discriminative response decreases as a geometric function of the number of flashes between it and the end of the sequence (Killeen, 2001b). Shepard (1987) found analogous exponential[4] proximity gradients along properly scaled stimulus dimensions (see Figure 2 of Killeen, 1992). Johansen et al. (2009) gave rats a variety of locations to nose-press, and found that the probability of nosing in the same hole on the next trial was an exponentially decreasing function of the delay between that response and its reinforcement on the prior trial.

Memorial distance is the distance of the nth prior event—enhanced by marking or other proximities, reduced by intervening events—that represents its associability with reinforcement. Logan's hybrid model (1979) posited a mixture of memorial traces; this leads to a power function of time (when the decay is exponential) or of log time (when the decay is itself a power function; Killeen, 2001b). Both Wixted and Ebbeson (1991, 1997) and Rubin and Wenzel (1996) liked the power function for the decay of memory (m):

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[4]Exponential gradients are the limiting form of geometric gradients as step size approaches zero.

$$m = (t_0 + kt)^{-\lambda}. \tag{3}$$

Equation 3 is the familiar hyperboloid of discounting fame, where the x-axis offset, t_0, is typically set to 1 (cf. Killeen, 2019a). It was adopted by Killeen and Grondin (2022) in their general theory of time perception, which was based on memory traces decaying according to Equation 3. If memory decays exponentially with variation over trials or participants in the rate parameter, the resulting memorial function will morph into Equation 3 (Murre & Chessa, 2011), and it and its integral may then be a better model for the exponential function used throughout the rest of this paper (e.g., Equation 4).

In sum, incentives draw behavior through spatial/temporal/stimulus/response/memorial space into consequential regions in which the approach to the goal may be consummated (Killeen, 1992) and do so as a function of that behavior's multidimensional proximity to signs and sources of the goal. The direction afforded may be blocked by intervening stimuli or responses. It may be facilitated by affordances for responses that are preferred, ones that are typically more proximal to the goal. This is not so very different from the directive theories of Davison and associates (Cowie & Davison, 2016; Cowie et al., 2016; Davison & Baum, 2007) and Shahan (2010, 2017).

Response rate as the dependent variable

Why focus this fundamental analysis on response *rate*? According to Skinner, response *strength* "describes the state of the reflex with respect to all its static properties at once" (1938, p. 15), and "rate of responding is the principal measure of the strength of an operant" (1938, p. 58). Killeen and Hall (2001) tested that assertion by exposing pigeons to 10-s trials containing FI, VI, and VR schedules of various lengths, under conditions of satiation, extinction, and control. They used a principal components analysis to assay how many factors would be necessary to explain the data. For the FI data, a single component accounted for 93% of its variance. A single factor also sufficed for three of the four pigeons under the VI schedules; a second factor was necessary for one pigeon for whom there was a negative correlation between latency and running rate. Assuming a single underlying factor of *response strength* (viz. *arousal*, A), they then used confirmatory factor analyses to describe how well those static properties measure the underlying construct. For the FI schedule they found that the properties—overall response rate B, probability of responding on that trial p, running rate (response rate after the pause) b, and the latency, or delay to the first response of the trial L,[5] were correlated

with the construct strength. Factor loadings were 1.0 for B, .96 for p, .83 for L, and .78 for b. For the VI schedules and those same properties, the loadings were 1.0 (B), .97 (p), .83 (L), and .96 (b). Skinner's assertion is thus validated: Response rate is the best single measure of an underlying factor of strength. In many other articles Skinner argued that it is the "business of a science of behavior to evaluate this probability [of a response] and to explore the conditions that determine it" (Skinner, 1950, p. 198). Here, and in general, he took response rate to be the best predictor/correlate of probability. Killeen and Hall's analyses also validate that intuition, with probability having almost as high a loading on strength as does response rate, with a median correlation between B and p of around 0.97.

In a companion article, the authors (Killeen et al., 2002) decomposed response rate into IRT distributions, and modeled those with Markov models and stochastic machines called clocked Bernoulli processes, which provided excellent accounts of the distributions. They were able to provide a map between response rate and probability—a Holy Grail of Skinner (their Equation 3 and Figure 17)—which provided an excellent account of those authors' data (their Figure 18).

Applying the Principles

Arousal, competition, and direction, realized above, are the three principles. Equation 2b tells us the total amount of behavior we have on hand; just how, and how much of that behavior is realized as target responses depends on the contingencies of reinforcement. A crucial construct in that respect is *coupling*, the proximity of the response to the goal. In most of the work on schedules, this is realized as proximity on the temporal or response dimensions, and in studies of discrimination and generalization, as stimulus proximity; outside of the box, however, other dimensions play an important role (Killeen, 2014a, 2019b). The *coupling coefficient* tells us what proportion of the induced responses appear as the measured target response. Originally a distinction was made between a theoretically derived model of the coefficient, ζ (zeta), and an empirically derived free parameter C. Because the latter is easier to recognize, write, and remember, I use it here throughout. In the first paradigms considered here, C plays no role, but it plays a central one in the subsequent paradigms.

SIMPLE SCHEDULES

Periodic feeding

Adjunctive responses

Incentives induce a panoply of behaviors (Roper, 1978; Segal, 1972; Staddon & Simmelhag, 1971): "restlessness,

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[5]Latency, or delay, is a measure of weakness not strength. To develop a measure out of it to measure strength, the authors converted latencies to proportion of the trial spent responding, $1 - L / T$, where T is the trial duration (10 s).

varied movements, effort, search" (Craig, 1918, p. 101, as cited in Innis et al., 1983). Killeen and Pellón (2013) showed how these can be captured by reinforcement, which can reach back at least 30 s (Lattal & Gleeson, 1990). Some responses are more memorable than others either because of natural susceptibilities of those action patterns to reinforcement (Shettleworth, 1981; Shettleworth & Juergensen, 1980); or because they are primed by the behavioral module they are involved in (e.g., in a search module, some responses, such as differentially scanning dense undergrowth for prey, might be more memorable at the time of sighting a prey than such responses would be during the chase or capture modules; (Timberlake, 2002; Timberlake & Lucas, 2019); or because they are more extended, or effortful; or because they have distinctive environmental consequences. Such responses can be strengthened at a greater remove from the reinforcer (Williams, 1999). Without a necessary contingent response, this paradigm generates a flux of interim and terminal responses, competing and collaborating (Reid et al., 1993), sufficiently orderly, nonetheless, to be described by these mathematical principles (Pellón & Killeen, 2015). Adjunctive responses follow the same timescale of acquisition as traditional operants, have similar delay of reinforcement gradients, and, like operants, are subject to blocking and marking (Patterson & Boakes, 2012). They lack reliable proximity to reinforcement, which causes them to often suffer in competition with the contingent responses of pecking, pressing, and magazine entry. They are operants nonetheless, however loosely coupled to reinforcement, and they follow from these principles (Killeen & Pellón, 2013).

Autoshaping/Sign tracking

Put a stimulus into the above scenario. If it comes after a US such as food, and therefore signals the absence of food, it will be avoided (Wasserman et al., 1974); other stimuli in the environment that are not discriminative of nonreinforcement will be approached, especially if those are focused. A prime example is a stimulus occurring just before the delivery of a US. Approach to it is called *sign tracking*. The hopper or dipper or magazine is also a sign of reinforcement. Approach to it is called *goal tracking* (Costa & Boakes, 2009), but it is the same thing: approach to signs of reinforcement that have different proximities to it, in time and in space (Mazur & Sanabria, 2011), moving the animal closer to consummation.

If the memory trace of a stimulus decays exponentially, its average strength (S) is

$$S = \frac{1 - e^{-\lambda t}}{\lambda t}, \quad \lambda t > 0, \tag{4}$$

finding a limit of 1 as $t \to 0$. Lambda (λ) is the rate of memory decay; it is the twin of β in the geometric

gradient. Equation 4 is the integral of the exponential memory trace divided by its duration. Its plot is almost indiscriminable from that of the familiar hyperbola (Equation 3 with $\lambda = 1$; Killeen, 2001a, § 3.4.2, and Figure 4). Directive strength tends to increase with t, as more of the temporal elements of the stimulus are conditioned (the numerator); but the denominator prevails, leading to a hyperbolic decrease in average strength as a function of t. Instantiating Equation 4 for CS and context and taking their ratio gives the attractive strength of the CS relative to that of the context[6]:

$$S_{CS/Con} = \lambda' \frac{T_{Con}}{T_{CS}} \left(\frac{1 - e^{-\lambda_T T_{CS}}}{1 - e^{-\lambda_C T_{Con}}} \right), \quad \lambda'$$
$$= \lambda_{Con}/\lambda_{CS}, \quad T_{Con}, T_{CS} > 0. \tag{5}$$

Here T_{Con} is the duration of the context (the *cycle time*, from one US to the next), and T_{CS} is the duration of the CS. In the sign-tracking paradigm, both the context and the CS are acquiring discriminative strength, although prior hopper-training has given a substantial head start to context (Balsam & Tomie, 1985; Killeen, 1984). For large values of the exponents, the parenthetical term goes to 1, leaving the relative strength of the CS proportional to T_{Con}/T_{CS}.

To predict the standard datum in sign tracking, the number of trials to acquisition, Gibbon and Balsam (1981, p. 241) embedded the asymptotic relative strength of the CS, proportional to T_{Con}/T_{CS} (as given asymptotically by Equation 5), in a classic "error-correction" learning model (Equation A20). Inserting that learning model into Equation 5, the expected number of trials before activating the manipulandum, n, is

$$n = k \ln \left(1 - \theta \frac{T_{CS}}{T_{Con}} \frac{1 - e^{-\lambda_{Con} T_{Con}}}{1 - e^{-\lambda_{CS} T_{CS}}} \right). \tag{6}$$

The free parameter k depends on the rate of learning; θ (theta) is the threshold strength required for a response; it absorbs λ' and keeps the parenthetical >0. These are nested models, so that Equation 6 gives the same excellent predictions as Gibbon and Balsam's model when the exponents are large, driving their ratio toward 1. The term $1 - e^{-\lambda_{CS} T_{CS}}$ in Equations 5 and 6 tells us how much the strength of a short CS (of duration T_{CS}), suffers from not using the tail of the delay of reinforcement gradient, which is truncated by that stimulus's brief tenure. This only makes a difference for short CS intervals, say < 5 s, where it improves the goodness of fit. Gibbon and Balsam excluded two data points involving $T_{CS} = 2$ s as outliers, requiring more trials than predicted by their

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[6]See Equation A19. Equation numbers prefaced by A appear in the appendices.

model but as predicted by Equation 6. At very short durations, the CS cannot accrue much attractive strength and loses the competition with the goal stimulus, and the animal moves to the hopper ("goal tracking"). One may infer that animals that are predisposed to being goal trackers (Silic et al., 2023) have larger values for λ_{CS}, but that is not known. The present approach is less parsimonious than Gibbon's and Balsam's (it has one additional parameter, λ, that predicts the effects of short CS values or two, λ_{CS} and λ_{Con}, if the CS is significantly more or less memorable than the context), but it is more principled: Equations 4–6 are derived from a theory of how delay of reinforcement gradients spread their directive effects over the stimuli, responses, and context that are their harbingers.

Instrumental responding as sign tracking

An animal's own behavior may be a sign of reinforcement: Consider the press of a rat's paw on a lever followed by a pellet. The rat will approach that sign by moving its paw to the lever and pressing it. Operant conditioning may thus be conceived as sign tracking. The above model for sign tracking can do double duty in predicting pausing in periodic schedules (Appendix E). With practice, the operant behavior itself becomes reinforcing, as seen in sports and hobbies. Stimuli that signal its availability become conditioned reinforcers. These conceptions address some of the anomalies seen in the behavior of organisms (see Killeen, 2014a, 2019b, for details).

Pausing on periodic schedules

Just as animals are attracted to signs proximal to reinforcement in the autoshaping paradigm, they are attracted to response states that are proximal to reinforcement, such as the terminal run state in periodic schedules. They are also attracted to other elements of the context that compete with it. To compute where those strengths are equal, set Equation 5 equal to 1. Appendix E solves this equation for the postreinforcement pause, which is predicted to be proportional to the duration of the context (viz. the cycle duration, or the interreinforcement interval; Equation A24). This result is called *proportional pausing*. The constant of proportionality reflects the attractiveness of the context relative to the CS or response state. The model may also account for the different epochs in a periodic schedule that are occupied by different (e.g., adjunctive, operant) response classes (Killeen & Pellón, 2013; Pellón et al., 2018; Reid et al., 1993) as they compete for expression (Pellón & Killeen, 2015; Reid & Dale, 1983). Proportional pausing will serve as a lemma in subsequent models of the postreinforcement pause; it is discussed more extensively below.

CLASSIC SCHEDULES

The fundamental equation of motion for operant "target" responses is an elaboration of Equation 2b:

$$b_{\text{Target}} = \frac{C}{\delta}\left(\frac{A}{1+A}\right), \quad \delta > 0. \tag{7}$$

This core model takes the bulk amount of behavior, b_{Total} in Equation 2b, and allocates a proportion C to the target response. As ever, $A = ar$. Exemplary treatments follow. Key equations are summarized in Appendix F.

Ratio schedules

An early study added ratio response requirements to the end of various fixed-interval (FI) schedules and found that rate of responding increased up to the highest ratio tested, $n = 12$ (Killeen, 1969). There was no change in rate of reinforcement, which disconfirmed the hypothesis that animals respond faster on ratio schedules than on interval schedules because it brings food faster on the former but not on the latter—that correlation does not control responding. The contingency, and the proximity to target responses that it engineered, controlled responding. Postreinforcement pause length varied with FI values but was invariant with changes in n and response rate, indicating that in both ratio and interval schedules pause length depends on the inter-food interval (Appendix E). These results were the original instigation for the development of the contingency analysis of MPR.

On typical ratio schedules the rate of reinforcement r depends on the rate of responding b. That dependency—the *schedule feedback function (sff)*—is $r = b/n$, with n the ratio requirement. For FR schedules, the measurement window for r is the single FR value; for a variable-ratio (VR) schedule, it is over all experienced ratio values.

Fixed-ratio schedules

In Equation 7 replace A with ar; then insert the sff, $r = b_{\text{Target}}/n$. Solve for b_{Target}:

$$b_{\text{Target}} = \frac{C_{FR}}{\delta} - \frac{n}{a}, \quad a, \delta > 0. \tag{8}$$

This lovely equation needs three amendments. The first is that it takes a floor of 0 at $n \geq n_{\text{ER}} = C_{FR}a/\delta$, a value known as the "extinction ratio." The coupling coefficient C_{FR} increases with the FR requirement and at n_{ER} is typically very close to 1. Therefore, $n_{\text{ER}} \approx a/\delta$, and that is the number of responses of duration δ that a single reinforcer of that type in that motivational context will

support. The value for n_{ER} thus increases proportionately with motivation (specific activation, a), and inversely with the response duration (δ, see bottom panel of Figure 2).

Coupling

The second touch is specification of the coupling coefficient for FR schedules, C_{FR}. As noted earlier, we assume that that each response absorbs the proportion β of the strengthening effect of reinforcement left by the ones more proximal to reinforcement. The sum of these strengthening effects (Bizo and Killeen, 1997, Equation 3) is

$$C_{FR} = 1 - (1 - \beta)^n. \tag{9}$$

Figure 2 shows that Equation 8 succeeds in describing experimental data. The parameters change in appropriate ways (e.g., differences in the bottom panel are due solely to different values for δ). While gratifying, it raises a significant issue. The top panel reports *running rates*, response rates after the postreinforcement pause. The other panels report overall response rate (the bottom panel is proportion of time spent responding, which is just overall response rate multiplied by δ).

Just what is Equation 8 about, run rate or overall rate? To answer this question a schedule feedback function for "break-and-run" responding on FR schedules is required to support models for both running response rate and the pause length, permitting the computation of overall response rate. This is accomplished in Appendices A–C.

Pausing

Why do animals pause on ratio schedules? They are hungry; why not just get down to work, the sooner the better? Food is a salient stimulus, so animals approach its location and linger afterward (Pellón & Killeen, 2015), a behavior called "area-restricted search" (Dorfman et al., 2022). Nontarget responses are elicited and may be captured by the subsequent reinforcer (Killeen & Pellón, 2013; Labajos et al., 2023). Reinforcers also predict future reinforcers, directing action toward their source (Cowie et al., 2011). The route toward them typically is through an operant response and is therefore in conflict with goal tending and later goal tracking (Killeen, 1989). Animals approach and enter the operant response state (Shull et al., 2002) only when its attraction is stronger than attraction toward nontarget responses and the goal. This analysis is consistent with Cowie and associates' theory (Cowie & Davison, 2016; Cowie et al., 2016), with Shahan's (2010) argument that conditioned reinforcers are directors, not strengtheners, and the analysis of Hatten and Shull (1983) who held that pause length is an (inverse) index of the attractiveness of the terminal response state. This analysis is quantified in Appendix E, where it is shown that pause length should be proportional to the interval between reinforcers.

In autoshaping the speed of conditioning depends on the ratio of the time between the incentives relative to the duration of the conditioned stimulus (Gibbon & Balsam, 1981; Killeen, 1984), as discussed above. The same principle governs the postreinforcement pause. Contextual cues are in competition with the discriminative strength of the operant response in its ability to predict the next reinforcer. Animals enter the response state
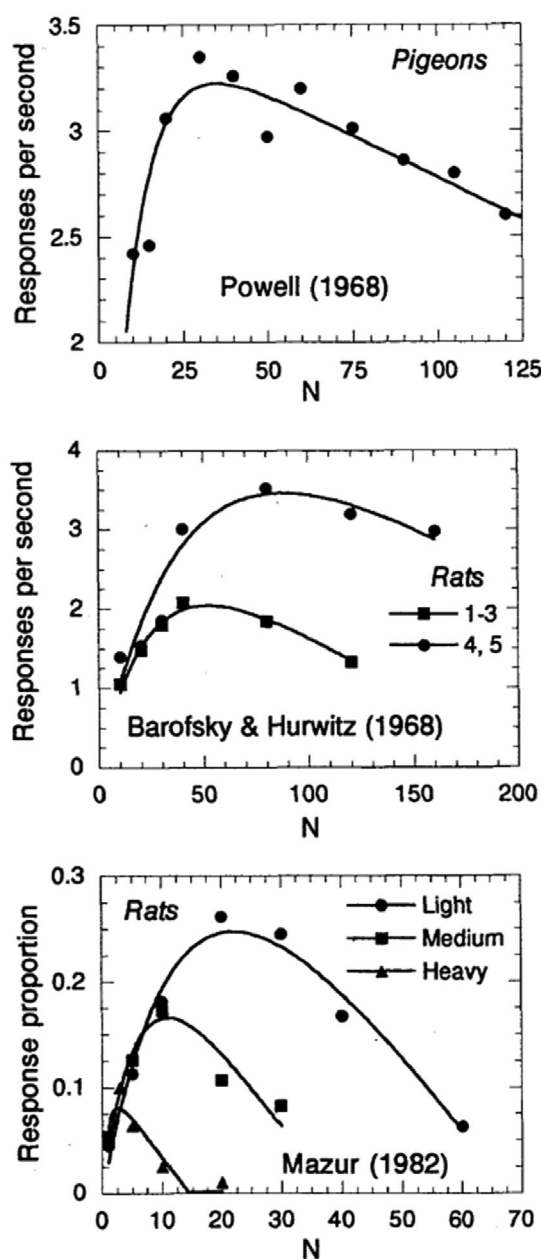

FIGURE 2 Data from three studies of fixed ratio responding. Reprinted from Killeen (1994) with permission of Cambridge University Press.

when it predicts a forthcoming reinforcer more strongly than does the fading trace of the last reinforcer and other contextual stimuli (Killeen et al., 2009).

A reinforcer is salient—it is memorable, and increasingly so as its magnitude increases. When it is replaced with a less salient stimulus, the following pause is shortened (Staddon, 1974), as it provides weaker competition against the terminal response state. The critical work on this issue was reported by Perone and Courtney (1992), who also provided an informative analysis of prior research. The authors varied reinforcer magnitude in conditions in which the magnitude of the forthcoming reinforcer was or was not signaled. In the latter case, larger magnitude reinforcers caused longer subsequent pauses: They provided stronger competition to the terminal state responses. When the magnitude of the forthcoming reinforcer was signaled, however, pausing was shorter when the forthcoming was of larger magnitude—the terminal responses were then stronger competition—but still covaried with the magnitude of the most recent reinforcer.

Anything that makes the context, conditioned to the US/reinforcer/PIE, more salient (e.g., a larger magnitude) will prolong the pause; anything that makes it less salient (e.g., a long intertrial interval; ITI; Bizo et al., Figure 7) will shorten the pause. Anything that makes the response more salient (e.g., a larger reinforcer that it signals, or marking it; Lieberman et al., 1979; Thomas et al., 1983, 1987) will shorten the pause. Some manipulations thus strengthen both target responding and desultory responding. Which of those has the advantage depends on the details of the paradigm, as shown by Perone and Courtney (1992). These opposing forces are further analyzed in Appendix E.

Cue competition between background and run state will generate "proportional pausing": The postreinforcement pause is a constant fraction of the interreinforcement interval (Appendix E). Bizo and Killeen (1997, Equation 11) used this law of behavior to develop the model for pausing on FR schedules (reprinted as Equation A10). It predicts a convex relation between FR value and pause length, as observed (their Figures 10–12). It leads, through Appendix A, to the following prediction for overall response rate:

$$b_{\text{Target}} = \frac{C_{FR}}{\delta} - \frac{n}{a(1-p)}, \quad \delta, a(1-p), > 0, \qquad (10/A9)$$

with p the constant of proportionality relating pause length to IRI.

Equation 10 is a surprising and appreciated outcome. The same model suffices for both running rates and overall response rates. Because a is a free parameter, we need not evaluate p but just recognize that the nominal value for a—call it a'—will be smaller for overall response rates than for running rates.

Cue competition elsewhere

The same competition that controls pausing on reinforcement schedules affects autoshaping and discrimination learning. Bouton and Hendrix (2011) observed quick differential conditioning of magazine entries when they used a long ITI as a discriminative stimulus for when a 10-s CS would be followed by food and a short ITI to signal when it would not be followed by food. But when they did the converse—a short ITI signaled food and a long one signaled no food—conditioning to the CS was much slower. This is because the control by the prior reinforcer—its prediction of food—was in opposition to that of the CS; a long ITI weakened directive strength by the prior reinforcer, making the CS a relatively more effective sign. A short ITI moved the prior reinforcer closer to the next feeding, enhanced its directive strength, and thus slowed acquisition of sign tracking to the CS.

Roberts and Kraemer (1982, 1984) varied ITI in a delayed matching-to-sample task and found that accuracy increased monotonically with ITI (as did Colares Leal et al., 2020). The logarithm of the ITI/delay ratio provided a near-perfect prediction of accuracy for their data. The S^D was in greater control when attention was not preempted by a recent prior reinforcer. This is similar to sign tracking, as the ITI/CS ratio is varied: Competition between the desultory control by the prior US and context and that of the more directed control by the S^D, by the CS or by the response, pervades these data. Salient stimuli control behavior: They may help it to navigate to the goal, or, in some cases, interfere with that direction, depending on their relative strengths (Equation 4). Models found in Appendix B capture pausing on both FR and VR schedules. Pauses are much shorter on the latter due to the occasional proximity of the first few responses to a reinforcer, which strengthen the attractiveness of responding soon after consumption of the prior reinforcer (Bizo et al., 1997, Figure 7).

Variable-ratio schedules

A VR schedule reinforces the ith response, where i is drawn from a distribution of values with mean n. If the distribution is exponential, it is called a *random ratio* schedule, which is implemented by reinforcing each response with probability $1/n$. To calculate the coupling coefficient, multiply the probability of a string lasting for i responses by the proximity of each response in that string to reinforcement, integrated from 1 to ∞. This results in a hyperbolic coupling coefficient (Killeen, 1994, Equation C16; Bizo et al., 2001, Equations 2 and 4), one that increases more slowly with n than does coupling on FR schedules:

$$C_{RR} = \frac{n}{n + (1-\beta)/\beta}, \quad \beta > 0, \qquad (11/A12)$$

Equation 7, with Equation 11 as the coupling coefficient, draws the curve through the data in Figure 3.

Interval schedules

Reinforcement rate on interval schedules is typically close to the reciprocal of their scheduled interval I but is less at very low rates of responding, due either to a small value of a or a large value of δ, and also at very high rates of reinforcement. Baum (1992) made a careful study of the feedback function on interval schedules. The most parsimonious function that provides an adequate account of the data is Staddon's (1980):

$$r = \frac{1}{I + 1/b}, \qquad (12)$$

where I is the interval value. Insert Equation 12 into Equation 7 for the model of arousal and constraint, and with obvious substitution, it becomes a familiar hyperbola:

$$b \approx \frac{C}{\delta}\left(\frac{a}{a+I}\right) = \frac{C}{\delta}\left(\frac{r}{1/a+r}\right) = \frac{kr}{1/a+r}. \qquad (13)$$

Equation A14 in Appendix D covers situations in which a small value of specific activation, a, due to minimal deprivation or an unappetizing reinforcer, will decrease the asymptote (McDowell & Dallery, 1999); in such cases the full Equation A14 should be used. It reduces to Equation 13 in most situations.

How should rate of reinforcement, r, be computed? As overall rate, the inverse of the interval value—or as the inverse of the time in the second, running state? The latter makes more intuitive sense, as the pause time and run time may be thought of as components of multiple

FIGURE 3 Average data of four rats, from Killeen & Sitomer (2003). Parameters are β = 0.76, δ = 0.25 s, and a = 350 resp/reinf. Reprinted with permission of Elsevier.

schedules. Because of proportional pausing, Equation 13 works for both instantiations of r; for the same reason, it will predict both overall and running rate (Appendix E). Equation 13 needs to be particularized for fixed and variable-interval schedules.

Pausing on interval schedules

The treatment of pausing on interval schedules follows that for ratio schedules. Animals approach the response state only when the memory trace of the prior reinforcer and the nontarget responses it evokes have abated. That will happen when the length of the pause[7] is proportional to the interval value (Fox & Kyonka, 2013; Hanson & Killeen, 1981; Schneider, 1969; Zeiler & Powell, 1994), with the constant of proportionality depending on the memorability of the prior reinforcer and the directive strength of the context relative to that of the response state (e.g., Lowe et al., 1974; Staddon, 1974), as is the case for ratio schedules.

Once in the run state, behavior will depend on the contingencies of reinforcement. On ratio schedules only target responses are reinforced, generating assiduous target responding. On interval schedules any behavior that precedes a target response is equally likely to be reinforced as is a target response, decreasing response rates. This also increases the likelihood that (premature) hopper approach will be reinforced, which would account for the downturn often seen in FI response rates just before reinforcement is due (Hanson & Killeen, 1981). Responding on many schedules is organized in bouts, with the break-run pattern being the most prominent. Motivational variables affect the rate of initiating bouts, whereas motoric variables and reinforcement contingencies affect responding within bout (Brackney et al., 2011).

Fixed-interval schedules

Equation 13 is the general equation of motion for responding on interval schedules. The coupling coefficient for interval schedules includes response rate, as the more target responses that occur before reinforcement, the stronger the coupling (see A15, A16). Coupling is reduced at very low response rates (as may be due to low motivation, a), and at very short intervals. For the usual case, however, it is more parsimonious to treat the coefficient in Equation 13, C/δ, as a single free parameter. In a classic study Schneider (1969) exposed six pigeons to six FI schedules ranging from 16 to 512 s in a Latin square design and reported the running rates shown in Figure 4.
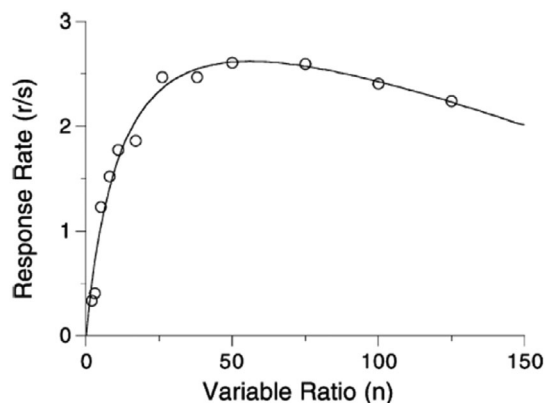
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[7]To be more precise, it is the *breakpoint*, when animals switch into the second state of terminal responding (Shull et al., 2001), and not the time to the first response, that is proportional. See Appendix E for the logic behind proportional pausing.
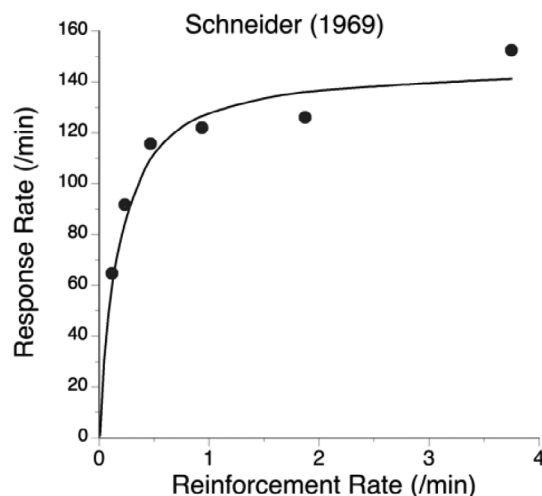
FIGURE 4 Average running rates of six pigeons on fixed-interval schedules. The curve is from Equation 13, with C/ δ = k = 147/min, and a = 414 s.

FIGURE 5 Response rates of three pigeons on VI schedules reported in Baum (1993). The curve was drawn by Equation 14, with parameters $k = 1.09$, $a = 0.55$ min/reinf., and $\lambda = 4.2/s$.

Variable-interval schedules

Equation 13 is equivalent to the classic hyperbola often used to fit VI schedules (see Equation A14). But that equation is not a complete model for VI schedules, as it lacks an articulated coupling coefficient. On variable schedules, some reinforcers follow soon after prior reinforcers; the first may block the reinforcing effectiveness of the second on responses occurring before the first (Killeen & Sitomer, 2003, Figures 17, 18). One needs to compute the probability of each interval and the extent to which responses within its (probabilistically truncated) range are able to strengthen the operant. A17 gives the resulting VI coefficient, which, inserted into Equation 7 gives

$$b = \frac{kr}{r+1/a} - \frac{r}{\lambda}, \quad k = 1/\delta; \quad \delta, a, \lambda, > 0. \quad (14/A18)$$

This has the semblance of Herrnstein's "single-key matching law" but with a subtrahend, r/λ. Lambda— $\lambda = -\ln(1 - \beta)$—is the rate of the decay of the memorial reinforcement gradient. The subtrahend r/λ comes from the truncation of the memory of strings of responses by a prior reinforcer, which displaces, or blocks, the memory of the responses that came before it (Bizo et al., 2001; Killeen & Smith, 1984). It predicts a downturn in Herrnstein's hyperbola at very high rates of reinforcement (its numerator), where the reach of a reinforcer is blocked by other reinforcers. Are there such downturns in the familiar hyperbola? Few studies have carried reinforcement rate high enough to see. One that did, while taking care to avoid satiation effects (Baum, 1993), reported the data shown in Figure 5.
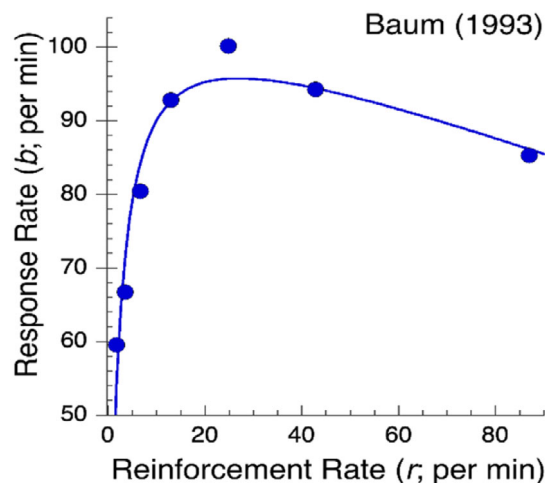
Sequential schedules

Progressive-ratio schedules

PR schedules are ratio schedules in which the response requirement (n) increments after each reinforcer. They are commonly used to assay the effects of disruptors, such as drugs, on behavior. PR schedules may progress arithmetically (e.g., 3, 6, 9, 12, 15, …) or geometrically (e.g., 3, 5, 7, 10, 15, …). The FR model provides an excellent account of the former (Killeen et al., 2009, Figures 1 and 2). That study showed a significant regression of *prp* on the prior component length (their Figure 3, $r^2 = .97$), with slope $k = 0.51$. This illustrates *linear* [proportional] *waiting* (Higa et al., 1991; Wynne & Staddon, 1988). We have assumed such proportional pausing in the earlier analyses of static schedules, and here it is also evident in dynamically changing schedules. Based on this regularity, Killeen and associates showed that *postreinforcement pause* was proportional to $n/(a - n)$, where a is specific activation; in their Figures 2–4 it accounted for the data precisely. The authors adduced a contextual conditioning account to explain the deviation of the geometric progressions from the arithmetic and tested it by differentially signaling the longer PR values. Signaling caused an increase in response rates on the shorter ratios, showing that they were indeed affected by context, including forthcoming rates of reinforcement. The inclusion of context permitted an excellent fit to the data. Yet such data called for a more general treatment, which was developed by Bradshaw and Killeen (2012). This instantiation provided a near-perfect fit to many data, and an explanation of the effects of various drugs and magnitudes of reinforcement on behavior in terms of interpretable changes in parameters. A spreadsheet is available on request.

Multiple schedules

Behavioral contrast

When distinctively signaled schedules are programmed to alternate, they inevitably interact. A prototypical example is a VI schedule alternating with another VI schedule or with extinction (EXT). Nevin and Shettleworth (1966) reported results from two pigeons shown in Figure 6. In their study a VI 6-min schedule alternated with a VI 2-min schedule. The depression in response rates at the onset of the leaner (VI 6) component and their elevation at the onset of the richer component is called local or transient contrast.

Following McSweeney (1987), Killeen (2014b) argued that it was competition from nontarget responses that caused contrast. Because the discriminative stimuli were often located near the operandum (typically on it in the case of key pecking), those target responses came under quick control by the stimuli. Nontarget responses were away from the operandum and thus away from good control by the discriminative stimuli. Following Equation 2b, they covaried with the rate of reinforcement, occurring at high rate in the richer schedule; after transition, they slowly adjusted to the new conditions (an exponential process called *hysteresis*), giving the gradual rise and fall seen in Figure 6.
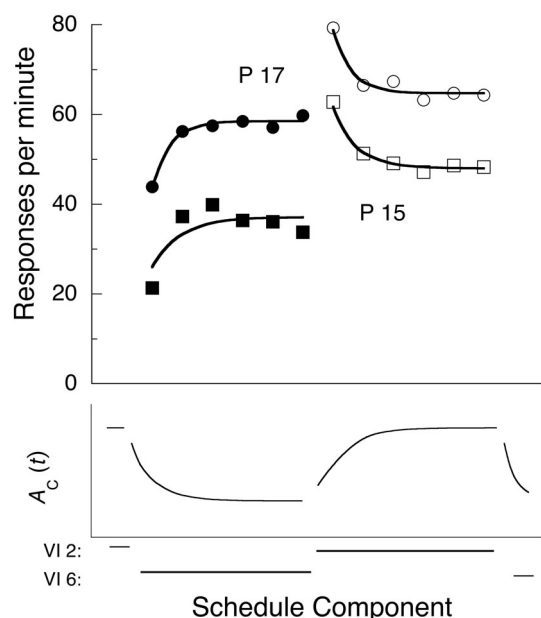

FIGURE 6 Response rates of two pigeons on MULT VI schedules reported by Nevin and Shettleworth (1966). The top panel shows an initial depression in response rates in transition from the VI 2 to the VI 6 and then a transient increase in transition back to the VI 2. The curves are from the theoretical model described in detail by Killeen (2014). The bottom panel shows the course of the hypothetical nontarget responses that cause the contrast by competing with the target response. Reprinted from Killeen (2014) with Wiley's permission.

For a target response to occur, it has to be elicited (Equation 7), and not be suppressed by a competing response:

$$b_{\text{Target}} = \frac{C_{VI}}{\delta}\left(\frac{A_T}{1+A_T}\right)\left(1 - \frac{A_C}{1+A_C}\right), \quad \delta > 0, \quad (15)$$

where $A_T = a_T r$ for the target response and $A_C = a_C r$ for the competing response. The first parenthetical is the propensity for a target response to be elicited, and the second the probability that a nontarget response will *not* be elicited at the same time. Nontarget responses often suffer a delay of reinforcement, as the last response before reinforcement is always the target response; this makes conditioning of them to the discriminative stimuli slower. The value for A_C therefore changes with time into the component, shifting exponentially from that of the prior component to that of the current one. After many sessions, however, animals eventually come under good stimulus control and transient contrast disappears.

If it is nontarget responses that cause contrast, then reinforcing them more directly—say, with a variable-time (VT) schedule in which reinforcers are delivered independently of target behavior—should generate even greater local contrast, as they increase competition from nontarget responses. Rachlin (1973) performed the experiment by adding free reinforcers every 15 s to one component of a multiple (VI 2, VI 2) schedule. The results shown in Figure 7 confirm this prediction. Other types of contrast, such as the "following schedule effect," admirably reviewed by Williams (2002), are addressed in Killeen (2014b).

Behavioral momentum

Nevin et al. (1983) perturbed responding on multiple schedules by providing interleaved free food between components and also by placing both components in EXT. In both cases they found that the pigeons' response rates on the richer components resisted the disruptions (responding continued) longer than did those on the leaner components. This began a long tradition of research, culminating in the synoptic article of Nevin and Grace (2000), with some continuing development (Nevin, 2015; Nevin et al., 2017) garnering 4,000 citations for "Behavioral Momentum" through 2022. Behavioral momentum theory (BMT) characterized this phenomenon as analogous to physical momentum. The chief dependent variable was the slope of the decrease in responding as a function of the disruptor—amount of intercomponent feeding, or time through extinction, satiation, etc. The slope was taken as an (inverse) measure of behavioral mass; moving bodies with large mass are less susceptible to disruption than those with small mass. The titular *momentum* was the area under the extinction curve or
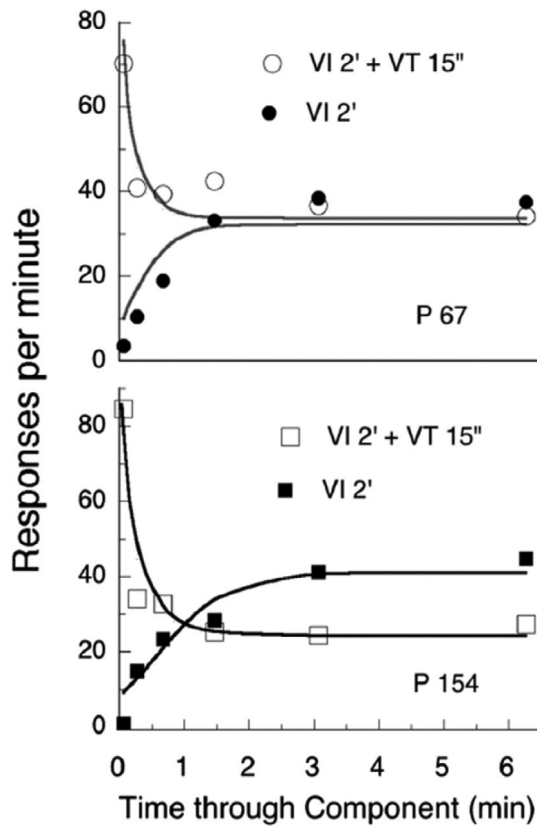
FIGURE 7 Response rates of two pigeons on MULT VI schedules reported by Rachlin (1973). The response rates demonstrate an initial transient elevation in response rates after transition from the VI 2 to the VI 2 + VT 15 s (unfilled symbols) and an initial transient depression after transition back to the VI 2. In both cases response rates recover to asymptote after a few minutes. The ascending functions are caused by the washing out of the competing behaviors that were reinforced in the component with the added reinforcers. The descending curves are due to the dearth of those competing responses in the simple VI 2 schedule, with the higher rate of responding after the transition due to the higher arousal which the higher rate of reinforcement entails—still focused on the target responses—until stimulus control abates and normal competition resumes. The curves are drawn by Equation 15; they are described in detail by Killeen (2014b) and are reprinted from that article with Wiley's noblesse oblige.

under the disruptor curve in general (Killeen, 2000). Specific activation, a, is the area under the extinction curve after a single reinforcer, so these concepts are closely affiliated.

The principles of MPR are applied to BMT by invoking Equation 15, as described below. The key insight is that when arousal (A in Equations 7 and 15) is decreased by a disruptor d, it decreases the propensity to respond, $A / (1 + A)$, in a nonlinear way. The rate of change of that propensity (its derivative) when A is degraded is proportional to $1 / (1 + A)^2 = (1 + ar)^{-2}$. For large values of A associated with high rates of reinforcement r, the rate of change is therefore quite small with disruption: propensity to respond, $A / (1 + A)$, decreases as an inverse function of the *square* of ar—for large values of r, not

much at all, for small values quite a bit. Strength is like a spring; it compresses easily at first and only with greater effort as it is further compressed, but it stores that potential energy. In disruption, high potential energy takes longer to dissipate. That reserve strength is essentially hidden in this simple nonlinear (hyperbolic) relation between response rate and arousal. Yet it is the foundational reason for the primary finding of BMT: Schedules with high rates of reinforcement (r) are more resistant to disruption than those with low rates of reinforcement.

Equation 13 suffices for the basic findings of BMT: the differential persistence of operants with different histories of reinforcement. Equation 15 adds competition from nontarget responses to it, whose quicker time course cause a bump in the first minutes of extinction as the target operant response is released from competition (Figure 8). Such waning competition is the same process that causes transient contrast in multiple schedules of reinforcement (Figures 6 and 7) and causes resurgence in those paradigms. Competing responses may have less or more "mass" than the target response, and thus dissipate more or less quickly. Assume an exponential decrease in arousal as a function of disruption—in particular, the disruption (d) caused by extinction:

$$d_x = e^{-\alpha_x t}. \tag{16}$$

Here x stands for either target or competing responses, each with their own time course α_T and α_C. In the analysis of contrast, this process was the decay of control by the previous component on the competing responses. Here it is the decay of persistence of the responses under disruption, such as experimental extinction. The alphas are (inverse) measures of their mass, as responses with more momentum decay more slowly. Then the evolution from the baseline (where $t = 0$ and $d_x = 1$) is

$$b_{\text{Target}} = \frac{C_{VI}}{\delta}\left(\frac{d_T A_T}{1 + d_T A_T}\right)\left(1 - \frac{d_C A_C}{1 + d_C A_C}\right). \tag{17}$$

This model captures most of the relevant data. As an example, Figure 8 shows the results of two studies of disruption by extinction, with the curves driven by Equation 17. These are normalized data, so they are predicted by the ratio of Equation 17 over Equation 17 with t (from Equation 16) = 0. This last gives the predicted response rate entering extinction. The transient increase in target responding seen in Figure 8 occurs because competing responses extinguish faster than target responses ($\alpha_C > \alpha_T$), momentarily releasing key pecking from competition. A similar explanation has been offered by Shahan (2022). It is the same mechanism that causes resurgence in general. Absent competition ($A_C = 0$) the component with the higher rate of reinforcement would still extinguish more slowly; it is an $A_C > 0$ in Equation 17 that causes the surgence above baseline rates.
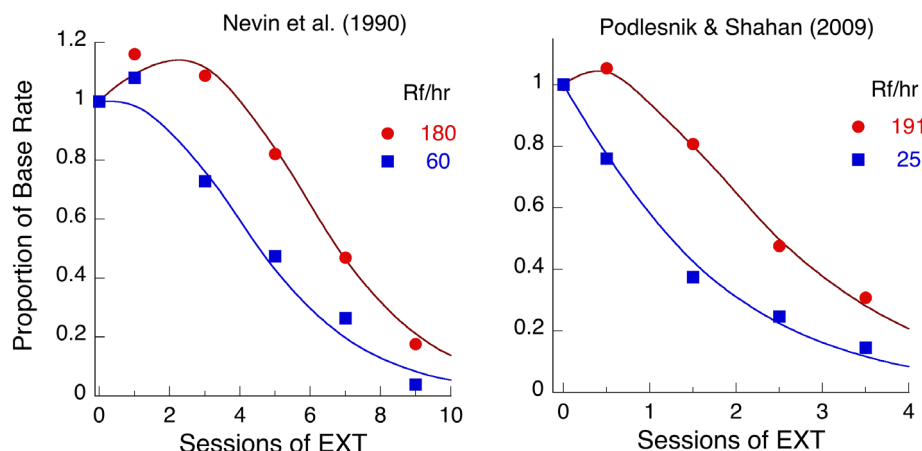
FIGURE 8 Response rates of three pigeons (left panel) and 10 pigeons (right panel) on multiple VI schedules undergoing extinction. The curves are from Equation 17, with four parameters total for all curves in each panel (a_T, α_T, a_C, α_C; the others cancel in the ratio). The bump is from the faster extinction of competing responses ($\alpha_C > \alpha_T$). The figure is from (Killeen & Nevin, 2018) reprinted from that article with permission of the authors and Wiley. See the original for details and parameter values.

The main message here is that behavioral momentum is intrinsic in Equation 7 and ramified by Equation 17. Schedules with higher rates of reinforcement confer greater momentum to their responses by virtue of those equations, which unwind their potential more slowly at high values of arousal than at low ones, due to the hyperbolic nature of Equation 7. Nevin and Killeen (2018) elaborate the physical analog in the appendix to their paper, to which intrepid readers may turn. See (Killeen, 2003; Killeen, Sanabria, et al., 2009) for other approaches to persistence in behavior.

Habits and actions

An intriguing line of research on the different characteristics of learned versus overlearned responding emerged from the laboratory of Tony Dickinson. In an early report, Adams (1980) trained rats on a VI schedule and made the food aversive by pairing it with lithium chloride. Lever pressing in extinction was not significantly reduced compared with nonpoisoned controls, even though the treatment profoundly retarded later consumption and reacquisition. This and other studies showed that instrumental performance can be independent of the current value of the reinforcer. Rats worked for reinforcers that, when delivered, they would reject. Such responding, under only weak control by the motivations that once engendered it, was called "habitual." It was not found with brief training, not so evident on VR schedules, and was due to a history of exposure to the reinforcer, not to the extent of response practice. Responses that were not habitual were called "actions." Dickinson formalized this research with his "dual process theory" (Dickinson, 2016; Perez & Dickinson, 2020).

Balleine and O'Doherty (2010, p. 49) provide a concise update of these different response characters:

goal-directed and habitual actions differ in two primary ways: (1) they differ in their sensitivity to changes in the value of the consequences previously associated with the action and (2) they differ in their sensitivity to changes in the causal relationship between the action and those consequences. … two kinds of experimental test have been used to establish these differences, referred to as outcome devaluation and contingency degradation, respectively.

It is evident these phenomena are, at least in part, manifestations of behavioral momentum—greater momentum called habits, lesser called actions. In Figure 7 the different momenta are due to different rates of reinforcement (as it was for Thrailkill et al., 2018, Figure 5), with higher rates increasing the value of A, and thereby slowing extinction. In the traditional dual-process literature, the differences are more often due to differences in length of training; over trials, the value of A continues to accumulate (Killeen et al., 1978). When tested in extinction, higher potential energy (A) from extended trials takes longer to unwind from its ceiling (Killeen, 1998). That reserve strength is latent in the simple nonlinear (hyperbolic) relation between strength and arousal (Equations 2b, 7). When a performance empowered by a high level of arousal has that devalued by satiation or taste aversion, the effect on persistence will be less than for the behavior that started with a smaller A.

Balleine and O'Doherty's (2010) second point was established through various studies. Dickinson et al. (1998) provided two levers on which rats could earn pellets. One of the levers interfered with random delivery of sucrose, the other did not. Rats with extended training

did not thereafter choose the lever that did not interfere, whereas those with limited training did. Variability is substantial in the early stages of skill acquisition, as it is under interval schedules (Boren et al., 1978; Eckerman & Lanson, 1969; Hoyert, 1992). Variability decreases with practice, as exploration narrows to exploitation. Combined with greater response strength (A) for the overtrained response, the results follow without invoking special classes for the responses. Dickinson (1985, Figure 7) reported the results of a study devaluing the reinforcer for VR and VI studies. For the VI condition, there was no difference in responses in extinction for the devalued and control groups. For the VR group, there was a large reduction in response rate for the devalued group—to a level that was not significantly different from the VI groups. What was different about the VR group was their much higher rates in the control condition, which we know is a characteristic attribute of such schedules. Why are VI schedules less likely to show devaluation effects? Equations 12 and 14 describe the competition between target responses and other responses. Unlike ratio schedules, interval schedules reinforce any response before the target response that takes time, and that is why they engender lower rates of target responding. But reduced motivation affects both target and competing responses, and the latter decrease offsets decreases in the target response. There are fewer competing responses in ratio performances, so the expected reduction in response rate is not masked by the concurrent reduction of competing responses.

Other experiments that have denatured the contingencies have done so by interposing additional noncontingent incentives into the schedules. Numerous such experiments have also been done to study behavioral momentum. The results are similar across subdisciplines. There have been several recent reviews of the literature (e.g., Balleine, 2019; Bouton, 2021), with more details about contextual conditioning and brain mechanisms. The present analysis of habits as responses with substantial momentum constitutes a different perspective on those provocative experiments.

Delay of reinforcement

The accumulation paradigm

On an FR 1 schedule rats will sometimes press the lever a couple times before collecting the pellets. Is that random, or is it rational (−ly explained)? The cost of food is the number of lever presses plus the trip to the hopper. The benefit is the number of lever-press delivered pellets. The benefit/cost ratio can be increased by increasing the number of lever presses before collecting them. Is something like this driving their behavior? That hypothesis may be tested by increasing the fixed cost of the trip to

the hopper, which should increase the number of accumulate lever presses before they are collected. This was tested by moving the hopper away from the lever for a group of rats (Killeen, 1974). At the distance of 8 ft. (2.44 m) the rats made an average of 7 accumulate responses before running to the hopper. This "cross-modality matching" experiment (Stevens & Marks, 1965), measures the value of effort/delay/distance in units of pellets. It was extended (Killeen & Riggsford, 1989; Killeen et al., 1981), with both studies showing that delay, not effort, was the controlling variable (also see, French & Reilly, 2017; Gaistardo & Bruner, 2021; McFarland & Lattal, 2001; Reilly et al., 2012).

Frame this as a choice between running to n pellets after n responses with a travel time of d s or making an additional lever press requiring δ s for a package of $n + 1$ pellets at a delay of $d + \delta$. For linear utility functions, the rats should run when the utility of $(n + 1) / (d + \delta) < n / d$. This may be solved for n: $n > d / \delta$: when n is greater than d / δ it is time to go (or, equivalently, when $n = d / \delta + \varepsilon$, where ε [epsilon] is ≤1). This predicts a linear relation between amount of compensation required as a function of distance d, as found (Killeen, 2019a). In this paradigm there are two routes by which to approach the goal: a shorter one directly to the hopper garnering n pellets and a slightly longer one, via an additional lever press, garnering $n + 1$—smaller sooner versus larger later. As n increases, the superiority of $n + 1$ to n gets too small to matter (ε) and the rat bolts for the hopper. Just as a specifies the amount of behavior a single reinforcer will support, n specifies the amount of reinforcement required to support a response at a delay of d s.

Notice that in this equation the distance d is divided by d_0, the time it takes to earn a single pellet. Increasing that time should decrease n for a fixed distance d. Killeen and Riggsford (1989) tested this prediction by putting the accumulate lever on a VR 4 schedule. Intuitions and behavioral economics suggest that the rats should have compensated by increasing lever presses a bit; but the above model predicts they should decrease lever presses. Lever pressing decreased.

At a nominal distance of $d = 0$, $n_0 = 1$, the minimum to get a pellet. At a distance of d, $n_d = d / \delta + \varepsilon$. Their ratio gives the value of the immediate pellet relative to the accumulation, discounted by the delay time: $n_0 / n_d = 1 / (\varepsilon + kd)$, where $k = 1 / \delta$. Look familiar? This is the classic "hyperbolic" relative value found with non-human animals in delay discounting tasks (Killeen, 2019a, 2020)—Equation 3 with λ and $\varepsilon = 1$. The rats were bidding on future payoffs (Kirby, 1997), specifying the amount of food it would take to justify their time investment to collect it. The accumulation paradigm provides an additional, efficient, way to measure delay discounting. It connects the discounting literature both to that of MPR and to that of central place foraging (Killeen et al., 1981).

Concurrent schedules

Concurrent schedules provide the best opportunity to see the third principle in action: Approach to better "patches" drives the basic law of concurrents, *matching* of response ratios to reinforcement ratios. Consider a two-armed bandit that pays off lever presses with probability 70% on the left and 30% on the right. You are unlikely to allocate your responses 70/30 (i.e., to match response proportions to reinforcement proportions). Pretty soon most of your responses would be to the left lever. But if these were concurrent VI schedules with VI 30 on the left and VI 70 on the right, you are likely to allocate your responses approximately 70/30 (i.e., to match). This is the original matching law (Herrnstein, 1961). What makes the difference? The way concurrent VI schedules are typically programmed is to "hold" a reinforcer once it has been set up until it is collected (Jensen & Neuringer, 2008). The longer you respond on one option, the more likely it is that reinforcement has set up on the other (MacDonall, 2005). As the hold is shortened, behavior shifts from matching to the nominal rates, to maximizing (Killeen & Shumway, 1971), driven by the positive feedback loop as animals approach ever greener pastures.

In traditional (unlimited hold) VI schedules, when the probability of reinforcement on an alternative patch has increased to the point it exceeds the probability for another response in the current patch, animals approach—change over to—the other patch (Dreyfus et al., 1982; Silberberg & Fantino, 1970). A yoked-chamber comparison of concurrent and multiple schedules showed that such approach drives relative time in the patches to equal the local rate of reinforcement in those patches (time matching). This in turn drives both the local rates of reinforcement in the patches and the local rates of responding in those patches, to equality (response matching; Killeen, 1972). This provides a negative feedback loop, keeping behavior near matching: Overpreference for one alternative will reduce the local rate of reinforcement there, causing more frequent approach to the alternative until equilibrium is restored. Yoked partners to the animals on the concurrent schedules also matched responding to obtained reinforcers. In a second experiment, the component durations were held equal, as in a traditional multiple schedule and local rates of reinforcement diverged from equality, with local rates of responding undermatching. Matching is ubiquitous when animals can approach better patches and leave them when other patches, perhaps ones improving with time like VI schedules, beckon, and it is largely restricted to situations with negative feedback.

This recognition that animals approach signs of (better probabilities of, or shorter delays to) reinforcement is called "melioration theory" (Bland et al., 2018; Herrnstein & Vaughan, 1980; Vaughan, 1981) and "local bookkeeping" (Heyman, 2023). A model of it

(in Killeen, 2015, Appendix A), predicts observed differences in arithmetic progression VI schedules and random-interval schedules (Elliffe & Alsop, 1996), the effects of changeover delays on choice (Stubbs & Pliskoff, 1969), and the "fix and sample" pattern of responding characterized by Aparicio and Baum (2006) and others (e.g., Baum et al., 1999). Killeen (1981a, Equation 25) developed Equation 2 for an early model of the generalized matching law. MacDonall (e.g., 2009) provides the best experimental analysis of the microcontingencies that drive responding in concurrent schedules (also see Gomes-Ng et al., 2018; McLean et al., 2018). These largely validate the third principle of approach to (better) signs of reinforcement.

Behavior at large

Through the course of his life Bill Timberlake constructed a synoptic account of behavior in its natural context. His *behavior systems theory* (Timberlake & Lucas, 2019) recognized that animals' perceptions and actions are predisposed by the particular motivational "modes" they are in. Such modes sharpen search templates, priming the recognition of relevant affordances in the environment. In the predation mode, animals are attuned to movements and scents and other signs that signal the potential presence of prey. Within that mode, different modules are induced by the environment: scanning the horizon, to chasing a prey once sighted, to attacking it once close. Within each module, such action patterns may carry animals into another module with greater proximity to reinforcement, thus constituting a stronger sign of it. Although naturally attracted to those responses (e.g., chasing a prey), they cannot be successfully approached absent particular stimuli (e.g., the sight of the prey).[8] The onset of stimuli correlated with the next module (e.g., vision of prey) directs the animal to the actions (e.g., coursing that prey) that will get them to a successive, more proximal, module and prepares their perceptions and motor patterns for it, as noted by Thorndike in his law of readiness (Thorndike, 1913). Those signs of transit act as both reinforcing and discriminative stimuli. This is what schedules look like outside of the box. They are gisted by the third principle: Animals approach signs that afford responses of higher probability, typically ones more proximal to reinforcement; within those successive contexts, relevant behaviors and perceptions are primed—ones that will best facilitate approach to the next module—and with transit, the enabling responses are reinforced.

Timberlake's second major contribution was *behavior regulation theory*, an extension of the Premack principle.

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[8]Although young animals will practice such responses as play and adults as "vacuum activities," parents may deliver stunned prey to offspring for part-task training. See Killeen (2019b) for a dynamic flow chart of wolves moving through such modules in attacking a herd of elk.

Both theories hold that it is behavior, not objects, that reinforce behavior: Eating is the reinforcer, not the food, which serves as an excellent sign for eating (Killeen & Jacobs, 2017a, 2017b). Premack posited that it was the opportunity to perform a more probable contingent behavior that reinforced instrumental responses that accessed it. Timberlake and Allison (1974) added to this account that it was deprivation of the opportunity to perform the contingent response below the rate occurring with free access that caused such probability differentials (Jacobs et al., 2019). Progress through a hierarchy of modules toward an ultimate goal of consummation is the engine that drives complex behavioral chains (Killeen, 2014a). Each transition gives opportunities to approach more reinforcing responses, and those strengthen and refine the responses in the prior module that triggered the transition through the law of effect. Seeing prey reinforces (and refines) search; approaching prey reinforces and attunes the sighting; chasing prey reinforces and refines the approach; capturing prey reinforces and improves the chase. In all cases animals approach stimuli and responses that are more reinforcing than are afforded by the current module. Timberlake's theories carry behavioral analysis into a broader ecological framework (Bowers et al., 2019; Cabrera et al., 2019) while connecting it to a more fundamental theoretical understanding of behavior (Silva et al., 2019). They carry MPR out of the experimental chamber with its minimal set of affordances to behavior at large, where the modes condition the salience of stimuli, the modules provide the repertoire of operants that are available, and where the reinforcers are not pellets but action patterns higher on a probability hierarchy.

Drugs, diseases, and brains

MPR's principled dissection of behavior by motivational, competitive, and directive effects have made it a useful tool in behavioral neuroscience (Osborne, et al. 1977; Reilly, 2003). It has been used to compare reinforcer types (Bizo & Killeen, 1997; Cabrera et al., 2010; Covarrubias & Aparicio, 2008; Olarte-Sanchez et al., 2015), magnitudes (e.g., Rickard et al., 2009; see Bradshaw & Killeen's, 2012, reanalysis), and operandum types (Bizo & Killeen, 1997). It has proven useful in analysis of motivational and motoric differences in Parkinsonian rats (Avila et al., 2009), inbred strains of mice (Hutsell & Newland, 2013), and chronic cocaine-exposed mice (Pope et al., 2016), as well as the effects of fluoxetine (Sanabria et al., 2008), stress (Kleen et al., 2006), lesions of the subthalamic nucleus (Bezzina et al., 2008), multidrug comparisons (den Boon et al., 2012), methylmercury exposure (Boomhower & Newland, 2016), and numerous other contrasts (e.g., Body et al., 2005; Ho et al., 2003; Kheramin et al., 2005; Killeen, Posadas-Sanchez, et al., 2009; Zhang et al., 2005).

Many of the above studies used PR schedules, but it was not until Bradshaw and Killeen (2012) that the PR model was finally perfected (see Olarte-Sanchez et al., 2015, for a synopsis). This model gave separate accounts for overall and running response rates that typically accounted for around 99% of the variance of the average data and provided parameters that reflected separate motor, memory, and specific activation effects of various interventions. The authors provided pseudocode for the analysis, along with a spreadsheet. This new PR model has been used to excellent effect (Olarte-Sanchez et al., 2015; Valencia-Torres et al., 2017; Wrenn et al., 2021). See Bezzina et al. (2015) for many elegant graphs and analyses validating this model and the leverage it provides on neuropharmacological issues.

Fellow travelers

Baum's multiscale molar behaviorism (2021) promulgates three "laws" (I would call them principles) of behavior analysis: induction, allocation, and covariance (2018b). Induction has much in common with the first principle of MPR, arousal. Arousal is a heightened state of an organism which, given environmental affordances relevant to its drive state (hunger, thirst, fear, etc.) will induce phylogenetically relevant behavior. Baum focuses on the behaviors that are induced. Baum's "allocation" concerns competition among responses for expression given temporal constraints. This is essentially the same as the second principle of MPR. His third principle, covariance, directs behavior, also the function of MPR's third principle—approach to signs of reinforcement. His third principle is halfway to a model, as it specifies a particular falsifiable mechanism, a correlation of behavior with phylogenetically important events. For MPR, the factor that directs approach is multidimensional proximity, each dimension with its own generalization gradient. In the case of the temporal dimension, that is the memorial gradient, a geometric decrease in the influence of a reinforcer over precedent responses (Killeen, 2011). Such a gradient has been documented many times throughout our literature (often pointing in the wrong direction, as though the reinforcer were acting only backward in time, Killeen, 2005, rather than on memorial traces of the response; or only in one direction, Cowie & Davison, 2020; Thorndike, 1933, Figures 4, 5).

Catania (2005) has realized many of the features of MPR in computer simulations. They are not ostensibly a simulation of MPR but rather of Skinner's (1938) *reflex reserve*. Skinner dropped the reserve when he realized it could not handle the effects of diverse schedules of reinforcement (Killeen, 1988). Catania saw how to remedy this by introducing the temporal delay of reinforcement gradient, with responses blessed as an increasing function of their proximity to reinforcement. Skinner's reserve

accumulated according to the same function as arousal and decreased similarly and in Catania's simulations had a similar spread of effect. Not surprisingly, but gratifyingly, he was able to simulate very credible records of performances on basic reinforcement schedules. Thanks to this modernization of the reserve (and his earlier work on gradients, which inspired MPR; Catania, 1971) we may see MPR as a realization of Skinner's hope for a unifying theory of reinforcement schedules. In the end, it is, of course, it is not their lineage but by how well the models of these theories provide a coherent, comprehensible, and accurate account of the data by which they must be evaluated.

CONCLUSION

The principles of MPR have stood for 30 years while the models they support have evolved.

1. Motivation—arousal/induction has not changed, whereas the connection to the "instigating effect" of incentive motivators, to "motivating operations," to Baum's "induction," and to Catania's "reserve," have been more clearly recognized. The extension to varying levels of motivation as a function of satiation by Bittar et al. (2012) enriches the domain of application of MPR.
2. Response competition is still fundamental. Although the single-response model (Equation 2) remains, added to it is the competition from other, unmeasured responses, Equation 15, which plays an important role in contrast and in behavioral momentum.
3. Direction—approach, and the contingencies—natural or artifactual—that arrange it, has itself evolved. In the original, the direction of induced responses was steered by temporal proximity of responses to reinforcers. That remains crucial for classic schedules, but other types of proximities are realized as important in the wider realm of behavioral control.

   In the original presentations, proximity was taken as fundamental; here it is recognized as a quality that induces approach. Animals approach and contact signs proximate to a reinforcer, with reinforcers being actions that are more probable—more desirable—whether it is the passage from coursing a prey to capturing it or from capturing it to consuming it. All of the contingency models in this paper concern the fine-tuning wrought by the law of effect within each of the behavioral modules, releasing new affordances for the next, more desirable module.

These are the fundamental principles of behavior, in both the laboratory and the field. The principles are robust, yet their models must evolve with continual reappraisal in light of new data. The principles themselves may be realized as implications of yet deeper principles.

Some of the evolution seen in the last decades include: the elaboration of the model for the cumulation of arousal when satiation or habituation is important; the development of a model for cue competition in a sign-tracking context, which yielded an ancillary model for pausing in periodic schedules (Appendix E); the extension of the response competition model (Equation 2) to contexts in which competition from unmeasured responses play an important role (Equation 15); the recognition that reinforcement is the obverse of blocking (to the extent that one event absorbs associative strength, less is available for other events, be they stimuli or responses); the justification of response rate as the dependent variable of choice; the recognition of operant responding as itself an instance of sign tracking; the validation of a model of blocking by reinforcers (Figure 5); the potential extension to delay discounting afforded by the accumulation paradigm; the inclusion of concurrent schedules as instances of approach (melioration); and reference to some of the many applications in behavioral neuroscience.

The main point of this article is to demonstrate that a coherent story can be written about how reinforcers invigorate, constrain, and direct behavior, all instantiating a general theory of reinforcement. The story is an edited collection of chapters with many contributing authors. Submissions are solicited for the next edition.

CONFLICT OF INTEREST STATEMENT
The author declares no conflicts of interest.

ETHICS APPROVAL
No organisms were used to produce this article; it is vegan.

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APPENDIX A: RATIO SCHEDULES

The schedule feedback function, *sff*, specifies the availability of reinforcement as a function of responding on a particular schedule. There is a substantial prerun pause (*prp*) on periodic schedules such as FR and FI schedules. That pause is approximately proportional to the interreinforcement interval (IRI; Schneider, 1969). This *proportional pausing* is derived in Appendix E. The IRI comprises the duration of the terminal run-state (for FR schedules, this is n/b_{Run}) plus the *prp*:

$$prp = p\left(\frac{n}{b_{Run}} + prp\right). \qquad (A1)$$

The parameter *p* is the constant of proportionality between the IRI and pause length. It is around ½, and is derived below. *N* is the response requirement, and b_{Run} is the running rate. Solve A1 for *prp*:

$$prp = \frac{p}{1-p}\frac{n}{b_{Run}}. \qquad (A2)$$

The time between reinforcers (the IRI, equal to *I*) is then the sum of the durations of the *prp* and terminal response state:

$$I = prp + \frac{n}{b_{Run}} = \frac{p}{1-p}\frac{n}{b_{Run}} + \frac{n}{b_{Run}}. \qquad (A3)$$

Collect terms:

$$I = \frac{n}{b_{\text{Run}}}\left(\frac{p}{1-p}+1\right) = \frac{n}{b_{\text{Run}}(1-p)}. \qquad (A4)$$

Reinforcement rate (r) is the reciprocal of I, as given by A4:

$$r = \frac{1}{I} = \frac{b_{\text{Run}}(1-p)}{n}. \qquad (A5)$$

Equation 7 is reprinted here:

$$b = \frac{C}{\delta}\left(\frac{A}{1+A}\right), \quad \delta > 0, \qquad (7)$$

and divide the parenthetical by A and expand that to ar:

$$b = \frac{C}{\delta}\left(\frac{1}{1/A+1}\right) = \frac{C}{\delta}\left(\frac{1}{1/ar+1}\right). \qquad (A6)$$

Multiply each side by the denominator of the right-hand side (rhs), and use Equation A5 to replace r with $b(1-p)/n$:

$$b\left(\frac{1}{a\frac{b(1-p)}{n}}+1\right) = \frac{C}{\delta}. \qquad (A7)$$

Multiply the lhs through by its coefficient b:

$$\frac{n}{a(1-p)}+b = \frac{C}{\delta}. \qquad (A8)$$

Isolate b, and for these FR schedules, specify the coupling coefficient as C_R for ratio schedules:

$$b = \frac{C_R}{\delta} - \frac{n}{a(1-p)}. \qquad (A9)$$

This is the fundamental equation for ratio schedules. For terminal response rate (and for VR schedules), set p to 0. For overall rate, either establish p empirically, or, most easily, let its work be done by a, which is a free parameter. In that case, the difference in terminal response rates and overall response rates will be explained by differences in a alone. When I is continually changing, as in progressive-ratio schedules, set the *prp* proportional to I from the prior ratio (Bradshaw & Killeen, 2012; Wynne & Staddon, 1988). (For simplicity of notation, there has been an elision of subscripts in this development. That is because, as shown in Appendix C, the local and global response rates are scalars of each other, leaving the basic results coherent.)

APPENDIX B: PAUSING ON RATIO SCHEDULES

Although Equation A2 gives us a model for pause length, it involves the running rate. To derive an independent prediction of *prp*, insert b from A9 (with p in A9 = 0, as we desire running rate here) into A2:

$$prp = \frac{p'n}{\frac{C_{FR}}{\delta} - \frac{n}{a}}, \qquad (A10)$$

where for succinctness, $p' = p/(1-p)$, now for the ratio as a whole, not for the run state. Bizo and Killeen (1997) showed that Equation A10 successfully described pausing in several studies. For PR schedules, Bradshaw and Killeen (2012) showed that a simpler form sufficed: $prp = pn/(a-n)$, $n < a$. This can be derived under simplifying assumptions from Equation A10, and works as well when the PR schedule is reduced to an FR schedule. In the section on state transitions, below, the value of p is interpreted as the relative attraction of background activities.

APPENDIX C: COUPLING COEFFICIENTS FOR RATIO SCHEDULES

Given the basic equation of motion (Equation 7), all that is left is to insert the coupling coefficient for FR or VR schedules. The former is

$$C_{FR} = 1 - (1-\beta)^n, \qquad (A11)$$

as described in the text.

The latter (VR coefficient) is a hyperbolic function of the response requirement. For the prototypical case of a Random Ratio schedule, this is:

$$C_{RR} = \frac{n}{n + (1-\beta)/\beta}. \qquad (A12)$$

Its form comes from the geometrically decaying influences of reinforcement over prior responses, integrated over the probability of reinforcement occurring after a string of a given length, over all lengths (Bizo & Killeen 1997; Killeen 1994).

APPENDIX D: INTERVAL SCHEDULES

Staddon (cf. Silveira et al., 2023; 1980) gave the schedule feedback function for interval schedules with a programmed Interreinforcement Interval (I) as:

$$r = \frac{1}{I + 1/b}. \qquad (A13)$$

Insert this into Equation 4 for a model of arousal and constraint in the context of interval schedules:

$$b = \left(\frac{\frac{C}{\delta}a - 1}{a + I}\right) \approx \frac{C}{\delta}\left(\frac{a}{a + I}\right) = \frac{kr}{r + 1/a}, \quad k = C/\delta. \quad \text{(A14)}$$

The -1 in the first parenthesis arises from the inclusion of the schedule feedback function (Equation A13). Its median value from the data reported in (Killeen, 1994) $\approx 1/100$. In the Schneider (1969) data (Figure 4) it only affected the predictions in the third decimal place. Although it adds no free parameters, its work can easily be accomplished by a very slightly inflated value of δ, and so for visual simplicity it is dropped (the second parenthesis). When animals are near satiated, however, it can make an important difference (McDowell & Dallery, 1999) and should be included.

Fixed time schedules. The coupling coefficient for FT schedules of duration I is

$$C_{FT} = 1 - e^{-\lambda bI}. \quad \text{(A15)}$$

This is the influence of reinforcement distributed over prior responses with a memory decay rate $\lambda = -\ln(1 - \beta)$. On fixed- and variable-time schedules, no response is required for reinforcement. Therefore, if rate $b = 0$, so is coupling, and rate will remain 0. But sometimes responding occurs for reasons other than reinforcement: Behavior is often induced by the delivery of incentives. These *adjunctive* responses can be captured and amplified by reinforcement (Killeen & Pellón, 2013; Pellón & Killeen, 2015). Equation A15 embodies a positive feedback loop, so performances on free reinforcement schedules will generally be unstable (as Reid et al., 1993 demonstrated).

Fixed interval schedules. The addition of the response requirement on FI schedules helps to stabilize response rates, and is modelled as

$$C_{FI} = \left(1 - e^{-\lambda\delta}\right) + e^{-\lambda\delta}\left(1 - e^{-\lambda\delta bI}\right)$$
$$= 1 - e^{-\lambda\delta(1 + bI)}. \quad \text{(A16)}$$

The first parenthetical is the influence of the reinforcer over the duration of the final response. In the right addend, the remaining unblocked effect multiplies the sum of the effect of reinforcement on the prior responses. This simplifies to the second line. For all but the shortest FI schedules, the exponential term is close to 0 and the coupling coefficient reaches its asymptote. It is economical, therefore, to treat C / δ as a single free parameter rather than compute Equation A16, except for very short intervals (e.g., <10 s) where coupling will decrease, and rates be lower than predicted by Equation 13.

Variable interval schedules. The coupling coefficient for VI schedules is derived by integrating the product of the probability of an interval of duration I by the exponential trace of responses that it would strengthen, over all values of I (Killeen, 1994, Equation C28):

$$C_{VI} = \frac{\lambda bI}{1 + \lambda bI}. \quad \text{(A17)}$$

Insert A17 into the rhs of A14 and solve for b:

$$b = \frac{kr}{r + 1/a} - \frac{r}{\lambda}, \quad k = 1/\delta, \text{ with } \delta, a, \lambda, > 0. \quad \text{(A18)}$$

Here, $r = 1 / I$. Writing $1 / a = r_0$ gives a version of Herrnstein's hyperbola, predicting response rates on VI schedules. What differs is the subtrahend, which occurs because on short intervals fewer responses are reinforced than might have been, due to blocking the provenance of one reinforcer by another (Killeen & Smith, 1984). Equation A18 is a simplification, as it assumes a feedback function of $r = 1 / I$—that is, it works from the middle parenthesis of Equation A14, based on the scheduled interval. For low-motivational conditions, A17 should be inserted into the first parenthesis of A14, but here that adds only clutter and is omitted.

APPENDIX E: STATE TRANSITION AND PAUSING ON PERIODIC SCHEDULES

Attraction is the key concept in this treatment of contingencies. When conditions make one state more attractive than another, animals approach it. This is the core of Timberlake's behavioral systems theory, as well as Thorndike's (Bowers et al., 2019) and Premack's (Killeen, 2014a). In the natural environment there are many signals that make one state more attractive than another (Killeen, 2019b). In the Skinner box fewer, key among them discriminative stimuli, which may include the animal's own behavior.

E.1. | Sign tracking

From Equation 4 in the text, compute the relative strength of the CS compared to the context:

$$S_{CS/Con} = \left(1 - e^{-\lambda_{CS}T_{CS}}\right)\lambda_{CS}T_{CS} \Big/ \left(1 - e^{-\lambda_{Con}T_{Con}}\right)\lambda_{Con}T_{Con}$$

$$= \frac{\left(1 - e^{-\lambda_{CS}T_{CS}}\right)\lambda_{Con}T_{Con}}{\left(1 - e^{-\lambda_{Con}T_{Con}}\right)\lambda_{CS}T_{CS}}$$

$$= \lambda' \frac{T_{Con}}{T_{CS}}\left(\frac{\left(1 - e^{-\lambda_{CS}T_{CS}}\right)}{\left(1 - e^{-\lambda_{Con}T_{Con}}\right)}\right),$$

$$\lambda' = \lambda_{\text{Con}}/\lambda_{\text{CS}}, \lambda_{\text{CS}}, T_{\text{Con}}, T_{\text{CS}} > 0. \qquad \text{(A19)}$$

Here T_{Con} is the duration of the context (the *cycle time*, from one US to the next, written as I in earlier sections) and T_{CS} is the duration of the CS. In the tracking paradigm, both the context and the CS acquire discriminative strength. For sign-large values of the exponents, the parenthetical term goes toward 1 and the relative strength of the CS is proportional to $T_{\text{Con}} / T_{\text{CS}}$.

Gibbon and Balsam (1981, p. 241) used a classic "error-correction" learning model with the asymptotic relative strength of the CS equal to $T_{\text{Con}} / T_{\text{CS}}$, to predict trials to acquisition. Solve that equation for n, the number of trials required for the CS to reach threshold strength θ (theta) for CS responding to start:

$$n = \frac{\ln\left(1 - \theta\frac{T_{\text{CS}}}{T_{\text{Con}}}\right)}{\ln(1 - \alpha)}, \qquad \text{(A20)}$$

where α is the speed of approach to asymptotic conditioning. In Equation 6 in the text, $1 / \ln(1 - \alpha)$ is written as k. Using this model with the asymptotic strength given by Equation A19 gives[9]:

$$n = \frac{\ln\left(1 - \theta\frac{T_{\text{CS}}}{T_{\text{Con}}}\frac{1 - e^{-\lambda_{\text{Con}}T_{\text{Con}}}}{1 - e^{-\lambda_{\text{CS}}T_{\text{CS}}}}\right)}{\ln(1 - \alpha)}. \qquad \text{(A21)}$$

Equation A21 provides a fine rendition of the number of trials to acquisition in sign-tracking experiments.

E.2. | Pausing

The verge of the first target response occurs when the forces of attraction to context and to CS are equal—when Equation A19 equals 1:

$$S_{\text{CS}/C} = \lambda'\frac{T_{\text{Con}}}{T_{\text{CS}}}\left(\frac{1 - e^{-\lambda_{\text{CS}}T_{\text{CS}}}}{1 - e^{-\lambda_{\text{Con}}T_{\text{Con}}}}\right) = 1,$$

$$\lambda' = \lambda_{\text{Con}}/\lambda_{\text{CS}}, \lambda_{\text{CS}}, T_{\text{Con}}, T_{\text{CS}} > 0. \qquad \text{(A22)}$$

Except for very small values of T_{CS}, the ratio of exponentials approximates 1. Setting it to that,

$$\lambda'\frac{T_{\text{Con}}}{T_{\text{CS}}} \approx 1; T_{\text{CS}} \approx \lambda'T_{\text{Con}}. \qquad \text{(A23)}$$

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[9]It is the inverse of Equation 5, $S_{\text{Con/CS}}$, that enters because the model predicts how *slow* conditioning is (n), and this increases with increases in the strength of the context.

The duration of the CS (T_{CS}) that is equally attractive as a context of duration T_{Con} is approximately proportional to T_{Con}; the $T_{\text{CS}} / T_{\text{Con}}$ ratio is the invariant λ'. T_{CS} is the duration of a CS that an animal just approaches in a sign-tracking experiment when the context has duration T_{Con}. (Recent research suggests that for very long CS times this may no longer be the case; Thrailkill et al., 2020).

Now think of T_{CS} as the response (running) state that the animal will just approach in a periodic schedule of duration T_{Con}. The prerun pause (*prp*) is the duration of the context/ interval length T_{Con} minus the duration of the response state; T_{CS}: $T_{\text{Con}} - T_{\text{CS}}$. It follows from that and Equation A23 that

$$\begin{aligned}\text{prp} &= T_{\text{Con}} - T_{\text{CS}} = T_{\text{Con}} - \lambda'T_{\text{Con}}\\ &= T_{\text{Con}}(1 - \lambda')\lambda' \leq 1.\end{aligned} \qquad \text{(A24)}$$

The predicted duration of the *prp* is thus proportional to the duration of the context, the interval value in periodic schedules. Strength of conditioning (Equation 4) varies inversely with λ, the rate of memory decay; slower decay embraces more of the target responses. As the memorial strength of the CS or response state increases (perhaps by being marked with a more salient stimulus), (a) λ_{CS} decreases and (b) $\lambda' = \lambda_{\text{Con}}/\lambda_{\text{CS}}$ increases, causing (c) $(1 - \lambda')$ to decrease, thus making the *prp* a smaller fraction of the interval. Animals speed to the response state or CS when that is more memorable or marked. Conversely, increases in the strength of the signal for the interval onset (e.g., a more marked or longer or more valued reinforcer) will lengthen the *prp*. Increases in the value of the reinforcer will also decrease λ_{CS} and thus strengthen the attraction to the response state. These opposing effects are manifest in the literature (Bradshaw & Killeen, 2012, Figures 2 and 3; Perone & Courtney, 1992).

E.2.1. | Computing local versus global rates of reinforcement and responding

To what rate of reinforcement does r in Equation 13 refer? To the inverse of the time between reinforcers I, or to the rate of reinforcement in the terminal run state? Call the former r_{G} for global rate and the latter r_{L} for local rate. These two epochs may be seen as components of a multiple schedule so that it makes more intuitive sense to use the local rate in that computation. That may be calculated as $r_{\text{L}} = 1 / (I - prp)$: One reinforcer over the duration of the run state. Because of proportional pausing, this is $r_{\text{L}} = 1 / (I - pI) = 1 / [I(1 - p)]$, where p is the constant of proportionality, and $r_{\text{G}} = 1 / I$. The last equation above may be written as $r_{\text{L}} = r_{\text{G}}/(1 - p)$. Multiply the numerator and denominator of Equation 13/A14 by $(1 - p)$, which restores it to its original form, but with a replaced by $a' = a / (1 - p)$; that is, by an adjustment in the value of a free parameter.

Thus, the more convenient $r = r_G$ may be used in Equation 13 as a proxy for r_L.

To what rate of responding does b in Equation 13 refer? To the number of responses (n) divided by the time between reinforcers I or, for periodic schedules, to the number of responses divided by the time in the terminal run state? Call the former b_G for global rate, and the latter b_L for local rate. The global rate is $b_G = n / I$. The local rate is $b_L = n / (I - prp) = n / [I(1 - p)]$. Solve the former for n, $n = Ib_G$. Substitute into the latter, and rearrange, to get: $b_G = b_L(1 - p)$: Global response rate is thus proportional to local response rate. To predict b_G multiply both sides of Equation A14 by $1 - p$, or simply adjust the value of the free parameter k. *Equation 14 thus predicts both running rates and overall rates of responding on FI schedules.*

APPENDIX F: KEY EQUATIONS

Arousal (the potential for action, A, Equation 1) is proportional to the rate of incitement (r):

$$A = ar.$$

Equation 2b is the foundation for many of the models in this paper:

$$b_{\text{Total}} = \frac{1}{\delta}\left(\frac{A}{1+A}\right), \quad \delta > 0.$$

It governs activation under temporal constraints. It requires a model of direction, which depends on the particular reinforcement schedule, to predict target responding. A takes ar from Equation 1.

Equation 4 gives the average area under an exponential density.

$$S = \frac{1 - e^{-\lambda t}}{\lambda t}, \quad \lambda t > 0.$$

It tells how strong the attraction will be to a continuous stimulus of duration t that presages reinforcement. When data are averaged across sessions or subjects where λ varies, Equation 3 and its integral may be the more accurate model.

Equation 2b is directed by the coupling coefficient, C, particularized for the various reinforcement schedules:

$$b_{\text{Target}} = \frac{C}{\delta}\left(\frac{A}{1+A}\right), \quad \delta > 0. \tag{7}$$

Instantiated for FR schedules and their feedback function, this becomes Equation 10:

$$b_{\text{Target}} = \frac{C_{\text{FR}}}{\delta} - \frac{n}{a(1-p)}, \quad \delta, a(1-p) > 0.$$

The equation of motion on interval schedules is Equation 14:

$$b = \frac{kr}{r + 1/a} - \frac{r}{\lambda}, \quad k = 1/\delta; \ \delta, a, \lambda > 0.$$

For FI schedules and all but the shortest VI schedules, the right subtrahend may be dropped.

For multiple schedules Equation 15 is the basic form:

$$b_{\text{Target}} = \frac{C_{\text{VI}}}{\delta}\left(\frac{A_T}{1+A_T}\right)\left(1 - \frac{A_C}{1+A_C}\right), \quad \delta > 0.$$

A_C is the arousal available to responses that compete with the target response. For the study of contrast, A_C spills over into the following component, fading exponentially and causing local contrast. In the study of behavioral momentum, both A_T and A_C dissipate exponentially during extinction or other disruption, the latter most quickly. In the study of resurgence, both forces dissipate during extinction, fighting each other on the way down. Then competition to the target response is removed, and its residual strength is manifest in target responses.