Agent-Based Computational Economics: Growing Economies From the Bottom Up

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Abstract Agent-based computational economics (ACE) is the computational study of economies modeled as evolving systems of autonomous interacting agents. Thus, ACE is a specialization of economics of the basic complex adaptive systems paradigm. This study outlines the main objectives and defining characteristics of the ACE methodology and discusses similarities and distinctions between ACE and artificial life research. Eight ACE research areas are identified, and a number of publications in each area are highlighted for concrete illustration. Open questions and directions for future ACE research are also considered. The study concludes with a discussion of the potential benefits associated with ACE modeling, as well as some potential difficulties.

Keywords
agent-based computational economics, artificial life, learning, evolution of norms, markets, networks

1 Introduction

Decentralized market economies are complex adaptive systems, consisting of large numbers of adaptive agents involved in parallel local interactions. These local interactions give rise to macroeconomic regularities such as shared market protocols and behavioral norms that in turn feed back into the determination of local interactions. The result is a complicated dynamic system of recurrent causal chains connecting individual behaviors, interaction networks, and social welfare outcomes.

This intricate two-way feedback between microstructure and macrostructure has been recognized within economics for a very long time [40, 71, 83, 86]. Nevertheless, for much of this time economists have lacked the means to model this feedback quantitatively in its full dynamic complexity. The most salient characteristic of traditional quantitative economic models supported by microfoundations has been their top-down construction. Heavy reliance is placed on extraneous coordination devices such as fixed decision rules, common knowledge assumptions, representative agents, and imposed market equilibrium constraints. Face-to-face personal interactions typically play no role or appear in the form of tightly constrained game interactions. In short, agents in these models have had little room to breathe.

Slowly but surely, however, advances in modeling tools have been enlarging the possibility set for economists [8, 14, 25, 32, 41, 54, 82, 110]. Researchers can now quantitatively model a wide variety of complex phenomena associated with decentralized market economies, such as inductive learning, imperfect competition, endogenous trade network formation, and the open-ended coevolution of individual behaviors and economic institutions.

One branch of this work has come to be known as *agent-based computational economics* (ACE), the computational study of economies modeled as evolving systems of autonomous interacting agents.[1] ACE researchers rely on computational laboratories[2] to study the evolution of decentralized market economies under controlled experimental conditions. Two basic concerns drive this study. One concern is descriptive, focusing on the constructive explanation of emergent global behavior. Why have particular global regularities evolved and persisted in real-world decentralized market economies, despite the absence of top-down planning and control? How, specifically, have these global regularities been generated from the bottom up, through the repeated local interactions of autonomous interacting agents? And why these particular regularities and not others? The second concern is normative, focusing on mechanism design. Given a particular economic entity, whether existing or simply envisioned, what are the implications of that entity for the performance of the economy as a whole? For example, how might a particular market protocol or government regulation affect economic efficiency?

As in a culture-dish laboratory experiment, the ACE modeler starts by constructing an economy with an initial population of agents. These agents can include both economic agents (e.g., traders, financial institutions, etc.) and agents representing various other social and environmental phenomena (e.g., government, land, weather, etc.). The ACE modeler specifies the initial state of the economy by specifying the initial attributes of the agents. The initial attributes of an agent might include type characteristics, internalized behavioral norms, internal modes of behavior (including modes of communication and learning), and internally stored information about itself and other agents. The economy then evolves over time without further intervention from the modeler. All events that subsequently occur must arise from the historical timeline of agent–agent interactions. No extraneous coordination devices are permitted. For example, no resort can be made to the offline determination and imposition of market-clearing prices through fixed point calculations.

This culture-dish methodology is also the methodology of artificial life (ALife) [107]. ACE and ALife researchers share a desire to demonstrate *constructively* how global regularities might arise from the bottom up, through the repeated local interactions of autonomous agents. Both sets of researchers use computational models as descriptive tools for understanding existing phenomena and as normative tools for the design and testing of alternative possibilities; and both sets of researchers share a desire to develop coherent theories that are comprehensive in scope rather than fractured along outmoded disciplinary boundary lines.

On the other hand, as stressed in [73], ALife researchers adopting the "strong" definition of ALife view their models as *syntheses* of actual life in computers, machines, and other alternative media. In contrast, until recently, ACE researchers have generally viewed their models as *representations* of existing or potential economic processes rather than as actual economic processes of intrinsic interest in their own right. As will be clarified in the next section, this sharp distinction between ACE modeling as representation and (strong) ALife modeling as synthesis is beginning to blur. The recent development of more powerful computational tools has led to increased efforts to automate economic markets, particularly Internet markets. Automation means that the market protocols permit price and quantity offers to be generated by computational agents (e.g., shopbots) as well as, or in place of, human agents. As a result, a growing number of ACE researchers are now involved in the design and testing of automated markets and computational agents for direct practical application, particularly on the Internet. Eventually, then, ACE researchers might need to address the same ethical challenges already confronting the ALife community regarding the autonomous proliferation and evolution of artificial life forms [15].

1 See http://www.econ.iastate.edu/tesfatsi/ace.htm for extensive resources related to the ACE methodology.
2 The felicitous phrase "computational laboratories" is adopted from Dibble [27].

Additional comparisons between ACE and ALife research will be made in Section 3 after a more detailed discussion of ACE research is given in Section 2. Before proceeding, an important disclaimer is in order. The primary objective of this survey is modest in scope: to introduce, motivate, and illustrate through concrete examples the potential usefulness of the ACE methodology by highlighting selected publications in eight research areas that I believe represent interesting and substantial contributions. The number of researchers now making use of the ACE methodology is large and growing, however, and the number of issues addressed in this literature is expanding rapidly. Inevitably, then, some important work will have been overlooked in this survey.[3] Moreover, although efforts are made to identify and pay tribute to the earliest studies within each covered research area, reader accessibility has been a primary concern. Consequently, published versions of papers have generally been cited in favor of working paper versions.[4]

2 Illustrative ACE Research Areas

Three special ACE journal issues have recently appeared that include a fairly diverse sampling of current ACE research [97–99]. The topics addressed in these special issues roughly divide into eight research areas: (a) learning and the embodied mind; (b) evolution of behavioral norms; (c) bottom-up modeling of market processes; (d) formation of economic networks; (e) modeling of organizations; (f) design of computational agents for automated markets; (g) parallel experiments with real and computational agents; and (h) building ACE computational laboratories.[5]

These eight research areas will be used below to illustrate the potential usefulness of the ACE methodology. Since all of the articles included in the special ACE journal issues went through a careful review process in which readability and accessibility were stressed, along with quality of content, a number of these articles will be highlighted in this discussion.

2.1 Learning and the Embodied Mind

ACE researchers and other computationally oriented social scientists have used a broad range of algorithms to represent the learning processes of computational agents. The earliest application of genetic algorithm learning in economics appears to be by Miller [67], and genetic algorithm learning continues to be used in many economic applications [23]. Chattoe [20] provides an excellent discussion of the use and misuse of genetic algorithms, genetic programming, and other forms of evolutionary learning representations in the modeling of social processes. Additional types of learning algorithms that have been used include action-based reinforcement learning algorithms [33, 88], Q-learning [92, 104], classifier systems [41], and various forms of learning algorithms that have been adapted for use in automated markets [38, 91].

Many of these learning algorithms were originally developed with optimality objectives in mind, so caution must be used in applying them to social processes. For computational models of automated economic processes, learning algorithms are appropriately motivated on the basis of optimality criteria. In this case, the investigator might reasonably employ a global learning scheme in which the current strategies of the computational agents are jointly coevolved on the basis of some type of exoge-

3 See http://www.econ.iastate.edu/tesfatsi/ace.htm for extensive annotated lists of pointers to the home pages of individual researchers and research groups now active in ACE-related research. Suggestions for additional pointers are most welcome.

4 If a history of ACE ever comes to be written, one difficulty will be that many of the pioneering studies in the 1980s and early 1990s either were published after long delays or remain as working papers.

5 See http://www.econ.iastate.edu/tesfatsi/aapplic.htm for pointers to introductory resource pages for each of these ACE research areas.

nous fitness criterion (e.g., market efficiency). On the other hand, for computational models of real-world economic processes with human participants, the learning algorithms used for the computational agents will generally need to incorporate the salient characteristics of actual human decision-making behavior if predictive power is to be attained. In this case it might be more appropriate to permit local learning schemes in which different "neighborhoods" of agents (e.g., firms within different industries) separately coevolve their strategies on the basis of some type of endogenous fitness criterion (e.g., relative firm profitability).

Gintis [37] echoes this caution. Departing from the traditional view of game theory as a formal study of rational behavior among strategically interacting agents, Gintis instead provides a problem-centered introduction to evolutionary game theory. He emphasizes the need for a better modeling of agent behavior in view of the numerous anomalies discovered in laboratory experiments between actual human-subject behaviors and the behaviors predicted by traditional rational-agent theories. In particular, Gintis takes an embodied-mind approach. He views games as strategic interaction problems embedded in natural and social processes. As agents repeatedly grapple with these problems over time, they ultimately evolve the ability to play these games effectively. Moreover, in Gintis' view, this evolution typically results less from cognitive processes than from various forms of imitation, such as those underlying cultural transmission.

Aware of these concerns, ACE researchers are increasingly moving away from the unconsidered adoption of off-the-shelf learning algorithms and toward a more systematic investigation of the performance of learning algorithms in various economic decision contexts. For example, Dawid [23] undertakes a systematic study of dynamic multi-agent economic models in which genetic algorithms are used to implement the evolution of individual strategies. He shows that particular aspects of this implementation (e.g., the precise configuration of parameter settings) can strongly influence the set of potential long-run outcomes. This work has had a substantial impact on ACE researchers, since genetic algorithms have been widely used by these researchers as learning representations for their economic agents.

The learning study by Rust et al. [81] has also had a substantial impact on ACE researchers. The authors report a comparative analysis of 30 computational trading algorithms submitted to a double-auction tournament held at the Santa Fe Institute between 1990 and 1991. The submitted algorithms ranged from simple rules of thumb to sophisticated learning algorithms incorporating ideas from artificial intelligence and cognitive science. The winner of the tournament turned out to be one of the simplest algorithms submitted, a "sniping" (last-minute bidding) strategy roughly describable as follows: *Wait while others do the negotiating, then jump in and steal the deal when the bid and ask prices get sufficiently close.* It is interesting to note that sniping has become an increasingly popular bidding strategy in Internet auctions such as eBay with hard closing rules (fixed end times), despite forceful attempts by auction managers to discourage the practice. Indeed, there is now an Internet company eSnipe that, for a fee, will automate this bidding strategy for any eBay participant [47].

Another learning study that has been highly influential among ACE researchers and economists in general is by Gode and Sunder [39], who report on continuous double-auction experiments with computational agents. A *continuous double auction* is an auction for standardized units of a real or financial asset in which offers to buy and sell units are posted and matched on a continuous basis. Continuous double auctions are a common form of trading institution for many real-world commodity and financial markets. Examples include the commodity trading pit of the Chicago Board of Trade and the New York Stock Exchange [50]. Gode and Sunder find that the allocative efficiency of their continuous double auction derives largely from its structure, independently of learning effects. More precisely, they find that market efficiency levels close to 100%

are attained even when their traders have "zero intelligence," in the sense that they submit random bids and asks that are subject only to a budget constraint.

A study by Vriend [103], which focuses on the importance of the *level* of learning for computational agents, is also attracting quite a bit of attention. Vriend conducts ACE experiments within the context of a standard Cournot oligopoly game: Namely, multiple seller firms compete by individually choosing their quantity levels of production, which in turn jointly determine the market price for the good produced. Two different genetic algorithm specifications for learning are considered: (a) individual level, in which each firm learns exclusively on the basis of its own experiences; and (b) population level, in which each firm learns from the experiences of other firms as well as its own. Vriend finds that population learning systematically results in an aggregate output level close to the socially desirable competitive level whereas individual learning does not. He traces this difference to "spite effects"—choosing actions that hurt oneself but hurt others even more. Spite effects operate under population learning to drive aggregate output toward the competitive level, but spite effects are not operational under individual learning.[6]

Other ACE researchers are attempting to calibrate their learning algorithms to empirical decision-making data. One interesting example of this type of research is a study by Marks [64], who takes up two important but immensely challenging questions. Do the perceptions and information usages of market participants evolve during the course of a market process? If so, how?

Marks first formally sets out an analytical market framework within which these questions can be rigorously posed and examined. The participants in this market are permitted to evolve their information-processing capabilities over time. More precisely, they are permitted to evolve the degree to which they partition their state spaces into distinguishable regions for the purposes of determining state-conditioned actions. For example, a crude partitioning may mean that a seller only pays attention to two possible price actions of its rival sellers (low price, high price) whereas a finer partition may mean that the seller pays attention to three possible price actions of its rival sellers (low price, intermediate price, high price).

Marks then focuses on the particular case in which the only available state information consists of the prices set by rivals. He considers two different measures for the information loss accruing to a price-space partition of a given crudeness: number of perceived states, and Claude Shannon's well-known entropy measure. He then incorporates price-space partitioning into an ACE model of a retail coffee market. Historical data are used to calibrate this computational model to actual historical market circumstances. The following specific question is posed: How much information do actual coffee brand managers choose to use in their repeated interactions over time? To investigate this question, Marks conducts a range of experiments under variously specified partitioning structures for the price space: dichotomous partitioning in level; dichotomous partitioning in first differences; and terchotomous partitioning in levels. Two historical scanner data sets from two different supermarket chains are separately examined. Marks' key finding, based on the range of tested partitioning models, is that the dichotomous partitioning model in first differences provides the most informative fit to the examined historical data. The implication is that actual coffee brand managers appear to home in on one particular aspect of their rivals' pricing strategies: namely, did these rivals change their prices last period or not?

6 As a cautionary note, the particular finding by Vriend that social learning dominates individual learning in terms of achieving market efficiency seems special to his market, in which the seller firms are all identically structured. In contrast, Nicolaisen et al. [69] find that a substantially higher level of market efficiency is consistently obtained under individual reinforcement learning than under genetic algorithm population-level learning in the context of a restructured electricity market in which the seller firms (generators) have differential costs.

Another interesting development in ACE studies of learning is the use of human-subject experimental data to calibrate the learning of computational agents. This work is discussed in Subsection 2.7, below.

2.2 Evolution of Behavioral Norms

The concept of a "norm" has been defined in various ways by different researchers. Axelrod [11] advances a behavioral definition, as follows: "A norm exists in a given social setting to the extent that individuals usually act in a certain way and are often punished when seen not to be acting in this way" (p. 47). He justifies this definition on the grounds that it makes the existence of norms a matter of degree, which permits one to study the growth and decay of norms as an evolutionary process. Using agent-based computational experiments, he then demonstrates how mutual cooperation can evolve among self-interested non-related agents through reciprocity with little or no explicit forward-looking behavior on the part of the agents. This seminal work has been extraordinarily influential among economists and game theorists alike. In particular, it has vastly enlarged the traditional scope of noncooperative game theory by encouraging the consideration of bounded rationality and evolutionary dynamics.

Another researcher whose work on behavioral norms has profoundly influenced economists is Thomas Schelling. Working with familiar examples from everyday life, and without the aid of sophisticated computational tools, Schelling [83] shows how patterned social behavior can arise as the unintended consequence of repeated local interactions among agents following simple behavioral rules. For example, he demonstrates how segregation by race can arise through local chain reactions if some agents prefer to avoid minority status by having at least half of their neighbors be of the same race as themselves.

Building on the work by Schelling, Epstein and Axtell [32] use agent-based computational experiments to investigate how various collective behaviors might arise from the interactions of agents following simple rules of behavior. In a subsequent study, Axtell et al. [13] study the emergence and stability of equity norms in society. In particular, using both analysis and computational experiments, they show how intrinsically meaningless "tags" associated with agents can acquire social salience over time, such as when tag-based classes emerge. This study has interesting connections with the work on tag-mediated interactions by Holland and Riolo [42, 78], who show that introducing even very simple tag-choice schemes in interacting-agent systems can dramatically change the course of evolutionary outcomes. Another related study is by Arifovic and Eaton [6], who study how computational agents learn to use tags (truthfully or deceptively) to signal their types.

More recently, Epstein [31] uses an agent-based computational model to study an important observed aspect of behavioral norm evolution: namely, that the amount of time an individual devotes to thinking about a behavior tends to be inversely related to the strength of the behavioral norms that relate to this behavior. In the limit, once a norm is firmly entrenched in a society, individuals tend to conform their behavior to the norm without explicit thought. Epstein's innovative model permits agents to learn how to behave (what norm to adopt), but it also permits agents to learn how much to think about how to behave.

2.3 Bottom-Up Modeling of Market Processes

The self-organizing capabilities of specific types of market processes is now one of the most active areas of ACE research. For example, articles included in the special ACE issues [98, 99] investigate the following types of markets: financial; electricity; labor; retail; business-to-business; natural resource; entertainment; and automated Internet exchange systems. To give the general flavor of this research, an early influential study by Robert Marks will first be reviewed. This will be followed by a discussion of several recent studies focusing on financial markets and restructured electricity markets, two of the most highly active and topical research areas for ACE market studies.

2.3.1 An Early Study by Marks

Robert Marks was one of the first researchers to use an ACE framework to address the issue of market self-organization. His research highlighted for economists—in compelling constructive terms—the potential importance of history, interactions, and learning for the determination of strategic market outcomes. Specifically, in [63], Marks used an ACE model of an oligopolistic market (i.e., a market with a small number of sellers) to investigate how price-setting seller firms might successfully compete. His model made use of a genetic algorithm to model his firms as boundedly rational inductive learners. Specifically, mutation and recombination operations were repeatedly applied to the collection of pricing strategies in use by firms as a way of permitting the firms both to experiment with new ideas (mutation) and to engage in social mimicry (recombination) by adopting aspects of the strategies used by more profitable firms.

One outcome observed by Marks in his experiments was the emergence of globally optimal "joint maximization" pricing across firms without any explicit price collusion. At the time, this type of bottom-up evolution-of-cooperation outcome was new to many economists, since few had yet encountered the seminal work by Axelrod [10] on this topic. Not surprisingly, then, Marks stressed this finding in his article. Nevertheless, in retrospect, an equally interesting finding is that the evolution of cooperation across firms was not assured. Rather, in many of the experimental runs, different configurations of niche strategies emerged that were successful only against a particular collection of competitors. Thus, firms were coevolving their strategies in an intricate dance of path-dependent interactions. Chance mattered for the determination of the final outcomes, as did the behavioral quirks that individual firms evolved in response to their own particular interaction histories. An important implication of this type of path-dependent coevolution is that the "optimal" pricing strategy evolved by a firm in any one particular run of a market experiment might in fact perform very poorly if simply inserted into the pool of pricing strategies evolved in a different run of the same market experiment.

2.3.2 Financial Markets

Conventional models of financial markets based on assumptions of rational choice and market efficiency are extremely elegant in form. Unfortunately, no single model to date has proved capable of explaining the basic empirical features of real financial markets, including fat-tailed asset return distributions, high trading volumes, persistence and clustering in asset return volatility, and cross-correlations between asset returns, trading volume, and volatility.[7]

Due in great part to these well-known difficulties, financial markets have become one of the most active research areas for ACE modelers. Indeed, ACE financial market models have been able to provide possible explanations for a variety of observed regularities in financial data [34, 43, 60, 61]. Several of the earliest ACE financial market studies are surveyed in detail in LeBaron [56], including the highly influential Santa Fe artificial stock market study by Arthur et al. [9]. The latter study develops a dynamic theory of asset pricing based on heterogeneous stock market traders who update their price expectations individually and inductively by means of classifier systems [41]. Several more recent studies are outlined below to give the flavor of the current literature.

Tay and Linn [90] conjecture that better explanatory power might be obtained in financial models by allowing the agents to form their expectations in accordance with

7 See http://www.econ.iastate.edu/tesfatsi/sources.htm for extensive resources related to financial markets.

the way investors form their expectations in real life: namely, in fuzzy terms using inductive reasoning. They argue that these features can be faithfully captured by a genetic-fuzzy classifier system, a modification of Holland's basic classifier system [41]. To test their claim, they modify the Santa Fe artificial stock market model [9] by permitting traders to form their expectations inductively using a genetic-fuzzy classifier system and by modifying the manner in which traders decide which prediction rules to rely on when making demand decisions. They report experimental findings that show that the asset prices and returns generated by their model exhibit characteristics, including measures of kurtosis, that are very similar to actual data.

LeBaron [57] is similarly interested in obtaining a better model fit to empirically observed regularities for financial markets. He calibrates an agent-based computational stock market model to aggregate macroeconomic and financial data. All investors use past performance to evaluate the performance of their trading rules, but different investors have memories of different length. A genetic algorithm is used to coevolve the collection of trading rules available to the agents. The model is calibrated to the growth and variability of dividend payments in the United States. LeBaron is able to show that the calibrated model generates return, volume, and volatility features remarkably similar to those characterizing actual financial time series data.

Foreign exchange markets have also proved to be extremely difficult to model with any predictive power using conventional modeling approaches. Izumi and Ueda [45] propose a new agent-based approach to the modeling of foreign exchange markets. They use field data (dealer interviews and questionnaires) to construct behavioral rules governing agent interactions and learning in a multi-agent foreign exchange model. The agents in their model compete with each other to develop methods for predicting changes in future exchange rates, with fitness measured by profitability. The objective of the authors is to provide a quantitative microfoundations explanation for empirically observed macroregularities in foreign exchange markets. They are able to show that their model provides a possible explanation for the emergence of the following three empirical features: peaked and fat-tailed rate change distributions; a negative correlation between trading volume and exchange rate volatility; and a "contrary opinions" phenomenon in which convergence of opinion causes a predicted event to fail to materialize.

Chen and Yeh [21] argue that social learning in the form of imitation of strategies is important in stock markets, along with individual learning, but that standard stock market models do not include the mechanisms by which such social learning actually takes place. They construct an ACE framework for the analysis of artificial stock markets that includes an additional social learning mechanism, referred to as a *school*. Roughly, the school consists of a group of agents (e.g., business school faculty members) who are competing with each other to supply publicly the best possible models for the forecasting of stock returns. The success (fitness) of school members is measured by the current forecasting accuracy of their models, whereas the success of traders is measured in terms of their wealth. Each trader continually chooses between trading in the market and taking time off to attend the school and test a sample of the forecasting models currently proposed by school members in an attempt to discover a model that is superior to the one he is currently using. The school members and the traders coevolve over time in an intricate feedback loop. To test the implications of their stock market model, Chen and Yeh conduct an experiment consisting of 14,000 successive trading periods. One key finding is that market behavior never settles down; initially successful forecasting models quickly become obsolete as they are adopted by increasing numbers of agents. Another key finding is that individual traders do not act as if they believe in the efficient market hypothesis even though aggregate market statistics suggest that the stock market is efficient.

As a final example, Howitt and Clower [44] use an ACE model of a decentralized market economy to study the potential emergence of a generally accepted medium of exchange (i.e., money). The authors are particularly interested in the possible role of "trade specialists" in supporting the emergence of money. A *trade specialist* is a trader who can reduce the costs of search, bargaining, and exchange by setting up a trading facility that enables nonspecialist traders to come together to trade on a regular basis. The authors use a stylized model of a decentralized market economy in which customers use simple behavioral rules to determine their economic activities and customers can only trade with each other through the intermediation of specialist trading facilities, called shops. Starting from an initial situation in which no institutions that support economic exchange exist, the authors find that a "fully developed" market economy emerges in just over 90% of all runs, in the sense that almost all agents are either in a profitable trade relationship or they own a shop. Moreover, over 99% of these fully developed runs exhibit a unique money, in the sense that one commodity and only one commodity is being used as a money to facilitate trades.

2.3.3 Restructured Electricity Markets

To date, most auction research has focused on one-sided auctions with a fixed number of agents who bid competitively for single units of an item in a single trading period. In reality, however, many auctions involve small numbers of buyers and sellers, asymmetric in size, who meet repeatedly and frequently and who determine their price and quantity offers strategically in an effort to exploit market power opportunities [50, 51].

As a case in point, auctions being designed for restructured wholesale electricity markets typically involve price and quantity offers for the sale of large amounts of bulk electricity by small numbers of electricity generators, some of whom have relatively large market shares.[8] The resulting market processes are extremely complex, rendering difficult the application of traditional analytical and statistical tools. Consequently, researchers are beginning to explore the possible use of agent-based computational frameworks. Some of this work is outlined below.

Bower and Bunn [17] use an ACE framework to study the following issue for the England and Wales wholesale electricity market: How would prices for bulk electricity be affected by the government-proposed change from a *uniform-price* auction, in which a single unit price is set for all units sold, to a *discriminatory-price* auction, in which a distinct unit price is set for each matched seller and buyer as a function of their bid and ask prices? The market is modeled as a sequential game among electricity generators (sellers) with market share and profit objectives. In each trading period each generator submits to the auction a supply function expressing its price and quantity offers. Each individual power plant for each generator is represented as a separate autonomous adaptive agent capable of evolving its supply strategy by means of a simple reinforcement learning algorithm. In contrast, agents on the demand side of the market are assumed to be passive price takers; their buying behavior is modeled by a fixed aggregate demand curve reflecting a standardized daily load profile corresponding to a typical winter day.

A key experimental finding of the authors is that, when supply function offers are not publicly available, the proposed change from a uniform-price to a discriminatory-price auction design permits larger generators to increase their profits relative to smaller generators. Larger generators have a significant informational advantage over smaller generators under the discriminatory auction because they submit more offers and therefore can learn more precisely about the current state of the market. The uniform-price

8 See http://www.econ.iastate.edu/tesfatsi/epres.htm for an extensive collection of resources related to restructured electricity markets.

auction mitigates this advantage by letting smaller generators share in the industry's collective learning by receiving the same market price for their electricity as any other generator. The authors conclude that, under certain circumstances, the choice of the auction design may actually be less important than simply ensuring that all auction participants have equal access to information, regardless of their size.

Bunn and Oliveira [18] construct an ACE model of a wholesale electricity market to explore the possible effects of the New Electricity Trading Arrangements (NETA) introduced in the United Kingdom in March 2001. Their model incorporates the following critical features of the NETA market design: strategically interacting market participants (electricity generators and energy purchasers for end-use customers); a system operator; interactions between a bilateral market, a balancing mechanism, and a settlement process; determination of day-ahead mark-ups on previous-day price offers by means of reinforcement learning; and daily dynamic constraints. The authors apply this NETA computational model to the full electricity system of England and Wales as it existed in summer 2000. They then use their experimental findings to provide insights about possible market equilibria under NETA as a function of both market structure and agent characteristics.

Nicolaisen et al. [69] construct an ACE model of a restructured wholesale electricity market in which prices are set by means of a discriminatory-price *double* auction, that is, an auction in which the sellers (generators) and the buyers (electricity purchasers for end-use customers) both actively make price offers. These price offers are determined adaptively in each successive auction round by means of a reinforcement learning algorithm developed on the basis of human-subject experimental data. The authors investigate three different specifications for the learning parameters. For each specification, they study the effects of differing capacity and concentration conditions on market power (distribution of profits) and market efficiency (total profits). Their findings show that the attempts by sellers and buyers to exercise *strategic* market power are largely ineffective; opportunistic ask and bid price offers offset each other due to the symmetry of the double auction design. On the other hand, the relative market power of sellers and buyers is well predicted by *structural* market power, that is, by the market power outcomes implied by structural market conditions under the assumed absence of opportunistic ask and bidding behavior. In addition, high market efficiency is generally attained. As a cautionary note, however, the authors also show that other forms of learning (e.g., social mimicry learning via genetic algorithms) can result in seriously degraded market efficiency.

2.4 Formation of Economic Networks

An important aspect of imperfectly competitive markets with strategically interacting agents is the manner in which agents determine their transaction partners, which affects the form of the transaction networks that evolve and persist over time.[9] Transaction networks are now frequently analyzed by means of transaction cost economics [109]. To date, however, this literature has not stressed the dynamics of learning, adaptation, and innovation, nor the development of trust. Instead, it is assumed that optimal forms of organization or governance will arise that are suited to the particular characteristics of agent transactions, such as the need for transaction-specific investments.

One particular type of transaction network that is attracting increased attention from economists on the basis of its potential optimality properties is a *small-world network* [105]. A small-world network is a connected network with two properties: (a) each node is linked to a relatively well-connected set of neighbor nodes; and (b) the presence

9 See http://www.econ.iastate.edu/tesfatsi/netgroup.htm for pointers to individual researchers and research groups who are currently studying economic and social network formation.

of shortcut connections between some nodes makes the average minimum path length between nodes small. Such networks have both local connectivity and global reach.

Wilhite [108] uses an ACE model of a bilateral exchange economy to explore the consequences of restricting trade to small-world trade networks. He focuses on the trade-off between market efficiency and transaction costs under four types of trade networks: (a) completely connected trade networks (every trader can trade with every other trader); (b) locally disconnected trade networks consisting of disjoint trade groups; (c) locally connected trade networks consisting of trade groups aligned around a ring with a one-trader overlap at each meeting point; and (d) small-world trade networks constructed from the locally connected trade networks by permitting from one to five randomly specified shortcut trade links between members of non-neighboring trade groups. Given each type of trade network, traders endowed with stocks of two goods seek out feasible partners, negotiate prices, and then trade with those who offer the best deals. A key finding is that small-world trade networks provide most of the market-efficiency advantages of the completely connected trade networks while retaining almost all of the transaction cost economies of the locally connected trade networks. His findings also suggest that there exist microlevel incentives for the formation of small-world trade networks, since the traders who use this type of network tend to do well relative to the traders who do not.

A natural extension of Wilhite's work with fixed trade networks is to consider how networks among trade partners initially form and subsequently evolve. Early ACE studies focusing on the endogenous formation of trade networks include Albin and Foley [1], Kirman [48], Tesfatsion [93, 94], Vriend [102], and Weisbuch et al. [106]. In each of these studies, a key concern is the emergence of a trade network among a collection of buyers and sellers who determine their trade partners adaptively, on the basis of past experiences with these partners.

More recent ACE research on the endogenous formation of trade networks has tended to focus on specific types of markets. Tesfatsion [95, 96] focuses on labor markets. An ACE labor market framework is used to study the relationship between market structure, worker–employer interaction networks, worksite behaviors, and welfare outcomes. Workers and employers repeatedly participate in costly searches for preferred worksite partners on the basis of continually updated expected utility, engage in worksite interactions modeled as prisoner's dilemma games, and evolve their worksite strategies over time on the basis of the earnings secured by these strategies in past worksite interactions. Any dissatisfied worker can quit working for an employer by directing his future work offers elsewhere, and any dissatisfied employer can fire a worker by refusing to accept future work offers from this worker.

Specially constructed descriptive statistics are used to study experimentally determined correlations between market structure and worker–employer network formations, and between network formations and the types of labor market outcomes that these networks support. Two aspects of market structure are studied as treatment factors: job concentration (number of workers to number of employers); and job capacity (total potential job openings to total potential work offers). One key finding is that, holding job capacity fixed, changes in job concentration have only small and unsystematic effects on attained market power levels. A second key finding is that interaction effects are strong. For each setting of the treatment factors, the resulting network distribution exhibits two or three sharp isolated peaks corresponding to distinct types of worker–employer interaction networks, each of which supports a distinct pattern of worksite behaviors and welfare outcomes.

Tassier and Menczer [89] focus on an interesting puzzle regarding the prominent role of job referral networks in U.S. labor markets. The authors note that a robust finding for U.S. labor markets is that approximately 50% of workers at any given time have obtained their jobs through referral-based hiring. For referral-based hiring to be this effective, the referral networks must be efficiently transferring job information between employers and potential workers. On the other hand, most job referrals in the U.S. labor market come from friends, relatives, or other social contacts, not from contacts chosen specifically for job referral. Why, then, do these socially determined networks also perform so well as referral networks?

Tassier and Menczer construct an ACE labor market model in which workers engage in both direct job search and social network formation. Workers survive and reproduce if they are able to acquire enough resources through wages (net of search and network maintenance costs) to meet a survival requirement. The authors study the properties of the social networks that evolve in order to establish the extent to which these networks transfer job information efficiently. Their model yields two main results. First, the evolved social networks have small-world network properties, in the sense that they are both very clustered (locally structured) and yet have global reach. These properties enhance the ability of the social networks to perform as job referral networks. Second, as evolution progresses, agents nevertheless ultimately expend more energy on direct job search and network formation than is socially efficient. This loss in social efficiency corresponds to an increase in individual-agent survival time. More precisely, there is a trade-off between the global efficiency of the labor market and the local robustness of the agents in terms of their ability to survive job losses.

Kirman and Vriend [49] construct an ACE model of the wholesale fish market in Marseilles that captures in simplified form the structural aspects of the actual fish market. Their objective is to understand two persistently observed features of the actual fish market: price dispersion, and widespread buyer loyalty to sellers in the form of repeat business. Each buyer and seller must make multiple decisions during each trading day regarding price, quantity, choice of trading partner, and treatment of trading partner (e.g., should a seller offer better deals to his more loyal buyers). Each of these decisions is separately modeled for each individual agent using a version of Holland's classifier system [41]. The authors report that, in experimental runs with their model, price dispersion and loyalty emerge as a result of the coevolution of buyer and seller decision rules. For example, regarding loyalty, buyers learn to become loyal as sellers learn to offer a higher payoff to loyal buyers, while these sellers, in turn, learn to offer a higher payoff to loyal buyers as they happen to realize a higher payoff from loyal buyers. The authors provide a detailed discussion of the dynamic processes that underlie this emergence of price dispersion and loyalty.

Klos and Nooteboom [52] use an ACE model to explore how transaction networks develop among buyer and supplier firms who repeatedly choose and refuse their transaction partners on the basis of continually updated anticipations of future returns. These anticipations depend in part on trust, where trust increases with the duration of a relationship, and in part on profitability. Buyer firms face a "buy or make" decision: they can search for suppliers to obtain components for the production of differentiated products to be sold in a final goods market, or they can choose to produce these components themselves. Supplier firms engage in both specific and general-purpose asset investment tailored to the collection of buyer firms with whom they are transacting. Buyer firms can increase revenues by selling more differentiated products, and supplier firms can reduce input costs for buyer firms by generating learning-by-doing efficiencies for the buyer firms with whom they are in longer-term relationships.

The Klos and Nooteboom model permits an assessment of the efficiency of resulting profit outcomes as a function of trust and market conditions. The authors report illustrative computational experiments with alternative settings for the degree of differentiation among the buyers' products. As predicted by transaction cost economics, more product differentiation favors "make" relative to "buy" decisions due to higher switching costs and scale effects. Nevertheless, the path dependencies and uncertainties that arise for firms due to the ability to make and break relationships on the basis of past experience result in profit outcomes that are not always efficient.

Rouchier et al. [80] are motivated by a field study focusing on seasonal mobility ("transhumance") among nomadic cattle herdsmen in North Cameroon. The field study explores the conditions that determine the access that nomadic herdsmen have to pasture lands. A key finding is that the grazing patterns and individual relationships established among herdsmen, village leaders, and village farmers tend to be very regular. In an attempt to better understand these observed regularities, Rouchier et al. use an ACE framework to model the dynamics of the relationships among three agent types: nomadic herdsmen who need both water and grass for their cattle and who seek access to these resources from village leaders and farmers in return for access fees; village leaders who provide herdsmen with either good or poor access to water depending on their order of arrival; and village farmers who own pasture land that they may or may not permit the herdsmen to use for cattle grazing. Herd sizes evolve as a function of the agreements that are reached.

Rouchier et al. test two different models of reasoning for their agents: a "cost priority" model based on ideas from transaction cost economics [109] under which agents care only about minimizing their costs; and a "friend priority" model based on ideas from institutional theory [70] under which agents also care directly about the stability of their relationships. Experiments are conducted in which the land in some villages randomly becomes unavailable for use as pasture for short periods of time, so that the farmers in these villages refuse all access requests from herdsmen during these periods. The authors show that the cost-priority and friend-priority models of agent reasoning result in dramatically different experimental outcomes. In particular, the global efficiency of the cost-priority model is surprisingly low relative to the friend-priority model, leading in some cases to the disappearance of herds. In explanation, the authors note that the cost-priority model tends to result in less-flexible agent behavior, and this in turn results in less robustness to land disruption shocks and more overgrazing of pasture lands. In reality, nomadic herdsmen are careful to sustain an extended social network of friends across a wide variety of villages through repeated interactions, and only the friend-priority model produced such a pattern.

A different kind of network problem is posed by information transmission over time. An *information cascade* is said to occur when agents ignore their own private information and simply imitate the selections of the agents who selected before them. Two well-known examples of information cascades within economics are bank panics and stock market crashes. Observing that others are withdrawing their funds from some financial institution (e.g., a bank or a stock market), agents might lose confidence in the institution and run to withdraw their own funds.

De Vany and Lee [26] construct an ACE framework within which they explore the existence and fragility of information cascades under a variety of alternative structural specifications. Their framework differs from standard information cascade models in two basic respects. First, each decision can involve a selection from among more than two options. Second, agents can receive local quality signals from neighboring agents in addition to global quantity information about the proportion of agents who have selected each option to date. For concreteness, De Vany and Lee apply their model to the study of the dynamics of motion picture box office revenues. The authors' main finding is that multiple cascades can coexist in an intermittent pattern in which two or more intertwined cascades are observed to alternate repeatedly over time as the dominant cascade pattern. This intermittence makes it difficult to isolate individual cascades and to predict which if any of the competing cascades will ultimately win out. The authors argue that the complex dynamical patterns observed in their computational experiments resemble the irregular dynamics observed in actual time-series data for motion picture box office revenues.

2.5 Modeling of Organizations

Within economics, a group of people is considered to constitute an *organization* if the group has an objective or performance criterion that transcends the objectives of the individuals within the group [101]. The computational modeling of organizations began at least as far back as the 1950s, when Nobel laureate Herbert Simon first encountered computers at the RAND Corporation capable of imitating intelligence symbolically, not just numerically [85, Chap. 13]. As detailed in Prietula et al. [75], however, progress was slow until the recent development of object-oriented programming (OOP). OOP is particularly "organization friendly" since explicit use of analogies to organizational phenomena have been used in the design of various OOP languages, such as Smalltalk.

The studies collected together in [75] view organizations as complex adaptive systems, and most make use of OOP. A broad range of organizational issues is addressed, including firm organization. This work, led by the efforts of Kathleen Carley's group at Carnegie Mellon University, has been a driving force in the recent surge of interest among social scientists in agent-based computational modeling in general and computational organization theory in particular. Although few economists are directly involved in this work at present, this could change in the near future. For example, Van Zandt [101, Sect. 4.1] explicitly calls for more attention to be paid to agent-based computational modeling in his survey of economic organization theory. Consequently, modeling of organizations is primarily included here as a potentially fruitful research area for future ACE work.

As seen in [75], agent-based computational studies of firms in organization theory have tended to stress the effects of a firm's organizational structure on its own resulting behavior. In contrast, as seen in Subsections 2.3 and 2.4, ACE market studies have tended to stress the effects of particular types of firm behavioral rules on price dynamics, growth, and market structure. Dawid et al. [24] strike out in an interesting new direction by combining these two perspectives. They use a stylized ACE market model to explore how the structure of the market and the internal organization of each participant firm affect the form of the optimal behavioral rules for the participant firms.

Specifically, Dawid et al. consider a collection of firms participating in an industry (i.e., a market for a closely related collection of goods, such as soft drinks). At the beginning of every time period, each firm chooses whether to produce an existing product variety or to introduce a new product variety. The demand for each product variety dies out after a stochastically determined amount of time, hence each firm must engage in some degree of innovation to sustain its profitability. Firms differ in their ability to imitate existing product varieties and in their ability to design new product varieties due to random effects and to "learning by doing" effects that alter the organizational structure of each firm. Each firm has an innovation rule determining its choice to innovate or not, and the firms coevolve these rules over time on the basis of anticipated profitability. The authors conduct systematic experiments to explore how, for optimal profitability, the innovation rule of a firm should adapt both to the structure of the industry as a whole and to the organizational structure of the individual firms of which it is comprised.

2.6 Design of Computational Agents for Automated Markets

In addition to saving labor time, automated contracting through computational agents can increase search efficiency in certain problem applications. For example, computational agents are often more effective at finding beneficial contractual arrangements in market contexts, which tend to be strategically complex multi-agent settings with large strategy domains. Consequently, a large number of researchers are now involved in the design of computational agents for automated markets. To date, much of this work has focused on important but nitty-gritty implementation, enforcement, and security issues.

For example, the contracts used in automated markets have generally been binding contracts that limit the ability of the computational agents to react to unforeseen events. Recently the concept of a "leveled commitment contract" has been proposed that permits agents to decommit from contracts by paying a monetary penalty to the contracting partner, but the efficiency of the resulting contracts depends heavily on the structuring of the penalties. Andersson and Sandholm [2] use an ACE model of an automated negotiation system to study experimentally the sensitivity of leveled commitment contractual outcomes to changes in penalty structurings and to changes in the design of the computational agent negotiators. Four types of penalties are considered: fixed, percentage of contract price, increasing based on contract start date, and increasing based on contract breach date. Agents differ by amount of look-ahead and by degree of self-interested behavior. Multiple task-allocation problem instances are tested, with five negotiation rounds permitted for each instance. In all tested settings, the authors find that choosing relatively low but positive decommitment penalties works best. Surprisingly, however, the authors also find that self-interested myopic agents achieve a higher social welfare level, and more rapidly, than cooperative myopic agents when decommitment penalties are low. Although a look-ahead capability improves agent performance, over short ranges of penalty parameters myopic agents perform almost as well.

In a provocative article, Kephart [47] attempts to clarify the broader implications of this ongoing work on automated markets. He argues that the higher search efficiency of computational agents in automated markets means that humans are on the verge of losing their status as the sole economic species on the planet. As evidence that this trend is already well under way, he points to the growing use of computational agents in automated auction markets on the Internet. To illustrate the higher efficiency of computational agents in the latter setting, he reports findings for auction experiments in which human bidders pitted against computational bidding agents are consistently outperformed. He concludes with the prediction that the information economy will become the largest multi-agent economic system ever envisioned, comprising billions of adaptive strategically interacting computational agents.

2.7 Parallel Experiments with Real and Computational Agents

Human-subject experimentation has become an important economic research tool [79]. One problem with human-subject experimentation, however, is that it is never possible to know exactly why a human subject is making a particular choice. Rather, the human subject's beliefs and preferences must be inferred from his choices. In contrast, in ACE experiments with computational agents, the modeler sets the initial conditions of the experiment. As the computational agents then coevolve their behavioral rules over time, the modeler can attempt to trace this evolution back to its root causes. A possible difficulty, however, is the realism of the evolutionary learning process.

This suggests a possible synergetic role for parallel human-subject and computational-agent experiments. Human-subject behavior can be used to guide the specification of learning processes for computational agents. Conversely, computational-agent behavior can be used to formulate hypotheses about the root causes of observed human-subject behaviors. Within economics, the earliest use of parallel experimentation appears to have been the pioneering study by Miller and Andreoni [68]. Other early studies include Andreoni and Miller [3], Arifovic [4, 5], Arthur [7], and Chan et al. [19]. Two recent examples are outlined below for illustration.

Building on an earlier study by Marimon et al. [62], Duffy [28] uses parallel experiments with human subjects and computational agents to examine the possible emergence of a generally accepted medium of exchange (i.e., money). Parallel experiments are conducted using similar versions of a search model of money. Type i agents desire to consume good i but produce good $i+1$. In each period, agents are randomly paired and must decide whether to exchange goods. An agent can accept a good in trade either because it is directly desired as consumption or because the agent plans to store the good for use in later trades. Goods have different storage costs. The key issue is whether the agents will converge on the use of some particular good as money that they are willing to accept in trades even though it has no direct consumption value. The behavioral rules used by the computational agents to conduct their trades are modeled on the basis of evidence obtained from the human-subject experiments. The computational agents adaptively select among their feasible behavioral rules by means of a simple form of reinforcement learning. Duffy reports that the findings for the computational-agent experiments match basic features of the findings for the human-subject experiments.

Duffy then uses the findings from the computational-agent experiments to predict what might happen in two modified versions of the search model of money that are designed to encourage greater speculative behavior by certain player types. Speculative behavior occurs when an agent accepts a good in trade that is costlier to store than a good he is already storing because his expectation is that the higher-cost good will prove to be more generally acceptable to other agents in future trades. Based on theoretical considerations, Duffy's key prediction for each of the modified versions of the model is that the speed with which the players learn to adopt speculative strategies will increase, which in turn will increase the likelihood of convergence to the speculative equilibrium. Actual experiments are then run for the two modified versions of the model using human subjects, with encouraging results: the findings from the experiments with human subjects are roughly similar to those predicted by the computational-agent experiments.

Pingle and Tesfatsion [74] conduct parallel experiments with human subjects and computational agents for a labor market with incomplete labor contracts. A distinctive feature of this experimental employment study relative to previous theoretical studies is that matches between workers and employers are determined endogenously on the basis of past worksite experiences rather than randomly in accordance with some exogenously specified probability distribution. In each time period, workers either direct work offers to preferred employers or choose unemployment and receive the nonemployment payoff, and employers either accept work offers from preferred workers (subject to capacity limitations) or remain vacant and receive the nonemployment payoff. Matched workers and employers participate in a risky employment relationship modeled as a prisoner's dilemma game. Both the computational agents and the human subjects evolve their partner preferences and worksite behaviors over time on the basis of past matching and worksite experiences.

In both types of experiments, increases in the nonemployment payoff result in higher average unemployment and vacancy rates while at the same time encouraging cooperation among the workers and employers who do form matches. On the other hand, given a high nonemployment payoff, an increasing number of the computational workers and employers learn over time to coordinate on mutual cooperation and avoid coordination failure, so that overall efficiency increases as well. This potentially important "longer run" policy effect is not clearly evident in the necessarily shorter trials run with human subjects. This difference raises challenging issues both for human-subject experimentalists wishing to conduct social policy impact studies and for computational experimentalists who wish to use human-subject experiments to validate their computational findings.

2.8 Building ACE Computational Laboratories

Many economists have advocated the systematic use of computational models for the testing of economic theories. For example, Nobel laureate Robert Lucas [59] writes: "[A theory] is not a collection of assertions about the behavior of the actual economy but rather an explicit set of instructions for building a parallel or analog system—a mechanical, imitation economy. [Our] task as I see it [is] to write a FORTRAN program that will accept specific economic policy rules as 'input' and will generate as 'output' statistics describing the operating characteristics of time series we care about, which are predicted to result from these policies" (pp. 272, 288). Taking advantage of the recent advent of more powerful computational tools, Lane [55] explicitly advocates the use of *agent-based* computational models. Specifically, he asks the reader to "imagine an Artificial Economy as an experimental environment in which users can easily tailor models designed to suit their own particular research agendas. Object-oriented programming techniques can be used to construct such an environment, which would consist of a library of different kinds of modeled institutions and agent types, together with an interface that makes it easy for users to combine different items from this library to make particular economic experiments" (p. 106).

A current drawback of agent-based computational modeling for many economists, however, is the perceived need for strong programming skills. Easily learned languages such as Starlogo are not powerful enough for many economic applications. General programming languages such as C++ and Java and authoring tools such as AgentSheets, Ascape, RePast, and Swarm provide useful repositories of software for constructing agent-based model economies, but their main appeal is to experienced programmers.

A *computational laboratory* (CL) provides a potentially useful middle way to avoid these difficulties. A CL is a computational framework that permits the study of systems of multiple interacting agents by means of controlled and replicable experiments [27]. CLs with a clear and easily manipulated graphical user interface can permit researchers to engage in serious computational research even if they have only modest programming skills. In particular, researchers can use a CL to test the sensitivity of a system to changes in a wide variety of key parameters without the need to do any original programming. On the other hand, a CL can be designed to be both modular and extensible. Thus, as users gain more experience and confidence, they can begin to experiment with alternative module implementations to broaden the range of system applications encompassed by the CL.[10]

For example, McFadzean et al. [65] have developed a CL designed specifically for the study of trade network formation in a variety of market contexts. This CL, referred to as the *Trade Network Game* (TNG) *Lab*, comprises buyers, sellers, and dealers who repeatedly search for preferred trade partners, engage in risky trades modeled as non-cooperative games, and evolve their trade strategies over time. The evolution of trade networks is visualized dynamically by means of real-time animations and real-time performance chart displays. The authors explain the architecture of the TNG Lab and demonstrate its capabilities and usefulness by means of illustrative labor market experiments. The primary objective of the authors, however, is to use the example of the TNG Lab to encourage the routine construction and use of CLs for social science applications.

3 Open Issues and Future Research Directions

A key open issue for ACE research area (a)—learning and the embodied mind—is how to model the minds of the computational agents who populate ACE frameworks.

10 See http://www.econ.iastate.edu/tesfatsi/acecode.htm for pointers to a wide variety of CLs, authoring tools, and general programming languages that are currently being used to design and/or test multi-agent systems.

Should these minds be viewed as logic machines with appended data filing cabinets, the traditional artificial intelligence viewpoint [35]? Or should they instead be viewed as controllers for embodied activity, as advocated by evolutionary psychologists [22]? If the focus of an ACE study is the design of a fully automated market, there is no particular reason why the minds of the computational agents should have to mimic those of real people—indeed, this could be positively detrimental to good market performance. On the other hand, if the focus is on the modeling of some real-world economic process, then mimicry might be essential to ensure predictive content.

An interesting related issue is the extent to which the learning processes of real-world market participants are maladapted to market institutions, leaving room for improvement from the application of optimization tools. Conversely, to what extent have existing market protocols evolved or been designed to avoid the need for any great rationality on the part of market participants? The former issue is considered by Kephart [47] and the latter issue is considered by Gode and Sunder [39] and Nicolaisen et al. [69].

Also, with what degree of flexibility should agent learning in ACE frameworks be specified? Many ACE studies tend to rely on learning algorithms in the form of relatively simple updating equations with fixed parameterizations. The evidence accumulated for these algorithms strongly suggests that no one algorithm performs best in all situations, nor does any one algorithm match best to observed human decision-making behavior under all conditions. A better way to proceed, then, might be to permit the agents in ACE frameworks to learn to learn. For example, each agent could be permitted to evolve a repertoire of behavioral rules or modes that the agent selectively activates depending on the situation at hand. Examples of such learning-to-learn representations include classifier systems [41], the "adaptive toolbox" approach advocated by Gigerenzer and Selten [36], and the evolvable neural network approach developed by Menczer and Below [66]. In addition, as stressed by Marks [64], it might be desirable to permit agents to evolve their information-processing capabilities along with their rule sets.

Parallel concerns arise regarding the modeling of learning in ALife frameworks. For example, Bedau et al. [15] pose, among others, the following three open problems for ALife researchers (p. 365):

> 7. Determine minimal conditions for evolutionary transitions from specific to generic response systems.
> 10. Develop a theory of information processing, information flow, and information generation for evolving systems.
> 11. Demonstrate the emergence of intelligence and mind in an artificial living system.

Point 7 refers to the acquisition of sensing and responding capabilities at a basic biological level, for example, the development of mechanisms of defense against molecular invasion. The learning-to-learn issues addressed by points 10 and 11 more closely parallel those presented above for ACE research. For example, Bedau et al. note that point 11 requires a consideration of the connection between life and mind, and whether it is more productive to study mind as embodied cognition or as a device for logical information processing.

Finally, what about the connection between individual-agent learning and the evolution of agent populations through the birth and death process? Some ACE researchers have made initial steps toward addressing this issue by examining individual-agent learning in the context of an "overlapping generations" economy in which successive generations of agents are born, have children, and die [5, 82]. On the whole, however, most ACE studies to date have assumed that agents are infinitely lived learners. In contrast, ALife researchers have devoted considerable attention to learning agents in evolving populations, although their focus has primarily been on biological rather than cultural evolution [16]. A question that has particularly vexed ALife researchers is individual *plasticity*, that is, the range of variability that individual agents are capable of expressing during their lifetimes in response to environmental variations, and the extent to which this plasticity is subject to evolutionary pressures.

An important issue for ACE research area (b)—the evolution of behavioral norms—is how mutual cooperation manages to evolve among economic agents even when cheating reaps immediate gains and binding commitments are not possible. What roles do reputation, trust, reciprocity, retaliation, spitefulness, and punishment play? More generally, how do exchange customs and other behavioral norms important for economic processes come to be established, and how stable are these norms over time? Are these behavioral norms diffusing across traditional political and cultural boundaries, resulting in an increasingly homogeneous global economy?

As detailed in Gintis [37], the evolution of behavioral norms has also been studied using classic game theory. In the latter, the approach has been to explain this evolution on the basis of individual rationality considerations, such as anticipations of future reciprocity. In contrast, Gintis and many ACE researchers (e.g., Epstein [31]) have tended to place equal or greater stress on peer emulation, parental mimicry, and other socialization forces thought to underlie the transmission of culture. Socialization forces are also emphasized in ALife studies of social organization, although much greater emphasis is placed on understanding the possible connections of these forces with biological evolution through genetic inheritance [15, Point 13].

A potentially fruitful area for future ACE research along these lines is the evolution of behavioral norms in collective action situations, such as the collective usage of common-pool resources. Many of the factors that can make these problems so difficult for standard economic modeling—for example, face-to-face communication, trust, and peer pressure—can easily be modeled within an ACE framework. Moreover, as seen in [58, 72], an extensive body of evidence on collective decision making has been gathered from human-subject experiments and field studies that ACE researchers could use as both guidance and validation for agent-based computational experiments.

An important issue driving ACE research area (c)—the bottom-up modeling of markets—is how to explain the evolution of markets and other market-related economic institutions. Although many ACE researchers are now actively researching this issue, much of this analysis focuses on the evolution of "horizontal" institutional structures, for example, trade networks and monetary exchange systems. In contrast, real-world economies are strongly hierarchical. Indeed, as pointed out by Simon [84, pp. 193–230], hierarchies appear to be essential to help individuals sort information in a complex world. Can an ACE approach be used to study the emergence of a hierarchically ordered economic system from an economic world with an initially horizontal structure?

A similar issue arises for ALife in trying to model the emergence of hierarchical organization for biological systems. Bedau et al. [15, p. 365] pose the following open problem to ALife researchers:

8. Create a formal framework for synthesizing dynamical hierarchies at all scales.

Holland's Echo framework [42] incorporates mechanisms intended to permit the emergence of hierarchical structure in a model of an ecological system, but it appears that much more still remains to be done [87]. Are there any valid analogies between social hierarchy and biological hierarchy that might productively be exploited to help ACE and ALife researchers understand and model the emergence of hierarchical structures?

The primary issue driving ACE research area (d)—the formation of economic networks—is the manner in which economic interaction networks are determined through deliberative choice of partners as well as by chance. Moreover, an interaction might consist of some kind of game situation in which the interacting partners have to choose actions strategically. Consequently, the payoff that will result from any given choice of partner might not be knowable in advance. This results in a highly complicated feedback process in which current partner choices are influenced by past action choices and current action choices are influenced by past partner choices. In contrast, in ALife studies, attention is typically focused on biological agents (e.g., ants, cells, plants) whose interaction patterns are assumed to be determined by a combination of random events, genetic features, and neighborhood topology rather than deliberative choice. Moreover, the payoffs associated with any particular interaction are often assumed to be known in advance.

Another important issue driving ACE research area (d) is the extent to which interaction networks are important for predicting market outcomes [77]. If interaction effects are weak, as in some types of auction markets [39, 69], then the structural aspects of the market (e.g., numbers of buyers and sellers, costs, capacities) will be the primary determinants of market outcomes. In this case, each different market structure should map into a relatively simple *central-tendency* output distribution that can easily be recovered by observing empirically or experimentally determined market outcomes in response to varying structural conditions. If interaction effects are strong, as in labor markets [96], then each different market structure might map into a *spectral* distribution of possible market outcomes with outcomes clustered around two or more distinct “attractors” corresponding to distinct possible interaction networks. Moreover, strong interaction effects might also affect the speed of convergence to these attractors, increasing this speed in the case of highly connected networks and impeding or inhibiting convergence if networks are sparsely connected or disconnected.

The main questions traditionally driving research area (e)—the modeling of organizations—have largely been normative. What is the optimal form of organization for achieving an organization’s goals? More generally, what is the relationship between environmental properties, organizational structure, and organizational performance? As illustrated by Dawid et al. [24], the increased use of ACE modeling in this research area might ultimately permit a significant widening of this traditional scope by permitting the quantitative study of organizations within broader economic settings, for example, the study of intrafirm organization for multiple firms participating within a market. A corresponding analogy for ALife might be the study of organ development for multiple organs coexisting within the context of a body.

ACE research area (f)—the design of computational agents for automated markets—has largely been driven by the quest for optimal agent designs in specific problem contexts. Nevertheless, Kephart [47] takes a broader tack, focusing instead on the increasing ubiquity of artificial life forms that this trend toward automation entails. This trend raises concerns for ACE researchers just as it does for ALife researchers. Bedau et al. [15, p. 365] highlight two open problems in this regard:

12. Evaluate the influence of machines on the next major evolutionary transition of life.
14. Establish ethical principles for artificial life.

A number of challenging issues remain unresolved for ACE research area (g)—parallel experiments with real and computational agents. Chief among these is the need to make the parallel experiments truly parallel, so that comparisons are meaningful and lead to robust insights. One major hurdle is the need to ensure that the salient aspects of an experimental design as perceived by human participants are captured in the initial conditions specified for the computational agents. Experience suggests this can be difficult to achieve, because the perceptions of human participants regarding the design and purpose of an experiment can differ systematically from the perceptions of the investigator [79].

Another major hurdle is that experiments run with human participants generally have to be kept short to prevent boredom among the participants and to keep within the budgetary constraints of the investigators.[11] This means that the "shadow of the past" might be strongly affecting experimental outcomes for individual human participants in ways not understood and controlled for by investigators. For example, participants might come to an experiment with idiosyncratic preconceptions regarding the reliability and generosity of strangers. In contrast, experiments with computational agents can be run for many generations to diminish dependence on initial conditions. An important question, then, is which type of horizon, short run or long run, provides the best approximation for real-world economic processes. Do real-world economic agents essentially move from one new economic situation to the next, never having a chance to settle into long-run behavior? Or do these agents participate repeatedly in economic situations with enough similarity that they are able to use long-run learned (or inherited) behaviors to deal effectively with these situations?

The basic issue for ACE area (h)—building ACE computational laboratories—is the need to construct CLs that permit the rigorous study of complex distributed multi-agent systems through controlled experimentation. Should a separate CL be constructed for each application, or should researchers strive for general multipurpose platforms? How can experimental findings be effectively communicated to other researchers by means of descriptive statistics and graphical visualizations without information overload? How might these findings be validated by comparisons with data obtained from other sources?

A particularly important unresolved issue for area (h) is the need to ensure that findings from ACE experiments reflect fundamental aspects of a considered application problem and not simply the peculiarities of the particular hardware or software used to implement the experiments. Clearly this issue is equally relevant for ALife researchers. Using a portable cross-platform language such as Java helps to ensure independence of the hardware platform, but not independence of specific software implementation features. To address the latter issue, one possible approach is model *docking*, that is, the alignment of different computational models to enable them to model the same application problem [12]. Another possible approach is to have at least two independently programmed versions of a computational model (e.g., a C++ and a Java version) and to run cross-program replication experiments on different hardware platforms (e.g., a personal computer and a UNIX workstation). This cross-platform replication would require, for example, the encapsulation of pseudo-random number generators and the saving of pseudo-random number seed values, good programming practice in any case. Regardless of the approach taken, however, an essential prerequisite is that source code be openly disseminated to other researchers for replication purposes.

A general question that has not yet been addressed is what constitutes the most suitable scale of analysis for ACE modeling? Most of the illustrative ACE studies outlined in the previous section can be categorized as *intranational economics*, the study of multi-agent economic processes that occur within the borders of a single country. Indeed, many of these studies focus on single markets or small collections of markets, the traditional purview of the field of industrial organization. On the other hand, some ACE researchers have undertaken ACE studies of open economies or international economic systems [5, 45, 53, 76]. How useful will ACE modeling be for addressing issues at

11 Internet-enabled experiments could potentially lower the organizational costs associated with running human-subject experiments. However, as detailed in [79], another contributing factor to the cost of running human-subject experiments is that participants typically receive monetary payments proportional to their experimentally determined net profit earnings to ensure their incentives mimic the incentives they would face in corresponding real-world economic situations.

this more macro level of analysis in comparison with other methodologies that are currently being developed for the same purpose, such as statistical mechanics approaches [29, 30]?

A related question concerns the time horizons assumed in ACE modeling. The ACE studies illustrated in Section 2 might be classified as intermediate-run studies, in that they focus on evolutionary processes taking place over many time periods but not over infinitely many time periods. In contrast, authors such as Kandori et al. [46] and Young [110] are interested in the probability with which different kinds of behavioral norms and institutions emerge from the interactive decisions of adaptive individuals in the very long run, as the number of time periods approaches infinity. By focusing on the very long run, these authors are able to bring to bear powerful analytical tools and concepts (e.g., ergodicity) developed for the asymptotic study of stochastic processes. Due to their constructive nature, ACE models cannot be used directly to confirm or reject the long-run distributional predictions of these analytical studies. However, ACE models could be used to examine the practical usefulness of these predictions by testing for speeds of convergence.

Finally, what about the direction of causality between individuals and social groupings? Does ACE have anything to say about this ancient social science debate? For anyone having actual hands-on experience constructing ACE models, it is difficult to imagine how this debate could be viewed as anything but a total chimera. Within any ACE model, the correct answer to the question "Which must come first, individuals or social groupings?" is "Neither." As in the real world, individuals and social groupings coevolve together in an intricate dance through time. Nevertheless, ACE researchers are only just beginning to exploit the power of ACE frameworks to model this complex two-way feedback process.

4 Summing Up the Potential Benefits and Costs

Over the past 50 years a great divide has opened up between economic theorists and other social scientists as economic theorists have increasingly resorted to mathematical systems of equations to model economic processes. These systems now routinely consist of stochastic nonlinear difference or differential equations, which many social scientists find either impenetrable or incredible as descriptions of social reality.

In contrast, the defining characteristic of ACE model economies is their constructive grounding in the interactions of autonomous adaptive agents, broadly defined to include economic, social, and environmental entities. ACE agents are necessarily constrained by the initial conditions set by the modeler. However, the dynamics of the ensuing economic process are governed by agent–agent interactions, not by exogenously imposed systems of equations, and the state of the economy at each point in time is given by the internal attributes of the individual agents that currently populate the economy. This type of dynamical description should have direct meaning for economists and other social scientists, thus increasing the transparency and clarity of the modeling process. Indeed, growing computational evidence suggests that simple individual behaviors can generate complex macro regularities. To the extent this evidence receives empirical support, even further improvements in clarity can be expected from ACE modeling.

The use of ACE model economies could also facilitate the development and experimental testing of integrated theories that build on theory and data from many different fields of social science. In particular, ACE frameworks could encourage economists to address growth, distribution, and welfare issues in a more comprehensive manner embracing a variety of economic, social, political, and psychological factors, thus restoring the broad vision of early political economists [86].

Moreover, as seen in the work by Sargent [82], ACE model economies can be used to test economic theories developed using more standard modeling approaches. Can agents indeed learn to coordinate on the types of equilibria identified in these theories and, if so, how? If there are multiple equilibria, which equilibrium (if any) will turn out to be the dominant attractor, and why? ACE model economies can also be used to test the robustness of these theories to relaxations of their standard assumptions, such as common knowledge, rational expectations, and perfect capital markets. A key question in this regard is the extent to which evolutionary forces might substitute for the high degree of individual rationality assumed in standard economic theories. Finally, ACE model economies can be used to test for observational equivalence, that is, for the possibility that multiple distinct microstructures are capable of supporting a given macro regularity.

To realize this potential, however, ACE researchers need to construct computational laboratories that encompass issues of recognized importance to economists and other social scientists. They need to use these computational laboratories to test clearly articulated hypotheses by means of controlled and replicable experiments. They need to report statistical summaries of their findings that convey the import of these findings in a transparent and rigorous way. They need to increase confidence in these statistical summaries by systematic comparisons with data collected by other researchers using other means. And they need to ensure that robust findings cumulate over time, so that each researcher's work builds appropriately on the work that has gone before.

As documented in [107], similar requirements for success are perceived by ALife researchers. It is certainly not easy for any one person to meet these requirements. One possible answer would seem to be interdisciplinary collaboration. However, experience suggests that communication across disciplinary lines can be very difficult if the individuals attempting the collaboration have little or no cross-disciplinary training. Consequently, perhaps the most important task that those of us involved in ACE and ALife research can undertake is to communicate to our students, along with our excitement, the absolute importance of acquiring basic computational and statistical skills together with suitable training in the desired application domains.

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