2 STL for Hybrid Systems

2.1 Hybrid Time STL

2.1.1 Hybrid System Solutions

Solutions to hybrid systems are defined on ordinary and discrete time:

1. Ordinary Time: $t \in \mathbb{R}_{\geq 0} := [0, \infty)$
2. Discrete Time: $j \in \mathbb{N} := \{0, 1, 2, ...\}$

Definition 2.1 (Hybrid Time Instance). A *hybrid time instance* is given by

$$(t, j) \in \mathbb{R}_{\geq 0} \times \mathbb{N}$$

Definition 2.2 (Compact Hybrid Domain). A set $E \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$ is a *compact hybrid domain* if it can be written as

$$E = \bigcup_{j=0}^{J-1} ([t_j, t_{j+1}] \times \{j\}) \tag{6}$$

for a finite sequence of times $0 = t_0 \leq t_1 \leq t_2 \leq ... \leq t_J$.

Definition 2.3 (Hybrid Domain). A set $E \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$ is a *hybrid domain* if for each $(T, J) \in E$

$$E \cap ([0, T] \times \{0, 1, ..., J\}$$

is a compact hybrid time domain.

Definition 2.4 (Compact Hybrid Shift). Given a compact hybrid domain E and (t^*, j^*), the forward compact hybrid shift of E by (t^*, j^*) is

$$(t^*, j^*) + E = \bigcup_{j=0}^{J-1} ([t_j + t^*, t_{j+1} + t^*] \times \{j + j^*\} \tag{7}$$

and the backward hybrid shift of E by (t^*, j^*) is

$$E - (t^*, j^*) = \bigcup_{j=0}^{J-1} ([t_j - t^*, t_{j+1} - t^*] \times \{j - j^*\} \tag{8}$$

for a finite sequence of times $0 = t_0 \leq t_1 \leq t_2 \leq ... \leq t_J$ and E satisfying (6).

Definition 2.5 (Compact Hybrid Interval). A *compact hybrid interval* is defined by two hybrid time instances (t_A^*, j_A^*) and (t_B^*, j_B^*), where $t_A^* + j_A^* \leq t_B^* + j_B^*$, and an hybrid domain E, over the range $[t_A^*, t_B^*] \times \{j_A^*, ..., j_B^*\}$, by

$$\mathcal{I} := (t_A^*, j_A^*) + E = \bigcup_{j=0}^{J} ([t_j + t_A^*, t_{j+1} + t_A^*] \times \{j + j_A^*\}) \tag{9}$$

Where $J = j_B^* - j_A^*$ and $t_0 = 0 \leq t_1 \leq t_2 \leq ... \leq t_{J+1} = t_B^* - t_A^*$.

Definition 2.6 (Open Hybrid Interval). An open hybrid interval is defined by two hybrid time instances, (t_A^*, j_A^*) and (t_B^*, j_B^*), on hybrid domain E, over the range $[t_A^*, t_B^*] \times \{j_A^*, ..., j_B^*\}$, by the following cases:

1. Left open interval

   $+t_1$? this set should not include !

   $$\mathcal{I} := ((t_0 + t_A^*, t_1 + t_A^*] \times \{j_A^*\}) \cup \{(t_A^*, j_A^*) + E\} = ((t_0 + t_A^*, t_1 + t_A^*] \times \{j_A^*\}) \cup \bigcup_{j=1}^{J} ([t_j + t_A^*, t_{j+1} + t_A^*] \times \{j + j_A^*\} \tag{10}$$

   Where $J = j_B^* - j_A^*$ and $t_0 = 0 \leq t_1 \leq t_2 \leq ... \leq t_{J+1} = t_B^* - t_A^*$;

2. Right open interval

   Same here.

   $$\mathcal{I} := \{(t_A^*, j_A^*) + E\} \cup ([t_J + t_A^*, t_B^*) \times \{j_B^*\}) = \bigcup_{j=0}^{J-1} ([t_j + t_A^*, t_{j+1} + t_A^*] \times \{j + j_A^*\} \cup ([t_J + t_A^*, t_B^*) \times \{j_B^*\}) \tag{11}$$

   Where $J = j_B^* - j_A^*$ and $t_0 = 0 \leq t_1 \leq t_2 \leq ... \leq t_J = t_{j_B^*} - t_A^*$;

3. Open interval ...