2 STL for Hybrid Systems

2.1 Hybrid Time STL

2.1.1 Hybrid System Solutions

Solutions to hybrid systems are defined on ordinary and discrete time:

1. Ordinary Time: $t \in \mathbb{R}_{\geq 0} := [0, \infty)$
2. Discrete Time: $j \in \mathbb{N} := \{0, 1, 2, ...\}$

Definition 2.1 (Hybrid Time Instance)**.** A *hybrid time instance* is given by

$$(t, j) \in \mathbb{R}_{\geq 0} \times \mathbb{N}$$

Definition 2.2 (Compact Hybrid Domain)**.** A set $E \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$ is a *compact hybrid domain* if can be written as

$$E = \bigcup_{j=0}^{J-1} ([t_j, t_{j+1}] \times \{j\} \qquad (5)$$

For a finite sequence of times $0 = t_0 \leq t_1 \leq t_2 \leq ... \leq t_J$.

Definition 2.3 (Hybrid Domain)**.** A set $E \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$ is a *hybrid domain* if for each $(T, J) \in E$

$$E \cap ([0, T] \times \{0, 1, ..., J\}$$

~~can be written as~~ is a compact hybrid time domain.

Definition 2.4 (Compact Hybrid Shift)**.** Given a compact hybrid domain E, and $(t^*, j^*) \in \mathbb{R}_{\geq 0} \times \mathbb{N}$ ~~a compact~~ the forward hybrid shift of E by (t^*, j^*) is ~~denoted by a ordinary time shift t^* and a discrete time shift j^* as~~

$$(t^*, j^*) + E = \bigcup_{j=0}^{J-1} ([t_j + t^*, t_{j+1} + t^*] \times \{j + j^*\} \quad \cancel{\text{or}} \tag{6}$$

and the backward hybrid shift of E by (t^*, j^*) is

$$E - (t^*, j^*) = \bigcup_{j=0}^{J-1} ([t_j - t^*, t_{j+1} - t^*] \times \{j - j^*\} \tag{7}$$

$$\tag{8}$$

For a finite sequence of times $0 = t_0 \leq t_1 \leq t_2 \leq ... \leq t_J$ and E satisfying (5).

Definition 2.5 (Compact hybrid Interval)**.** A *compact hybrid interval* is defined by two hybrid time instances, (t_A^*, j_A^*) and an ? (t_B^*, j_B^*), ~~on~~ $t_A^* + j_A^* \leq t_B^* + j_B^*$ original hybrid domain E, over the range $[t_A^*, t_B^*] \times \{j_A^*, ..., j_B^*\}$, by

$$\mathcal{I} := (t_A^*, j_A^*) + E' \tag{9}$$

Why can't E' be from (0,0) onward, namely a standard htd?

With

defining E?

$$E' := (E - (t_A^*, j_A^*)) \cap L = \bigcup_{j=0}^{j_B^*} ([t_j - t_A^*, t_{j+1} - t_A^*] \times \{j - j_A^*\}) \cap L \tag{10}$$

Where L captures the lower and upper bounds of the shifted interval.

$$L := [0, t_B^* - t_A^*] \times \{0, ..., j_B^* - j*_A\} \tag{11}$$

For a finite sequence of times $0 = t_0 \leq t_1 \leq t_2 \leq ... \leq t_{j_B^*}$.

Definition 2.6 (Open Hybrid Interval)**.** An open hybrid interval is defined by two hybrid time instances, (t_A^*, j_A^*) and (t_B^*, j_B^*), on original hybrid domain E, over the range $[t_A^*, t_B^*] \times \{j_A^*, ..., j_B^*\}$, by

$$\mathcal{I} := (t_A^*, j_A^*) + E' \tag{12}$$
$$E' := (E - (t_A^*, j_A^*)) \cap L \tag{13}$$

With L defined by the following cases

1. Interval open on the left, $L := (0, t_B^* - t_A^*] \times \{0, ..., j_B^* - j*_A\}$
2. Interval open on the right, $L := [0, t_B^* - t_A^*) \times \{0, ..., j_B^* - j*_A\}$