Minimum Jerk Motion Planning

Using 5th Order Polynomials

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Introduction: Minimum Jerk Paths

- Jerk: The 3th derivative
- Important consideration in physical systems

Necessary Condition for Minimum Jerk

$$p^{(6)}(t) = 0 \tag{1}$$

Candidate Function: 5th Order Polynomial

$$p(t) = a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \tag{2}$$

How can we use 5th Order Polynomials?

- Generate minimum jerk motion primitives by interpolation
- Take a path $P(t) : [0, T] \mapsto \mathbb{R}^3$ with $P(t) = \begin{bmatrix} p(t) \\ p'(t) \\ p''(t) \end{bmatrix}$

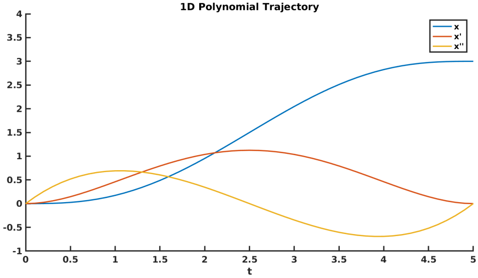
$$p(t) = a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \tag{3}$$

- Take two states $x_n, x_{n+1} \in \mathbb{R}^3$
- Set as end points $P(0) = x_n, P(T) = x_{n+1}$
- The coefficients are found simply by solving a linear system

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ T^5 & T^4 & T^3 & T^2 & T & 1 \\ 5T^4 & 4T^3 & 3T^2 & 2T & 1 & 0 \\ 20T^3 & 12T^2 & 6T & 2 & 0 & 0 \end{bmatrix} * \begin{bmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} p(0) \\ p'(0) \\ p''(0) \\ p(T) \\ p'(T) \\ p''(T) \end{bmatrix} = \begin{bmatrix} x_n \\ x_{n+1} \end{bmatrix} \tag{4}$$

Example: Trajectory on a line

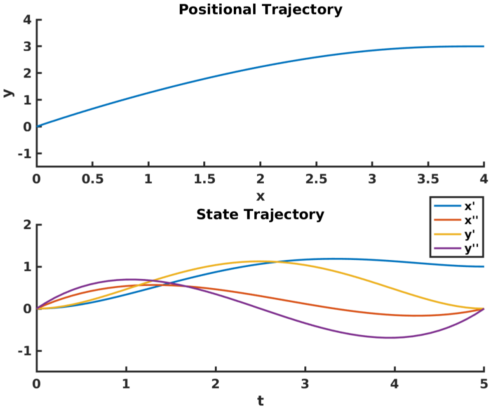
$$x_n = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, x_{n+1} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$



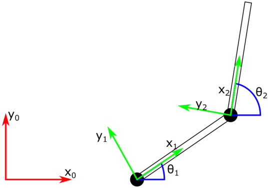
Example: Trajectory on a plane

1D trajectory generation can be extended to 2D trajectory generation.

$$X_n = [x_n, y_n] = \begin{bmatrix} 0,0 \\ 0,0 \\ 0,0 \end{bmatrix}, X_{n+1} = [x_{n+1}, y_{n+1}] = \begin{bmatrix} 4,3 \\ 1,0 \\ 0,0 \end{bmatrix}$$



Example: Motion Profiles for Manipulators



2-Joint, 2-link System

General Manipulator Dynamics

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$

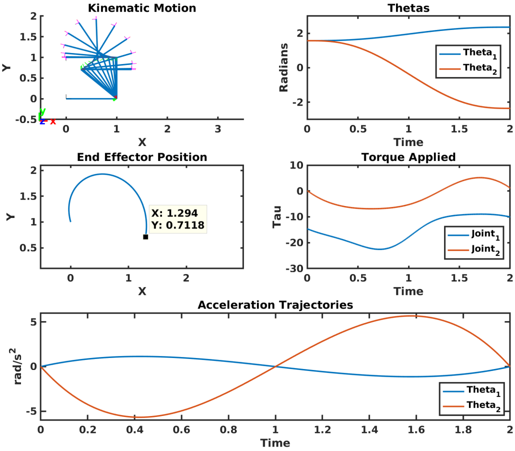
Torque is computed to cancel non-linearities

$$\tau = M(\theta)u + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) \quad (5)$$

$$M(\theta)\ddot{\theta} = M(\theta)u$$
$$\implies u = \ddot{\theta}$$

Example: Motion Profiles for Manipulators

- Trajectory generation in the join-space



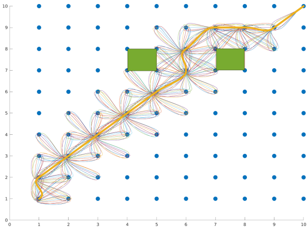
2-Joint, 2-Link Example

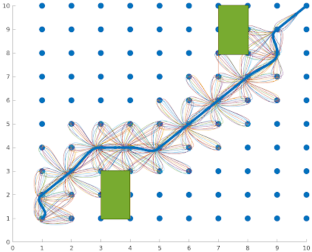
$$\theta_n = \left[\theta_n^1, \theta_n^2\right] = \begin{bmatrix} \frac{\pi}{2}, \frac{\pi}{2} \\ 0, 0 \\ 0, 0 \end{bmatrix},$$

$$\theta_{n+1} = \left[\theta_{n+1}^1, \theta_{n+1}^2\right] = \begin{bmatrix} \frac{3\pi}{4}, \frac{-3\pi}{4} \\ 1, 0 \\ 0, 0 \end{bmatrix}$$

Example: A^* with polynomial paths

- Augment A^* with polynomial sections
- A^* chooses the next node n that minimizes the cost $f(n) = g(n) + h(n)$
- $h(n)$ is a heuristic cost that encourages the search to stay pointed towards the goal
- Idea: Incorporate polynomial path-length into $g(n)$.





Future Work

- Can minimum jerk polynomials be used for reference generation?