Regulation and Trajectory Tracking for Robotic Manipulators

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Abstract

This paper examines regulation and trajectory tracking methods for a two link robotic manipulator. First kinematics and dynamics are derived for the manipulator. With the dynamics in hand, computed torque control is used in joint and work space coordinates to track a reference trajectory generated by a 5^{th} order polynomial.

Keywords: Trajectory Generation, Linear Systems, Robotic Manipulators, Computed Torque

1. Introduction

Multi-link robotic manipulators are a well studied topic. Many papers describe how the nonlinear dynamics can be controlled for end-effector regulation and trajectory tracking.(1)(6)(5) There are various methods to linearize the an element of the controls problem. Such methods include small angle approximations, Taylor approximations, and feedback linearization. (4) (2). Certain elements of the robotic manipulator problem can be formulated as linear problems. Our linear methods focus on the trajectory generation which in which a linear system is solved to fit a function between two points.(7) Work-to-be-done includes decoupling the dynamics of each link to achieve linear control over the manipulator.

1.1. Outline

The paper begins with a derivation of the 2-joint, 2-link manipulator's kinematics. The workspace-end-effector transform allows the position of the end-effector to be written as a function of the joint angles. Additionally, the position of each link's center of mass can be written as a function of joint angles. This is needed to derive gravitational force matrix. From the kinematics, differential motion equations can be derived. These allow us to write out the manipulator jacobian which is useful for analyzing the dynamics of the end-effector for work-space control. Next, an overview is given of joint-space and work-space computed torque control. A state trajectory is required for these methods. The following section explains curve fitting between two functions and gives an example in 2D space. In section 3.3 an in-progress method for linear end-effector control is presented. Finally simulation results are presented for two scenarios. These scenarios present join-space computed torque control for end-effector regulation.

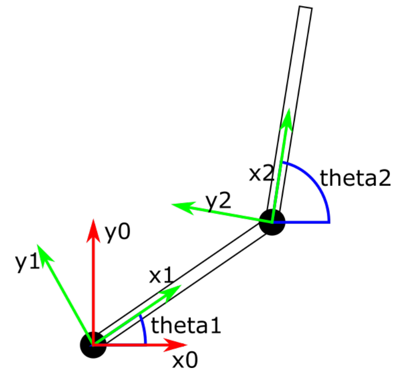

Figure 1: 2-Joint, 2-link System

2. Formulation of a Two Joint, Two Link System

2.1. *Notation*

(Images need updating) We denote ${}^{A}\vec{v} \in \mathbb{R}^2$ as a vector measured relative to coordinate system $A \subset \mathbb{R} \times \mathbb{R}$. Given a coordinate system $B \subset \mathbb{R} \times \mathbb{R}$, ${}^{A}T_B \in \mathbb{R}^3$ is a homogeneous transform from coordinate system A to B.

U and R represent the universal(workspace) and robotic coordinate systems. J_1 and J_2 represent the joint coordinates. D_R represents the the distance of the robot to the workspace origin while L_1, L_2 represent link lengths; M_1, M_2 are the masses; C_1, C_2 are the center of mass; each for the respective links. The joint space state vector is given by: $\theta = (\theta_1, \theta_2)^\top$. The workspace state vector is given by $z = (x, y)^\top$

2.2. *Direct Kinematics*

(6) The transform from the workspace origin to the hand is given by

$$ {}^{U}T_H = {}^{U}T_R \cdot {}^{R}T_{J_0} \cdot {}^{J_0}T_{J_1} \cdot {}^{J_1}T_H \tag{1} $$

From ${}^{U}T_H$ we can express the cartesian position of the hand as a function of the joint space:

$$ x_H(\vec{\theta}) = D_R + L_1 \cos(\theta_1) + L_2(-\sin(\theta_1)\sin(\theta_2) + \cos(\theta_1)\cos(\theta_2)) \tag{2} $$

$$ y_H(\vec{\theta}) = L_1 \sin(\theta_1) + L_2(\sin(\theta_1)\cos(\theta_2) + \sin(\theta_2)\cos(\theta_1)) \tag{3} $$

The transform giving the height of the center of mass of link-1 and link-2 are given by $h_1(\vec{\theta})$ and $h_2(\vec{\theta})$, respectively. These will be used in the dynamic analysis. Let $\mathbb{C}_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ and $\mathbb{R}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

$$ h_1(\vec{\theta}) = \frac{L_1 \sin(\theta_1)}{2} = \mathbb{R}_2 \cdot {}^{U}T_R \cdot {}^{R}T_{J_0} \cdot {}^{J_0}T_{C_1} \cdot \mathbb{C}_3 \tag{4} $$

$$ h_2(\vec{\theta}) = L_1 \sin(\theta_1) + \frac{L_2 \sin(\theta_1 + \theta_2)}{2} = \mathbb{R}_2 \cdot {}^{U}T_R \cdot {}^{R}T_{J_0} \cdot {}^{J_0}T_{J_1} \cdot {}^{J_1}T_{C_2} \cdot \mathbb{C}_3 \tag{5} $$

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2.2.1. *Differential Motion*

We can express the velocities of the hand as:

$$\dot{x_H}(\vec{\theta}) = \dot{\theta}_2\left[-L_2\sin(\theta_1)\cos(\theta_2) - L_2\sin(\theta_2)\cos(\theta_1)\right] + \dot{\theta}_1\left[-L_1\sin(\theta_1(t)) - L_2\sin(\theta_1)\cos(\theta_2) - L_2\sin(\theta_2)\cos(\theta_1)\right] \tag{6}$$

$$\dot{y_H}(\vec{\theta}) = \dot{\theta}_2\left[-L_2\sin(\theta_1)\sin(\theta_2) + L_2\cos(\theta_1)\cos(\theta_2)\right] + \dot{\theta}_1\left[L_1\cos(\theta_1(t)) - L_2\sin(\theta_1)\sin(\theta_2) + L_2\cos(\theta_1)\cos(\theta_2)\right] \tag{7}$$

Which can be written in terms of the jacobian $J(\theta) = \begin{bmatrix} J_1(\theta) & J_2(\theta) \\ J_3(\theta) & J_4(\theta) \end{bmatrix}$

$$\dot{z} = \begin{bmatrix} \dot{x_h} \\ \dot{y_h} \end{bmatrix} = J(\theta)\dot{\theta} = \begin{bmatrix} J_1(\theta) & J_2(\theta) \\ J_3(\theta) & J_4(\theta) \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{8}$$

Where

$$J_1 = -L_1\sin(\theta_1(t)) - L_2\sin(\theta_1)\cos(\theta_2) - L_2\sin(\theta_2)\cos(\theta_1) \tag{9}$$

$$J_2 = -L_2\sin(\theta_1)\cos(\theta_2) - L_2\sin(\theta_2)\cos(\theta_1) \tag{10}$$

$$J_3 = L_1\cos(\theta_1(t)) - L_2\sin(\theta_1)\sin(\theta_2) + L_2\cos(\theta_1)\cos(\theta_2) \tag{11}$$

$$J_4 = -L_2\sin(\theta_1)\sin(\theta_2) + L_2\cos(\theta_1)\cos(\theta_2) \tag{12}$$

It follows that

$$\dot{\theta} = J^{-1}\dot{x} \tag{13}$$

$$\ddot{\theta} = J^{-1}\ddot{x} + \frac{d}{dt}(J^{-1})\dot{x} \tag{14}$$

This will be useful when deriving linear control for the manipulator.

2.3. *Manipulator Dynamics and Control*

The dynamics of an open chain manipulator can be written as(5):
(Derivation in the works)

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau \tag{15}$$

Where τ is a torque vector and M,C,N describe the dynamics of the system. In the 2-link, 2-joint case $\tau = (\tau_1, \tau_2)^\top$.

3. Method of solution

3.1. *Manipulator Open Loop Joint-Space Computed Torque Control*

Given a path $\ddot{\theta}_d(t)$ in the joint space, the torque is readily computed through the manipulator equation:

$$M(\theta)\ddot{\theta}_d + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau \tag{16}$$

The computed torque cancels out the external forces of the manipulator while $\ddot{\theta}_d(t)$ overcomes the inertia of the manipulator. Substituting the computed torque of equation (16) into equation (15) gives us

$$M(\theta)\ddot{\theta} = M(\theta)\ddot{\theta}_d$$
$$\implies \ddot{\theta} = \ddot{\theta}_d$$

Since we have control of $\ddot{\theta}$, and if there are no disturbances, and $z(0) = z_d(0)$ and $\dot{z}(0) = \dot{z}_d(0)$ then the manipulator will follow $\theta_d(t)$.

3.2. *Trajectory Generation*

Generation of $z_d(t)$ can be performed using linear system methods. Given a initial state $z_d(0)$, final state $z_d(T)$ and a final time T, a 5^{th} order polynomial can be fit between the two state points. The results is a path $P(t)$:

$$P(t) = \begin{bmatrix} z_d(t) \\ z_d'(t) \\ z_d''(t) \end{bmatrix} \tag{17}$$

$$z_d(t) = a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \tag{18}$$

$P(t)$ connects states $z_d(0)$ and $z_d(T)$ if $P(0) = z_d(0)$ and $P(T) = z_d(T)$ where T is a chosen time of completion. The coefficients of $z_d(t)$ can be solved in terms of the two states. Specifically, the coefficients must satisfy the relation:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ T^5 & T^4 & T^3 & T^2 & T & 1 \\ 5T^4 & 4T^3 & 3T^2 & 2T & 1 & 0 \\ 20T^3 & 12T^2 & 6T & 2 & 0 & 0 \end{bmatrix} * \begin{bmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} p(0) \\ p'(0) \\ p''(0) \\ p(T) \\ p'(T) \\ p''(T) \end{bmatrix} = \begin{bmatrix} x_n \\ x_{n+1} \end{bmatrix} \tag{19}$$

3.2.1. *Trajectory Generation Example*

1D trajectory generation can be extended to 2D trajectory generation.

$$X_n = \begin{bmatrix} x_n, y_n \end{bmatrix} = \begin{bmatrix} 0,0 \\ 0,0 \\ 0,0 \end{bmatrix}, X_{n+1} = \begin{bmatrix} x_{n+1}, y_{n+1} \end{bmatrix} = \begin{bmatrix} 4,3 \\ 1,0 \\ 0,0 \end{bmatrix}$$

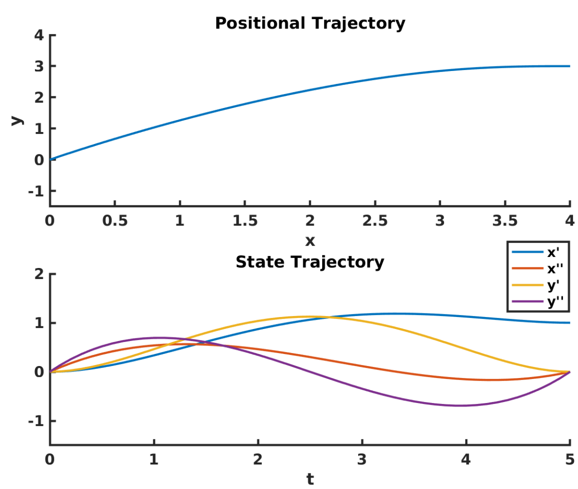


Figure 2

3.3. *Linearizing the End Effector*

Using the techniquie of GCT as shown in(3) we are able to linearize the behavior of the end effector with respect to the outside observer. This allows us to move the nonlinearaties to the torques and a change of coordinates. The required torque and nonlinear change of base are

$$\tau = C - MJ^{-1}\dot{J}\dot{q} + MJ^{-1} \tag{20}$$

$$\begin{bmatrix} y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} h(q) \\ J(\dot{q}) \end{bmatrix}; \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} h^{-1}(y) \\ J^{-1}(\dot{y}) \end{bmatrix} \tag{21}$$

With the resulting linear system

$$x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix} \tag{22}$$

$$\dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} u \tag{23}$$

In the above equation $u = \ddot{y} = J\ddot{q} + \dot{J}\dot{q}$
(3)

4. Simulation Results

4.1. *Computed Torque For Open Loop Joint-Space Control*

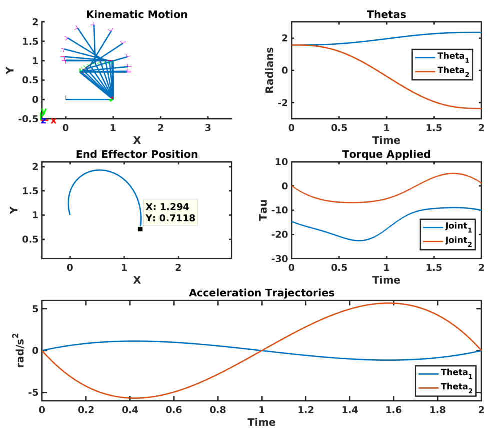
Two scenarios were simulated using a 5^{th} order polynomial trajectory generation. The acceleration trajectories are used to compute the torque required to move the manipulator from one position to another.

4.1.1. *Scenario One*

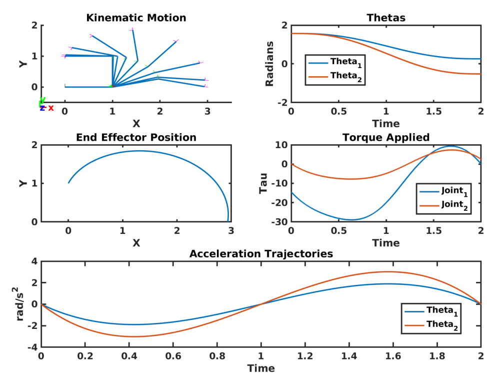
Initial angles are given as $\theta_1(0) = \frac{\pi}{2}, \theta_2(0) = \frac{\pi}{2}$ and are at rest. Final angles are given as: $\theta_1(T) = \frac{3\pi}{4}, \theta_2(T) = -\frac{3\pi}{4}$ and are are at rest. Path time length is $T = 2$ seconds.

4.1.2. *Scenario Two*

Initial angles are given as $\theta_1(0) = \frac{\pi}{2}$, $\theta_2(0) = \frac{\pi}{2}$ and are at rest. Final angles are given as: $\theta_1(T) = 0.26$rad, $\theta_2(T) = -0.42$rad and are at rest. Path time length is $T = 2$ seconds.



((a)) Scenario 1



((b)) Scenario 2

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5. APPENDIX

5.1. *Manipulator Open Loop Work-Space Computed Torque*

Substituting equation (13) into equation (15) gives:

$$\tilde{M}(\theta)\ddot{z} + \tilde{C}(\theta,\dot{\theta})\dot{z} + \tilde{N}(\theta,\dot{\theta}) = F \tag{24}$$

Where

$$\tilde{M} = J^{-\top} M J^{-1} \tag{25}$$

$$\tilde{C} = J^{-\top}\left(C J^{-1} + M\frac{d}{dt}(J^{-1})\right) \tag{26}$$

$$\tilde{N} = J^{-\top} N \tag{27}$$

$$F = J^{-T}\tau \tag{28}$$

Suppose there is a path, $z_d(t)$ given in the workspace. The open loop method of following the path is to compute the joint torques τ that cancel out the external forces and overcome the inertia of the manipulator.

This control torque τ is computed as

$$F = \tilde{M}(\theta)\ddot{z} + \tilde{C}(\theta,\dot{\theta})\dot{z} + \tilde{N}(\theta,\dot{\theta}) \tag{29}$$

Where $\tau = J^{\top} F$.

As with the joint space computed torque, if there are no disturbances and $z(0) = z_d(0)$ and $\dot{z}(0) = \dot{z}_d(0)$ then the manipulator will follow $z_d(t)$.

5.2. *Matrix Definitions*

$$^{U}T_{R} = \begin{bmatrix} 1 & 0 & D_R \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{30}$$

$$^{R}T_{J_0} = \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) & 0 \\ -\sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{31}$$

$$^{J_0}T_{J_1} = \begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & L_1 \\ -\sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{32}$$

$$^{J_1}T_{H} = \begin{bmatrix} 1 & 0 & L_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{33}$$

$$^{U}T_{C_1} = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & DR + \frac{L_1\cos(\theta_1)}{2} \\ \sin(\theta_1) & \cos(\theta_1) & \frac{L_1\sin(\theta_1)}{2} \\ 0 & 0 & 1 \end{bmatrix} \tag{34}$$

$$^{U}T_{C_2} = \begin{bmatrix} -\sin(\theta_1)\sin(\theta_2) + \cos(\theta_1)\cos(\theta_2) & -\sin(\theta_1)\cos(\theta_2) - \sin(\theta_2)\cos(\theta_1) & DR + L_1\cos(\theta_1) + \frac{L_2(-\sin(\theta_1)\sin(\theta_2)+\cos(\theta_1)\cos(\theta_2))}{2} \\ \sin(\theta_1)\cos(\theta_2) + \sin(\theta_2)\cos(\theta_1) & -\sin(\theta_1)\sin(\theta_2) + \cos(\theta_1)\cos(\theta_2) & L_1\sin(\theta_1) + \frac{L_2(\sin(\theta_1)\cos(\theta_2)+\sin(\theta_2)\cos(\theta_1))}{2} \\ 0 & 0 & 1 \end{bmatrix} \tag{35}$$

$$\mathrm{ROT}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{36}$$

$$\mathrm{TRAN}(a, b) = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \tag{37}$$